Additivity of deflated input-output tables in national accounts

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Abstract

Input-output tables deflated by chained prices indices are not additive over product rows. The paper discusses the reasons and suggests a remedy. The method is based on a distinction, in concept, between "real value", on the one hand, and change of "volume", on the other, the first correcting for the monetary variation of the unit of account resulting from inflation of the general price level, while the latter isolates the variation of one product price relative to the other products, caused by the forces of supply and demand on each individual commodity market. An example of the resulting growth analysis is compiled for the Dutch input-output tables between years 1990 and 2000.

1. Introduction

It is long and well established practice, to compile national accounts, and input-output tables as their integral part, not only in nominal, but also in real terms. While the monetary unit furnishes "the only common denominator which can be used to value the extremely diverse transactions recorded in the accounts and to derive meaningful balancing items" (ESA 1995, para. 10.01) the problem is that this unit is not stable over time. Its variation may be small when controlled by monetary authorities, but may also reach a range where not even monthly data can be compared and added without an adjustment correcting for the monetary effect. In short, as "there is not in nature any correct measure of value" (Ricardo 1952, p. 387), a statistical method must be constructed in order to distinguish whether an increase in the nominal value of a national accounts aggregate signifies an intrinsic increase in its value, or whether it results as a "blowing up" from monetary inflation, only (Reich 2005). The formula Laspeyres invented in year 1871 served its purpose for more than a hundred years, at least in Germany. And over that time, additivity of the resulting aggregates has not been a problem, because it is naturally implied there. It is only now with the advent and standardization, of chained indices that the resulting aggregates cannot be added meaningfully over products, and balances for industries be compiled, after deflating them.

The reaction to this new experience is diverse. Some do not care. Declining that additivity is an important feature of a system of accounts at all, they find that it is not even desirable, anymore (Ehemann et al. 2002, p. 40). The majority carries on both shoulders. While admitting that aggregates should add up and balance, one also sees the need to re-base the indices, when in the course of time, the pattern of relative prices tends to become progressively less relevant to the economic situation of later periods "to the point at which they become unacceptable" (SNA1993, para. 16.31). Although "it must be recognised" that the lack of additive consistency can be a serious disadvantage for many types of analysis, the "preferred measure" of volume and price change is a chain index (ESA1995 para.10.65). Only a few take the issue serious (Casler 2006, Tödter 2006). Realising that both qualities, time relevance, and additivity, are essential characteristics of a system claiming to be "a system of accounts", one questions the general opinion that the two qualities are incompatible, by nature (Reich 2003). True, chain indices, as they have been invented and come into use are not additive, but are they the only ones possible?

Index number theory has mostly been studied and developed with view to change of prices, and prices are not additive variables. This quality, therefore, never figured as an important axiom, or "test" there. But for national accounting tables, and input-output tables, in particular, volumes are the prominent variable, and additivity of entries plays an entirely different role there. The naïve view that if you get your prices right you get your volumes right, by themselves, may not be altogether true. It seems the floor of discussing old habits in the light of new requirements must be opened, and possible solutions be sought and weighed against each other. This has not really been done, yet, and it is time to start, before some sudden political issue connected to the problem may take over and arouse public opinion.

2. Problem analysis: formulas and concepts

When the US Senate Boskin Committee visited the construction of statistical price measurement, it fell over one particular stone of the longstanding edifice of methods, and this obstacle has been duly removed, since. But just like when changing tyres of your car you discover that the wheels and brakes may need an overhaul, too, the repair of the Laspeyres index reveals more than just a failure of being up to date with your aggregation weights. Certainly, weighing present transactions by means of past prices with the result that present growth rates depend on the choice of some past base year chosen arbitrarily is hardly defendable, once it appears in public, and chaining is the appropriate remedy. But there is more substance to this renovation, and part of it is new, and has never been observed, because it was well, and unconsciously, taken care of within the old Laspeyres system. The problem of additivity of chain indices is not only one of index number formula, but also of their conceptual content. We must dig more deeply, in order to settle us well again, and ask questions that have not been asked before. Let us begin by trying a standard definition of the problem:

"The creation of an integrated system of price and volume indices is based on the assumption that, at the level of a single homogeneous good or service, value (v) is equal to the price per unit of quantity (p), multiplied by the number of quantity units (q), that is

$$v = p \times q \quad . \qquad (1)$$

Price is defined as the value of one unit of a product, for which the quantities are perfectly homogeneous not only in a physical sense but also in respect of a number of other characteristics…To be additive in an economic sense, quantities must be identical and have the same unit price. For each aggregate of transactions in goods and services shown in the accounts, price and quantity measures have to be constructed so that

value index = price index × volume index.

This means that each and every change in the value of a given flow must be attributed either to a price change or to a change in volume or to a combination of the two". (ESA1995, paras. 2.12, 2.13).

The text reads a bit clumsy, which is explained, perhaps, by a desire to be precise. A careful analysis of the passage sheds light on some severe conceptual stumbling blocks, hidden under this seemingly spotless surface. There are three problems involved. Named in the order of decreasing professional awareness they are

a) the relationship between homogeneity of a product and its aggregation,
b) the decomposition of a value aggregate into a price and a volume component,
c) the difference in meaning between individual product price and general price level.

Regarding the first, we read that price is only defined for a product, for which the quantities are “perfectly homogeneous”, not distinguishable from each other. Aggregates are inhomogeneous, by definition, so that the problem arises what the concepts of quantity and price mean for them. Logically, it would follow that they do not apply, given the previous definition. They are applied, although in a different disguise. They are applied as indices which are dimensionless and one of them is renamed and “preferred to be called” (SNA) “volume index”. The question is then, how does “volume” relate to “quantity”, in concept.

In the ESA the question is answered implicitly by the rest of the chapter following the quoted introduction, where the methods of index construction are described. Let us try an explicit summary! A valid price index for national accounts and input-output tables is the result of cooperation between two different and independent statistics, national accounts and price statistics, the latter establishing the individual price series, the first providing the weights for aggregation. The homogeneous product is the concept, indeed, on which price measurement is based. The price observer must certify that between two times of observation the product from which she takes the price has not changed in any of its characteristics. The postulate of homogeneity applies to, and is meaningful as a guideline for, the so called “price representative”, i.e. the specific good chosen to represent a whole class of similar, but not identical products. The class itself is inhomogeneous.

Thus in equation 1, the definition runs actually the other way around: *v*, the value of the inhomogeneous transaction aggregate is given by the national accounts (an elementary class of consumer expenditure, for example), the price index *p* pertaining to one specific homogeneous product selected as representing all others in its class is given by the price department and divided into the transactions value. The result is entered back into national accounts. The defining variables in this equation are thus *v* and *p*, while the variable to be defined (definiendum) is not *v,* but *q*

$$q = \frac{v}{p} \ . \qquad (2)$$

It is necessarily inhomogeneous, and not a homogeneous quantity at all. One calls it “volume” in order to express the distinction. Volumes may change not only because quantities change, but also because the quality of the goods contained in the aggregate change, or, because the different qualities comprised in the aggregate assume different weights. The SNA tells a nice story of why the volume of automobile sales may change, without change in any prices merely by shifting the weights of the components (SNA1993 para. 16.11). It seems to imply that it is only after aggregation in the national accounts that heterogeneity occurs. But heterogeneity is already there at the very elementary level of product classes. “In practice, even the lowest possible level of aggregation will still in many cases involve heterogeneous product groups and the level of detail will be greater in some parts of the national accounts than in others, depending on the sources available.” (Al, P.G et al. 1986 , p. 364), Although product classifications used in statistical offices contain hundreds of classes, this is far from creating the “perfect homogeneity” called for when measuring an actual price. Each class stands for thousands of products. As a consequence, the variable *q* expresses volume (quantity plus quality) rather than quantity even at the very elementary level of a price statistical product classification[1]. The SNA explicitly prefers the term “volume index” over “quantity index”, in order to express the distinction, and avoid misunderstanding.

Secondly, how to construct the decomposition of a transactions aggregate? If it is a volume and not a quantity index that we enter into the rows of our input and output tables which form are we to use? The ESA95 answers by resorting an abstraction at this point. Having introduced a mathematical formula where it is simple (equation 1) it shies away from the same precision when things become complicated, and rightly so, because the index number problem has probably employed more people over the years than have worked at the ESA95. The question is answered verbally by stating that “the preferred measure of year to year changes in volume is a Fisher volume index” (ESA93 para.10.61), without further explanation or comment, except for the one about lacking additivity already quoted above. Again we must dig more deeply.

The common term in the aggregate equation quoted above, after the homogeneous quantity equation, is the word “index”. Why must three variables which carry well defined dimensions at the microeconomic level, namely money unit (€), quantity unit (piece, kg etc.) and their ratio (€/piece, €/kg, etc.) loose all of this colour when aggregated to the macro-level, and assume the shade of dimensionless figures? It seems to be a habit rather than a need, and in actual tables we do find dimensions. Volumes are often presented as value figures such as GDP “in prices of the previous year”, or prices of some fixed base year. It seems a small matter, but it makes a difference in understanding the step from quantity to volume when you conceive the latter not as a dimensionless index, but as a value figure, expressed in monetary units. It is a different value than the nominal one, of course, but still a value, and by speaking in monetary units you naturally avoid the illusion that volumes are quantities. The resort to mere indices is a step away from constructing a true accounting table[2].

Another point is equally subtle, and also important. The resort to indices has a reason, namely that price and volume are undetermined when you consider only one year. You cannot take the tables of year 0 and study its prices of cars against those of textiles. Although for each of them the price index is set at 100, the base year, that does not entail that these prices are equal, in any sense. The same is then true for volumes, of course. Volumes are equal to nominal values in the base year, and this holds for any year you choose for this purpose. The working with index numbers tends to make people forget the indeterminateness in absolute terms and create the illusion that aggregates may be decomposed into a price and a volume component in the same way as in an individual transaction under equation 1. The ESA93 knows better, demanding that not values but “each and every change in the value of a given flow” must be decomposed into a price and a volume change. It probably would agree if we added that even in case the value itself does not change (in nominal terms), a mutually compensating change in the two direction of price, on the one hand, and volume, on the other, ought to be recorded. Change (velocity, in physics) is what we can measure, not absolute levels (size) of volumes and prices.

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[1] One may go even one step further. Looking at an individual price index series one will discover that the product observed today is not the same as its counterpart ten years ago. The link has been established by estimating, in a more or less sophisticated manner, the quality change that had to be taken into account in between. The “perfect homogeneity” claimed by the ESA is a reasonable ideal, but it exists only in the mathematical world of general microeconomic equilibrium.

[2] Publishing indices instead of value figure has been a well-known method of communists governments used in order to conceal accounting information, as some of the elder input-output experts may recall.

The third issue entailed in the ESA presentation of “general principles of price and volume measurement” is not directly found in the passage quoted above. It is discovered when this passage is read together with the one quoted in the beginning of this paper, and has to do with the problem of measuring unit. Values, and volumes, as we have agreed to call them now, are measured in money units. Other than in the natural sciences this measuring unit is in itself not invariable. Values between two different years are comparable only if we may assume that the money in which they are measured has itself not varied in value. It is generally understood that this condition implies an inflation rate of zero, in the strict sense. But we can compare nominal figures for different years if we correct for inflation in between.

It has not been noticed, yet, in the profession that the two paragraphs 10.01 and 10.13 of ESA95 address two different economic phenomena. The one speaks about money and the consequences its variation in purchasing power entails for its use as a measurement unit. The other addresses prices of goods and services where it is understood, implicitly, that these prices are measured in a constant measurement unit lest they could not be compared among each other. While the distinction seems very clear in concept, it is not followed very well in the every day language of economic statistics. The different phenomena are all assembled, and confounded under the expression “real”.

The current crisis about additivity induces us to distinguish more clearly than before between “real values”, on the one hand, and “volumes”, on the other (Reich 2001). The distinction is not only useful in concept, but also simple in method. Real values are obtained by correcting all nominal entries of input-output tables, and national accounts in general, for the common rate of inflation. You arrive at accounts not in current money units (nominal values), but in money units of a base year (real values), which are then mutually comparable and additive over time, in contrast to nominal values, expressed in current, i.e. varying measurement units. If in addition you also correct for the specific relative price change of each product, wherever this concept is applicable (product transactions), you arrive at each product’s volume change, as explained before. The distinction between real value (nominal values corrected for change in the measurement unit) and volume (real values corrected for change in relative prices of products, in addition to inflation) is the pivot point from which it is possible to construct an additive, chained decomposition of value changes of transactions aggregates into price and volume components.

The problem appears now for the first time, because under the old Laspeyres regime, both variations were eliminated congruently. In fixing a base year of prices you also fix the measurement unit, implicitly. Laspeyres index means measurement at constant prices in constant currency units of the same base year. With chaining there is no constancy of base year any more, and we must treat each phenomenon separately. The following formula describes the distinction. Write

$$\tilde{p} = \frac{p}{\Lambda}, \qquad (3)$$

where Λ denotes the general price level of an economy. Its logarithmic differential $\delta \ln \Lambda / \delta t$ is the rate of inflation, if it is positive, or, of deflation in the opposite case. Then $\tilde{p}$ is the price a good carries relative to the others, or, as we may then call it, its “real price”[3]. As a consequence, we may also define the “real value” of a transaction by

$$\tilde{v} = \frac{v}{\Lambda} = \tilde{p}q \qquad (4)$$

in what we feel is a necessary complement and clarification to equation 1. It expresses the recognition of the fact that money, in its function as a unit of measuring economic value, is not constant over time.

3. Problem solution: numerical integration

The set of concepts developed above for dealing with changing prices in an accounting framework clarifies the issue of what is meant by additivity of accounting entries. Homogeneous quantities are additive over time, not over products, and have no economic meaning. Prices are also not additive, neither over products nor over time. It is only values, the combination of the two that yield additivity across products. They would also be additive over time if the unit of account remained stable. In order to achieve full additivity over products and over time, the decomposition of a value change in money (nominal) must thus not result in two, but in three components, one for the change in volume, expressing an increase or decrease in production of the specific good or service in question, one for the change in the specific price of the commodity relative to other products, expressing a change in the forces of supply and demand on the corresponding commodity market, and finally, a component correcting for general inflation, or deflation, as the change in the unit of measurement.

Having thus stated the theory of our problem in conceptual terms, its mathematical formulation turns out as an ordinary task of numerical integration. Let *V(t)* be an aggregate of product transactions, composed of sub-aggregates *v(t)*, both measured in Euros,

$$V(t) = \sum v(t) \text{ [Euros].} \qquad (5)$$

All are unknown functions of time, observed only by means of discrete time measurements. The sub-aggregates *v* have price indices *p* attached to them, yielding corresponding volumes *q* according to equation 2. All these are also pictured as continuous functions of time. The question is how to disentangle the price and the volume component in the aggregate *V*. Is it possible to find aggregate variables P and Q so that equation 6 holds,

$$V(t) = P(t) \times Q(t) \text{ [Euros] ?} \qquad (6)$$

As said before, the problem stated in this way is unsolvable, because for a single year, taken by itself, the decomposition is undetermined. The price index may assume any number, the number for the volume index follows suit by equation 6. It is only the change in these variables that allows the desired decomposition. The change, written in differentials, is given by

$$\delta V = P\delta Q + Q\delta P \text{ [Euros].} \qquad (7)$$

[3] To avoid a common misunderstanding, the actual, observed price is the nominal price, of course. All so-called real variables are bookkeeping constructs produced in order to further analysis of the actual nominal price, no more.

The differentials are also functions of time. The time changes of the sub-aggregates *v* can be decomposed in a similar way,

$$\delta v = p\delta q + q\delta p \text{ [Euros]}, \tag{8}$$

because of definition 2. Decomposition 8 is obviously additive between the two components, and if we want additivity throughout the accounting system, compiling higher and higher aggregates, we must define

$$\begin{aligned} P\delta Q &= \sum p\delta q \\ Q\delta P &= \sum q\delta p \end{aligned} \text{ [Euros]} \tag{9}$$

as decomposition of the aggregate. Summing the volume changes of the sub-aggregates weighted by their prices yields the volume change - in Euros - of the aggregate, while summing the price changes of the sub-aggregates weighted by their volumes yields the change in value of the aggregate, which is caused by price changes of its elements, again in Euros. The decomposition respects the fact that neither prices nor volumes are additive by themselves, but only their combination is, values.

Simple, almost trivial as it is, this presentation of the aggregation problem, is nevertheless unusual. One is not accustomed, in index number theory, to think in Euros, or any other actual monetary unit of measurement, for that matter. Doing so, however, is not only essential of national accounting but also helps finding the link from index number theory to index number practice. The link is almost there, when you convert differentials to differences:

$$\begin{aligned} P\Delta Q &= \sum p\Delta q \\ Q\Delta P &= \sum q\Delta p \end{aligned} \text{ [Euros]} \tag{10}$$

The formula is imprecise in that it does not specify the time schedule, which needs to be defined, because the index is now discrete. Doing so yields

$$\begin{aligned} P^{t-1}(Q^t - Q^{t-1}) &= \sum p^{t-1}(q^t - q^{t-1}) \\ Q^t(P^t - P^{t-1}) &= \sum q^t(p^t - p^{t-1}) \end{aligned} \text{ [Euros].} \tag{11}$$

This is the well-known system of Laspeyres- and Paasche formulas, the first being assigned to the volume change, the second to the price change of the aggregate. Both changes add up to the total nominal change of the variables. Equation 11 reflects the fact that aggregates compiled in previous year prices are additive over products. The question is whether one has lost additivity over time, in exchange.

To answer it we return to continuous functions. Here additivity over time means integration over time, which is customarily performed by means of the so-called Divisia index:

You divide, for example, the first of equations 9 by V and write

$$\int_{t_0}^{t_1} \frac{P\delta Q}{V} = \int_{t_0}^{t_1} \frac{\delta Q}{Q} = \int_{t_0}^{t_1} \sum w \frac{\delta q}{q} \tag{12}$$

where

$$w = \frac{pq}{V} \tag{13}$$

are the weights of each product class in the aggregate. This is a possible, and not a necessary way of integrating the observed value changes. It follows the tradition of dimensionless indices, which are not easy to reconcile with economic accounts in money. In particular, adding volumes of different aggregates that have been compiled this way implies using the weights that existed between them in the base period. “It is not possible with this system to produce additively consistent tables in deflated values.” (Al, P.G.et al. 1986, p. 358). A more refined approach may be to integrate equations 9 directly as they stand, and write

$$\begin{aligned} V_q &= \int_{t_0}^{t_1} P\delta Q = \int_{t_0}^{t_1} \sum p\delta q \\ V_p &= \int_{t_0}^{t_1} Q\delta P = \int_{t_0}^{t_1} \sum q\delta p \end{aligned} \quad \text{[Euros]} \tag{14}$$

yielding the finite changes in volume and in prices of the aggregate in question. In practice, a chain index represents an approximation to the line integral (Al, P.G et al.1986, p. 357) Definitions 14 are additive, because the operations of integration and summation may be exchanged. They are commutative algebraic operations.[4]

But as explained above, an observed nominal price change may result from two causes, which are naturally distinguished in theoretical economics, but have not found their suitable expression in statistics, yet. A commodity’s price may change either because the condition on its specific market, demand and supply of the product change, or because money, in its function as means of payment, is disturbed, which also reflects on its functioning as a unit of measurement. More precisely, attributing an increase in price observed at a specific product to that product implies the assumption that the unit of measurement, money, has not changed. In contrast, interpreting it as an expression of inflation implies the relative price of the product not to have changed. Since the value of money is measured by the average of all prices (or rather its inverse), the two phenomena are easily confounded. However, everybody agrees that when we observe a general price rise for all products by the same amount this has nothing to with the value of those products, but reflects a pure devaluation of the currency, while when observing one specific price change, the general price level remaining constant (i.e. all other price slightly falling) we have a clear revaluation of the product in question vis-à-vis the others. It is in order to catch this distinction in the causes of a change of nominal value, - we repeat, - that we distinguish between nominal price, real price, and the general price level, as their bridge (definitions 3 and 4).

Integrals 14 are thus not well defined, because the unit of measurement in which they are expressed as quantitative variables varies itself over time and the integral (14) will not

[4] For an index theoretical foundation of this approach see (Balk and Reich 2006).

disentangle the two forces contained in the variation of the nominal prices *p*. For the purpose of separating the real from the monetary movements we must introduce the general price level as a third element of decomposition and write

$$\begin{aligned} \tilde{V}_q &= \int_{t_0}^{t_1} \tilde{P}\,\delta Q = \int_{t_0}^{t_1} \sum \tilde{p}\,\delta q \\ \tilde{V}_p &= \int_{t_0}^{t_1} Q\,\delta\tilde{P} = \int_{t_0}^{t_1} \sum q\,\delta\tilde{p} \end{aligned} \qquad \text{[Euros of year 0]} \qquad (15)$$

using definitions 3 and 4 of real values and real prices. The Euros in which the changes are measured are now constant and those of year 0.

In definitions 3 and 4 we assumed the general price level Λ as given. In order to compile real values we must now specify how this variables is to be measured. As said before, this is not a new statistics, but simply the index used for measuring inflation. Let vector $q_A(t)$ be the commodity basked used for the purpose, so that we can then define

$$\log \Lambda = \int_{t^0}^{t^1} \sum \frac{q_A \delta p}{\sum q_A p} \qquad (16)$$

The corresponding finite difference equations may be[5]

$$V^T{}_q = \sum_{t=1}^{T} \frac{p^{t-1}}{\Lambda^{t-1}} (q^t - q^{t-1}) \qquad (17)$$

$$V^T{}_p = \sum_{t=1}^{T} q^t \, \frac{p^t - p^{t-1}}{\Lambda^{t-1}} \qquad (18)$$

$$\Lambda^T = \prod_{t=1}^{T} \frac{\sum q_A{}^t p^t}{\sum q_A{}^t p^{t-1}} \qquad (19)$$

In the following example we will apply these index number formulas to a set of actual input-output tables.

[5] Others are also possible, of course, but we do not go into details here (see Hillinger 2000, Balk 2003).

4. Example: Applying the method to GDP expenditure

If incorporating additivity in chain index formulas appears simple, expressing it in tables may even be more so. We take the yearly input-output tables of the Netherlands between years 1995 and 2000 as a convenient case to prove the method. The following tables show gross domestic product for these years, and its decomposition into imports, exports, consumption of households and private organisations serving them, consumption of general government, and gross capital formation (GCF), in current prices (table 1), and in those of the previous year (table 2). They furnish the data from which to start our test.

Table 1: Nominal values
(in current prices and mill. current Euros.)

	Imports	Exports	H.holds, P.Org.	Gen. Gov.	GCF	GDP
1995	155927	173879	148238	72624	63419	302233
1996	164622	182712	157064	72861	67044	315059
1997	184361	204152	164996	76420	72518	333725
1998	197027	216207	175977	80440	78597	354194
1999	209471	225712	187593	85526	84710	374070
2000	250802	271819	200642	91288	89344	402291

Table 2: Values in prices and mill. Euros of previous year

	Imports	Exports	H.holds, P.Org.	Gen. Gov.	GCF	GDP
1995	--	--	--	--	--	--
1996	162735	181820	154166	72325	65839	311415
1997	180326	198857	161770	75177	71674	327153
1998	200099	219224	172969	79163	76984	348241
1999	208435	227272	184315	82443	82751	368346
2000	231513	251246	194221	87247	85837	387038

Source: Statistics Netherlands (2004)

Neither of these data are additive over time. Adding imports of year 1995 to those of year 2000 would be unacceptable, not because prices have changed, but because the unit in which they are measured has. Similarly for the data of table 2, building on prices of the respective previous years.

Table 3 shows the traditional method of chaining the data over time. One multiplies the ratios of aggregates in prices of previous years over their nominal value in the previous year, and applies them to the value of the base year in order to reach a number in Euros. The corresponding formula is

$$Q^t = V^{1995} \times \frac{\sum p^{1995} q^{1996}}{\sum p^{1995} q^{1995}} \times \ldots \times \frac{\sum p^{t-1} q^t}{\sum p^{t-1} q^{t-1}} \quad . \tag{20}$$

These time series are not additive, as the deflated components do not add up to deflated GDP. Table 3 shows the balance between the two numbers. Its size depends on the speed of change in relative prices, which is small here. Years 0 and 1 show a pure balance of zero, the first because it is in nominal values, and the second because it is in the form of the old Laspeyres constant price index which is naturally additive, as said before. The problem begins with year 1997.

Table 3: Chained volumes, traditional method
prices 1995=100

	Imports	Exports	H.holds, P.Org.	Gen. Gov.	GCF	GDP	sum comp.	balance
1995	155927	173879	148238	72624	63419	302233	302233	0
1996	162735	181820	154166	72325	65839	311415	311415	0
1997	178259	197886	158785	74625	70385	323369	323423	-54
1998	193476	212496	166458	77303	74720	337435	337502	-67
1999	204678	223371	174345	79228	78669	350918	350936	-18
2000	226216	248640	180505	80823	79716	363083	363468	-385

Source: Own calculation

An additive deflation proceeds in two steps, of which the first is to correct the nominal flows for the mere change in the unit of measurement, i.e. inflation of the currency value. The result depends on the choice of the commodity basket against which the currency is being gauged. Two of those are in use, the consumer price index, and the implicit GDP deflator. We opt for the second, because this choice places the two key variables of economic analysis, growth and inflation, in a coherent accounting relationship. The real value of GDP is thus equal to its volume, by definition, or, put the other way around, its real price is always equal to one[6]. The same is not true for any other aggregate or sub-aggregate, except by accident. Table 4 is thus compiled by using the price index of GDP as the general price level and divide it into all nominal entries of table 1. In contrast to table 1 these real values, expressed in Euros of year 1995, are comparable and additive over time. The formula is repeated from equations 3 and 4

$$\tilde{V}^t = \frac{V^t}{\Lambda^t} \tag{21}$$

where Λ^t is defined as

$$\Lambda^t = \frac{\sum p^{1996} q_{GDP}{}^{1996}}{\sum p^{1995} q_{GDP}{}^{1996}} \times \ldots \times \frac{\sum p^t q_{GDP}{}^t}{\sum p^{t-1} q_{GDP}{}^t} \tag{22}$$

in line with equation 19. This is a conventional chained Paasche index compiled on the basis of GDP as underlying product basket. Additivity over components has not been shown to balance in table 4, because it is obvious from equation 21 in connection with table 1.

Table 4: Real values
(in mill. Euros of 1995)

	Imports	Exports	H.holds, P.Org.	Gen. Gov.	GCF	GDP	Price level
1995	155927	173879	148238	72624	63419	302233	100
1996	162718	180599	155248	72018	66269	311415	101,17
1997	178640	197817	159876	74049	70268	323369	103,20
1998	187705	205977	167651	76634	74878	337435	104,97
1999	196506	211742	175982	80233	79467	350918	106,60
2000	226358	245327	181087	82391	80636	363083	110,80

[6] It is the „gold“ of former times, the standard of value.

The second step of analysis concerns what is nicely called “Preisbereinigung” in German, the actual separation of a change in the relative and specific price of a product as compared to the others (has the product become more expensive?), from its change in volume (has more of it been produced?). Table 5 and 6 demonstrate the compilation.

Table 5: Volume change from previous year
(in mill. Euros of 1995)

	Imports	Exports	H.holds, P.Org.	Gen. Gov.	GCF	GDP
1995	--	--	--	--	--	--
1996	6808	7941	5928	-299	2420	9182
1997	15522	15959	4652	2290	4576	11954
1998	15250	14605	7725	2658	4327	14066
1999	10868	10541	7943	1908	3957	13482
2000	20678	23954	6218	1614	1057	12165
sum	69125	72999	32466	8172	16338	60850
perc.' 95	44,3	42,0	21,9	11,3	25,8	20,1

Table 6: Price change from previous year
(in mill. Euros of 1995)

	Imports	Exports	H.holds, P.Org.	Gen. Gov.	GCF	GDP
1995	--	--	--	--	--	--
1996	-17	-1221	1082	-307	430	0
1997	400	1259	-23	-259	-577	0
1998	-6185	-6445	49	-73	283	0
1999	-2067	-4777	388	1690	631	0
2000	9174	9631	-1113	544	112	0
sum	1306	-1551	383	1595	879	0
perc.' 95	0,8	-0,9	0,3	2,2	1,4	0,0

Source: Own calculation

The formula for table 5 is

$$\Delta V^t{}_q = \frac{p^{t-1}q^t - p^{t-1}q^{t-1}}{\Lambda^{t-1}} \ , \tag{23}$$

which is just another way of writing equation 17. It says that the movement in volume of an aggregate is calculated by its change in prices of previous years, corrected for change in the measurement unit. The price change formula for table 6 reads as the corresponding complement in Paasche form, namely

$$\Delta V^t{}_p = \frac{p^t q^t - p^{t-1}q^t}{\Lambda^{t-1}} \tag{24}$$

Table 5 shows that all changes in volume of the components add up to the volume change of GDP, and table 6 shows how the different relative price changes cancel, when GDP is chosen as the product basket for the general price level index. Tables 5 and 6 sum to table 4.

5. Answering to critique

The problem of non-additivity of deflated input-output tables has not commanded much attention in the profession, yet. Nevertheless, some stiff resistance to any possible solution has been voiced, already. It is necessary therefore, after having explained what we consider a workable solution, to address these criticisms, and weigh their validity in light of the theory exposed above.

Let it be said in the beginning, and in order to reduce possible tension that under circumstances of slow price changes, as they are normally observed today, practical consequences of the discussion are limited. Whether you believe in additivity and choose a deflation method in accordance with it, or whether you don't and are satisfied with distributing the discrepancies over the tables, the resulting differences in treatment lie within the statistical margins of error of the tables, so that users neither in politics nor in econometrics must expect different results of their analyses. The issue is one of the theory and the aesthetics of national accounting, with not much practical relevance for the short and medium term horizon (see table 3).

We hold the firm theoretical position in this paper, nevertheless, that additivity of entries is an essential quality of any accounting system designed for producing economically meaningful balances. But additivity is not the only requirement such a system is to meet, of course. It may happen that in order to meet it other goals must be sacrificed, which is the general opinion, indeed. Ehemann et al. (2002) have studied the problem in this direction, and as their object of investigation is a method, which has much in common with the one presented here, their argument must be reconsidered.

Ehemann et al. (2002) look at a method proposed by Hillinger (2000), and find that while preserving additivity under deflation the method produces negative volumes, which seems not a reasonable outcome for an accounting method. As a brief answer we have pointed out above that volumes are not quantities and do not describe a state, but a change of state of a market or an industry in a certain direction. You cannot look at the input-output table of some year and distinguish its volumes from its prices, there. Volume is a variable of change, or of movement, between two years in the direction of product growth, in contrast to the movement of prices, which expresses the terms of exchange of those products. Both movements take place jointly. Integration of only one component is an artificial analytical device, but not a description of reality.

More precisely, the question Ehemann et al. pose is whether it is reasonable to have

$$p^0q^0 + p^0\Delta q^1 \leq 0 \qquad (25)$$

for a specific sub-aggregate. It is not so, obviously, because the left-hand side of equation 25 may be transformed into

$$p^0q^0 + p^0q^1 - p^0q^0 \geq 0 \qquad (26)$$

which must necessarily be positive under the normal conditions of positive quantities and prices. However, if we extend the time series by one more year, the question is whether

$$p^0 q^0 + p^0 \Delta q^1 + p^1 \Delta q^2 \leq 0 \tag{27}$$

is an unreasonable outcome, too. And this need not be. For extending equation 27 yields

$$p^0 q^0 + p^0 q^1 - p^0 q^0 + p^1 q^2 - p^1 q^1 \leq 0\,, \tag{28}$$

which again transforms into

$$p^0 q^1 + p^1 q^2 \leq p^1 q^1 \tag{29}$$

or

$$\frac{p^0}{p^1} + \frac{q^2}{q^1} \leq 1 \tag{30}$$

There is no reason why a change of this sort may not occurr, why the price change over the first period and the volume change over the second one may not together be smaller than one. The full decomposition of the change between year 0 and year 2 is given by equation 31,

$$p^0 q^0 + p^0 \Delta q^1 + q^1 \Delta p^1 + p^1 \Delta q^2 + q^2 \Delta p^2 = p^2 q^2 \tag{31}$$

Some of these terms are positive, others may be negative. Also the sum of changes in volume may turn out more negative than the initial value is positive. But we are not adding quantities which are economically meaningless in national accounts, but valued quantities. If you begin with a sale of a few computers at high price (p^0q^0), increasing the quantity at decreasing prices ending up selling many at a low price (p^2q^2), the accumulated growth in quantity valued at its respective price ($p^0 \Delta q^1 + p^1 \Delta q^2$) may well be higher than the final sales value so that subtracting it from the final value, or the first, results in a negative figure. But this figure has no meaning in itself, except as a comparison of a change of state with a state (growth rate). If the volume change is so high, the accompanying price change must necessarily be equally high but go into the opposite direction.

An analogy may illustrate the argument, at last. If you fly from Vienna to London you go 1000 km West. Yet, you don't expect to see Paris on the way, because you cover 500 km to the North, at the same time, also without visiting Berlin. Those two distances are virtual movements if taken by themselves, and only their combination describes the actual flight. In the same line, adding up volume changes separately from their accompanying price changes is made possible by the mathematical feature of integration as being an additive operation, but does not describe the actual movement which always takes place in terms of nominal values, and not just volumes alone.

6. Additive growth analysis of the Dutch economy 1990 - 2000

Economic growth of a country may be studied from two sides. Looking at the expenditure side of GDP one distinguishes between different categories of final demand (table 1), and deflating each of them, one arrives at the contribution to growth each one makes. Additivity over products is an essential for arriving at a coherent analysis of these data, and it is given within the system of calculating the components of GDP in current and in previous year prices (Tödter 2006). The other approach comes from the production side. One is interested in determining what is the contribution to national growth of each industry. This may be answered by studying its output, but it is generally agreed that a better variable of investigation is each industry's value added.

Value added is determined as a balance of output and intermediate input, so that additivity is even more essential for coherence here than on the expenditure side. However, in contrast to output, value added has no intrinsic quantity component. While for output the idea of underlying quantities, and corresponding prices is at least not far fetched, no such construction makes sense for value added. Deflation of value added is therefore compiled on an implicit assumption, namely that its volume change may be derived as the balance of volume changes of output and intermediate input, and for the price change similarly. change. Double deflation, as it is called, requires additivity, if it is to arrive at a meaningful result. Ans additive deflation produces interesting results if it is applied to the value added of industries within the input-output framework. We choose the development of the Dutch economy in the last decade as a case of interest.

Table 7 is the result of additive deflation of value added of 25 industries for the period 1990 to 2000. Output and intermediate consumption are also shown, which are deflated first, and deflated value added is derived thereafter. A welcome feature of additivity is its consistency in aggregation, which means that one may either deflate output and intermediate consumption separately, and derive deflated value added thereafter, or find the price index of value added by balancing the price changes of output and intermediate consumption first, and then apply that index to value added. The result must be the same. Table 7 shows varying developments of the different industries.

Agriculture, forestry and fishing, to choose industry (1), has kept its output more or less constant over the years, if corrected for inflation (real values), but within this level the industry has produced 13,3 percent more in volume while loosing 18,3 percent to its customers through lower prices. It has produced the additional 2760 mill. Euros of output with almost no additional input (342 mill. €1995). But the increased productivity of the sector has not been retained, but more than fully passed on to the other industries of the economy through a lowering of prices (-3790 mill. €1995). Industry (21), business activities, in contrast, has doubled its output in volume, gaining an additional 16.2 percent of the1990 level through higher prices. Value added follows this pattern, for both industries, which seems to be a general rule. Output and value added move rather proportionately. Calculating the coefficient of determination yields 0.936 for the correlation between volume changes of output and values added, and of 0.815 for the price changes.

Figures 1 and 2 provide a condensed picture of the development of value added of the industries. Visibly, there is no correlation between the movement in volume, representing the growth of product of an industry and its implicit real price, which represents its terms of trade within the economy. Figure 1, however, allows disaggregation of certain groups. There is a group of six industries, namely (15) construction, (19) financial services, (20) real estate activities, (22) health and social services, (23) other services, (24) general government where a growth in production by roughly 5000 mill. €1995 goes hand in hand with an improvement of their terms of trade of equal magnitude, although (15) construction, for example, has used proportionately more inputs than before (40.6 percent of 1990 as against 30.9 percent) and thus become less productive. There is the outlier of (21) business services already mentioned whose growth of roughly 20000 €1995 is accompanied by an improvement of real prices of 3500 €1995.

Another group consists of industries which have lost in real prices while they have gained in volume, namely (1) agriculture, (7) chemical products, (18) transport, (16) and trade. Industry (7), chemical products owes the loss to the input side where it had to pay an additional 1086 mill. €1995 , cutting the gain from its production growth (+2438 mill. €1995) almost in half. Since terms of trade are a relational variable, one may say that the second group has nourished growth of the first one by passing part of its production gains on to it through the price mechanism. All other industries are crowding around the origin in figure 1, which indicates they have neither grown very much in their product nor gained or lost much on their markets. In conclusion, the period has been less one of growth than of restructuring of the Dutch economy.

Table 7
Growth and trade of gross value added
Netherlands 1990 - 2000

		year	1990	2000	change	1990	to	2000
			real values Λ=0,9845	real values Λ =1,108	volume	volume	real prices	real prices
			min €1995	min €1995	min €1995	percent of 1990	min €1995	percent of 1990
1	Agriculture,	Output	20762	19732	2760	13,3	-3790	-18,3
	forestry and fishing	Interm. cons.	10561	10469	342	3,2	-433	-4,1
		GVA	10201	9263	**2418**	23,7	**-3356**	-32,9
2	Mining and	Output	8684	11094	1280	14,7	1130	13
	quanrrying	Interm. cons.	1484	2291	656	44,2	150	10,1
		GVA	7200	8803	624	8,7	980	13,6
3	Food products,	Output	34979	39550	7713	22	-3142	-9
	beverages and	Interm. cons.	27552	29657	4964	18	-2859	-10,4
	tobacco	GVA	7427	9893	2749	37	-283	-3,8
4	Textile and leather	Output	4491	4175	-12	-0,3	-304	-6,8
	products	Interm. cons.	3010	2885	66	2,2	-190	-6,3
		GVA	1481	1290	-78	-5,3	-113	-7,7
5	Paper products,	Output	13686	16917	4128	30,2	-898	-6,6
	publishing and	Interm. cons.	8206	10176	2650	32,3	-680	-8,3
	printing	GVA	5480	6741	1478	27	-217	-4
6	Petroleum products	Output	8982	15821	1057	11,8	5782	64,4
		Interm. cons.	7964	14469	1487	18,7	5017	63
		GVA	1018	1352	-431	-42,3	765	75,1
7	Chemical products	Output	22652	31263	9003	39,7	-393	-1,7
		Interm. cons.	15987	23637	6565	41,1	1086	6,8
		GVA	6665	7625	**2438**	36,6	**-1478**	-22,2

8	Rubber and plastic products	Output	4149	5168	1510	36,4	-492	-11,9
		Interm. cons.	2671	3489	1049	39,3	-231	-8,7
		GVA	1478	1679	461	31,2	-261	-17,6
9	Metal products	Output	14894	18258	4340	29,1	-975	-6,5
		Interm. cons.	9500	12222	3284	34,6	-563	-5,9
		GVA	5394	6036	1055	19,6	-413	-7,7
10	Machinery	Output	8779	13592	4953	56,4	-140	-1,6
		Interm. cons.	5640	9235	3594	63,7	0	0
		GVA	3139	4357	1359	43,3	-140	-4,5
11	Electrical and optical equipment	Output	12902	17495	5540	42,9	-946	-7,3
		Interm. cons.	8220	12406	4686	57	-500	-6,1
		GVA	4682	5089	853	18,2	-446	-9,5
12	Transport equipment	Output	9626	12634	2981	31	27	0,3
		Interm. cons.	7680	9891	2264	29,5	-53	-0,7
		GVA	1946	2744	717	36,9	80	4,1
13	Other manufacturing	Output	11217	15469	3819	34,1	433	3,9
		Interm. cons.	6005	8597	2639	43,9	-47	-0,8
		GVA	5212	6873	1181	22,7	480	9,2
14	Electricity, gas, water supply	Output	12016	16470	4180	34,8	274	2,3
		Interm. cons.	7256	11577	3934	54,2	386	5,3
		GVA	4760	4894	245	5,2	-112	-2,3
15	Construction	Output	37322	54372	11549	30,9	5500	14,7
		Interm. cons.	23471	35076	9528	40,6	2077	8,9
		GVA	13852	19296	**2021**	14,6	**3423**	24,7
16	Trade and repair	Output	50327	74492	25538	50,7	-1373	-2,7
		Interm. cons.	18422	30084	10874	59	789	4,3
		GVA	31906	44408	**14664**	46	**-2162**	-6,8
17	Hotels, restaurants	Output	8206	13139	3796	46,3	1136	13,8
		Interm. cons.	4157	6616	2129	51,2	330	7,9
		GVA	4049	6523	1667	41,2	807	19,9
18	Transport, storage and communication	Output	28893	50322	23994	83	-2564	-8,9
		Interm. cons.	12838	25863	12645	98,5	380	3
		GVA	16055	24459	**11349**	70,7	**-2944**	-18,3
19	Financial activities	Output	17165	35876	13332	77,7	5380	31,3
		Interm. cons.	6400	14726	7248	113,2	1078	16,8
		GVA	10765	21151	**6084**	56,5	**4302**	40
20	Real estate activities	Output	21197	34040	6928	32,7	5916	27,9
		Interm. cons.	4393	7202	2214	50,4	595	13,5
		GVA	16803	26838	**4713**	28,1	**5321**	31,7

21	Business activities	Output	31513	69467	32859	104,3	5094	16,2
		Interm. cons.	13023	29181	14608	112,2	1549	11,9
		GVA	18491	40286	**18251**	98,7	**3545**	19,2
22	Health and social work	Output	42767	55672	8249	19,3	4656	10,9
		Interm. cons.	12698	17912	3983	31,4	1231	9,7
		GVA	30069	37761	**4267**	14,2	**3425**	11,4
23	Other services	Output	22172	32768	6195	27,9	4402	19,9
		Interm. cons.	5647	8380	2566	45,4	168	3
		GVA	16525	24388	**3629**	22	**4234**	25,6
24	General government	Output	15790	25933	7022	44,5	3121	19,8
		Interm. cons.	7548	12795	4356	57,7	891	11,8
		GVA	8242	13138	**2666**	32,4	**2230**	27,1
25	Goods and services n.e.c.	Output	950	1140	183	19,3	7	0,7
		Interm. cons.	950	1140	183	19,3	7	0,7
		GVA	0	0	0	0	0	0
	Total economy	Output	464121	684860	192898	41,6	27842	6
		Interm. cons.	231284	349975	108515	46,9	10176	4,4
		GVA	232837	334885	84383	36,2	17665	7,6

Source: Statistics Netherlands and own calculations

Figure 1

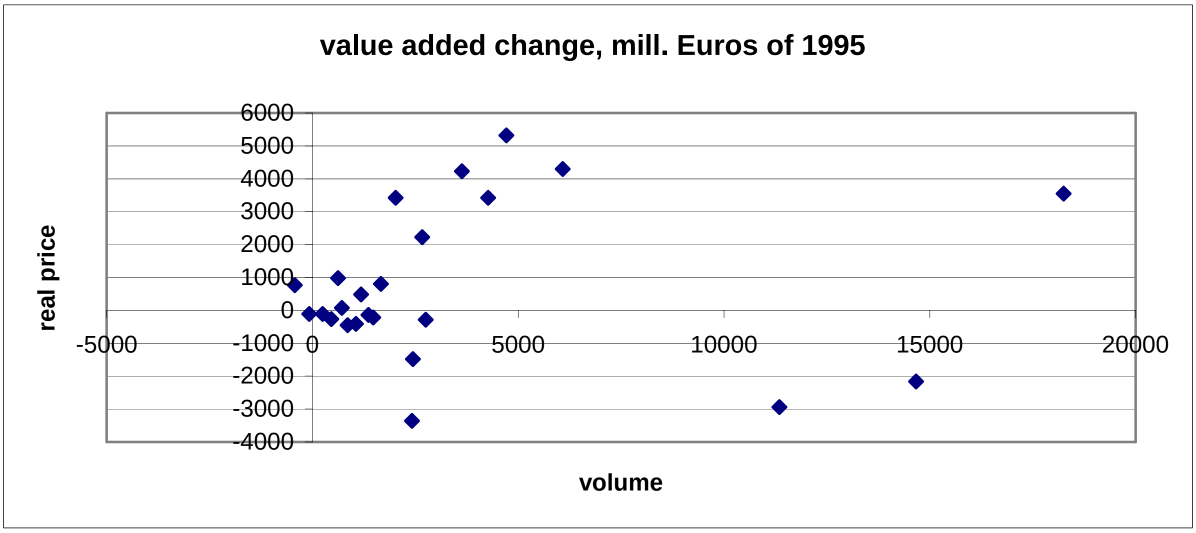
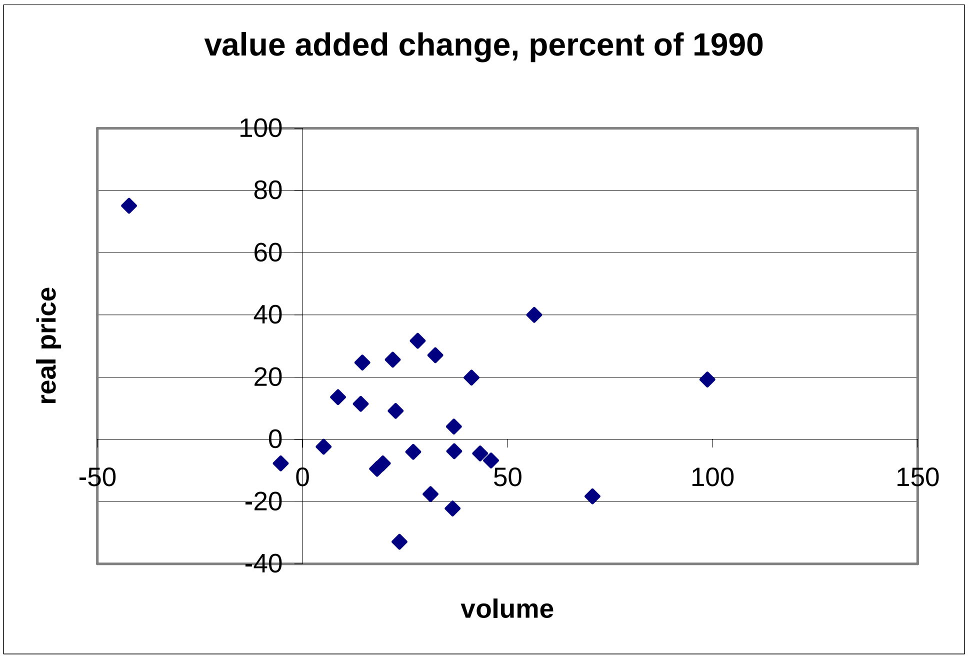


Figure 2



7. Conclusion

Let us resume the compilation procedure drawn out in equations17 to 19, illustrated in tables 4 to 6, and applied in table 7, verbally. You begin by deflating the nominal tables by means of the uniform GDP deflator, thus arriving at tables in constant Euros (real values). By subtracting, for each year, the nominal figures from the figures of the following year in prices of the previous year you arrive at the growth (and growth rate, if you wish) in nominal Euros, as is done under the conventional chaining method. But instead of multiplying the successive growth rates you add the absolute growth differences after having them made comparable deflating each by the corresponding uniform GDP deflator. The balance between the volume changes derived in this way and the changes in real value yield the complementary change in real prices.

Additivity of deflating procedures may be ignored, or even rejected, for different reasons. Only one is attacked in this paper, namely that additivity is impossible to achieve in connection with chain indices. Or, putting it the other way around, we hold the position that it is not a necessary implication to pay for up-datedness of price and value data by giving up additivity. The remark by Ehemann et al. that “interestingly, additivity was not mentioned as a desirable property of the estimates” when the decision to adopt chain indices was taken (Ehemann et al. 2002, p. 37), may be interpreted in two directions, either that it was not considered to be important, so the authors; or that its loss was self-understood as being an inevitable cost, which we repeat it is not.

In this light, the rejection of additivity as a desirable quality of input-output tables seems premature and reads more like a rationalisation of a decision already taken, than a rationale for deciding about the future. The quantitative effect of switching from the present non-additive to an additive method may be within the range of error of national accounts figure, generally, as long as the economy is in equilibrium. So there is no need to recalculate past time series, and the econometric results drawn from them will not have to be revised. But compilation and control, as well as marketing and analysis of the deflated series will be more in line with economic common sense than the under non-additive chaining rule. Economics takes place in Euros and Dollars, and not in indices. The yearly growth and inflation rates are identical in both methods, anyway.

Input output-tables, and national accounts in general, require additivity of their entries, in contrast to price statistics, for two reasons. One is the axiom of complete economic circuit, meaning that no value can get lost within the system, which is expressed by the fundamental equation that input equals output. The other is connected to the first in saying that the balance of output and intermediate input measures the value added to existing product (while consuming it in production of a new product). It is fairly reasonable to postulate these axioms not only for nominal tables, but for those in real terms as well. Otherwise one might turn the spear around: why claiming additivity for nominal tables, if for real ones it is not deemed desirable? Value added at constant prices was naturally additive, otherwise it would hardly have been accepted as an analytical variable. So why should value added at previous years prices not also be additive, especially as additivity is already given for each consecutive pair of years, and the only problem is to compose a long time series out of these elements?

The paper argues that the long term time series may be constructed by observing the fact that price statistics measure two effects in one, in that an individual price series contains, besides a particular market behaviour, the price movement due to general monetary effects, which have little to do with those particular market conditions. Using GDP, or the consumer price index, as is also customary, as a measure for the latter, it is possible to make the Euros of different years comparable, and additive, and thus construct an additive time series of price and volume movements.

Finally, the problem of additivity of deflated input-output tables may be discussed under two perspectives. One is speculative in nature. You re-assess and possibly renovate the theoretical concepts accompanying deflating procedures and demonstrate that within this theory the distinction between monetary phenomena and real phenomena has not been spelled out sufficiently, yet. This is our approach here. But you may also consider the issue from a purely practical point of view, accepting the theory of non-additive index numbers, and look only for a simple method of distributing the margins over the tables. In the latter case, the method suggested here may be just as convenient one as any other.

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