Human Capital within a Sraffian Fixed Capital Framework

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ABSTRACT *Although the classical economists identified human capital analogous to any other stock of physical capital, skills have not been considered explicitly; instead, the problem of heterogeneous labour was set aside by reducing different abilities to one kind of labour. Relative prices are then determined on the basis of a uniform wage rate. In the 1950s Gary Becker and Jacob Mincer dealt with the impact of education and training on earnings within a neoclassical model. The worker is paid according to his marginal productivity so that wage differentials occur due to different levels of qualification. On the basis of age-productivity profiles, which show the development of skills over the individuals lifetime, a Sraffian model is presented in which commodities are produced by means of commodities. Human capital is introduced in analogy to fixed capital, i.e. in each stage of production a finished commodity is jointly produced with an employee at different age. This allows the implementation of education as well as on-the-job training. From the pattern of depreciation it is possible to derive an efficiency-based wage regime, and by truncation the optimal length of employment can be determined. Moreover, the rate of return on education and the wage premium for the employee can be calculated so that the investigation of human capital is extended by the individual dimension.*

1 Introduction

Most attempts in educational economics have been done in a neoclassical framework, in which human capital is assigned to the role of being the prime force for economic growth. They all have in common that human capital is treated as an aggregate

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quantity of distribution in a one-commodity economy. This paper presents an alternative way of introducing knowledge in a production model by following the classical approach. Adam Smith not only recognized the economic advantages of division of labour, but already outlined the idea of treating human capital analogous to fixed capital. His successors Ricardo and Marx adopted his notion without dealing with it in detail. Since then the concept seems to have fallen in oblivion or at least has never been explicitly worked out. The present paper attempts to close this gap by applying the fixed capital theory to human capital. The stock of skills and productive knowledge embodied in men is thus treated like a durable means of production by taking into account the length of employment over some production periods. In order to follow the formation of the ideas, the paper is subdivided into five sections. Section 2 encompasses the notions of human capital by Smith, Ricardo and Marx. Although the classical economists assumed the wage rate to be at subsistence level, they were well aware of the existence of qualitatively different kinds of labour. Unlike the neoclassical tradition, which *a priori* acts on the assumption that knowledge is measurable and can thus be aggregated, the classical economists dealt with heterogeneous labour and its effects on the wage rate, and tried to establish rules that justify and legitimate its homogenization. Section 3 is devoted to the implications of education at the individual level, as investigated by Gary Becker and Jacob Mincer. The conceptions of general training and specific training are illustrated. Reverting to studies in economic and psychological area, typical age-productivity profiles are demonstrated, whose shape reflects the learning capability of an individual. A comparison with the age-earnings curve shows that delayed wage-contracts, i.e. increasing payments by age, diverge with the actual productivity of the worker and would lead to substantial welfare losses on the macroeconomic level.

Section 4 presents a human capital model that is elaborated on the basis of Sraffa's fixed capital approach. The economy under consideration is characterized by a long run position, i.e. it is abstracted from dynamic aspects, which allows to constrain the analysis to the price system. The model contains three sectors, of which only one utilizes skilled labour in the production of its commodity. The education and the ICT-producing sector are formally interrelated by introducing human capital as durable means of production. The asset is treated as under construction in the first periods, before being employed in the production of the ICT-commodity in the remaining periods. Referring to the concept of vertical integration, the prices of the finished goods are calculated. In a subsequent step, the value of human capital is determined, which allows to build a wage-regime based upon the efficiency pattern of the worker. Furthermore, two indicators for the profitability of education on the individual level are proposed. Thus, the effects of knowledge are investigated from the point of view of both contracting parties, of the employer as well as of the employee. In order to show the applicability of the model, a numerical example is given in the appendix in which prices of finished commodities and the value of human capital are calculated for a simple economic system. The last section reflects on the results of this paper,

discusses its legitimation and suggests some areas for further research.

2 Historical notes

The classical economists Adam Smith, David Ricardo and Karl Marx were aware of the impact of skills and abilities on the production system and on economic progress as a whole. However, they *a priori* abstract from the problem by reducing different types of qualification to homogeneous labour, so that the resulting uniform wage rate does not reflect the level of knowledge. What is remarkable is the analogous treatment of human and fixed capital. This analogy can already be found in the writings of Sir William Petty, who compared the loss of armaments and machinery of warfare with the loss of human life (Petty 1986, I). Adam Smith already identified the improvement in skills of the worker as a fundamental source of economic progress and of increasing economic welfare and demonstrated for the fist time how investments in human capital affect personal incomes and the structure of wages. His focus mainly lies on determining the causes of the wealth of a country. Thus, the distinction between fixed and circulating capital does not play such a major role as in Marxs theory. Considering the economy at the macro-level, Smith draws a clear distinction between private capital, that is the stock of commodities owned by a person and social fixed capital i.e. the general stock of a country. The acquisition of abilities thereby belongs to the fixed capital of society, (Spengler 1977, p.33). In the following, Smith gives a clear instruction of how to deal with human capital embodied in the labourer:

> The improved dexterity of a workman may be considered in the same light as a machine or instrument of trade which facilitates and abridges labour, and which, though it costs a certain expense, repays that expense with a profit (Smith 1937, pp.265 f.).

The costs of acquiring skills will be offset by the future profits which the worker yields of being more productive than the average labourer. The different knowledge enhanced either through investment in education or through learning by doing thus explains wage differentials among the labour force. According to Smith, natural born talents play a minor role, as

> the differences between the most dissimilar characters, [...], seem to arise not so much from nature, as from habit, custom, and education (Smith 1937, pp.15 f.).

The advanced economy is characterized by the division of labour, which leads to rising labour productivity. Smith is nevertheless aware of the possible danger of a strong diversification of the working process: The intellectual endowment of the labourer is reduced to the single working step that he is supposed to execute. The break up of the process inevitably leads to a decay of his potential abilities. The worker himself becomes a machine that, though being highly specialized in the sector it is recruited,

can not be transferred to other activities without severe losses. The educational institutions are the means for offsetting these adverse effects, and should be supplied or at least supported by the government. Education alone can guarantee a basic common knowledge of the labour force, which is important for the progress of society by narrowing the discrepancies in the individual endowment of human capital. It is a mean of a society to prevent from superstition, fundamentalism and other dangerous currents. However, education may not have the character of a public good; in fact, tuition fees, an elaborative reward system, competition among the teaching institutions work as an incentive system and lead to an optimal allocation of knowledge within the economy (Spengler 1977, p.35). Hence, Smith's invisible hand gets visible once again in the accumulation of human capital.

It is interesting that he notes that

> the general stock [of abilities] of any country or society is the same with that of all its inhabitants or members (Smith 1937, p.264).

Thus, the value of human capital does not rise if considered on the macro-economic level, so that positive external effects induced by human capital do not occur.

David Ricardo goes along with Adam Smith that human capital has a strong impact on economic growth:

> The natural price of all commodities, excepting raw produce and labour, has a tendency to fall, in the progress of wealth and population; for though, on one hand, they are enhanced in real value, from the rise in the natural price of the raw material of which they are made, this is more than counterbalanced by the improvements in machinery, by the better division and distribution of labour, and by the increasing skill, both in science and art, of the producers (Ricardo 1951, p.71).

Skills have a complementary character since countries that are rich in resources can not optimally utilize them unless a stable government and an appropriate educational system are provided (Ricardo 1951, p.76). In chapter 31, *On Machinery*, Ricardo revised some of his previous considerations that labourers would equally benefit from technical progress as capitalists (Ricardo 1951, p.378). Increasing utilization of fixed capital leads to a rise in the gross and net produce of an economy. However, higher incomes do not necessarily imply advantages for all classes:

> I now, [however], see reason to be satisfied that the one fund, from which landlords and capitalists derive their revenue, may increase, while the other, that upon which the labouring class mainly depend, may diminish, and therefore it follows, if I am right, that the same cause which may increase the net revenue of the country, may at the same time render the population redundant, and deteriorate the condition of the labourer (Ricardo 1951, p.379).

The negative effect on the labour class can be compensated if the additional revenues are used to accumulate more capital, which in turn requires employing some labour. Thus, workers who were discharged due to the introduction of the machine, may be needed in the production of other commodities. In the best case, labourers are just transferred to other areas of work, while the aggregate labour demand remains the same. In contrast to his predecessors, Marx focused on labour as the main source of economic wealth. He strictly differentiated between labour power and commodity inputs in the production. The labourer behaves rational, he starts and regulates his work by making use of commodities and natural ressources. Unlike produced goods, which have a unidirectional relation to the environment as they may modify it, the worker does not only change it but changes himself in the interaction with nature. Marx does not see labour entirely as a further factor of production, but emphasizes its assignment to human beings as the only 'intellectual' creatures:

> What distinguishes the worst architect from the best of bees is this, that the architect raises his structure in imagination before he erects it in reality. At the end of every labour-process, we get a result that already existed in the imagination of the labourer at its commencement (Marx 1976, I,p.198).

Marx's strong distinction therefore constitutes the valorisation process (Verwertungsprozess) and the labour process (Arbeitsprozess). The first leads to the determination of exchange value and prices respectively, which then defines the income distribution, whereas the latter founds the process of creation. The two different processes form the distinction of constant and variable capital (Meacci 1989, p.13). Accordingly, constant capital presents a transfer of value (preservation of value), while the variable part itself contributes to the creation of surplus value (Marx 1976, I, p.238-40). Thus, variable capital contains the wage goods (subjective factors) which are the only source of exploitation of labour power and therefore of surplus in Marxs theory. The constant capital is composed of the "instruments of labour" , i.e. other means of production (objective factors), whose composition reflects the social conditions at the workplace. Constant capital denotes both, circulating capital, which vanishes after the end of the period as it is completely used up in the production process, and fixed capital like machinery, buildings etc. Constant capital is only of value for production if the worker is able to deliver the objects from their "death sleep" (Marx 1976, I,p.201) and transform them into commodities which supply the wants of the consumers. Though Marx illuminates the role of the human being in the production process, he did not consciously discriminate between different skills embodied in it. The labour of a spinner does not differ from the labour engaged in the production of any other good since the question is not to deal

> [...] with the quality, the nature and the specific character of the labour, but merely with its quantity. And this simply requires to be calculated (Marx 1976, I, p.236).

Wage differentials

Although in general the classical approach treats human capital analogous to any other stock of physical capital, skills – as an own class of capital goods – are not introduced explicitly. Instead, the problem of heterogeneous labour is *a priori* left aside by reducing different abilities to one kind of labour, which leads to a uniform wage rate in the determination of prices (Schefold 1989, p.247).

Adam Smith was already well aware that labour can not only be measured in time dimension; it also deviates in the intensity of work:

> It is often difficult to ascertain the proportion between two different quantities of labour. The time spent in two different sorts of work will not always alone determine this proportion. The different degrees of hardship endured, and of ingenuity exercised, must likewise be taken into account (Smith, *Inquiry into the Nature and Causes of the Wealth of Nations*, qtd. in (Kurz and Salvadori 1995, p.323)).

A standard measure of such subjective factors like ingenuity and efforts is hardly to be found. However, according to Smith such differences in labour are reflected in the prices of the commodities whose production they entered as inputs (Kurz and Salvadori 1995, p.323). Moreover, he considers the proportion of different labour in the production as relatively stable and constant over time, even if relative prices of commodities change [1]. Ricardo adopts the notion of the stability of relative labour inputs in order to reduce different types of work to a common unit, a procedure which Schefold calls "weak homogeneity", i.e. labour power, being physiologically measurable, does not differ, no matter in which processes it is utilized. (Schefold 1989, p.250). In contrast, Marx's approach of abstract labour is based upon the so called "strong homogeneity":

> All labour of a higher or more complicated character than average labour is expenditure of labour-power of a more costly kind, labour-power whose production has cost more time and labour, and which therefore has a higher value, than unskilled or simple labour-power. This power being of higher value, its consumption is labour of a higher class, labour that creates in equal times proportionally higher values than unskilled labour does. Whatever difference in skill there may be between the labour of a spinner and that of a jeweller, the portion of his labour by which the jeweller merely replaces the value of his own labour-power, does not in any way differ in quality from the additional portion by which he creates surplus-value. [] We therefore save ourselves a superfluous operation, and simplify our analysis, by the assumption, that the labour of the workman employed by the capitalist is unskilled average labour (Marx 1976, I,p.237).

[1] This assumption is empirically not evident, as Erreygers (1994, p.48) points out.

Schefold derives Marx's physiological measure from a normal level of effort as the arbitrary origin of variable intensive utilization of labour power. This "normal effort" then allows the comparison between two kinds of labour which have been occupied in the production process for the same time (Schefold 1989, p.250). In *Production of Commodities by Means of Commodities*, Sraffa adopts the classical conception of homogenization of the labour force:

> we assume differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labour receives the same wage (Sraffa 1960, pp.9-10).

Proceedings in the theory of economic growth, especially those in the twentieth century, however force the treatment of knowledge as a main and self-contained source of progress. Since then, there have been many and mainly fruitful attempts to introduce heterogeneous labour in a classical production model. Schefold did so in differentiating between several types of labour in the scope of an "artificial segmentation" (Schefold 1989, pp.246-56, and pp.313-23). One kind can be regarded as the set of jobs which bear a higher level of disadvantages than the average, e.g. Smith's public executioner. Another kind is formed by produced skills. If the costs of education are known, relative wages can be calculated in the same manner as relative prices of commodities, even though certain difficulties arise (e.g. how to treat skills which are used in the non-commodity producing sector like in a household). The next class are natural talents which by definition have the attribute of scarcity, and are thus treated in the scope of the theory of rents. Another approach to the introduction of heterogeneous labour gives Parrinello. He considers the problem of wage stickiness by differentiating between two different types of labour characterized by two different wage regimes. The wages of the second regime thereby partially reflect the scarcity in special inputs of the secondary labour vector. The solution can then be found by the optimal choice of techniques (Parrinello 1990). A similar conception is represented by the Sraffian model of Guido Erreygers, who introduces different kinds of labour which are not available in abundance so that relative wages operate as a signal of scarcity. Unlike in the theory of land, unskilled labour nevertheless gains a positive reservation price equal to the subsistence wage level. Not until one type of labour is scarce in the sense that there is an excess demand for it, the worker endowed with these specific skills is able to yield an extra wage (Erreygers 1994, pp.47-66). Gerhard O. Orosel (Orosel 1973, pp.251-79) uses a linear production model of von Neumann type to illustrate human capital as an input to the production system of the economy as well as an output of educational institutions and of the commodity sector in terms of informal training on the job. As the assumption of constant returns to scale has been retained, the model can not explain structural dynamics caused by the formation of human capital. It is more likely to show in which way abstract services may enter the classical von Neumann model.

3 The neoclassical perspective

In the 1950s economic theory (re-)discovered the role of human capital for the wealth and progress of a country. The economic analysis of education has been based upon the principles of capital theory and its adaptation to human agents of production (Rosen 1987, p.681). Technically speaking, human capital is regarded as the stock of skills embodied in the agents of production. Investments in human capital either within the commodity market in form of training on the job or in a separate education sector, extend the knowledge of the individuals, which further lead to increasing earning power. Almost all attempts in educational economics have been fitted into a neoclassical framework by linking the computation of knowledge value with the marginal product approach; revolutionary work in this field has been done by Gary Becker and Jacob Mincer.

From a general empirical background, Becker deduced an intuitive model of education, focussing on the individual optimization problem of demand for human capital. Accordingly, agents that are assumed to be identical in their abilities make their consumption decisions dependent on the expected incomes net of costs of education. Thus, young people are more likely to invest in human capital, since they can collect its return over more years. In a competitive economy, higher wages corresponding to a higher level of education reflect the greater productivity of labour (Mincer 1991, p.1). Paying attention to the fact that the growing size of markets enforces specialization, Becker worked out an appropriate classification of on-the-job training, distinguishing between human capital that is general and can be transferred among firms and sectors, and knowledge which can only be applied to a special firm. Other forms of education such as schooling can then be assigned to one of these two main distinctions.

General training

Since general training increases productivity in all firms equally, firms do not have incentives to pay for the educational services,though they provide them. Thus, it is up to the single worker whether he accepts the training offer and bears its costs or maintains at the basic level of skills that he supplied to the firm when he entered it (Becker 1964, p.12). Unlike the entrepreneur, the labourer has a motivation to extend his skills, knowing that he will yield higher wages in the future. These higher wages, reflecting the marginal product of labour, presents a compensation for the reduction of his current earnings w_t by the costs of training k.

Since intertemporal equilibrium requires that

$$\sum_{t=0}^{T-1} \frac{MP_t}{(1+i)^{t+1}} = \sum_{t=0}^{T-1} \frac{w_t}{(1+i)^{t+1}} \tag{1}$$

we obtain with respect to the training period,

$$MP_0 = w_0 + k$$
$$w_0 = MP_0 - k$$

Hence, the net earnings during the training period correspond to the difference between the income on the potential marginal product and the costs of training.

A similar operation is used in accounting for the treatment of material goods, which emphasizes the connection to physical capital theory (Becker 1964, p.13). In Becker's conception the investments contain not only direct outlays, but also the opportunity costs of training, i.e. the foregone earnings that could have been acquired elsewhere. The most obvious case can be found in schooling, where the student balances the advantages of being employed and receiving a wage against the activities of pure accumulation of human capital.

Specific training

Specific training means that skills elaborated during the training period can only be applied to the firm which provides the training, as the benefits, i.e. the increasing productivity of the worker, can not be transferred to other industries. Hence, the entrepreneurs have incentives to invest in education while the labourers face a constant wage rate independent of the sector where they are occupied. All the returns from training are collected by the firms in the form of larger profits resulting from higher productivity (Becker 1964, p.20). The training will be provided as long as the present value of returns exceeds their costs. Moreover, a competitive equilibrium requires in the long run the equality of gains, i.e. the difference of the marginal product and the uniform wage rate, and investment outlays.

$$MP_0^{'} + \sum_{t=0}^{T-1} \frac{MP_t}{(1+i)^{t+1}} - \sum_{t=0}^{T-1} \frac{w_t}{(1+i)^{t+1}} = w_0 + C(2)$$

MP_0 denotes the opportunity marginal product of trainees (i.e. the potential product), and C the costs of training to the firm. In full equilibrium, the return from training equals costs.

The provision of specific training is more risky than the supply of general training: If the worker drops out (knowing that he will get paid the same wage everywhere), the expenses of education are sunk costs to the firm and an equally trained person may not be available. To anticipate such a turn over, the employer is therefore willing to offer a premium for each man-hour.

In reality, cases of pure general training as well as of completely specific training are unlikely to occur; many firms offer both types of educational services, of course in different proportion to each other, dependent on the sector in which they operate.

Becker's classification can also be extended to educational organizations as well as to other factors which positively effect productivity. School for instance, can be considered as a special institution entirely associated with production of training. It is more or less a single production system, whereby a firm providing general training is mainly involved in the production of commodities and considers the training itself as a joint product. Furthermore, improvements in mental and physical health, mostly supported by the government, increase overall productivity and can therefore be treated like general training. If a firm invests in better working conditions, this equals an offer of specific training. The premium the workers gain need not necessarily be measured in monetary terms; most of the campaigns aim at binding the labourer by increasing his solidarity to the firm. The estimate of total costs requires of course *a priori* some knowledge and specification of the number of periods invested in education. While the period covered by formal schooling is easily determined, the period covered by on-the-job training is much more difficult to calculate.

4 Human Capital within a Fixed Capital Framework

In the following, a general production model is presented, in which education and training are accounted for by referring to Sraffa's fixed capital theory. Piero Sraffa himself regarded the utilization of fixed capital, e.g. machines, as one of the main characteristics of modern industries (Sraffa 1951, p.63). Durable means of production are regarded as inputs to the process of production which are not used up within one period of time but may be remained in the production system for a sequence of periods. Thus, the problem arises of how to define the relative efficiency of a machine which is used to produce the same commodity by means of commodity and labour inputs over its lifetime. Although the process involves the same composition of inputs, it can not be assumed that they also enter each period of production in equal quantities, since the machine may work with variable efficiency (Baldone 1980, p.89). For the determination of productivity and depreciation quotas of the machine, a joint production approach has proved to be useful in order to determine its quantitative contributions; what is left of the fixed capital part at the end of one period, is thereby considered as a joint product of the finished commodity produced in the respective process: Wear and tear of fixed capital is taken account for by handling a machine with the physical lifetime T at different ages as qualitatively different goods each produced in a different process. The heterogeneity of the capital goods in question has therefore a temporal dimension. The set of processes involved in the production of one finished good will be called a group (Schefold 1980, p.147) or an industry respectively. Thus, the industry is subdivided into T many processes to calculate the relative prices of the finished goods as well as of the intermediate goods, i.e. the old machines. The difference in the prices of the machine in two consecutive years is nothing else than its depreciation over time. Unlike in the neoclassical models, the depreciation rate is not an exogenous construct

based upon the technical data of a machine, but is determined within the system according to the efficiency pattern of the machine during its utilization. Truncation of processes leads to the calculation of the economic lifetime, which does not necessarily have to coincide with the physical life time of a machine.

The idea of treating human capital within a fixed capital model was outlined by Kurz and Salvadori (Kurz and Salvadori 1995, pp.327-34). They distinguish between two kinds of accumulation of human capital. The first is the problem of training offered by an educational sector, e.g. schooling. Institutions operating in this sector do not "produce" any other commodities apart from the trained person. The second case is on-the-job training, which is provided within the commodity sector, so that the specific skills of a person appear as a joint product in the respective production processes. Although giving clear instructions of how to deal with education in a classical scheme, they did not elaborate them within a theoretical framework. The present paper therefore attempts to close this gap, by presenting a model which encompasses both problems of general and specific training. The production system under consideration contains three interrelated subsystems. The first one is the production of commodities by means of non-durable commodities (for the sake of simplicity we rule out fixed capital utilization in this sector). The second subsystem is the set of processes involved in the production of one type of skilled labour and will be called the education sector. Since schooling takes time, a fixed capital approach is applied, where in all intermediate stages of production only a student of different age and endowed with increased knowledge is produced, while the degree he obtains at the end of schooling appears as the marketable output in the last production period. The graduate student then enters the third subsystem, which uses finished commodities and non durable-means of production together with unskilled labour and professional knowledge to produce finished goods of high quality. The processes involved in training are part of this system in so far as they constitute the primary as well as some of the first secondary processes. When the training is finished, the person maintains in the same industry as an expert in his working field, which leads to positive net returns in the remaining set of processes.

The industrial structure of an economy can be differentiated according to the level of skill-intensive production. Various studies[2] show that the demand for specific skills is highest in manufacturing and service industries producing innovative communication technology (ICT), while ICT using industries and non-ICT sectors predominantly employ unskilled labour. In the following, these denotations are adopted in order to distinguish between the two branches. The production system will therefore consist of $n-2$ industries forming the basic manufacturing sector, of the general education sector and of one industry operating in the ICT producing sector in which the training is off shored. The whole technique produces n finished commodities, one of which is the academic degree.

[2]See for example (Prskawetz, Lindh, et al. 2006)

For the time being, it is assumed that the economy is stationary so that a long-period analysis is presented. We take final demand as given and represented by the consumption vector $\mathbf{c}$. With regard to the labour force, it is distinguished between unskilled labour $\mathbf{l}^1$ and skilled labour $\mathbf{l}^2$. The resulting labour matrix L contains the number of working hours of both types necessary in each process of production.

In order to avoid an unnecessary level of complexity, the basic manufacture sector, in the following only called the base system[3], is considered as a single production system, i.e. only one finished good is produced in each process. This assumption of course rules out the cases of joint production and fixed capital. Inputs of non-durable means of production produced in the different sectors together with some unskilled labour are involved in the production of $n-2$ finished products, which then are utilized as inputs in their own production as well as in the processes of the other subsystems.

A representative single process can thus be stated as follows:

$$(1+r)\,\mathbf{a}_i\mathbf{p} + w_1 l_i = b_i p_i, \quad i = 1, ...n-2. \tag{3}$$

Subindex i stands for the ith process employed in the base system producing a single commodity i.

The whole subsystem contains $n-2$ processes and is denoted by:

$$(1+r)\,A_B\mathbf{p} + w_1\mathbf{l^1} = B_B\mathbf{p} \tag{4}$$

$A_B = \begin{bmatrix} \mathbf{a}_1 \\ \vdots \\ \mathbf{a}_{n-2} \end{bmatrix}$ is the input matrix of dimension $(n-2)\times n$ of the base system. The row vector $\mathbf{a}_i = (a_{i1}, ..., a_{in})\,, i = 1, ...n-2$, presents the inputs necessary to produce one unit of the ith output.

B_B denotes the output matrix of the base system. Because of single production and by the normalization of output, it is an identity matrix of order $(n-2)$. $\mathbf{l}^1 = \left(l_1^1, ..., l_{n-2}^1\right)$ is the column vector of unskilled labour, measured in number of man-hours demanded.[4] $\mathbf{p} = (p_1, ..., p_n)$ is the vector of prices of the n finished commodities. As usual, r and w_1 refer to the rate of profits and the wage rate for unskilled labour respectively.

Industry $(n-1)$ relates to the public education sector which is characterized by a fixed capital system of production. In each year of schooling, the person accumulates

[3] Note that the base sector – though here assumed to be basic – does not entail any condition about the decomposability of the subsystem. It is only an expression for the *class* of commodities produced in this sector, not for their *function* in the production system.

[4] Since only unskilled labour is employed in the subsystem under consideration, vector $\mathbf{l}^2$ denoting skilled labour is a zero vector, which appears later on in the presentation of the whole production system.

general knowledge by means of finished commodities and unskilled labour.[5] For sake of simplicity, it is also assumed that the government pays the costs of schooling out of tax revenues, such that the concept of compulsory education is applied. The related tax regime will not be considered in the scope of the model.

To simplify the notation, the greek letters α, β will be used to indicate these industries that utilize and produce human capital, i.e. α, β refers to the education sector and the ICT-sector respectively. Thus, schooling can be described by a group of T_α processes, where T_α is given and denotes the length of education. In each year, the student enters the production process as the main input, since she has to work hard to extend her knowledge, and leaves the process one year older. Each process t, $t = 1, ..., T_\alpha$, can be stated as follows:

$$(1+r)\left\{\mathbf{a}_\alpha(t)\,\mathbf{p} + h_\alpha(t-1)\,p_{\alpha,t-1}\right\} + w_1 l_\alpha^1(t) = h_\alpha(t)\,p_{\alpha,t} \tag{5}$$

$\mathbf{a}_\alpha(t)$ denotes the column vector of inputs in the tth education period, $\mathbf{p}$ is the price vector of the n finished commodities. $h_\alpha(t)$ stands for the t year old student. $p_{\alpha,t}$ denotes the value of her skills in the corresponding year. Note that in the primary process, when the 'fresh' student enters school, human capital in the sense of the acquisition of knowledge is not yet available and therefore can not be utilized as an input, hence $h_\alpha(0) = 0$.

The system under consideration does not entail the modus that a student who fails the class has to repeat the education period. If she is not able to meet the required level of qualification, she has to drop out. Otherwise students with different educational backgrounds appear at the input side of the following processes, which would amount to joint utilization of machines in the fixed capital approach.

Since the output is positive only in the last period and corresponds to the graduate person, it is evident that

$$\sum_{t=1}^{T_\alpha} b_{n-1}(t) = b_{n-1}(T_\alpha) = 1$$

The complete subsystem can be presented as:

$$(1+r)A_\alpha\mathbf{p} + w_1\mathbf{l^1} = B_\alpha\mathbf{p} \tag{6}$$

After the completion of her studies, the person enters the ICT-producing commodity sector endowed with a certain level of general knowledge. This implies that in the last

[5] The assumption that the teachers are treated like unskilled workers is restrictive; but in the model considered, skilled labour entirely refers to the knowledge used in the production of the ICT-commodity. Moreover, the target is to derive a wage system for the utilization of human capital that is based upon the efficiency in producing a finished good. Since the school sector is entirely involved in the education of students, no commodity is produced.

year of education the output referring to the student $h_\alpha\,(T_\alpha)$ is equal to zero since he does not attend school any longer.

To simplify matters, the ICT-producing sector entails only one manufacturing industry producing a single finished telecommunication good of high quality. Since its production necessarily requires specific skills, the graduate student is getting trained on the job in the first years of her occupation, which in the following are called the set of training processes. As already pointed out in section 3, the skills elaborated by this kind of training can only be applied to the firm under consideration and can not be transferred to other industries. In contrast to Becker's conception, it is assumed that the worker herself partly pays the costs of training by getting earnings below the minimum wage rate. After having finished, the worker receives an additional premium according to her efficiency.

In order to be able to use the features of the fixed capital theory, the main assumptions with regard to the utilization of non-durable means of production shall be adopted:

1. No joint production of finished goods means that only one consumption good is produced in the nth industry.

2. Non transferability of intermediate goods holds, if the trainee/ trained person can not be hired in a different industry from the one where she has got trained. This is not at all an artificial axiom, since specific training restricts the employee to that working area in whom she already attained the necessary level of qualification.

3. The ICT-producing process only utilizes finished goods as inputs and perhaps one worker of a certain age.

4. The person engaged in the labour process does not present a consumption good. This is evident by matter of human capital incorporated in an individual.

5. It is assumed that the firm does not have to pay any additional rewards to the employee at the end of his career, so that the concept of free disposal is applied.

6. Furthermore, differences in natural talents are not taken into consideration since they belong to the set of exhaustible resources whose scarcity can only be accounted for – like land – in the rent theory.

7. The institutional lifetime of skilled labour is given and coincides with the years of work necessary in order to be allowed to retire.

Since it is assumed that the employee pays indirectly for her training, the costs of education reduce the basic wage rate by a fixed amount in all years of employment. The payments below productivity in the training periods are offset by the additional wage premium in the remaining years. For matter of convenience it is assumed that the wage rate of skilled labour is a constant fraction of the basic wage rate. Hence,

$$\begin{aligned} w_2 &= cw_1 \\ c &\leq 1 \end{aligned}$$

The rate of costs of training C is then the difference between the wage rates:

$$C = w_1 - w_2 = w_1 - cw_1 = w_1 (1 - c)$$

From the firm's point of view the person does not immediately contribute to a positive net return because the skills have not yet been attained. However, as the specific knowledge is indispensable for the production of an ICT- commodity, the trainee will not be dismissed, no matter which type of efficiency pattern she shows in each year of training $t, t = 1, ...\tau$. The lower wage rate should therefore recompense the firm for the losses in the first years. Afterwards the productivity of the employee varies with respect to experience and age.

The group of the T_β processes of the nth industry contains – apart from some finished commodities as inputs – the person who has been recruited from the education sector, together with some unskilled labour l_n^1 and the number of hours l_n^2 the skilled person is employed. Each process $t, t = 1, ..., T_\beta$, can be denoted by:

$$(1 + r) \{\mathbf{a}_\beta \mathbf{p} + h_\beta (t - 1) p_{\beta,t-1}\} + w_1 l_\beta^1(t) + w_2 l_\beta^2(t) = b_\beta(t) p_\beta + h_\beta (t) p_{\beta,t} \qquad (7)$$

$\mathbf{a}_\beta$ denotes the row vector of inputs of the ICT-producing industry, $\mathbf{p}$ is the price vector of the n finished commodities. $h_\beta (t)$ stands for the trainee employed in the ICT-producing industry for t years. $p_{\beta,t}$ denotes the value of the skills the trainee provides in year t of her occupation.

As the professional drops out at the end of period T_β, $h (T_\beta) = 0$.

We can normalize the output vector of the ICT-product such that each output in a different stage of production presents a proportion of the overall output in the industry:

$$\sum_{t=1}^{T_\beta} b_n(t) = 1$$

The complete subsystem can be presented as:

$$(1 + r) A_\beta \mathbf{p} + L_\beta \mathbf{w} = B_\beta \mathbf{p} \qquad (8)$$

where the vector $\mathbf{w}^T = [w_1, w_2]$ denotes the vector of wage rates and the labour matrix L_β contains the number of hours of both labour types.

The economy

Two conditions have to be satisfied in order to get a solvable equation system:

1. The *existence condition* requires that the number of processes in the total production system is larger or equal to the number of commodities involved.
2. The *uniqueness condition* requires that matrix $(B-(1+r)A)$ is regular.

The technique is thus presented by the indecomposable square matrix $A_{m\times m}$, the output matrix $B_{m\times m}$, and the labour matrix $L_{m\times 2}$. Since m denotes the number of processes engaged in the system, it is the sum of the n finished commodities, the $T_\alpha - 1$ periods of general education, and the $T_\beta - 1$ years during which the skilled person is employed in the ICT-sector.

$$A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{0} \\ \mathbf{a}_2 & \vdots \\ \vdots & \vdots \\ \mathbf{a}_{n-2} & \mathbf{0} \\ \mathbf{a}_\alpha(1) & \mathbf{h}_\alpha(0) \\ \vdots & \vdots \\ \mathbf{a}_\alpha(T_\alpha) & \mathbf{h}_\alpha(T_\alpha) \\ \mathbf{a}_\beta(1) & \mathbf{h}_\beta(0) \\ \vdots & \vdots \\ \mathbf{a}_\beta(T_\beta) & \mathbf{h}_\beta(T_\beta - 1) \end{bmatrix} \quad B = \begin{bmatrix} \mathbf{b}_1 & \mathbf{0} \\ \mathbf{b}_2 & \vdots \\ \vdots & \vdots \\ \mathbf{b}_{n-2} & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_\alpha(1) \\ \vdots & \vdots \\ \mathbf{b}_\alpha(T_\alpha) & \mathbf{h}_\alpha(T_\alpha) \\ \mathbf{b}_\beta(1) & \mathbf{h}_\beta(1) \\ \vdots & \vdots \\ \mathbf{b}_\beta(T_\beta) & \mathbf{0} \end{bmatrix} \quad L = \begin{bmatrix} l_1^1 & 0 \\ l_2^1 & 0 \\ \vdots & \vdots \\ l_{n-2}^1 & 0 \\ {l_\alpha}^1(1) & 0 \\ \vdots & \vdots \\ {l_\alpha}^1(T_\alpha) & 0 \\ l_\beta^1(1) & l_\beta^2(1) \\ \vdots & \vdots \\ l_\beta^1(T_\beta) & l_n^2(T_\beta) \end{bmatrix}$$

To ensure the viability of the production system, the aggregate demand for each finished commodity must not exceed its aggregate supply. Since we normalized each output to 1, the following must hold:

$$\sum_{i=1}^{m} a_{ij} \leq 1$$

With regard to human capital, some considerations shall be pointed out: Since the base sector only employs and produces finished goods, each sub vector of human capital in the input and output matrix is a zero vector. Each process involving human

capital employs one and only one person, either a student in the education period or a professional (in training) afterwards. Hence, each $\mathbf{h}_{\mathrm{i}}(t), i = \alpha, \beta,\ t = 1, ..., T_i.$, in the input and the output matrix is a different unit vector.
The variables that have to be calculated are the prices of the finished goods, the value of human capital by age and the wage rates for unskilled and skilled labour:

$$\mathbf{p} = \begin{bmatrix} p_1 \\ \vdots \\ p_n \\ p_{\alpha,1} \\ \vdots \\ p_{\beta,T_\beta - 1} \end{bmatrix}$$

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

To solve the price system, we assume according to Sraffa the rate of profit to be given and to be non-negative. Furthermore,the wage rate for unskilled labour is chosen as the *numeraire*, i.e

$$\hat{\mathbf{p}} = \mathrm{w}_1{}^{-1}\mathbf{p}$$

and since w_2 is equal to cw_1, we define

$$\Omega^T = [1, c]$$

Hence,the number of unknowns is equalized to the number of equations, so that the system is solvable:

$$(1+r)\,A\hat{\mathbf{p}} + L\boldsymbol{\Omega} = B\hat{\mathbf{p}} \tag{9}$$

The long run period position for $r \geq 0$ is yield if $(\mathbf{x}^*, \mathbf{p}^*, q^*)$ is the solution to the following equation system:

$$[B - (1+r)\,A]\,\hat{\mathbf{p}} \leq L\boldsymbol{\Omega} \tag{10}$$

$$\mathbf{x}^T\,[B - (1+r)\,A]\,\hat{\mathbf{p}} \leq \mathbf{x}L\boldsymbol{\Omega} \tag{11}$$

$$\mathbf{x}\ [B - (1+r)\,A] = \mathbf{c} \tag{12}$$

$$\mathbf{x} \geq \mathbf{0}, \hat{\mathbf{p}} \geq \mathbf{0} \tag{13}$$

The first equation is the result of cost minimizing choice of techniques, i.e. all processes

in operation do neither gain extra profits nor yield extra costs.
Equation (11) and (12) express that in a stationary state each process have to be activated to such an extent that reproduction of the requirements of investment as well as the requirements of use is ascertained. Since only finished products can be consumed, consumption vector $\mathbf{c}$ is a vector whose first n components are positive, and the others equal to zero. Since the graduate student does either not present a consumption good, the $(n-1)$st coefficient is zero as well. The last inequalities indicate that the activity vector and the prices in terms of a a positive wage rate are non-negative in the long run.

Since skilled employees have the characteristics of non-basic commodities, the value of human capital need not be calculated simultaneously with the prices of the finished goods and the wage rates. A common way to reduce the system is to eliminate the intermediate stages of production from the system. In the case of the base sector, where each good is produced in one period of time, the subsystem of $n-2$ processes enters the reduced system without any transformation. With regard to the education sector and the ICT-producing sector , each tth equation of the group of processes $i, i = \alpha, \beta$, is multiplied by a factor$(1+r)^{T_i - t}$, and summed up over t, $t = 1, ..., T_i$.:

$$\tilde{\mathbf{a}}_i(r) = \sum_{t=1}^{T_i} (1+r)^{T_i - t} \mathbf{a}_i(t)$$

$$\tilde{\mathbf{b}}_i(r) = \sum_{t=1}^{T_i} (1+r)^{T_i - t} \mathbf{b}_i(t)$$

$$\tilde{\mathbf{l}}_i(r) = \sum_{t=1}^{T_i} (1+r)^{T_i - t} \mathbf{l}_i(t)$$

Consequently, the education sector is aggregated to a single process, containing the input and output vectors as well as the labour requirements at different stages of production of the referring sector. By the same token, it is able to reduce the processes running in the ICT-industry to one integrated process.
The associated price equations form a vertically integrated system (or system of core processes), in which human capital is masked out:

$$(1+r)\,\tilde{A}\,(r)\,\mathbf{p}_1 + \tilde{\mathrm{L}}\mathbf{w} = \tilde{\mathrm{B}}\,(\mathrm{r})\,\mathbf{p}_1 (14)$$

where

$$\tilde{A}\,(r) = \begin{bmatrix} \mathbf{a}_1 \\ \vdots \\ \tilde{\mathbf{a}}_n \end{bmatrix} \quad \tilde{B}\,(r) = \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \tilde{\mathbf{b}}_n \end{bmatrix} \quad \tilde{\mathbf{l}}\,(r) = \begin{bmatrix} \mathbf{l}_1 \\ \vdots \\ \tilde{\mathbf{l}}_n \end{bmatrix} \quad \mathbf{p}_1\,(r) = \begin{bmatrix} \hat{p}_1 \\ \vdots \\ \hat{p}_n \end{bmatrix} \quad \mathbf{w}\,(r) = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

This technique only presents the finished products, i.e. the system is reduced to n equations.

The triplet $\left(\tilde{A}(r), \tilde{B}(r), \tilde{L}(r)\right)$ is dependent on the prevailing rate of profit; the larger the number of production periods T_i, the more the prices are influenced by variations in the rate of profit.

The value of human capital

Since human capital is treated analogeously to intermediate capital goods in the fixed capital approach, its value does not present a market price. While finished commodities are demanded in different industries, one and the same person is occupied within the same firm and is engaged in the production of the same commodity. Once the prices of the finished commodities and the wage rates are determined, it is easy to calculate the value of human capital by reverting to the group of processes formed by the education and the ICT-producing sector respectively [6]. The T_i processes

$$(1+r)\left\{\mathbf{a}_i\mathbf{p}_1 + h_i(t-1)\,p_{i,t-1}\right\} + \mathbf{l}_i(t)\mathbf{w} = \mathbf{b}_i(t)\mathbf{p}_1 + h_i(t)\,p_{i,t}$$
$$i = \alpha, \beta; t = 1, ..., T_i.$$

can be reordered in such a way that the value of skilled labour employed and produced in the respecting process stands on the left and all other costs on the right hand side of the equation:

$$p_{i,t-1}(r) - p_{i,t}(r) = \frac{1}{(1+r)}\left[\mathbf{b}_i(t) - (1+r)\mathbf{a}_i(t)\right]\mathbf{p}_1(r) - \mathbf{l}_i(t)\mathbf{w}$$
$$t = 1, 2, ..., T_i.$$

where $h_i(t)$ is left aside as it is a unit vector anyway.

To simplify the notation, we again define

$$y_{i,t}(r) = \left[\mathbf{b}_i(t) - (1+r)\mathbf{a}_i(t)\right]\mathbf{p}_1(r) - \mathbf{l}_i(t)\mathbf{w}$$
$$t = 2, ..., T_i - 1.$$

The first and the last year of employment in the ICT-sector have to be considered separately. With respect to the primary process, the net return $y_{\beta,1}$ must be reduced by the value of the graduate student, already calculated in the integrated system from the side of costs of production. Then it is possible to determine the same value by discounting all future net returns:

$$p_{\beta,0}(r) = p_{n-1} = \frac{y_{\beta,1}}{(1+r)} + \frac{y_{\beta,2}}{(1+r)^2} + \cdots + \frac{y_{\beta,T_\beta}}{(1+r)^{T_\beta}}$$

[6] The derivation follows Schefold's considerations in (Schefold 1989, pp.159-75).

Thus, the costs of schooling equals the discounted future net returns. This result is similar to the conclusion Becker arrived at, but is different in two aspects: Becker deduced the formula as the equilibrium condition at the individual level, and thus also takes opportunity costs,i.e. foregone earnings into his consideration. Here, the equality is automatically provided by the concept of vertical integration, if the rate of profit remains uniform in each period and reflects the efficiency criterium from the point of view of the employer. Hence, opportunity costs, which are interpersonally hardly comparable are not (and need not to be) involved.

In the last year the professional drops out at period T_β, so that the related value of human capital is zero. Thus, we have at once

$$p_{H,T_\beta-1}(r) = \frac{1}{1+r} y_{\beta,T_\beta}(r)$$

By use of matrix $N(r)$

$$N(r) = \begin{bmatrix} 1 & -(1+r)^{-1} & 0 & 0 & 0 \\ 0 & 1 & -(1+r)^{-1} & \vdots & \vdots \\ 0 & 0 & 1 & -(1+r)^{-1} & 0 \\ \vdots & \vdots & 0 & 1 & -(1+r)^{-1} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

the values of human capital are derived as the solutions to the following matrix equation:

$$N(r)\mathbf{p}_i = \frac{1}{(1+r)} \mathbf{y}_i\,(r) \tag{15}$$

Wage premium

The wage premium is paid to the employee in the ICT-sector to recompense her effort at work. It therefore serves as an incentive to raise the productivity of skilled labour. What we assume is that the annual charge on human capital is paid out to the worker after having finished the initial training programm. The annual charge can be stated as

$$z_{\beta,t} = (1+r)p_{\beta,t-1} - p_{\beta,t}$$

or in matrix notation

$$\mathbf{z}_\beta = (\mathbf{1}+\mathbf{r})\,\mathbf{N}(\mathbf{r})\mathbf{p}_\beta$$

From (15) it immediately follows, that

$$\mathbf{z}_\beta = \mathbf{y}_\beta$$

Thus,the wage premium is equal to the annual net return. In the following it is proposed that as long as the net return is negative, the employed person is getting paid the common wage rate w_2. Only if $y_{\beta,t}$ is positive, it is transferred in full amount to the skilled worker, since the increase in productivity can be traced back to the use of her skills and the raise of her effort respectively:

$$z_{\beta,t} = \begin{cases} 0 \text{ for } y_{\beta,t} \leq 0 \\ y_{\beta,t} \text{ for } y_{\beta,t} > 0 \end{cases}$$

The total annual wage payments the skilled labourer earns, denoted by $\pi_{\beta,t}$, are:

$$\pi_{\beta,t} = w_2 l^2(t) + z_{\beta,t}$$

where w_2 corresponds to the minimum wage rate for skilled labour, and works as a reservation price which is paid to the worker independently of her performance.

In the training periods characterized by negative returns the premium equals to zero so that the trainee earns the wage rate of skilled labour for each man-hour employed. As soon as $z_{\beta,t}$ becomes positive, she is getting paid above the reservation price, i.e.

$$\begin{aligned} &\pi_{\beta,t} = w_2 l_{\text{ß}}^2(t) + z_{\beta,t} \\ &= w_2 l_{\text{ß}}^2(t) + [\mathbf{b}_\beta(t) - (1+r)\,\mathbf{a}_\beta(t)]\,\mathbf{p}_1(r) - w_1 l_{\text{ß}}^1(t) - w_2 l_{\text{ß}}^2(t) \\ &= [\mathbf{b}_\beta(t) - (1+r)\,\mathbf{a}_\beta(t)]\,\mathbf{p}_1(r) - w_1 l_{\text{ß}}^1(t) \end{aligned}$$

What is left at the right hand side of the equation is the net return at time t net of wage payments for skilled labour. Thus, from the worker's point of view, the output produced is the counting element, but not the time, i.e. the hours spent on the job, since the fixed part of the earnings cancels out. If the working-hours per day are increased, the wage payments rise only if the net returns also climb up. Hence, the constructed wage-regime is entirely based on the efficiency of the professional so that a lower effort would harm herself. Furthermore, labour saving progress is also of advantage for the skilled person, provided that the amount of unskilled labour is reduced.

Age-efficiency profile

Various studies have been investigating the effects of cognitive and physical abilities on labour performance, using either supervisor's ratings, piece-rates or employer-employee matched data sets for their analysis (Prskawetz, Lindh, et al. 2006, p.25).[7] They all show that individual productivity increases in the first years of occupation and is then remaining constant before it decreases at the end of the career. For a close examination of the efficiency pattern that the skilled worker shows during his career, we divide in the first set of T_α education processes, the τ training processes, which run until the

[7] Further references are also found in (Skirbekk 2003).

person has attained the required level of qualification, and the set of $T_{\beta-\tau}$ performance processes starting from then onwards until the age of retirement. During the periods of general schooling the student does not produce any commodity output. It follows that up to period $T_{\alpha-1}$ the net output is certainly negative and corresponds to the value of annual inputs that are used in the education sector:

$$\mathbf{b}_\alpha(1)(r), ..., \mathbf{b}_\alpha(T_\alpha - 1)(r) = \mathbf{0} \rightarrow y_{\alpha,1}(r), ..., y_{\alpha,T_\alpha-1}(r) < 0$$

Since the educational system is paid out of tax revenues and the production in the ICT-sector requires skilled labour, the negative returns do not lead to the truncation of the processes, no matter which efficiency profile the student shows during schooling.Only by assumption that his efficiency increases from each year to next, his value would be positive throughout the whole period of general education.

Training processes refer to the length of period in which the trainee can not fully apply her skills to production. While the length of schooling is fixed, the number of training periods does not only depend on the learning ability of the employee but also on the rate of profit, because the net return serves as the ruling indicator. Empirical studies come to different results concerning the length of the training period and the optimum date for its supply. Depending on the type of occupation, the number of periods varies between three years for manufacture activities and ten years in analytic fields (see (Illmakunnas, Maliranta, and Vainiomki 1999) and (Ericsson and Lehmann 1996)). From the employer's point of view, the investment in further on-the-job training is less profitable with regard to older workers, since the returns can only be collected for fewer years. Since a positive net output at the given rate of profits indicates the completion of training, the net returns of human capital are negative in each training period $t = 1, ..., \tau$, where $\tau < T_\beta$ denotes the end of the training period:

$$y_{\beta,1}(r), ..., y_{\beta,\tau}(r) < 0$$

If formal training implies that the trainee can increase her skills continuously, she is of rising efficiency already at the beginning of her career, so that the firm has no reason to dismiss her. But even if the net returns do not increase from one year to the next, the person would still be employed, as there is no alternative way to produce the ICT-commodity. After having completed the training, the skilled labourer will produce a positive output of commodity n no later then in period $\tau + 1$. From then onwards, the net output is increasing by a diminishing rate. After period ψ the output will fall again until T_β.[8]

Hence we have,

$$0 < y_{\beta,\tau+1}(r) < y_{\beta,\tau+2}(r) <, ..., < Y_{\beta,\psi}(r)$$
$$\tau < \psi < T - 1$$

[8]The case of constant efficiency is ruled out since it can hardly be sustained with respect to human capital.

i.e. the professional is of rising efficiency at a given rate of profit r, and has a positive value in the time interval $[\tau + 1, \psi]$.
Though education and on-the-job training can compensate a reduction in physical and mental abilities, their positive effect can only be extended to a certain stage in the individual career, since receptivity is declining with the age of the worker. This also affects the success of retraining since it is getting harder for the employee to get used to new tasks and a changing working environment.Moreover, whereas the effects of learning are distinctive in different scale according to the kind of work, the decline in cognitive abilities such as reasoning, speed and memoring that starts by the age of 50 is independent of gender, national labour market conditions, and qualification (Skirbekk 2003, pp.4-6). Physical abilities show the same peak pattern for the age group of 50 and above, though they are strongly correlated with the type of work. [9]

Thus, in the last years of his career, the person is of falling efficiency, and therefore the net returns are monotonically falling in that range. As soon as the net output returns negative, the production process becomes unprofitable, so that the employee would not be occupied in the ICT-producing sector any longer, even if he has not already attained the age of retirement.

$$y_{\beta,\psi+1}(r) > y_{\beta,\psi+2}(r) > ...y_{\beta,T_\beta}(r) \geq 0$$

Figure (4.1) shows the age-efficiency profile of skilled labour derived from the considerations above.

Age-earnings profile

While studies on productivity generally show an inverted U-shaped curve of marginal labour product whose inflection point is located between the age of 30 and 40 (Skirbekk 2003, p.13), wage-payments tend to peak at relatively high ages, at the end of the professional career. This in turn causes serious problems for the sustainability of an economy: Since the population grows older, the financing of a seniority based wage system that is in conflict to the actual productivity of the workers, obstruct economic progress in several ways. On the one hand, the propensity to consume is tendentiously lower for older people whereas young persons are likely to spend more. On the other hand, such a wage system stimulates the level of unemployment of old workers especially if the rate of technological change is high (Bartel and Sicherman 1998). These problems do not arise in the elaborated model, since the derived wage regime is based upon the individual efficiency. While attending school the person does not apply his skills for the production of a commodity, and since he has not yet acquired the basic level of knowledge he formally belongs to the unskilled labour force. The effort-based wage payments thus only occur in the ICT-sector and depend

[9] With the rise of the rate of technological change, physical characteristics become a less important factor for productivity.

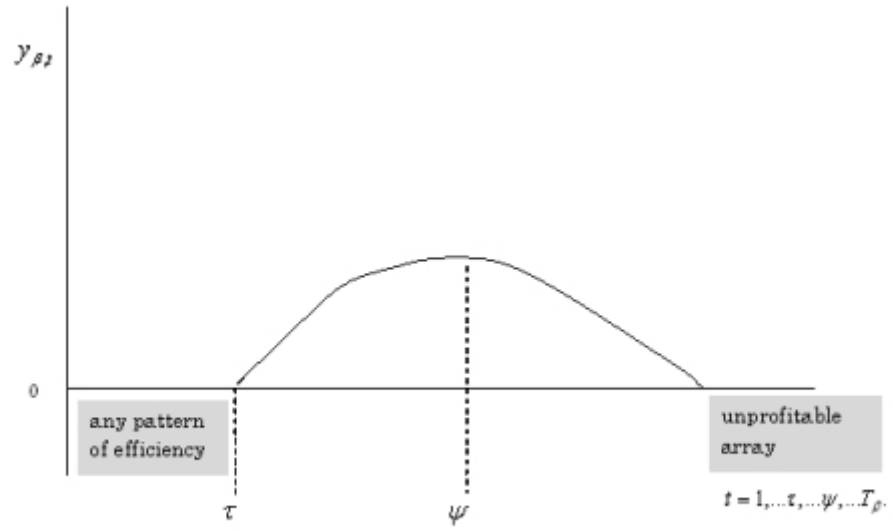

Figure 1: age-efficiency profile

on the efficiency of the employee. As the efficiency in turn reflects the development of the annual net returns, the derived age-earnings profile has the same shape as the age-efficiency curve.
In the first years of employment $y_{\beta,t}(r)$ it is assumed to be negative, i.e.

$$Y_{\beta,t}(r) \equiv [\mathbf{b}_\beta(t) - (1+r)\mathbf{a}_\beta(t)]\,\mathbf{p}_1(r) - \mathbf{l}_\beta(t)\mathbf{w} < 0$$
$$t = 1, ..., \tau.$$

It follows, that

$$[\mathbf{b}_\beta(t) - (1+r)\mathbf{a}_\beta(t)]\,\mathbf{p}_1(r) - w_1 l\beta^1(t) < w_2 l^2_\beta(t)$$
$$t = 1, ..., \tau.$$

Hence, the trainee produces a net output per manhour below the current wage rate. Reverting to statement (4), the wage premium in the respective years is equal to zero.

From τ onwards the net returns are positive, i.e.

$$[\mathbf{b}_\beta(t) - (1+r)\mathbf{a}_\beta(t)]\,\mathbf{p}_1(r) - w_1 l^1_\beta(t) > w_2 l^2_\beta(t)$$
$$t = \tau + 1, ..., T_\beta.$$

In this case, the net profits per unit of skilled labour are higher than the wage rate, and the professional is compensated by exactly that difference.
Fig. (4.2) shows the age-earnings curve based upon the efficiency considerations above.

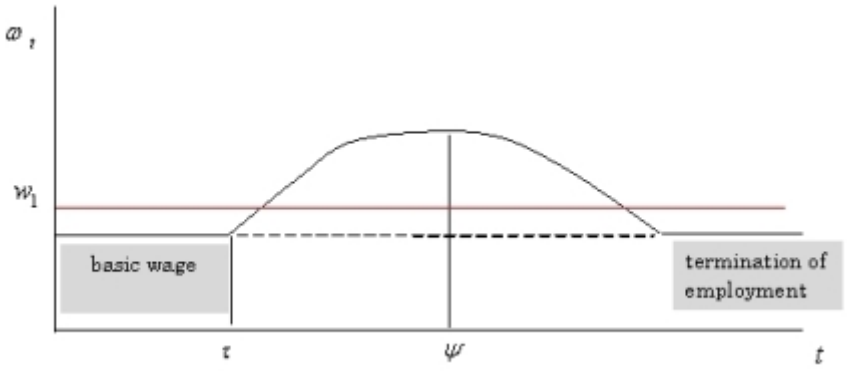

Figure 2: age-earnings profile

The end of the training period is indicated by the intersection of the wage line and the curve of net returns. Thus, the duration of education is endogenously determined and is dependent on the efficiency pattern, on the given rate of profit as well as on the rate of costs of education.

Rates of return on human capital

At the individual level, the returns of training can be evaluated by introducing two indicators which are commonly used in empirical studies[10].

The rate of return of education measures profitability of skilled labour and is the relation between the returns of education, i.e. the wages, to the investment costs, both discounted to the present value. Reconsidering the wage payments for skilled labour, we have

$$\pi_{\beta,t} = w_2 l^2(t) + z_\beta, t$$

If $\pi_{\beta,t}$ is summed up over the length of employment and discounted to the present value, then

$$\Pi = \sum_{t=1}^{T_\beta} \pi_{\beta,t} \left(1+r\right)^{-t} \tag{16}$$

The costs of training can be stated as

$$X = \sum_{t=1}^{T_\beta} C l^2 \left(1+r\right)^{-t} = \sum_{t=1}^{T_\beta} \left(1-c\right) w_1 l^2 \left(1+r\right)^{-t} \tag{17}$$

[10]See for example (Mincer 1991)

Thus, the returns of education over all periods of employment are

$$\rho(r) = \frac{\sum_{t=1}^{T_\beta} \left(cw_1 l^2 + z_\beta, t\right)(1+r)^{-t}}{\sum_{t=1}^{T} (1-c)\, w_1 l^2 (1+r)^{-t}} \tag{18}$$

If ρ is smaller than one, the person would prefer maintaining at his basic level of knowledge, since the overall costs of education exceed the expected revenues.
The educational premium compares the annual earnings due to education with the expected payments from an alternative employment and therefore relates the wage premium to the wage rate of unskilled labour for each period t:

$$\sigma_{\beta,t} = \frac{\pi_{\beta,t}}{w_1 l^2} \tag{19}$$

The premium thus measures the current returns of education for the skilled worker. In the training periods the employee is only paid the basic wage rate w_2. Hence, for $t = 1, ..., \tau$ the educational premium reduces to:

$$\sigma_{\beta,t} = \frac{\pi_{\beta,t}}{w_1 l_t^2} = \frac{cw_1 l_t^2 + z_{\beta,t}}{w_1 l_t^2} = c \tag{20}$$

Only if the net return is sufficiently positive,σ_t will be higher than one.

Length of employment

The question arises for how many years working agreements should be terminated, given the efficiency profile of the employee.

As already mentioned, the decline of cognitive and physical abilities constitutes an inverted U-shaped age-efficiency profile where the curve is monotonically falling from a certain age onwards until it intersects the zero line. At that time, the person would be dismissed since then the net returns become negative, implying negative values of her skills. The period of dismissal does not necessarily have to coincide with the required age of retirement. Thus, the worker will be unemployed in the mean-time.

Given the technical data of a production system, i.e. its input and output composition, the length of employment in the ICT-producing industry depends on the following variables:

1. The given rate of profits
2. The costs of education
3. The existence of alternative methods of production

The efficiency profile varies with the given rate of profits. A person may be of rising efficiency throughout his lifetime at one rate of profit and falling at another, no matter, in what standard prices are measured. Again the following formula is considered:

$$(1+r)\,p_{\beta,t-1}(r) - p_{\beta,t}(r) = y_{\beta,t}(r) \tag{21}$$

If the value of human capital does only change moderately in two consecutive years, we get

$$r = \frac{y_{\beta,t}}{p_{\beta,t-1}} \tag{22}$$

If the rate of profit is equal to zero, then the value of the professional employed in the respective year is equal to the net return generated by her. Note that the price of human capital reaches its maximum before the net return (Schefold 1980, p.157). If the rate of profit increases, the saddle point will be displaced backward; it follows *ceteris paribus*, that skilled labour works at highest efficiency at later ages and the length of employment will be stretched.

The given costs of education also play a main role. Fig.(4.2) shows that the lower the costs, the more the wage line of skilled labour approaches the line of unskilled labour. This in turn implies that the training period is extended, while the length of professional career is reduced.

From the previous considerations it is obvious, that seniority-based wage regimes – characterized by continuously rising wages over the individual's career– would lead to a truncation of the technique so that the corresponding length of employment would always be lower than in the case of efficiency-based wage regimes.

Different types of qualification employed in one industry

In the presented model, only one kind of qualification – the IS-professional – is considered. If more types of skilled labour are required for the production of a commodity, they form a set of qualifications that contain different occupations which are hired for the primary process in analogy to the acquisition of new machines. Since the institutional lifetime is supposed to be given and equal for all professionals, they will leave the firm simultaneously. In contrast to the employment of only one professional, the efficiency-based wage rate will then depend on the performance of the team of skilled workers regarded as the composite human capital asset of the firm. Thus, the annual charge for its utilization is distributed equally between all professionals, since the entrepreneur can not elicit the effort of the single worker. This in turn might offset the incentives of the skilled person to give her best in the job, but would otherwise strengthen her capacity for teamwork. By choice of the cost-minimizing combination of different qualifications, the optimum length of employment can be determined by truncation. What still must hold is the axiom of non-transferability of human capital among various industries because only the relative value of the team, but not of each

constituent member, can be calculated. Furthermore, the concept can be applied to the analysis of property rights, as Kurz and Salvadori (1995,p.329) suggested. Unlike in the presented approach, the professional might not have exclusive property rights on his knowledge, since specific skills are provided by the firm. Thus the annual wage premium is distributed between the employer and his employee. While the composition of the requirements for consumption still does not influence long run prices in this concept, the growth rate might play a role in the determination of the cost minimizing processes, so that the dual system should be integrated into the model.

5 Concluding remarks

The concept presented in this paper may be a first approach to apply fixed-capital theory to the treatment of human capital and should be considered as a skeletal structure for further research in this area. Some arguments may challenge critics, particularly the fact that from a theoretical point of view men are not distinguishable from machines. The author intends in no way to suggest the reification of the human being.Although in economics skills and abilities have ever since been analogously treated to physical capital, one has to be aware of the differences. Unlike machinery, human capital does not only depend on technical conditions and can be at best standardized at the level of working agreements. Whereas the type of education and training can be interpersonally compared, many further features of human capital can not. Differences in talents, social competence and learning ability as well as family and environmental background are mostly not accounted for in labour contracts. Even if different persons dispose of an equal level of qualification, and the same skills and abilities, they need not be paid equally. Wage gaps can be traced back to many factors, such as gender and race discrimination. The abstraction from social impacts on the acquisition and utilization of knowledge has entirely been done for the purpose of arriving at a general and simple model of human capital. But fitting in the model in a social context would certainly enrich the spectrum of conclusions.

What has been shown is just one side to the coin – the price side – while the dual quantity system has not been considered. Especially with regard to the various effects of knowledge on economic development a dynamic analysis is almost indispensable. It is questionable if a von Neumann approach of proportional growth is satisfactory since some goods may be more demanded than others. In this case it is useful to apply the hyper-vertical integration approach by Luigi Pasinetti. He correctly recognizes the difference in the interrelation of demand side and the price system on the one hand and the quantity system on the other hand (Pasinetti 2003). While prices of finished commodities remain the same no matter if they are used as means of production or as consumption goods, the quantities used in production and demanded by the consumers vary for each subsystem. Thus it is more appropriate to assume that the consumption demand grows by different rates according to the preferences of the people. But this

inevitably leads to a divergence in prices: While only one price can be assigned to each final good used for consumption purposes, the set of prices of the means of production differ in the various growing subsystems, which would make the analysis far more complex.

The benefit of investigating skills within the Sraffian approach rests *inter alia* in the fact, that most of the extensions which have been developed in fixed capital theory so far can easily be adapted for the case of human capital.

Full time, part time and shift work

The model only dealt with the question of how many periods a person shall be employed on the basis of the given technical alternatives. By the same token it is possible to determine the optimum number of working hours per day. An interesting field of study is the case of shift work, where the pattern of utilization can not always be chosen in respect to economical considerations, but is often dependent on institutional, physical and economic factors. If a physician works during day and night, he yields a premium to his ordinary wage and needs more inputs than during day, as his physical characteristics may require more strength. While the prices of commodity inputs behave time invariant, the wage rate per hour increases by the premium wage. Usually an entrepreneur utilizes a machine in such a dimension that cost minimization occurs; in medical services such a condition is restrictive; thus medical institutions often work by losses, which have to be paid by the public hand.

Skill-based technical progress

If the composition of inputs changes in such a way that specific skills are necessary in the production of the good in the basic manufacture sector, then the existing process(es) are replaced by a set of methods that contain the utilization of human capital. This can be done by implementing a roundabout process (Schefold 1989, pp.236-240 and pp.179-215), in which the required knowledge is accumulated before it can be applied to production. The increasing employment of skilled workers would then lead to a decline in the demand for unskilled labour, which is already empirically evident for many branches.

Settlement on dismissal

If the working agreement is cancelled due to the truncation of the processes referring to the last years, the skilled worker can be recompensed by a dismissal pay. This matter of fact can be introduced by the introduction of costly disposal. Persons who drop out are regarded to be fully engaged directly or indirectly in the reproduction of goods they are produced with. The output matrix is then divided in three submatrices: the finished commodity matrix, the non market outputs and the professionals at different

age. The vertically integrated subsystem has to be expanded by a further process which uses commodity inputs and the professional to produce nothing. The net returns are negative in this case, and will lead to negative prices of the professional which are treated as additional costs. To internalize them, the group of processes becomes more expensive. If it is up to the government to support the unemployed, the settlement of dismissal is centralised by introducing a further institution which is responsible for the distribution of the support payments. A further subsystem will be installed whose production is running over one period, indicating a (single) lump sum, or as a group of processes which amount to annual payments. Which of the systems will be implemented, either a decentralized or a centralised, is dependent upon the choice of the cost-minimizing technique.

The previous remarks give rise for future work in this area. The paper only represents a first attempt to indicate an alternative direction of educational economics, starting from the classical nucleus. Once more Sraffa's approach proved to be of considerable value in the discussion of the prevailing economic theories.

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