Input-Output Analysis
For Multi-location Supply Chain Management Control:
A Theoretic Model
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Abstract: Economic globalization has forced and is still forcing enterprises to develop new global manufacturing and distribution concepts. A growing number of products are produced in multiple plants dispersed around the globe. This paper designs and discusses an enterprise input-output (EIO) model of international supply chain which leads to a better understanding of the complex process flows within a multi-location enterprise’s production network. Different from its previous counterparts, the EIO system of this paper is grounded on the analysis of plants dispersed in different locations. Besides the consumption during production activities, this system also takes into account the consumption caused due to the dispersed feature of the supply chain, i.e., the transportation costs. After that, a mixed-integer linear programming model based on input-output account is applied to find the optimal solutions of the production and distribution decisions for the multi-location supply chain. Conclusions are given in the end of this paper.

Key words: multi-location supply chain, enterprise input-output analysis, mixed-integer linear programming tool

1. Introduction
A supply chain can be defined as an integrated process consists of a number of various business entities including suppliers, manufacturers, distributors, and retailers. They work together in an effort to acquire raw materials, convert them into specified final products, and finally deliver these final products to retailers\textsuperscript{3}. In the literature on logistics for civil and industrial engineering projects, supply chain concept is used to compare different transportation alternatives, to make investment decisions and regulatory planning. In the management context, the literature focuses on the coordination of all supply chain components for reducing costs while maintaining a high level of customer-service\textsuperscript{4}. Recently, due to a number of changes in the manufacturing environment, there has been an increasing attention placed on the performance, design, and analysis of the supply chain as a whole. Indeed, the rising

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\textsuperscript{4} Albino et al., Input-output models for the analysis of a local/global supply chain, Int.J. Production Economics 78 (2002) 119-131
costs of manufacturing, shrinking resources of manufacturing bases, shortened product life cycles, and also the globalization of market economies, have made the supply chain management issues more significant and challenging than ever before. This paper mainly discusses the supply chain management issue caused by the forces of economic globalization, that is, the management control of a multi-location supply chain.

Economic globalization has forced enterprises to develop new global manufacturing and distribution concepts. A growing number of products, ranging from garments to computers, are produced in multiple plants dispersed around the globe. Products previously manufactured in a single location are nowadays produced in a complex network of facilities separated by thousands of miles. As a result, more and more enterprises have international supply chains, upon which primary activities including purchasing, manufacturing and distribution are not restricted to one particular region but take place on a world-wide scale.

However, this geographically dispersed supply chain exhibits conflicting goals and objectives, because each plant in this supply chain is confronted with alternative production decisions, multiple optional vendors, and multiple target markets. It is quite possible that any two of the plants encounter decision conflicts in production, procurement or distribution, accordingly inducing the multi-purpose disequilibrium within the whole supply chain. Therefore it makes sense to build up a comprehensive process/material flow network to depict the complicated internal mutual relationship between plants, external suppliers and target markets. The main purpose of this paper is to explore a theoretic method for making optimal production and distribution decisions for the multi-location supply chain, subject to the constraints of production capacity, market demand, external materials supply, etc., therefore to realize multi-objective equilibrium and rational resources allocation within the supply chain. The enterprise input-output model this paper designs and discusses leads to a better understanding of the interrelated material and cost consumption within a multi-location enterprise’s production-distribution network. And a mixed-integer linear programming model based on input-output account is applied to reach the optimal solutions for production and distribution planning.

Two main contributions of this paper are: (1) the enterprise input-output system presented here breaks through the framework of the previous traditional counterparts, which were based on the analysis of production processes. This EIO system is grounded on the analysis of plants dispersed in different locations. Besides the consumption during production activities, this system also takes into account the consumption caused due to the dispersed feature of the supply chain, i.e., the transportation costs. (2) the linear programming tools have almost predominated the solution methods of modern supply chain management issues; however, the application of input-output coefficients in the mathematical model, which
comprehensively and conveniently describe the interrelations within the network, can solve the computational problem of linear programming models induced by large number of variables and constraints.

This paper is structured as follows: In this introductory section, the purpose of this paper of studying multi-location supply chain is presented given the economic globalization background. Section 2 describes the problem this paper studies. In Section 3, an enterprise input-output system is established for the multi-location international supply chain. In Section 4, a mixed-integer linear programming model based on input-output account is set up to make optimal production and distribution decisions. Conclusions and prospects are given in Section 5.

2. Problem description

The model presented in this paper is based on a multi-location supply chain of an international manufacturing enterprise. This supply chain is characterized by a location dispersed structure, where plants, vendor, and markets are located in different places. In this material flow system, the inputs to various manufacturing plants consist of raw materials that can be sourced from different vendors; the output of finished products can be transported and distributed to different markets (see Fig.1).

Major features of this supply chain are given below:

1. It consists of several manufacturing plants dispersed in different locations. To produce a final product, all the plants face the same production procedure. When a plant gets through the whole procedure, it implies this plant supplies the final product. However, in this supply chain, each plant can alter its production plan according to its goals. Besides producing the final product, each plant can also choose to just produce the components, by not getting
through the production procedure, and the components it produces would be delivered to other plants in this system as semi-product.

(2) Some material vendors are involved in this supply chain. A vendor can serve one or more plants in this system, and a plant can choose one or more vendors too. The procurement plan is generally based on transportation costs and production decisions.

(3) The final products produced by plants are finally delivered and distributed to several alternative target markets. Plants make their sale plans according to the demand constraint and distribution costs which involve the costs of packing, transportation and marketing.

Such a multi-location supply chain tends to exhibit conflicting goals and objectives, while the lagging information flow induced by dispersed structure makes it hard to realize multi-objective equilibrium among different facilities (plants), by which the production and distribution decisions are made to maximize the whole system’s profit, instead of any certain plant. The model that is proposed in this paper can be used at an operational setting and tackles production, procurement and consumer assignments to existing facilities, in order to optimize the production and distribution activities.

Admittedly, the planning and control of the supply chain seems to be mainly an operational issue since it is meant to steer the flow of goods and information in all kinds of purchasing, manufacturing and distribution activities. Thomas and Griffin (1996) reviewed previous models which address various kinds of coordinated planning and scheduling problems at the operational level. Because of the internationalization of a company’s supply, production, and distribution activities, a more strategic perspective is presented to solve such issues. Cohen and Mallik (1997) classify the literature on “global supply chain management strategy planning” into option valuation and network flow models. Characteristic examples of the first category are presented by Kogut and Kulatilaka (1994) and Huchzermeier and Cohen (1996) in which a firm’s global supply chain network is modeled in order to evaluate variations in global manufacturing strategies as a response to fluctuating exchange rates. An even more general model for the international operations of the firm is developed by Cohen et al (1989), who model the coordination of procurement, production, and distribution in the form of a rather comprehensive stochastic, dynamic optimization problem. Their framework is an extension of Cohen and Lee (1989) and covers a wide variety of international operations issues. From the survey by Vidal and Goetschalkx (1997), it follows that Cohen et al. (1989) “present the main features that differentiate an international supply chain from a single-country model”. The Cohen et al. paper, just like many other contributions, can be traced back to the seminal article by Geoffrion and Graves (1974), who presented a mixed integer programming

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(MIP) formulation for a multi-commodity, multi-plant, and multi-DC system. Since then, strategic models for global manufacturing and global supply chain management are often formulated as MIP problems, for example, by Lootsma (1994) and Arntzen et al. (1995), more recently, Jayaraman and Pirkul (2001), who study an integrated logistics model for locating production and distribution facilities in a multi-echelon environment and claim to present an efficient solution procedure provided all input data are known and available.

However, the use of conventional mixed-integer linear programming tools for solving supply chain coordination issues is limited due to the complexity of the problem and the large number of variables and constraints, particularly for realistically sized, even fairly small, problems. Therefore, it is unlikely that standard mathematical programming packages can find optimal solutions to all instances of this problem expending acceptable levels of computational effort. Input-output techniques, by comprehensively and conveniently describing the mutual technical relations among the network’s units, can alleviate the problem of complexity to some extent. In the following, a general framework of enterprise input-output model for multi-location supply chain is presented, after the enterprise input-output method is reviewed at first.

3. Enterprise Input-Output analysis for multi-location supply chain

3.1 literature review

From a physical point of view, a supply chain can also be considered as an input–output system (Storper & Harrison, 1992) that describes the product flows existing among production processes. The supply chain can be considered as an input-output system that produces a specific good, and the input-output system can involve many production units characterized by a specific work division. The presentation of an input-output system of a given product makes it possible to understand the relationships existing among the production units and also the governance structure of a production system. It uses a disaggregated level and considers the pattern of materials and energy flows amongst industry sectors, and between sectors and the final customer (Miller & Blair, 1985).

Enterprise input–output (EIO) accounts are useful to complement the managerial and financial accounting systems currently used extensively by firms, and in the recent decades, extensive work on enterprise input-output accounts and models has been done (Filipic, 1985, 1986; LaNoce and Latorre, 1988; Li, 1991; Li and Ruikun, 1990; Limere, 1982; Pimpao, 1979; Song, 1989; Tong, 1991; Zhang et al., 1991; Polenske, 1997; Lin and Polenske, 1998; Polenske and McMichael; Marangoni & Fezzi, 2002.). In particular, Lin & Polenske (1998) built a specific input–output model,

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input-output process-flow model (IOPM), for a steel plant, which was based on production processes rather than on products or branches. The model comprised issues of sustainability by focusing on economics-energy-environment interactions. For a given final product, the model computes output, materials, energy and waste (pollution) flows, thus providing a measure of the environmental impact of production processes. Changes either in the demand for the final product, or in the technologies adopted by each production process, could be covered and their effects on both the production network that characterized the industrial district and on the environment could be analyzed.

By extending and modifying the IOPM method, Polenske and McMichael (2002) studied Chinese coke-making sector. Using a similar approach, Tang et al. (1994) and Grubbstrom and Tang (2000) formulated models that analyzed the complex network of materials, energy and pollution flows that characterize the supply chain of a final product. Similar studies include. Albino et al. (2000) used Input–output techniques to develop a specific model for an industrial district, which was described as a set of supply chains, each as a network of production processes. Similar studies include Albino et al. (2002), Albino et al. (2003).

Chinese researchers on enterprise input-output technique have made unique contributions, with distinguished detailed investigation and analysis in specific industries. Among the most distinctive ones are the “enterprise input-output model with two factors” (Tong, 1991), the “oil piping input-output model”, the “machine manufacturing enterprise input-output model” (Tong 1992, Tong 1990), etc.

3.2 General framework

An international supply chain is usually characterized by a high work division and by several firms producing several final products, each of which consists of a sub-supply chain comprised of its own production process and alternative vendors. Modeling of a supply chain is essential for supply chain analysis, by showing either the mutual relationships or interdependencies of its elements. The purpose of this paper is to build up such an input-output system that accounts the internal complex production-distribution network of a multi-location international supply chain. As an accounting tool, it helps investigate material/cost flow related to a multi-location supply chain for a given period. As a design tool, combined with mathematical programming tools, it helps make optimal production decisions for multi-objective equilibrium within the whole supply chain.

3.2.1. Subscripts

Throughout this model description the following set of indices will be used:

- **n** plants: manufacturing plants within the enterprise, located globally.
- **q** main products: components of the final product. \( n' \) equals to \( n \times q \), for
simplification, denotes the total number of products all the n plants within the enterprise can produce.

k vendors: external material suppliers; for the production of one product there may be more than one vendor, and a vendor may serve more than one plant by supplying more than one type of materials.

r raw materials: raw materials

m target markets: markets that accept the products produced by this system; dispersed in different regions or even countries.

3.2.2. Assumptions

For simplification, the model is restricted to the following assumptions:

(1) The whole supply chain produces only one final product, while producing a number of components. All the plants face the same procedure of producing the final product.

(2) For each manufacturing plant, the production decisions are alternative. It can choose to produce the final product by getting through the procedure, or just several main products (components), or both; but in a limited number of the product types it produces.

(3) Component produced by any plant is delivered to a limited number of other plants in this system as semi-product for the final product.

(4) Any two facilities in this system has fixed transportation mode (road, rail or air) for products or material deliveries.

(5) This supply chain does not have inventories.

The enterprise input-output table for a multi-location supply chain is shown in Tab.1.

3.2.3. Explanations of the matrices

(1) \((x_{ij})_{n \times n'} = \) unit variable production cost for manufacturing plants, evaluates the unit production cost of product i for supporting the production of product j. Element 0 implies no consumption relationship between two products.

(2) \((T_{ij})_{n \times n'} = \) transportation cost of plant i for product j;

(3) \((F_{ij})_{(k \times r) \times n'} = \) unit variable production cost of material i for product j;

(4) \((C_{ij})_{k \times n} = \) transportation cost for delivering material i to product j (the plant);

(5) \((M_{ij})_{m \times n'} = \) distribution cost of final product j in market i, including packing, transportation and marketing costs.

(6) \((V_{ij})_{n \times n'}\) and \((S_{ij})_{n \times n'} \) denotes the wage and tax consumption during the production of product j;
Table 1 An input-output table of multinational supply chain (Unit: RMB)

<table>
<thead>
<tr>
<th>Production points</th>
<th>Final demand</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1, Plant 2, Plants n</td>
<td>Sales, Inventory, Others</td>
<td></td>
</tr>
</tbody>
</table>

### Production cost
- Plant 1: Main product 1, Main product q
- Plant 2: Main product 1, Main product q
- ... Plant n: Main product 1, Main product q

\[
X_i = \sum_{j=1}^{n} \sum_{q=1}^{q} x_{ij} \quad \text{for } i \in \{1, \ldots, q\}
\]

### Transportation cost
- Plant 1, Plant 2, Plant n

\[
T_{ij} = \sum_{k=1}^{n} \sum_{r=1}^{r} t_{ij}^{k,r} \quad \text{for } i \in \{1, \ldots, q\}
\]

### Materials cost
- Vendor 1, Vendor 2, ... Vendor k

\[
F_{ij} = \sum_{k=1}^{k} \sum_{r=1}^{r} f_{ij}^{k,r} \quad \text{for } i \in \{1, \ldots, q\}
\]

### Transportation cost
- Vendor 1, Vendor 2, ... Vendor k

\[
C_{ij} = \sum_{k=1}^{k} \sum_{i=1}^{i} c_{ij}^{k,i} \quad \text{for } i \in \{1, \ldots, q\}
\]
<table>
<thead>
<tr>
<th>Market points</th>
<th>Distribution cost</th>
<th>Market 1</th>
<th>Market 2</th>
<th>...</th>
<th>Market m</th>
<th>$(M_{ij})_{m \times n'}$</th>
<th>$Y_5$</th>
<th>$M_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary input</td>
<td>Labor wages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(V_j)_{1 \times n'}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Taxes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(S_j)_{1 \times n'}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$X_j$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(7) \( Y_i \ (i=1,\ldots,5) \) are all types of final demands in this system. Among them, vector \( Y_i \) denotes the upper value of the final demand for any product in all markets.

3.2.4 Main coefficients

Based on the balance relationship of input-output table in the arrow direction, the total costs for plants production and consumption conforms to the following equation:

**Intermediate consumption + final consumption = total consumption**

That is:

\[
\sum_{j=1}^{n} x_{ij} + Y_i = X_i, i = 1, 2, \ldots, n' \tag{1}
\]

Define \( a_{ij} \) the direct consumption coefficients between plants:

\[ a_{ij} = x_{ij} / X_j, (i, j = 1, 2, \ldots, n') \]

Thus

\[
\sum_{j=1}^{n} a_{ij} \cdot X_j + Y_i = X_i
\]

Then define:

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix},
\]

\[ Y = (Y_1, Y_2, \ldots, Y_n)^T \]

\[ X = (X_1, X_2, \ldots, X_n)^T \]

Therefore:

\[
A \cdot X + Y = X
\]

\[
X = (I - A)^{-1} \cdot Y
\]

Define \( B = (I - A)^{-1} \) the total demand coefficient matrix, and

\( C = (I - A)^{-1} - I \) the total consumption coefficient matrix, then it is easy to calculate other coefficients in this system, by multiply their corresponding direct consumption coefficients by matrix \( C \). Thus we obtain:

(1) \( T \) = total consumption coefficient matrix for transportation cost by product \( j \);

(2) \( F \) = total consumption coefficient matrix for material supply of vendor \( i \) by product \( j \);

(3) \( C \) = total consumption coefficient matrix for transportation cost of vendor \( i \) by product \( j \);

(4) \( M \) = total consumption coefficient matrix for distribution cost of market \( i \) by product \( j \);

(5) \( V \) and \( S \) are total consumption coefficient matrices of wage and tax by
product j.

4. A mixed-integer linear programming model

The purpose of the application of the mixed-integer linear programming model is to the optimal production and distribution planning at a strategic stage, i.e., which plant to produce, which product to be produced and how many, which vendor to choose, which market to focus on, etc.. Here our mathematical programming model is setup along the lines of the Cohen et al. (1989) framework, because this model will be simplified and adapted to better fit our purposes. Generally, the linear programming model and its implementation support several objective functions, the most important of which are: maximizing total sales, minimizing costs, and maximizing contribution margin. The MIP model in this paper chooses the minimizing costs scenario, based on input-output account presented in Section 3. To keep in line with the input-output account, the planning horizon of the MIP model is one month.

4.1. Decisions and variables

(1) \( x_{ij} \) : amount of product i produced for product j

(2) \( d_{rkj} \) : amount of raw material r supplied to product j by vendor k

(3) \( o_{jm} \) : amount of \( x_{ij} \) packed for market m

(4) \( w_{ij} \) : zero-one variable only equal to one if \( x_{ij} \) is positive (choice of production planning)

(5) \( p_{mj} \) : zero-one variable with respect to market alternatives for product j (choice of distribution planning)

(6) \( g_{kj} \) : zero-one variable only equal to one if \( d_{rkj} \) is positive (choice of procurement planning)

4.2. Costs

(1) \( b_{ij} \) = elements in matrix B, denotes the unit production cost of plants, restricted by the maximized final demand in markets.

(2) \( t_{ij} \) = elements in matrix T, denotes the transportation cost between plants

(3) \( f_{rkj} \) = elements in matrix F, denotes the unit production cost of material r produced by vendor k for product j

(4) \( c_{kj} \) = elements in matrix C, denotes the transportation cost between vendor k and product j
(5) \( m_{mj} \) = elements in matrix M, denotes the distribution cost in market m by product j

(6) \( \alpha_{ij} \) = fixed production cost for production of product i for product j

4.2. Objective function

Minimize

\[
Z = \sum_{i,j} w_{ij} [x_{ij} (b_{ij} + v_{ij} + s_{ij}) + t_{ij} + \alpha_{ij}] + \sum_{r,s,j} g_{kj} [d_{r kj} \cdot f_{r k} + c_{kj}] + \sum_{m,j} p_{mj} [o_{jm} \cdot m_{mj}]
\]

This model minimizes total costs. The objective function incorporates variable costs (continuous variables) as well as fixed cost (zero-one variables).

4.3. Constraints

(2) Item constraints: \( 0 \leq \sum_{j=1}^{n'} w_{ij} \leq m, m \leq n' \)

(3) Vendor capacity constraints: \( \sum_{j=1}^{\bar{D}_i} d_{ij} \leq \bar{D}_i \)

(4) Market demand: \( H_{mj} \leq \bar{H}_{mj} \leq \bar{H}_{mj}, \sum_{m=1}^{M} \bar{H}_{mj} = Y_1 \)

Obviously, owing to the introduction of input-output coefficients, the number of variables and constraints both diminishes to a great extent. That is because the coefficients are deduced from \( Y_1 \), the upper bound of final demand; in other words, all the coefficients in this MIP model indicate the technical constraint of the production activities, and that is why this MIP has comparably less variables and constraints than its counterparts. Theoretically, the stability and accuracy of the solutions of mathematical programming models is adversely affected due to the complexity of model and large number of variables and constraints. Therefore this problem can be abated to some degree by introducing the input-output account.
5. Conclusion

Our base model as specified above captures strategic issues like how to plan production, procurement and distribution decisions. This paper has made two major contributions. Firstly, the enterprise input-output system presented in the paper breaks through the framework of the previous traditional counterparts, which were based on the analysis of production processes. This EIO system is grounded on the analysis of plants dispersed in different locations. Besides the consumption during production activities, this system also takes into account the consumption caused due to the dispersed feature of the supply chain, i.e., the transportation costs. Secondly, the linear programming tools have almost predominated the solution methods of modern supply chain management issues; however, the application of input-output coefficients in the mathematical model, which comprehensively and conveniently describe the interrelations within the network, can solve the computational problem of linear programming models induced by large number of variables and constraints.

A number of research issues for extending the current model are already under investigation. A numerical experiment is going to be checked to test the feasibility of the model for application in reality. Besides, the extension to the proposed model of incorporating the idea of multitype-transportation modes has also been taken into consideration.

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