The input-output framework and modelling assumptions: considered from the point of view of the economic circuit

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Abstract - The immediate cause for this paper is another paper presented for the first time by Professor de Mesnard at the 14th International Input-Output Conference and published in the Journal of Regional Science in 2004. According to him input-output models derived from supply and use tables by means of the product technology assumption fail in terms of the economic circuit (the chaining from final demand to total output in a traditional Leontief input-output system). I have studied his paper(s) intensively and I have, for several reasons, to make serious considerations to his point of view. My paper is not merely a reply to de Mesnard but it tries also to give a more general approach of the SNA input-output framework in terms of the economic circuit. When considering explicitly make and use tables in the economic circuit make and use

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tables should be considered in every step of the economic circuit. The mathematical series of this make and use tables in every step should converge to the “total” make and use tables as given in the national accounts. When combining technology assumptions (product technology, industry technology) with output structure assumptions (constant product-mix, constant market shares) different versions of the economic circuit are obtained. The make, use and product-by-product tables obtained in every step will differ according to the chosen assumptions but the make and use tables have to converge to the tables integrated in national accounts. In this way different consistent versions of the economic circuit under the assumption of product technology can be obtained. Professor de Mesnard seems to reduce product technology to the special case of a constant product-mix but even when one considers this special case his statement raises questions. When the output structure assumptions are extended to sales structure assumptions (fixed industry sales structures and fixed product sales structures) industry-by industry tables can be obtained in every step of the economic circuit. This has given me some interesting insights. The industry-by-industry table based on the assumption of a fixed product sales structure does not appear to be invariant of the technology assumption (what some people claim) when it is used for impact analysis. Some attention is also given to the practical aspects of input-output compilation. Assumptions should not only be considered from a theoretical point of view but attention should also be paid to the statistical framework in which they are applied.

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Introduction

The choice of which input-output model to derive from underlying supply and use tables is usually made on the basis of theoretical and/or practical considerations.

The first choice is that between so-called product by product or industry by industry tables. Product by product tables describe the input structure of (analytically constructed) homogeneous branches in terms of product groups and value added and also the final uses by category of these product groups. Industry by industry tables describe market relations: the intermediary deliveries between statistical units grouped in industries according to their principal activity, the value added of these industries and their deliveries to the categories of final demand.

When product by product tables are chosen the next step is the choice of the technology assumption. Several modelling assumptions have been proposed over the last decades but the choice is mostly limited between two assumptions: product technology (a given product always has the same input structure irrespective in which industry it is produced) or industry technology (the input-structure of an industry remains invariant irrespective of its product-mix). There is a lot of disagreement about which of these two assumptions is the most preferable. There is considerably less controversy, in fact none, about which assumption is preferable when industry by industry tables are chosen. But the choice between product by product tables or industry by industry tables as official tables forming part of national accounts is the subject of an international controversy. The SNA 68 and its accompanying input-output manual (United Nations, 1968 and 1973) presented different versions of input-output tables but gave no preference to one of them. The ESA 70 and 79 national accounting systems of the EU (Eurostat, 1979) included input-output tables based on homogeneous branches (formally product by product tables) but they did not give any indication of how these tables should be constructed starting from basic data (these systems did not include supply and use tables).

Two Dutch academics, Kop Jansen and ten Raa, did put forward four axioms of desirable properties of input-output tables (Kop Jansen and ten Raa, 1990):

- material balance (supply = use)
- financial balance (output = costs)
- scale invariance (the technical coefficients should be invariant to proportional variations of the input requirements and outputs of the industries)
- price invariance (the constant price estimate of the input-output table should be invariant to price fluctuations)

As we understand these are the axioms on which traditional input-output analysis by means of Leontief equations is based. When considering input-output tables in a system of supply and use tables only the product technology fulfils the four desirable properties, industry technology fulfils only the first one. This means that if one wants to perform traditional input-output analysis product by product tables based on product technology should be compiled. It is logical to use a model that is in conformity with the axioms on which the analysis is based on, or to make an estimate that approaches this model as much as possible. An article published in the Economic Systems Research journal in 2003 treated this matter further but it did not contain any drastic changes compared with the first paper (ten Raa and Rueda-Cantucha, 2003).

The SNA 93 (United Nations et al., 1993) contains only product by product tables and showed preference for product technology, referring to the 4 axioms of desirable proportions put forward by ten Raa and Kop Jansen). The accompanying input-output manual (United Nations, 1999) presents different kind of input-output tables (like its predecessor) but also with a preference for product technology. The ESA 95 also only contains product by product tables. These are in fact the tables that the member states of the EU are (legally) obliged to transmit to Eurostat. But it showed no preference for any type of technology assumption. An accompanying input-output manual was written but unfortunately not published because of disagreements between the member states of the EU over which kind of input-output table is preferable as part of national accounts (Beutel, 2005).

At the 14th International Input-Output Conference a new critique of a theoretical nature against the product technology was formulated by Professor de Mesnard of the University of Dijon (the author of this paper did not attend this session). According to him the interpretation of the product technology fails in terms of the economic circuit (de Mesnard, 2002). A definitive version of his paper has been published in the Journal of Regional Science (de Mesnard, 2004b). The use of the term “economic circuit” should be interpreted as the chaining of intermediate demand caused by an initial impact on final demand in the traditional Leontief input-output system. He claims that product technology breaks the economic circuit because of the emergence of negative outputs at every step of the economic circuit and should be abandoned in favour of industry technology.

At the same conference more opposition to product technology could be heard from B. Thage of Statistics Denmark (Thage, 2002a). He claims that his critique of product technology is more of a pragmatic nature and based on a long established practice. Statistics Denmark has been in fact applying the SNA input-output system for decades with supply and use tables as the core of the national accounts (Thage, 1986). The objections of Statistics Canada against product technology are of the same nature (Lal, 1999). Both prefer industry by industry tables as part of national accounts for practical reasons but B. Thage welcomed de Mesnard's critique as an extra argument (Thage, 2002b).

This critique was also mentioned in a paper of the US Bureau of economic Analysis (Guo et al. ,2002) Up to 1992 at least, the US Benckmark input-output tables were partly calculated by a mix of a transfer method being like (but not exactly equal to) the product technology model and industry technology. The authors of this paper mentioned de Mesnard critique in a neutral way in a general overview of the input-output literature on technology assumptions.

The author did attend the presentation by de Mesnard of the second version of his paper at the intermediate input-output conference in Brussels in 2004 (de Mesnard, 2004a). The audience consisted mainly of CGE modellers (input-output and CGE modelling was the subject of the conference) clearly not acquainted with the methodological and practical aspects of the compilation of input-output systems and there was hardly any reaction. As far is I know the authors who have studied and defended up to a certain degree the product technology model in the past have not replied to de Mesnard critique. His statements seem to be taken automatically for granted or simply ignored, neither of which we consider being a good attitude.

The author is a member of the Belgian institute charged with the compilation of the "official" Belgian input-output tables (this is my principal activity, my secondary activity consist in using these tables). In this capacity the Belgian "input-output team" is rather a user and not a developer of methodologies but a user of methodologies should be interested in the theoretical back-ground of these methodologies. From this point of view I have an undertaken a study of de Mesnard's critique because:

- his paper does not contain elaborated examples
- by my knowledge people far more suitable to react have not done this (to my surprise until now not even one article in the ESR journal has mentioned de Mesnard statements[1]).

We have made exercises with different versions of the input-output systems considering supply, use, product by product and industry and industry tables under different modelling assumptions. We have only considered the Leontief model and not the Gosh model (as far as I know this model has never been used by my institution and it is in general considered as a curiosity). Before we show these exercises, let us first repeat how traditional Leontief input-output systems function.

[1] The article of Ten Raa and Rueda-Cantuche mentioned several papers presented at the Montreal input-output conference but not that one of de Mesnard.

1. Traditional Leontief input-output models

The representation of the economic circuit in case of the traditional Leontief input-output model is straightforward.

In the traditional Leontief input-output model, each industry produces only one product and each product is only produced by one industry. In other words, the use table is already an input-output table. The statement “each industry produces only one product and each product is only produced by one industry” should be toned down in practice. The pure concept of a product can only be reached by a level of detail of the underlying product and industry classifications that is not applicable in reality. The original statement should therefore be mitigated to: each industry is already a homogeneous branch; it produces only products that come under the activity corresponding to its label in the underlying industry classification.

Consider the following simple representation[2]:

$$\begin{array}{c c|c} X & f & q \\ v' & & \\ \hline q' & & \end{array}$$

- X : intermediate table (square matrix)
- f : final demand (for simplicity, we take it as a vector)
- v : value added (also taken as a vector for simplicity reasons)
- q : total output (vector)

Let us consider the following identity in the economic circuit:

[2] We ignore imports as in all didactic representations of input-output models.

$$X \cdot i + f = q \tag{1}$$

The technical coefficients are given by:

$$A = X \cdot \hat{q}^{-1} \tag{2}$$

These are supposed to be constant provided no price or technological changes.

Let us know consider the so-called economic circuit that should lead from f to X.

The initial effect on output is final demand $q_0 = f$ (step 0). Step 1 is the direct intermediate demand $q_1 = A \cdot f$, step 2 is the first phase of indirect demand $q_2 = A \cdot q_1$, and so on …

Each step of this simple economic circuit has an output vector $q_r = A^r \cdot f$ (r= 0, 1 2 …)

From step 1 on it also has an intermediate table $X_r = A \cdot \hat{q}_{r-1} = A \cdot \left(\widehat{A^{r-1} \cdot f} \right)$ (r=1, 2, …).

The two series of the cumulative result of this chaining are respectively equal to total output and the intermediate table:

$$\sum_{r=o}^{\infty} q_r = \sum_{r=0}^{\infty} A^r \cdot f = (I - A)^{-1} \cdot f = q \tag{3}$$

$$\sum_{r=1}^{\infty} X_r = \sum_{r=1}^{\infty} A^r \cdot \left(\widehat{A^{r-1} \cdot f} \right) = A \cdot \hat{q} = X \tag{4}$$

According to de Mesnard, a similar representation of the economic circuit is possible in the system of supply and use tables under the assumption of industry technology but not under the assumption of product technology. He claims that in the latter case, the economic circuit is broken because of the existence of negatives which he seems to consider as unavoidable.

Let us first consider what product and industry technology do really mean and how de Mesnard interprets them.

2. Basic meaning of product and industry technology

2.1. The framework

We use the symbols of the UN input-output manual (United Nations, 1999):

$$
\begin{array}{c|c}
M & q \\
\hline
g' &
\end{array}
\qquad\qquad
\begin{array}{cc|c}
U & f & q \\
va' & & \\
\hline
g' & &
\end{array}
$$

- M : make table (products x industries)
- U : absorption table (products x industries)
- f : final demand (by product, vector)
- va : value added (by industry, vector)
- g : total output of industries (vector)
- q : total output of products (vector)

We will use the example given in the UN input-output manual to illustrate our statements throughout this paper:

$$
\begin{bmatrix} M & q \\ g' & \end{bmatrix} = \left[\begin{array}{ccc|c} 156 & 24 & 0 & 180 \\ 9 & 80 & 0 & 89 \\ 0 & 0 & 62 & 62 \\ \hline 165 & 104 & 62 & \end{array}\right]
\qquad
\begin{bmatrix} U & f & q \\ va' & & \\ g' & & \end{bmatrix} = \left[\begin{array}{cccc|c} 19 & 28 & 10 & 123 & 180 \\ 29 & 18 & 8 & 34 & 89 \\ 7 & 7 & 3 & 45 & 62 \\ 110 & 51 & 41 & & \\ \hline 165 & 104 & 62 & & \end{array}\right]
$$

Three well-known matrices of coefficients derived from these tables are:

- The absorption coefficients matrix $B = U \cdot \hat{g}^{-1}$ (the input-structure of the industries in terms of products)
- The market shares matrix $D = M' \cdot \hat{q}^{-1}$ (the contribution of each industry to the total output of each product)

- The product-mix matrix $C = M \cdot \hat{g}^{-1}$ (the share of each product in the total output of each industry)

$$B = \begin{bmatrix} 11.5\% & 26.9\% & 16.1\% \\ 17.6\% & 17.3\% & 12.9\% \\ 4.2\% & 6.7\% & 4.8\% \end{bmatrix} \qquad D = \begin{bmatrix} 86.7\% & 10.1\% & 0.0\% \\ 13.3\% & 89.9\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix} \qquad C = \begin{bmatrix} 94.5\% & 23.1\% & 0.0\% \\ 5.5\% & 76.9\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

The make and use tables above can be considered as the matrices compiled for the national accounts. The matrices B ,C and D are not always constant by definition. Whether or not they are considered to be constant (to remain equal to the values derived from the national accounts) during each step of the economic circuit, or when performing impact analysis, depends on the assumptions made to derive symmetric input-output tables.

Product and industry technology are two different assumptions on the base of which two different product-by-product tables can be derived from the make and absorption tables.

2.2. Product technology

Product technology assumes that a given product always has the same input structure irrespective where it is produced. It means that a given product x product matrix of technical coefficients A is hidden behind the “observed” make and absorption tables according to which all industries produce their different (principal and secondary) products. This means that:

$$U = A \cdot M \tag{5}$$

This matrix A is in fact Leontief type matrix which is supposed to lie at the base of the make and absorption tables. So it is no more logical that the product technology model meets all the so-called Kop Jansen-ten Raa conditions without any further stipulations, while the other treatments of secondary products do not.

Equation (5) can also be written as:

$$B = A \cdot C \tag{6}$$

This formula seems to have caused a lot of confusion since some seem to suppose that the invariability of A rests on the invariability of matrices B and C while it is assumed to be constant by nature (Konijn, 1994).

A constant product-mix matrix (which is often erroneously given as the definition of product technology) is by no means necessary. If C varies A remains invariable by definition, and B adapts itself to the new product-mix. Richard Stone clearly did not mention a constant product-mix in part 3 of A Program for Growth (Stone et al., 1963). Neither the SNA 93, nor the accompanying input-output manual mentions a constant product-mix explicitly as a condition for product technology (they neither say that C has to be invariable, nor that it can be variable). The SNA 68 and its accompanying input-output manual (United Nations, 1973) did not either when they defined product technology. But they declared industry-by-industry tables characterised by a constant product-mix as "the industry-by-industry variant of product-technology". This is in the first place not right (see below) and seemed in the second place to have caused a lot of confusion.

Product technology was identified with a constant product-mix in the 1985 edition of Miller and Blair "Input-output analysis: foundations and extensions" (Miller and Blair, 1985). This book is considered as a standard work and de Mesnard has taken over their definition of product technology. Does his critique of product technology still holds if we accept the more general definition of product technology given by Konijn?

The pure concept of product technology can only be reached by a level of detail of the underlying product and industry classifications that is not applicable in reality. The strict definition is therefore in reality mitigated to: if an industry has a secondary production (it produces products that come under the activity corresponding to the label of another industry in the underlying industry classification) the input-structure of this secondary production is equal to the input-structure of the total principal production of this other industry. This means that strictly speaking not a product x product table but a homogeneous branch x homogeneous branch table is calculated. This is an approximation of a product x product table but if it is calculated by means of matrix calculation it continues to satisfy the Kop Janssen-ten Raa conditions.

A can be calculated by $U \cdot M^{-1}$ or $B \cdot C^{-1}$. Application of product technology requires the number of products and industries to be equal: mathematically this is necessary for the inversion of the matrices C or M, economically this means that the estimation of the input-structure of a homogeneous branch requires a corresponding industry that is the principal producer of the products characterising that homogeneous branch. The technology matrix of the Leontief system is namely homogeneous, symmetrical (this term is generally used while according to Almon "symphisic" is more appropriate, Almon, 2000) and square and the assumption of product technology is an attempt to implement input-output analysis by means of Leontief equations into a framework with supply and use tables. This means that the working format of the make and use tables has to be aggregated to square matrices with dimension equal to the number of industries.

In the SNA input-output manual A is equal to:

$$A = \begin{bmatrix} 10.3\% & 31.9\% & 16.1\% \\ 17.6\% & 17.2\% & 12.9\% \\ 4.1\% & 7.5\% & 4.8\% \end{bmatrix}$$

2.3. Industry technology

Industry technology means that the input-structure of an industry remains invariant irrespective of its product-mix. This means that B is matrix of constants given no price or technological changes. Without any additional condition an invariable matrix of product-by-product technical coefficients simply does not exist.

According to the assumption of industry technology, the input-structure of a product in terms of other products is a weighted average of the input-structure of the industries where it is produced as a principal or secondary activity:

$$B \cdot D = \begin{bmatrix} 13.6\% & 25.4\% & 16.1\% \\ 17.5\% & 17.3\% & 12.9\% \\ 4.6\% & 6.5\% & 4.8\% \end{bmatrix}$$

Since B is a matrix of constants by definition, $B \cdot D$ can only be invariable if D is a matrix of constants.

The derivation of a product by-product matrix of invariable technical coefficients under industry technology needs the additional assumption of constant market shares, while the assumption of product technology does not need the additional assumption of a constant product-mix (invariability of C).

The matrix multiplication $B \cdot D$ looks like a Leontief matrix but this is only apparent since the technology matrix at the base of the system is the invariable matrix B which is clearly not a Leontief matrix. So it is no wonder that the matrix $B \cdot D$ does not meet all the Kop Jansen-Ten Raa conditions.

The SNA 93 judges the industry technology as highly implausible, as Richard Stone et al. already did in 1963 (although they admitted that industry technology can be a better approximation if some industries are too aggregated in the working format of the supply and use tables, Stone et al., 1963). Almon did not mince his words and considers the official recommendation by international organizations and use by numerous statistical offices of industry technology “little short of scandalous” (Almon, 1998).

3. The economic circuit of the input-output framework

3.1. General conditions

If we extend the economic circuit of the traditional Leontief input-output model to the general system with supply and use tables, make and absorption tables should be considered at every step.

The starting point remains final demand by product f. The make table in step 0, M_0 is derived from this final demand vector according to the used output assumption (constant market shares or product mix).

Step 1 starts with the direct intermediate demand of the make table M_0. This is given by an absorption table U_1. How U_1 is derived from M_0 depends on the technology assumption. How the production of these inputs $U_1 \cdot i$ is distributed over the industries is given by the make table M_1 and depends once again on the output assumption.

This M_1 table causes the intermediate demand in step 2 given by the absorption table U_2.

The mathematical series of the make and absorption table in every step should convergence to the “total” make and absorption tables as given in the national accounts:

$$\sum_{r=0}^{\infty} M_r = M \tag{7}$$

$$\sum_{r=1}^{\infty} U_r = U \tag{8}$$

3.2. The economic circuit under the output assumption of constant market shares

3.2.1. Representation of the economic circuit

Imposing the assumption of constant market shares in the framework of the economic circuit implies that the contribution of each industry to total output of each product remains equal to the ratio in national accounts in every step of the circuit:

$$D_r = M'_r \cdot q_r^{-1} = D \tag{9}$$

It is obvious to combine this output assumption with the technology assumption of industry technology in order to obtain invariant product-by product coefficients but it can also be combined with the product technology assumption. How do product and industry technology behave through the economic circuit when they are combined with the constant market shares assumption?

product technology | **industry technology**

The initial effect on output in step 0 is equal to final demand:

step 0: (product technology)

$$f = \begin{bmatrix} 123 \\ 34 \\ 45 \end{bmatrix}$$

step 0: (industry technology)

$$f = \begin{bmatrix} 123 \\ 34 \\ 45 \end{bmatrix}$$

This has to be transformed into a make table by means of the market share matrix D:

$$M_0 = \hat{f} \cdot D' \tag{10}$$

$$\begin{bmatrix} M_0 & q_0 \\ g_0 & \end{bmatrix} = \left[\begin{array}{ccc|c} 106.6 & 16.4 & 0 & 123 \\ 3.4 & 30.6 & 0 & 34 \\ 0.0 & 0.0 & 45.0 & 45 \\ \hline 110.0 & 47.0 & 45.0 & \end{array}\right]$$

$$C_0 = M_0 \cdot \hat{g}_0^{-1} = \hat{f} \cdot D' \cdot \left(\widehat{D.f}\right)^{-1} \tag{12}$$

$$C_0 = \begin{bmatrix} 96.9\% & 34.9\% & 0.0\% \\ 3.1\% & 65.1\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

$$M_0 = \hat{f} \cdot D' \tag{11}$$

$$\begin{bmatrix} M_0 & q_0 \\ g_0 & \end{bmatrix} = \left[\begin{array}{ccc|c} 106.6 & 16.4 & 0 & 123 \\ 3.4 & 30.6 & 0 & 34 \\ 0.0 & 0.0 & 45.0 & 45 \\ \hline 110.0 & 47.0 & 45.0 & \end{array}\right]$$

$$C_0 = M_0 \cdot \hat{g}_0^{-1} = \hat{f} \cdot D' \cdot \left(\widehat{D.f}\right)^{-1} \tag{13}$$

$$C_0 = \begin{bmatrix} 96.9\% & 34.9\% & 0.0\% \\ 3.1\% & 65.1\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

In step 0 there is no difference between product and industry technology because only the output assumption is used.

Step 1 consists of the direct intermediate demand. This differs according to product or industry technology because the absorption table differs according the technology assumption. In the case of product technology the direct inputs of the industries are determined by the input-structure of the products they have delivered to final demand in step 0. Under the industry technology assumption the direct inputs of the industries are determined by the level of their total deliveries to final demand in step 0. From here on, product and industry technologies differ.

step 1:

$$U_1 = A \cdot M_0 = A \cdot \hat{f}.D' \tag{14}$$

$$\begin{bmatrix} U_1 & q_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 12.1 & 11.4 & 7.3 & 30.8 \\ 19.3 & 8.1 & 5.8 & 33.3 \\ 4.6 & 3.0 & 2.2 & 9.7 \end{array}\right]$$

step 1:

$$U_1 = B \cdot \hat{g}_0 = B.\left(\widehat{D.f}\right) \tag{15}$$

$$\begin{bmatrix} U_1 & q_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 12.7 & 12.6 & 7.3 & 32.6 \\ 19.3 & 8.1 & 5.8 & 33.3 \\ 4.7 & 3.2 & 2.2 & 10.0 \end{array}\right]$$

In the case of product technology the absorption coefficients matrix of step 1 can be calculated as follows:

$$B_1 = U_1 \cdot \hat{g}_0^{-1} = A.\hat{f} \cdot D' \cdot \left(\widehat{D \cdot f}\right)^{-1} \tag{16}$$

$$B_1 = \begin{bmatrix} 11.0\% & 24.4\% & 16.1\% \\ 17.6\% & 17.4\% & 12.9\% \\ 4.2\% & 6.3\% & 4.8\% \end{bmatrix}$$

B_1 and C_0 differ from B and C while B_1 equals $A \cdot C_0$ This illustrates that the invariability of A by itself is the basic assumption of product technology, and not the existence of invariable absorption and product-mix coefficients matrices.

Total output by product $q_1 = U_1 \cdot i$ in step 1 differs according to the technology assumption. In the case of product technology q_1 does not depend on the distribution of final demand by (delivering) industry in the make table of step 0, but only on the distribution of final demand by product. This is a logical consequence of the product technology assumption. Intermediate demand by product is only determined by final demand by product since each product has a unique input structure regardless of the industry where it is actually produced: $q_1 = A \cdot f$. Under the assumption of industry technology this is not the case: $q_1 = B \cdot D \cdot f$

The make tables of step 1 are equal to:

$$M_1 = \hat{q}_1 \cdot D' = \left(\overset{\wedge}{A.f} \right) \cdot D' \tag{17}$$

$$\begin{bmatrix} M_1 & q_1 \\ g_1 & \end{bmatrix} = \left[\begin{array}{ccc|c} 26.7 & 4.1 & 0.0 & 30.8 \\ 3.4 & 29.9 & 0.0 & 33.3 \\ 0.0 & 0.0 & 9.7 & 9.7 \\ \hline 30.1 & 34.0 & 9.7 & \end{array}\right]$$

$$M_1 = \hat{q}_1 . D' = \left(\overset{\wedge}{B.D.f} \right) \cdot D' \tag{18}$$

$$\begin{bmatrix} M_1 & q_1 \\ g_1 & \end{bmatrix} = \left[\begin{array}{ccc|c} 28.2 & 4.3 & 0.0 & 32.6 \\ 3.4 & 29.9 & 0.0 & 33.3 \\ 0.0 & 0.0 & 10.0 & 10.0 \\ \hline 31.6 & 34.3 & 10.0 & \end{array}\right]$$

$$C_1 = M_1 \cdot \hat{g}_1^{-1} = \left(\overset{\wedge}{A \cdot f} \right) \cdot D' \cdot \left(\overset{\wedge}{D \cdot A \cdot f} \right)^{-1} \tag{19}$$

$$C_1 = \begin{bmatrix} 88.8\% & 12.1\% & 0.0\% \\ 11.2\% & 87.9\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

$$C_1 = M_1 \cdot \hat{g}_1^{-1} = \left(\overset{\wedge}{B \cdot D \cdot f} \right) \cdot D' \cdot \left(\overset{\wedge}{D \cdot B \cdot D \cdot f} \right)^{-1} \tag{20}$$

$$C_1 = \begin{bmatrix} 89.3\% & 12.7\% & 0.0\% \\ 10.7\% & 87.3\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

Step 2 consists of the first phase of indirect demand. The absorption tables are equal to:

step 2:

$$U_2 = A \cdot M_1 = A \cdot \left(\overset{\wedge}{A.f} \right) . D' \tag{21}$$

$$\begin{bmatrix} U_2 & q_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 3.8 & 10.0 & 1.6 & 15.4 \\ 5.3 & 5.9 & 1.3 & 12.4 \\ 1.3 & 2.4 & 0.5 & 4.2 \end{array}\right]$$

step 2:

$$U_2 = B \cdot \hat{g}_0 = B . \left(\overset{\wedge}{D.f} \right) \tag{22}$$

$$\begin{bmatrix} U_2 & q_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 3.6 & 9.2 & 1.6 & 14.5 \\ 5.6 & 5.9 & 1.3 & 12.8 \\ 1.3 & 2.3 & 0.5 & 4.1 \end{array}\right]$$

In the case of product technology the absorption coefficients matrix of step 2 is equal to:

$$B_2 = U_2 \cdot \hat{g}_1^{-1} = A.\left(\widehat{A.f}\right) \cdot D' \cdot \left(\widehat{D \cdot A \cdot f}\right)^{-1} \qquad (23)$$

$$B_2 = \begin{bmatrix} 12.8\% & 29.3\% & 16.1\% \\ 17.6\% & 17.3\% & 12.9\% \\ 4.4\% & 7.1\% & 4.8\% \end{bmatrix}$$

B_2 and C_1 differ not only from B and C but also from B_1 and C_0, while $B_2 = A \cdot C_1$.

This illustrates once again that the invariability of A does not depend on the invariability of absorption and product-mix matrices but that it is given by definition.

Total output by product $q_2 = U_2 \cdot i$ is equal to $A^2 \cdot f$ under the assumption of product technology and $(B \cdot D)^2 \cdot f$ when assuming industry technology.

The make tables that give the distribution of this output are equal to:

$$M_2 = \hat{q}_2 \cdot D' = \left(\widehat{A^2.f}\right) \cdot D' \qquad (24)$$

$$\begin{bmatrix} M_2 & q_2 \\ g_2 & \end{bmatrix} = \left[\begin{array}{ccc|c} 13.3 & 2.1 & 0.0 & 15.4 \\ 1.3 & 11.2 & 0.0 & 12.4 \\ 0.0 & 0.0 & 4.2 & 4.2 \\ \hline 14.6 & 13.2 & 4.2 & \end{array}\right]$$

$$M_1 = \hat{q}_2.D' = \left[\widehat{(B.D)^2 \cdot f}\right] D' \qquad (25)$$

$$\begin{bmatrix} M_2 & q_2 \\ g_2 & \end{bmatrix} = \left[\begin{array}{ccc|c} 12.5 & 1.9 & 0.0 & 14.5 \\ 1.3 & 11.5 & 0.0 & 12.8 \\ 0.0 & 0.0 & 4.1 & 4.1 \\ \hline 13.8 & 13.4 & 4.1 & \end{array}\right]$$

$$C_2 = M_2 \cdot \hat{g}_2^{-1} = \left(\widehat{A^2 \cdot f}\right) \cdot D' \cdot \left(\widehat{D \cdot A^2 \cdot f}\right)^{-1} \qquad (26)$$

$$C_2 = \begin{bmatrix} 91.4\% & 15.5\% & 0.0\% \\ 8.6\% & 84.5\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

$$C_2 = M_2 \cdot \hat{g}_2^{-1} = \left[\widehat{(B.D)^2 \cdot f} \cdot D'\right] \left[\widehat{D \cdot (B \cdot D)^2 \cdot f}\right]^{-1} \qquad (27)$$

$$C_2 = \begin{bmatrix} 90.7\% & 14.4\% & 0.0\% \\ 9.3\% & 85.6\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

This process is very logically continued. Let us take a look at the formulas in step r of the circuit:

step r:

$$U_r = A\cdot M_{r-1} = A\cdot \widehat{\left(A^{r-1}\cdot f\right)}\, D' \qquad (28)$$

$$B_r = U_r\cdot \hat{g}_{r-1}{}^{-1} = A\cdot \widehat{\left(A^{r-1}\cdot f\right)}\, D'\cdot \widehat{\left(D\cdot A^{r-1}\cdot f\right)}^{-1} \qquad (30)$$

$$B_r = A\cdot C_{r-1} \qquad (31)$$

$$q_r = U_r\cdot i = A^r\cdot f \qquad (32)$$

$$M_r = \hat{q}_r\cdot D' = \widehat{\left(A^r\cdot f\right)}\, D' \qquad (34)$$

$$C_r = M_r\cdot \hat{g}_r{}^{-1} = \widehat{\left(A^r\cdot f\right)}\, D'\cdot \widehat{\left(D\cdot A^r\cdot f\right)}^{-1} \qquad (36)$$

step r:

$$U_r = B\cdot g_{r-1} = B\cdot \left[D\cdot \widehat{(B\cdot D)^{r-1}\cdot f}\right] \qquad (29)$$

$$q_r = U_r\cdot i = \left(B\cdot D\right)^r\cdot f \qquad (33)$$

$$M_r = \hat{q}_r\cdot D' = \widehat{\left[(B\cdot D)^r\cdot f\right]}\, D' \qquad (35)$$

$$C_r = M_r\cdot \hat{g}_r{}^{-1} = \widehat{\left[(B\cdot D)^r\cdot f\right]}\, D'\cdot \widehat{\left[D\cdot (B\cdot D)^r\cdot f\right]}^{-1} \qquad (37)$$

Let us look now at the convergence of the process. In both cases the series of the make tables of the successive steps adds up to the “general” make table of the national accounts:

$$\sum_{r=0}^{\infty} M_r = \sum_{r=0}^{\infty} \widehat{\left(A^r\cdot f\right)}\, D' = \hat{q}\cdot D' = M \qquad (38)$$

$$\sum_{r=0}^{\infty} M_r = \sum_{r=0}^{\infty} \widehat{\left[(B\cdot D)^r\cdot f\right]}\, D' = \hat{q}\cdot D' = M \qquad (39)$$

The convergence of the series of make tables suffices for the series of the absorption tables of the different steps to add up to the “general” absorption tables of the national accounts:

$$\sum_{r=1}^{\infty} U_r = \sum_{r=1}^{\infty} A\cdot M_{r-1} = A\cdot \sum_{r=0}^{\infty} M_r = A\cdot M = U \qquad (40)$$

$$\sum_{r=1}^{\infty} U_r = \sum_{r=1}^{\infty} B\cdot \hat{g}_{r-1} = B\cdot \sum_{r=0}^{\infty} \hat{g}_r = B\cdot \hat{g} = U\text{ [3]} \qquad (41)$$

3.2.2. Product x product tables

What do the product x product tables in each step of the economic circuit look like under both assumptions?

[3] $\sum_{r=0}^{\infty} M_r = M$ implies automatically $\sum_{r=0}^{\infty} g_r = g$.

When assuming product technology the product x product table in step r is simply equal to $A \cdot \hat{q}_r$. This is independent of the output assumption because both A and q_r are independent of it.

When assuming industry technology the part of u_r^{ji} (total input of product j in industry i in step r of the circuit) that is used for the production of product k is equal to $u_r^{ji} \cdot m_{r-1}^{ki} / g_{r-1}^{i}$ (simple proportional rule). If we summon over all industries i we do obtain $x_r^{jk} = \sum_i u_r^{ji} \cdot m_{r-1}^{ki} / g_{r-1}^{i} = \sum_i b_{ji} \cdot m_{r-1}^{ki}$. In matrix format this does give $X_r = B \cdot M'_{r-1}$. Since M_{r-1} depends on the output assumption X_r also does.

step 1:

$$X_1 = A \cdot \hat{f} \tag{42}$$

$$\begin{bmatrix} X_1 & q_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 12.7 & 10.8 & 7.3 & 30.8 \\ 21.6 & 5.9 & 5.8 & 33.3 \\ 5.0 & 2.6 & 2.2 & 9.7 \end{array}\right]$$

step 1:

$$X_1 = B \cdot M_0 = B \cdot D \cdot \hat{f} \tag{43}$$

$$\begin{bmatrix} X_1 & q_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 16.7 & 8.6 & 7.8 & 32.6 \\ 21.6 & 5.9 & 5.8 & 33.3 \\ 5.6 & 2.2 & 2.2 & 10.0 \end{array}\right]$$

step 2:

$$X_2 = A \cdot \hat{q}_1 = A \cdot \widehat{(A \cdot f)} \tag{44}$$

$$\begin{bmatrix} X_2 & q_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 3.2 & 10.6 & 1.6 & 15.4 \\ 5.4 & 5.7 & 1.3 & 12.4 \\ 1.2 & 2.5 & 0.5 & 4.2 \end{array}\right]$$

step 2:

$$X_2 = B \cdot M'_1 = B \cdot D \cdot \hat{q}_1 = B \cdot D \cdot \widehat{(B \cdot D \cdot f)} \tag{45}$$

$$\begin{bmatrix} X_2 & q_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 4.4 & 8.4 & 1.6 & 14.5 \\ 5.7 & 5.8 & 1.3 & 12.8 \\ 1.2 & 2.2 & 0.5 & 4.1 \end{array}\right]$$

The general formulas in step r are equal to:

step r:

$$X_r = \hat{A} \cdot q_{r-1} = A \cdot \widehat{(A \cdot f)} \tag{46}$$

step r:

$$X_r = B \cdot M_{r-1}{}' = B \cdot D \cdot \hat{q}_{r-1} = B \cdot D \cdot \left[\widehat{(B \cdot D)^{r-1} \cdot f}\right] \tag{47}$$

In both cases the series of the product x product tables of the successive steps adds up to the "general" product x product tables as directly derived from the make and use tables in the national accounts:

$$\sum_{r=1}^{\infty} X_r = \sum_{r=1}^{\infty} A \cdot \hat{q}_{r-1} = A \cdot \sum_{r=0}^{\infty} \hat{q}_r = A \cdot \hat{q} = X\text{ [4]} \quad (48) \qquad \sum_{r=1}^{\infty} X_r = \sum_{r=1}^{\infty} B \cdot M_{r-1}{}' = B \cdot \sum_{r=0}^{\infty} M_r{}' = B \cdot M' \quad (49)$$

$$X = \begin{bmatrix} 18.6 & 28.4 & 10.0 \\ 31.7 & 15.3 & 8.0 \\ 7.3 & 6.7 & 3.0 \end{bmatrix} \qquad X = \begin{bmatrix} 24.4 & 22.6 & 10.0 \\ 31.6 & 15.4 & 8.0 \\ 8.2 & 5.8 & 3.0 \end{bmatrix}$$

3.2.3. Industry x industry tables

Can we derive industry x industry tables at every step of this version of the economic circuit? An output assumption does not suffice to derive industry x industry tables. In order to do this we need to expand the handled output assumption to a so-called market assumption. Of course we need to use a market assumption related to the handled output assumption. A common market assumption that is an extension of the output assumption of constant market shares is the assumption of a so-called “fixed product sales structure”. Under the hypothesis of constant market shares one supposes that the share of each industry in the total output of a product remains constant: $m_{ji} = d_{ij} \cdot q_j$.

When assuming a fixed product sales structure one extends this assumption to each element of the use table: the part of u_{ji} supplied by industry i is equal to $d_{ij} \cdot u_{ji}$ and the part of f_j supplied by industry j to $d_{ij} \cdot f_j$. The assumption of a fixed product sales structure automatically implies constant market shares but the reverse is not necessarily true.

The industry x industry tables at every step of the circuit are in both cases (product and industry technology) given by the multiplication of D with the absorption table of each step. Let us call this tables E_r, in accordance with the symbols used in the previous UN

[4] $\sum_{r=0}^{\infty} M_r = M$ also automatically implies $\sum_{r=0}^{\infty} q_r = q$.

input-output manual (United Nations, 1973): $E_r = D \cdot U_r$. The industry x industry coefficients are equal to $E_r \cdot \hat{g}_{r-1}^{-1} = D \cdot B_r$. Since the absorption coefficients matrix remains constant under the assumption of industry technology the industry x industry coefficients are equal to $D \cdot B$ through the whole economic circuit. Under the hypothesis of product technology they are different in each step since B_r is different in each step.

step 1:

$$E_1 = D \cdot U_1 = D \cdot A \cdot \hat{f} \cdot D' \tag{50}$$

$$\begin{bmatrix} E_1 & g_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 12.5 & 10.7 & 6.9 & 30.1 \\ 19.0 & 8.9 & 6.2 & 34.0 \\ 4.6 & 3.0 & 2.2 & 9.7 \end{array}\right]$$

$$D \cdot U_1 \cdot \hat{g}_0^{-1} = D \cdot A \cdot \hat{f} \cdot D' \cdot \left(D \cdot \hat{f}\right)^{-1} = D \cdot B_1 \tag{52}$$

$$D \cdot B_1 = \begin{bmatrix} 11.3\% & 22.9\% & 15.3\% \\ 17.3\% & 18.8\% & 13.7\% \\ 4.2\% & 6.3\% & 4.\% \end{bmatrix}$$

step 1:

$$E_1 = D \cdot U_1 = D \cdot B \cdot \hat{f} \tag{51}$$

$$\begin{bmatrix} E_1 & g_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 12.9 & 11.8 & 6.9 & 31.6 \\ 19.1 & 9.0 & 6.2 & 34.3 \\ 4.7 & 3.2 & 2.2 & 10.0 \end{array}\right]$$

$$D \cdot U_1 \cdot \hat{g}_0^{-1} = D \cdot B \tag{53}$$

$$D \cdot B = \begin{bmatrix} 11.8\% & 25.1\% & 15.3\% \\ 17.3\% & 19.1\% & 13.7\% \\ 4.2\% & 6.7\% & 4.8\% \end{bmatrix}$$

step 2:

$$E_2 = D \cdot U_2 = D \cdot A \cdot \left(A \cdot \hat{f}\right) \cdot D' \tag{54}$$

$$\begin{bmatrix} E_2 & g_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 3.9 & 9.2 & 1.5 & 14.6 \\ 5.3 & 6.6 & 1.3 & 13.2 \\ 1.3 & 2.4 & 0.5 & 4.2 \end{array}\right]$$

$$D \cdot U_2 \cdot \hat{g}_1^{-1} = D \cdot A \cdot \left(A \cdot \hat{f}\right) \cdot D' \cdot \left(D \cdot \hat{A} \cdot f\right)^{-1} = D \cdot B_2 \tag{56}$$

$$D \cdot B_2 = \begin{bmatrix} 12.8\% & 27.1\% & 15.3\% \\ 17.5\% & 19.4\% & 13.7\% \\ 4.4\% & 7.1\% & 4.8\% \end{bmatrix}$$

step 2:

$$E_2 = D \cdot U_2 = D \cdot B \cdot \hat{g}_1 = D \cdot B \cdot \left(D \cdot B \cdot \hat{D} \cdot f\right) \tag{55}$$

$$\begin{bmatrix} E_2 & g_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 3.7 & 8.6 & 1.5 & 13.8 \\ 5.5 & 6.6 & 1.4 & 13.4 \\ 1.3 & 2.3 & 0.5 & 4.1 \end{array}\right]$$

$$D \cdot U_2 \cdot \hat{g}_1^{-1} = D \cdot B \tag{57}$$

$$D \cdot B = \begin{bmatrix} 11.8\% & 25.1\% & 15.3\% \\ 17.3\% & 19.1\% & 13.7\% \\ 4.2\% & 6.7\% & 4.8\% \end{bmatrix}$$

The general formulas in step r are equal to:

step r:

$$E_r = D \cdot U_r = A \cdot \left(\hat{A^{r-1}} \cdot f\right) D' \tag{58}$$

$$A \cdot \left(\hat{A^{r-1}} \cdot f\right) D' \cdot \left(D \cdot \hat{A^{r-1}} \cdot f\right)^{-1} = D \cdot B_r \tag{60}$$

step r:

$$E_r = D \cdot U_r = D \cdot B \cdot \hat{g}_{r-1} = D \cdot B \cdot \left[D \cdot (B \cdot \hat{D})^{r-1} \cdot f\right] \tag{59}$$

$$D \cdot U_r \cdot \hat{g}_{r-1}{}^{-1} = D \cdot B \tag{61}$$

Let us now look at the convergence of the process. Both product and industry technology, combined with the assumption of a fixed product sales structure, converge to the same ‘total’ industry x industry table:

$$\sum_{r=1}^{\infty} E_r = \sum_{r=1}^{\infty} D \cdot U_r = D \cdot \sum_{r=1}^{\infty} U_r = D \cdot U \tag{62}$$

Because in both cases the series of the absorption tables of the different steps add up to the “general” absorption tables of the national accounts: $\sum_{r=1}^{\infty} U_r = U$.

$$D \cdot U = \begin{bmatrix} 19.4 & 26.1 & 9.5 \\ 28.6 & 19.9 & 8.5 \\ 7.0 & 7.0 & 3.0 \end{bmatrix}$$

Does this mean that the industry x industry table derived under the assumption of a fixed product sales structure is independent of the technology assumption?

P. Konijn seems convinced this is the case in general with industry by industry tables (Konijn, 1994). From this he draws the conclusion that industry x industry tables describe only market relations and are consequently unsuitable for input-output analysis. They should, according to him, not be used for analysis about the technology of the economy.

B. Thage of Statistics Denmark also thinks that industry by industry tables do not involve (strong) technology assumptions but only weaker market assumptions. But from this he draws the opposite conclusion and prefers industry x industry tables, based on the assumption of a fixed product sales structure as the table which should be published as part of official statistics together with the supply and use tables.

Is this industry by industry table really independent of technology assumptions? Product and industry technology converge both to the same industry by industry table in the "base" version, the table derived from the supply and use tables in national accounts. But the convergence process is different in both cases. Is the table still independent of the technology assumption when this is used for input-output analysis? Let us trace this by means of a very simple example of impact analysis.

3.2.4. Impact Analysis

Let us consider a different final demand vector as starting point of the economic circuit (step 0):

$$q_0 = \begin{bmatrix} 150 \\ 50 \\ 60 \end{bmatrix}$$

Under the hypothesis of product technology the output vector q and the product x product table X that are the outcome of the process engendered by the final demand vector q_0 are only a function of the coefficients matrix A which is assumed to be constant by definition. The resulting make, absorption and industry x industry tables also depend on the market shares matrix D.

When assuming industry technology all these tables depend upon both the absorption coefficients matrix B (assumed to be constant by definition) and the matrix D.

$$q = (I - A)^{-1} \cdot q_0 \qquad (63)$$

$$q = \begin{bmatrix} 225.2 \\ 121.1 \\ 82.2 \end{bmatrix}$$

$$q = (I - B \cdot D)^{-1} \cdot q_0 \qquad (64)$$

$$q = \begin{bmatrix} 224.3 \\ 120.9 \\ 82.1 \end{bmatrix}$$

$$M = \hat{q} \cdot D' = \left[(I - A)^{-1} \cdot q_0\right]^{\wedge} D' \qquad (65)$$

$$\begin{bmatrix} M & q \\ g & \end{bmatrix} = \left[\begin{array}{ccc|c} 195.1 & 30.0 & 0.0 & 225.2 \\ 12.2 & 108.8 & 0.0 & 121.1 \\ 0.0 & 0.0 & 82.2 & 82.2 \\ \hline 207.4 & 138.9 & 82.2 & \end{array}\right]$$

$$M = \hat{q} \cdot D' = \left[(I - B \cdot D)^{-1} \cdot q_0\right]^{\wedge} D' \qquad (66)$$

$$\begin{bmatrix} M & q \\ g & \end{bmatrix} = \left[\begin{array}{ccc|c} 194.4 & 29.9 & 0.0 & 224.3 \\ 12.2 & 108.7 & 0.0 & 120.9 \\ 0.0 & 0.0 & 82.1 & 82.2 \\ \hline 206.7 & 138.6 & 82.1 & \end{array}\right]$$

$$U = A \cdot M = A \cdot \left[(I - A)^{-1} \cdot q_0\right]^{\wedge} D' \qquad (67)$$

$$U = \begin{bmatrix} 24.1 & 37.8 & 13.3 \\ 36.4 & 24.0 & 10.6 \\ 8.8 & 9.4 & 4.0 \end{bmatrix}$$

$$U = B \cdot \hat{g} = B \cdot \left[D \cdot (I - B \cdot D)^{-1}\right]^{\wedge} q_0 \qquad (68)$$

$$U = \begin{bmatrix} 23.8 & 37.3 & 13.2 \\ 36.3 & 24.0 & 10.6 \\ 8.8 & 9.3 & 4.0 \end{bmatrix}$$

$$X = A \cdot \hat{q} = A \cdot \left[(I - A)^{-1} \cdot q_0\right]^{\wedge} \qquad (69)$$

$$X = \begin{bmatrix} 23.3 & 38.6 & 13.3 \\ 39.6 & 20.9 & 10.6 \\ 9.1 & 9.1 & 4.0 \end{bmatrix}$$

$$X = B \cdot D \cdot \hat{q} = B \cdot D \cdot \left[(I - B \cdot D)^{-1} \cdot q_0\right]^{\wedge} \qquad (70)$$

$$X = \begin{bmatrix} 30.4 & 30.7 & 13.2 \\ 39.4 & 21.0 & 10.6 \\ 10.3 & 7.8 & 4.0 \end{bmatrix}$$

$$E = D \cdot U = D \cdot A \cdot \left[(I - A)^{-1} \cdot q_0\right]^{\wedge} D' \qquad (71)$$

$$E = \begin{bmatrix} 24.6 & 35.2 & 12.6 \\ 36.0 & 26.6 & 11.3 \\ 8.8 & 9.4 & 4.0 \end{bmatrix}$$

$$E = D \cdot U = D \cdot B \cdot \left[D \cdot (I - B \cdot D)^{-1} \cdot q_0\right]^{\wedge} \qquad (72)$$

$$E = \begin{bmatrix} 24.3 & 34.8 & 12.5 \\ 35.8 & 26.5 & 11.3 \\ 8.8 & 9.3 & 4.0 \end{bmatrix}$$

In the base situation we started from given make and use tables (in national accounts) and calculated:

- the product x product and industry x industry tables belonging to it respectively under the (technology) assumptions of product and industry technology and the output assumption of fixed market shares
- the industry x industry tables belonging to it respectively under the (technology) assumptions of product and industry technology and the market assumption of a fixed product sales structure

Here we look in first instance which make and absorption tables are generated under both assumptions given a hypothetical final demand. We notice that the make and absorption tables differ (slightly) according to the technology assumption (the output assumption is the same in both cases).

The industry by industry tables also differ (slightly) according the technology assumption. Not only the values but also the coefficients:

$$E \cdot \hat{g}^{-1} = D \cdot A \cdot \left[(I - A)^{-1} \cdot q_0 \right]^{\wedge} D' \cdot \left[D \cdot (I - A)^{-1} \cdot q_0 \right]^{\wedge -1} \quad (73)$$

$$E \cdot \hat{g}^{-1} = D \cdot B \quad (74)$$

$$E \cdot \hat{g}^{-1} = \begin{bmatrix} 11.8\% & 25.4\% & 15.3\% \\ 17.3\% & 19.2\% & 13.7\% \\ 4.3\% & 6.8\% & 4.8\% \end{bmatrix} \qquad E \cdot \hat{g}^{-1} = \begin{bmatrix} 11.8\% & 25.1\% & 15.3\% \\ 17.3\% & 19.1\% & 13.7\% \\ 4.2\% & 6.7\% & 4.8\% \end{bmatrix}$$

The coefficients of the industry x industry table based on a fixed product sales structure remain invariant under the industry technology assumption: they are the same as in the base situation (national accounts). This is logical since in this case one assumes B to be constant. Under the assumption of product technology the coefficients of the industry x industry table based on a fixed product sales structure are variable. This is also logical because the absorption coefficients matrix B is in general not constant under the assumption of product technology:

$$D \cdot A \cdot \left[(I - A)^{-1} \cdot q_0 \right]^{\wedge} D' \cdot \left[D \cdot (I - A)^{-1} \cdot q_0 \right]^{\wedge} \neq D \cdot A \cdot \left[(I - A)^{-1} \cdot f \right]^{\wedge} D' \cdot \left[D \cdot (I - A)^{-1} \cdot f \right]^{\wedge} \quad (75)$$

On the basis of this simple exercise we cannot agree with P. Konijn and B. Thage when they claim that the industry x industry table based on a fixed product sales structure does not involve technology assumptions.

$D.U$ is the industry x industry table based on a fixed product sales structure regardless of product or industry technology in the base situation: as an illustrative appendix of national accounts (although the supposed convergence process to this table is different in both cases). But when one assumes the coefficients of this table to remain invariable when performing impact analysis (this is what one usually does during input-output analysis) one combines implicitly the (weaker) market assumption of a fixed product sales structure with the (strong) assumption of industry technology.

Furthermore because of this we cannot agree with B. Thage when he claims that the (4) necessary assumptions for carrying out input-output analysis can be assumed to be valid how matter the input-output table has been constructed (Thage, 2005). Using the industry x industry table based on a fixed product sales structure for input-output analysis is implicitly based on the assumption of industry technology. During input-output analysis this table is related to the product by product table based on industry technology and constant market shares and we know that this table is in contradiction with 3 of the necessary assumptions of input-output analysis[5].

We have to mention that B. Thage advocates the industry by industry table which is directly derived from the rectangular supply and use tables (dimension mxn, m products, n industries, m>n). Only a product by product table based on industry technology and constant market shares with dimension m x m can be derived from the rectangular supply and use tables. Product and industry technology can only considered as counterparts after the supply and use tables are aggregated to square tables with dimension nxn.

It is not because the m x m product by product table based on industry technology has no product technology counterpart that the n x n industry by industry table based on a fixed product sales structure does not involve a technology assumption. The invariability of its coefficients require the combination of the assumption of a fixed product sales structure and industry technology but now formulated within the

[5] In the working paper published in 1986 the Danish input-output tables were presented as industry by industry tables based on industry technology (Thage, 1986).

framework of the rectangular make and use tables, not the square ones. Look at the formulas in the right-hand column in this paper and consider the M, U, B and D matrices to be rectangular: it is then immediately clear that this industry by industry table is related to the mxm product by product table based on industry technology and constant market shares when performing input –output analysis.

The product x product table with maximum dimension based on industry technology and constant market shares makes little sense. The input-structure of products belonging to the same industry differs only to the degree in which they are produced as a secondary activity by other industries (Konijn, 1994). Under the assumption of product technology the identification of a homogeneous branch requires a corresponding industry in the supply and use tables.

At last we would like mention that the SNA 68 was altogether not that very wrong when it proposed $D \cdot U$ as the industry by industry variant of industry technology. This table is invariant of the technology assumption (and thus solely dependent of the market assumption of a fixed product sales structure) only when it is considered as an illustrative appendix to the supply and use tables in the national accounts. But when its coefficients are used for impact analysis (I guess this is the main reason why input-output tables are constructed) it is based on the combination of industry technology and a fixed product sales structure.

3.2.5. Alternative versions of product technology

Product technology does not need the assumption of constant market shares to derive product x product tables with constant coefficients. These coefficients are invariable by definition. How does product technology perform within the framework of the economic circuit when one supposes other market shares matrices than the one derived from national accounts $D = M' \cdot \hat{q}^{-1}$?

What happens if we use a different matrix T with the characteristics of a market shares matrix ($i' \cdot T = i', t_{ij} \geq 0)$) in each step of the circuit?

The product x product tables in each step will remain equal to the ones in the base version ($X_r = A \cdot \left(\widehat{A^{r-1} \cdot f}\right)$) and consequently the output vector q_r too ($q_r = A^r \cdot f$).

Product x product tables based on product technology are well independent of the output assumption.

The make tables in each step of the circuit are equal to $M_r = \hat{q}_r \cdot T' = \left(\widehat{A^r \cdot f}\right) \cdot T'$. The series of the make tables M_r does not converge to the make table of the national accounts:

$$\sum_{r=0}^{\infty} \left(\widehat{A^r \cdot f}\right) \cdot T' = \hat{q} \cdot T' \neq M = \sum_{r=0}^{\infty} \left(\widehat{A^r \cdot f}\right) \cdot D' = q \cdot D' \tag{76}$$

Consequently the series of the absorption tables also does not converge to the national accounts absorption table $\sum_{r=1}^{\infty} U_r = \sum_{r=1}^{\infty} A \cdot M_{r-1} = A \cdot \sum_{r=0}^{\infty} M_r \neq A \cdot M = U$ but the product technology identity remains valid: $B_r = A \cdot C_{r-1} = A \cdot \left(\widehat{A^r \cdot f}\right) \cdot D' \cdot \left(D \cdot \widehat{A^{r-1} \cdot f}\right)^{-1}$.

Let us go one step further and use a different matrix shares matrix $i' \cdot T_r = i', t_r^{ij} \geq 0)$ in each step. The make tables in each step are given by $M_r = \hat{q}_r \cdot T_r' = \left(\widehat{A^r \cdot f}\right) \cdot T_r'$. In this case it is not impossible that the series $M_r = \hat{q}_r \cdot T' = \left(\widehat{A^r \cdot f}\right) \cdot T_r'$ converges to the make table M of national accounts but the author is mathematically not skilled enough to tract to which set of conditions the matrices T_r should satisfy in order to realize this convergence.

3.3. The economic circuit under product technology combined with the output assumption of a constant product-mix

3.3.1. Representation of the economic circuit

Imposing the assumption of constant product-mix in the framework of the economic circuit implies that the output composition each industry remains equal to the composition in national accounts in every step of the circuit:

$$C_r = M_r \cdot \hat{g}_r^{-1} = C \tag{77}$$

The initial effect on output in step 0 is once again equal to final demand by product:

$$f = \begin{bmatrix} 123 \\ 34 \\ 45 \end{bmatrix}$$

This has to be transformed into a make table by means of the product-mix matrix C. A difference with the assumption of a constant D is that the matrix C cannot transform f (or q_r in general) in a make table with a constant product-mix in one step. This has to be done in two steps. Firstly final demand by product has to be transformed into final demand by producing industry:

$$e = C^{-1} \cdot f \tag{78}$$

$$e = \begin{bmatrix} 121.4 \\ 35.6 \\ 45.0 \end{bmatrix}$$

C^{-1} has only to be used to calculate the column totals of the make table in each step, not the individual elements. For this the matrix C is used in a second phase:

$$M_0 = C \cdot \hat{e} = C \cdot \widehat{\left(C^{-1} \cdot f\right)} \tag{79}$$

$$\begin{bmatrix} M_0 & q_0 \\ g_0 & \end{bmatrix} = \left[\begin{array}{ccc|c} 114.8 & 8.2 & 0.0 & 123.0 \\ 6.6 & 27.4 & 0.0 & 34.0 \\ 0.0 & 0.0 & 45.0 & 45.0 \\ \hline 121.4 & 35.6 & 45.0 & \end{array}\right]$$

In this way the product-mix of M_0 is equal to the general product-mix C. Does the fact that this transformation has to be done in two steps and in particular that in the first step not C is used but its inverse pose conceptual problems to the presentation of the economic circuit under the assumption of a constant product-mix? De Mesnard thinks it does (see below).

The market shares matrix D_r varies now in each step:

$$D_0 = M_0' \cdot \hat{q}_0^{-1} = \left(\widehat{C^{-1} \cdot f}\right) C' \cdot \hat{f}^{-1} \tag{80}$$

$$D_0 = \begin{bmatrix} 93.3\% & 19.5\% & 0.0\% \\ 6.7\% & 80.5\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

The direct intermediate demand of step 1 is given by the product technology identity:

$$U_1 = A \cdot M_0 = A \cdot C \cdot \left(\widehat{C^{-1} \cdot f}\right) \tag{81}$$

$$\begin{bmatrix} U_1 & q_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 14.0 & 9.6 & 7.3 & 30.8 \\ 21.3 & 6.2 & 5.8 & 33.3 \\ 5.2 & 2.4 & 2.2 & 9.7 \end{array}\right]$$

Since C_r is assumed to be constant, B_r is forced to be constant: $B_1 = B$. The product technology identity is reduced to $B = A \cdot C$ in each step of the circuit.

The make table of step 1 is calculated in two steps. Firstly, total output by industry and secondly the individual elements:

$$g_1 = C^{-1} \cdot q_1 = C^{-1} \cdot A \cdot f \tag{82}$$

$$M_1 = C \cdot \hat{g}_1 = C \cdot \left(\widehat{C^{-1} \cdot A \cdot f}\right) \tag{83}$$

$$\begin{bmatrix} M_1 & q_1 \\ g_1 & \end{bmatrix} = \left[\begin{array}{ccc|c} 21.2 & 9.6 & 0.0 & 30.8 \\ 1.2 & 32.1 & 0.0 & 33.3 \\ 0.0 & 0.0 & 9.7 & 9.7 \\ \hline 22.4 & 41.7 & 9.7 & \end{array}\right]$$

The corresponding market shares matrix D_1 clearly differs from D_0 and D:

$$D_1 = M_1' \cdot \hat{q}_1^{-1} = \left(C^{-1} \cdot \widehat{A \cdot f}\right) C' \cdot \left(\widehat{A \cdot f}\right)^{-1} \tag{84}$$

$$D_1 = \begin{bmatrix} 86.7\% & 10.1\% & 0.0\% \\ 13.3\% & 89.9\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

Let us now proceed with step 2:

$$U_2 = A \cdot M_1 = A \cdot C \cdot \left(C^{-1} \cdot \widehat{A \cdot f}\right) \tag{85}$$

$$\begin{bmatrix} U_2 & q_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 2.6 & 11.2 & 1.6 & 15.4 \\ 3.9 & 7.2 & 1.3 & 12.4 \\ 1.0 & 2.8 & 0.5 & 4.2 \end{array}\right]$$

$$g_2 = C^{-1} \cdot q_2 = C^{-1} \cdot A^2 \cdot f \tag{86}$$

$$M_2 = C \cdot \hat{g}_2 = C \cdot \left(C^{-1} \cdot \widehat{A^2 \cdot f}\right) \tag{87}$$

$$\begin{bmatrix} M_2 & q_2 \\ g_2 & \end{bmatrix} = \left[\begin{array}{ccc|c} 11.9 & 3.5 & 0.0 & 15.4 \\ 0.7 & 11.7 & 0.0 & 12.4 \\ 0.0 & 0.0 & 4.2 & 4.2 \\ \hline 12.5 & 15.2 & 4.2 & \end{array}\right]$$

$$D_2 = M_2' \cdot \hat{q}_2^{-1} = \left(C^{-1} \cdot \widehat{A^2 \cdot f}\right) C' \cdot \left(\widehat{A^2 \cdot f}\right)^{-1} \tag{88}$$

$$D_2 = \begin{bmatrix} 77.1\% & 5.5\% & 0.0\% \\ 22.9\% & 94.5\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

The general formulas in step r are equal to:

$$U_r = A \cdot M_{r-1} = A \cdot C \cdot \left(C^{-1} \cdot \widehat{A^{r-1} \cdot f}\right) \tag{89}$$

$$g_r = C^{-1} \cdot q_r = C^{-1} \cdot A^r \cdot f \tag{90}$$

$$M_r = C \cdot \hat{g}_r = C \cdot \left(C^{-1} \cdot \widehat{A^r \cdot f}\right) \tag{91}$$

$$D_r = M_r' \cdot \hat{q}_r^{-1} = \left(C^{-1} \cdot \hat{A^r} \cdot f\right) C' \cdot \left(\hat{A^r} \cdot f\right)^{-1} \tag{92}$$

Let us look now at the convergence of the process:

$$\sum_{r=0}^{\infty} M_r = \sum_{r=0}^{\infty} C \cdot \left(C^{-1} \cdot \hat{A^r} \cdot f\right) = C \cdot \left(C^{-1} \cdot \hat{q}\right) = C \cdot \hat{g} = M \tag{93}$$

The series of the make tables of the successive steps adds up to the “general” make table of the national accounts. This automatically implies that the series of the absorption tables of the different steps to add up to the “general” absorption tables of the national accounts: $\sum_{r=1}^{\infty} Ur = \sum_{r=1}^{\infty} A \cdot M_{r-1} = A \cdot M = U$.

Notice that this version of the economic circuit can also be considered as a particular case of the economic circuit with differing market share matrices $T_r = \left(C^{-1} \cdot \hat{A^r} \cdot f\right) C' \cdot \left(\hat{A^r} \cdot f\right)^{-1}$. The convergence of series of the make tables of the successive steps adds up to the “general” make table of the national accounts can also be seen from a different angle:

$$\sum_{r=0}^{\infty} \left(\hat{A^r} \cdot f\right) \cdot T_r' = \sum_{r=0}^{\infty} \left(\hat{A_r} \cdot f\right) \cdot \left[\left(C^{-1} \cdot \hat{A^r} \cdot f\right) C' \cdot \left(\hat{A^r} \cdot f\right)^{-1}\right]' = \sum_{r=0}^{\infty} C \cdot \left(C^{-1} \cdot \hat{A^r} \cdot f\right) = M \tag{94}$$

3.3.2. Product by product and industry by industry tables

Regarding product by product tables, since these are independent of the output assumption when supposing product technology, the formulas are the same as given in the left-hand columns of page 20 and 21.

How do we have to derive industry x industry tables in each step of this version of the economic circuit? Once again we need to expand the handled output assumption to a so called market assumption. Just like in the case of constant market shares we need to use a market assumption related to the handled output assumption. A common market assumption that is an extension of the output assumption of a constant product-mix is the assumption of a so called "fixed industry sales structure". Under the hypothesis of a constant product-mix one supposes that the share of each product in the total output of an industry remains constant: $m_{ji} = c_{ji} \cdot g_i$. When assuming a fixed industry sales structure one extends this assumption to each element of the industry by industry table: the part of product j in E_{li} is equal to $c_{jl} \cdot E_{li}$ and the part of e_j taken in by product j is equal to $c_{jl} \cdot e_l$. The assumption of a fixed industry sales structure automatically implies a constant product-mix but the reverse is not necessarily true.

This means that: $u_{ji} = \sum_l c_{jl} \cdot E_{li}$ what is equal to $U = C \cdot E$ or $E = C^{-1} \cdot U$.

The industry x industry tables in each step of the circuit are then given by the multiplication of C^{-1} with the absorption table of each step: $E_r = C^{-1} \cdot U_r$. The industry x industry coefficients are equal to $E_r \cdot \hat{g}_{r-1}{}^{-1}$. Since the absorption coefficients matrix is forced to remain constant because A (product technology) and C (fixed industry sales structure) the industry x industry coefficients are equal to $C^{-1} \cdot B$ through the whole economic circuit.

step 1:

$$E_1 = C^{-1} \cdot U_1 = C^{-1} \cdot A \cdot C \cdot \left(\widehat{C^{-1} \cdot f}\right) \tag{95}$$

$$[E_1 \quad g_1] = \left[\begin{array}{ccc|c} 8.2 & 8.3 & 5.9 & 22.4 \\ 27.2 & 7.4 & 7.1 & 41.7 \\ 5.2 & 2.4 & 2.2 & 9.7 \end{array}\right]$$

$$E_1 \cdot \hat{g}_0{}^{-1} = C^{-1} \cdot A \cdot C \cdot \left(\widehat{C^{-1} \cdot f}\right) \cdot \left(\widehat{C^{-1} \cdot f}\right)^{-1} = C^{-1} \cdot B \tag{96}$$

$$C^{-1} \cdot B = \begin{bmatrix} 6.7\% & 23.4\% & 13.2\% \\ 22.4\% & 20.8\% & 15.8\% \\ 4.2\% & 6.7\% & 4.8\% \end{bmatrix}$$

step 2:

$$E_2 = C^{-1} \cdot U_2 = C^{-1} \cdot A \cdot C \cdot \left(C^{-1} \cdot \hat{A} \cdot f \right) \tag{97}$$

$$[E_2 \quad g_2] = \left[\begin{array}{ccc|c} 1.5 & 9.8 & 1.3 & 12.5 \\ 5.0 & 8.7 & 1.5 & 15.2 \\ 1.0 & 2.8 & 0.5 & 4.2 \end{array} \right]$$

$$E_1 \cdot \hat{g}_0^{-1} = C^{-1} \cdot A \cdot C \cdot \left(C^{-1} \cdot \hat{A} \cdot f \right) \cdot \left(C^{-1} \cdot \hat{A} \cdot f \right)^{-1} = C^{-1} \cdot B \tag{98}$$

Let us now look at the convergence of the process. The convergence of the series of the industry by industry tables of each step to the ‘total’ industry x industry table is self-evident:

$$\sum_{r=1}^{\infty} M_r = \sum_{r=1}^{\infty} C^{-1} \cdot U_r = C^{-1} \cdot \sum_{r=1}^{\infty} U_r = C^{-1} \cdot U \tag{99}$$

The SNA 68 described the table $C^{-1} \cdot U$ as the industry by industry variant of product technology. This was exaggerated but not entirely wrong. It is (among o. things, see below) the combination of product technology and a fixed industry sales structure.

We observe this if we perform again the impact analysis described on page 24-28 but now based on the combined assumptions of product technology and a fixed industry sales structure.

The total output product remains independent of the output assumption (product technology):

$$q = (I - A)^{-1} \cdot q_0 \tag{100}$$

$$q = \begin{bmatrix} 225.2 \\ 121.1 \\ 82.2 \end{bmatrix}$$

The total output by industry can consequently be derived:

$$g = C^{-1} \cdot q = (I - A)^{-1} \cdot q_0 \tag{101}$$

$$g = \begin{bmatrix} 203.3 \\ 143.0 \\ 82.2 \end{bmatrix}$$

The make table is calculated on the assumption of a constant product-mix:

$$M = C \cdot \hat{g} = C \cdot \left[C^{-1} \cdot (I - \hat{A})^{-1} \cdot q_0 \right] \tag{102}$$

$$\begin{bmatrix} M & q \\ g & \end{bmatrix} = \left[\begin{array}{ccc|c} 192.2 & 33.0 & 0.0 & 225.2 \\ 11.1 & 110.0 & 0.0 & 121.1 \\ 0.0 & 0.0 & 82.2 & 82.2 \\ \hline 203.3 & 143.0 & 82.2 & \end{array} \right]$$

Finally the absorption table can be derived by the product technology identity:

$$U = A \cdot M = A \cdot C \cdot \left[C^{-1} \cdot (I - \hat{A})^{-1} \cdot q_0 \right] \tag{103}$$

$$U = \begin{bmatrix} 23.4 & 38.5 & 13.3 \\ 35.7 & 24.7 & 10.6 \\ 8.6 & 9.6 & 4.0 \end{bmatrix}$$

The industry by industry tables are derived as:

$$E = C^{-1} \cdot U = C^{-1} \cdot A \cdot C \cdot \left[C^{-1} \cdot (I - \hat{A})^{-1} \cdot q_0 \right] \tag{104}$$

$$E = \begin{bmatrix} 13.7 & 33.4 & 10.8 \\ 45.5 & 29.8 & 13.0 \\ 8.6 & 9.6 & 4.0 \end{bmatrix}$$

The industry by industry coefficients have remained equal to $C^{-1} \cdot B$:

$$E \cdot \hat{g}^{-1} = C^{-1} \cdot A \cdot C \cdot \left[C^{-1} \cdot (I - \hat{A})^{-1} \cdot q_0 \right] \cdot \left[C^{-1} \cdot (I - \hat{A})^{-1} \cdot q_0 \right]^{-1} \tag{105}$$

This is very logical because the two matrices that form the matrix product are assumed to be constant: C by definition and the invariability of B follows from the combined invariability of C and A (by definition).

3.3.3. Industry technology and a constant product-mix

Combining product technology with a constant product-mix forces the absorption coefficients matrix B to be constant. Industry technology means that the B matrix is constant by definition. Is there then still any difference between product and industry technology when they are both combined with the output assumption of a product-mix or the related market assumption of a fixed industry sales structure?

Hardly: all the make, use and industry by tables are equal in every step of the circuit and consequently the total tables too, as well as in the base situation of the national accounts as when performing input-output analysis. Only the product by product tables differ and under the hypothesis of industry technology and they have now variable technical coefficients:

step 1:

$$X_1 = B \cdot M_0' \;\; = B \cdot D_0 \cdot \hat{f} \tag{106}$$

$$\begin{bmatrix} X_1 & q_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 15.4 & 8.1 & 7.3 & 30.8 \\ 21.6 & 5.9 & 5.8 & 33.3 \\ 5.4 & 2.1 & 2.2 & 9.7 \end{array}\right]$$

The coefficients are clearly different from $B \cdot D$:

$$B \cdot M_0' \cdot \hat{f}^{-1} = B \cdot D_0 \tag{107}$$

$$B \cdot D_0 = \begin{bmatrix} 12.5\% & 23.9\% & 16.1\% \\ 17.6\% & 17.4\% & 12.9\% \\ 4.4\% & 6.2\% & 4.8\% \end{bmatrix}$$

step 2:

$$X_2 = B \cdot M_1' = B \cdot D_1 \cdot \hat{q}_1 = B \cdot D_1 \cdot \left(B \cdot \hat{C}^{-1} \cdot f\right) \tag{108}$$

$$\begin{bmatrix} X_2 & q_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 5.0 & 8.8 & 1.6 & 15.4 \\ 5.4 & 5.8 & 1.3 & 12.4 \\ 1.5 & 2.2 & 0.5 & 4.2 \end{array}\right]$$

$$B \cdot M_1' \cdot \hat{q}_1^{-1} = B \cdot D_1 \tag{109}$$

$$B \cdot D_1 = \begin{bmatrix} 16.3\% & 26.4\% & 16.1\% \\ 17.5\% & 17.3\% & 12.9\% \\ 5.0\% & 6.6\% & 4.8\% \end{bmatrix}$$

The general formulas in step r are equal to:

$$X_r = B \cdot M_{r-1}' = B \cdot D_{r-1} \cdot \hat{q}_{r-1} = B \cdot D_{r-1} \cdot \left[\widehat{\left(B \cdot C^{-1}\right)^{r-1} \cdot f} \right] \tag{110}$$

$$B \cdot M_{r-1}' \cdot \hat{q}_{r-1}^{-1} = B \cdot D_{r-1} \tag{111}$$

The series of these product by product tables in each step converges to the same total product by table as under the combined assumptions of product technology and constant market shares:

$$\sum_{r=1}^{\infty} X_r = B \cdot \sum_{r=1}^{\infty} M_{r=1}' = B \cdot M' = B \cdot D \cdot \hat{q} \tag{112}$$

But when we perform again the same impact analysis we obtain a product by product table with different technical coefficients. This illustrates that this version of industry technology has no stable product by product coefficients.

$$X = B \cdot M' = B \cdot \hat{g} \cdot C' = B \cdot \left[\widehat{\left(I - C^{-1} \cdot B\right)^{-1} \cdot C^{-1} \cdot q_0} \right] \cdot C' \tag{113}$$

$$X = \begin{bmatrix} 31.0 & 30.9 & 13.3 \\ 39.5 & 21.0 & 10.6 \\ 10.4 & 7.9 & 4.0 \end{bmatrix}$$

$$X \cdot \hat{q}^{-1} = B \cdot M' \cdot \hat{q}^{-1} = B \cdot \left[\widehat{\left(I - C^{-1} \cdot B\right)^{-1} \cdot C^{-1} \cdot q_0} \right] \cdot C' \cdot \left[\widehat{C \cdot \left(I - C^{-1} \cdot B\right)^{-1} \cdot C^{-1} \cdot q_0} \right] \tag{114}$$

$$B \cdot D = \begin{bmatrix} 13.8\% & 25.5\% & 16.1\% \\ 17.5\% & 17.3\% & 12.9\% \\ 4.6\% & 6.5\% & 4.8\% \end{bmatrix}$$

Finally we like to close this part with the remark that the industry x industry table $C^{-1} \cdot U$ is also the combination of industry technology and a fixed industry sales structure.

3.4. An attempt to understand Mesnard's critique

As a starting-point we would like to repeat that de Mesnard's identifies product technology with the invariability of the B and C matrices. Consequently we can suppose that his critique is solely directed against this variant of product technology. According to the de Mesnard the beginning of the economic circuit with the transformation of final demand by product into final demand by industrial output $e = C^{-1} \cdot f$ (and in general the transformation $g_r = C^{-1} \cdot q_r$) is an illegal transformation. Only the transformation $q_r = C \cdot g_r$ remains true. We try to follow him here but we cannot follow him anymore when he illustrates his statement with an (although incomplete) example (at last).

If we translate his example into our exercise he seems to calculate the first make table as $M_0 = \hat{f} \cdot \left(C^{-1}\right)'$. In fact C^{-1} is used here as a market shares matrix! Just as $i' \cdot D = i$, $i' \cdot C^{-1} = i$ but C^{-1} (and therefore $C^{-1'}$) always contains negative elements which unavoidably leads to negatives in the make table M_0. Negative outputs of some products in some industries are of course impossible. It would mean a negative starting-point of the economic circuit (step 0).

$$\begin{bmatrix} M_0 & q_0 \\ g_0 & \end{bmatrix} = \left[\begin{array}{ccc|c} 132.4 & -9.4 & 0.0 & 123.0 \\ -11.0 & 45.0 & 0.0 & 34.0 \\ 0.0 & 0.0 & 45.0 & 45.0 \\ \hline 121.4 & 35.6 & 45.0 & \end{array}\right]$$

The product-mix matrix $C_0 = M_0 \cdot \hat{g}_0^{-1}$ of step 0 is equal to:

$$C_0 = \begin{bmatrix} 109\% & -26.4\% & 0.0\% \\ -9.0\% & 0.0\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

This is clearly not equal to the product-mix matrix C. The use of C^{-1} as a (wrong) market shares matrix does not guarantee the invariability of the product-mix coefficient matrix.

Continuing with step 1:

$$U_1 = A \cdot M_0 = A \cdot \hat{f} \cdot \left(C^{-1}\right)' \tag{115}$$

$$\begin{bmatrix} U_1 & q_1 \end{bmatrix} = \left[\begin{array}{ccc|c} 10.2 & 13.4 & 7.3 & 30.8 \\ 21.4 & 6.1 & 5.8 & 33.3 \\ 4.5 & 3.0 & 2.2 & 9.7 \end{array}\right]$$

$$B_1 = U_1 \cdot \hat{g}_0^{-1} = A \cdot \hat{f} \cdot \left(C^{-1}\right)' \cdot \left(\widehat{C^{-1} \cdot f}\right)^{-1} \tag{116}$$

$$B_1 = \begin{bmatrix} 8.4\% & 37.6\% & 16.1\% \\ 17.6\% & 17.1\% & 12.9\% \\ 3.7\% & 8.5\% & 4.8\% \end{bmatrix}$$

The actual absorption coefficients matrix of step 1 also differs from the general matrix B. The identity $B_1 = A \cdot C_0$ remains valid.

Just as M_0, the make table of step 1 $M_1 = \hat{q}_1 \cdot \left(C^{-1}\right)' = \left(\widehat{A \cdot f}\right) \cdot \left(C^{-1}\right)'$ contains negative elements, which is not allowed:

$$\begin{bmatrix} M_1 & q_1 \\ g_1 & \end{bmatrix} = \left[\begin{array}{ccc|c} 33.2 & -2.4 & 0.0 & 30.8 \\ -10.8 & 44.1 & 0.0 & 33.3 \\ 0.0 & 0.0 & 9.7 & 9.7 \\ \hline 22.4 & 44.1 & 9.7 & \end{array}\right]$$

$$C_1 = M_1 \cdot \hat{g}_1^{-1} = \left(\widehat{A \cdot f}\right) \cdot \left(C^{-1}\right)' \cdot \left(C^{-1} \cdot A \cdot f\right)^{-1} \tag{117}$$

$$C_1 = \begin{bmatrix} 148.0\% & -5.6\% & 0.0\% \\ -48.0\% & 105.6\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

C_1 differs clearly from C but also from C_0.

Let us now continue with step 2.

$$U_2 = A \cdot M_1 = A \cdot \left(\overset{\wedge}{A \cdot f}\right) \cdot \left(C^{-1}\right)' \tag{118}$$

$$\begin{bmatrix} U_2 & q_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 0.0 & 13.8 & 1.6 & 15.4 \\ 4.0 & 7.2 & 1.3 & 12.4 \\ 0.5 & 3.2 & 0.5 & 4.2 \end{array}\right]$$

$$B_2 = U_2 \cdot \hat{g}_1^{-1} = A \cdot \left(\overset{\wedge}{A \cdot f}\right) \cdot \left(C^{-1}\right)' \cdot \left(C^{-1} \cdot \overset{\wedge}{A \cdot f}\right)^{-1} \tag{119}$$

$$B_2 = \begin{bmatrix} 0.0\% & 33.1\% & 16.1\% \\ 17.8\% & 17.2\% & 12.9\% \\ 2.4\% & 7.7\% & 4..8\% \end{bmatrix}$$

$$B_2 = A \cdot C_1 \tag{120}$$

$$M_2 = \hat{q}_2 \cdot \left(C^{-1}\right)' = \left(\overset{\wedge}{A^2 \cdot f}\right) \cdot \left(C^{-1}\right)' \tag{121}$$

$$\begin{bmatrix} M_2 & q_2 \\ g_2 & \end{bmatrix} = \left[\begin{array}{ccc|c} 16.6 & -1.2 & 0.0 & 15.4 \\ -4.0 & 16.4 & 0.0 & 12.4 \\ 0.0 & 0.0 & 4.2 & 4.2 \\ \hline 12.5 & 15.2 & 4.2 & \end{array}\right]$$

$$C_2 = M_2 \cdot \hat{g}_2^{-1} = \left(\overset{\wedge}{A^2 \cdot f}\right) \cdot \left(C^{-1}\right)' \cdot \left(C^{-1} \cdot \overset{\wedge}{A \cdot f}\right)^{-1} \tag{122}$$

$$C_2 = \begin{bmatrix} 132.0\% & -7.7\% & 0.0\% \\ -32.0\% & 107.0\% & 0.0\% \\ 0.0\% & 0.0\% & 100.0\% \end{bmatrix}$$

In general $B_r = A \cdot \left(\overset{\wedge}{A^{r-1} \cdot f}\right) \cdot \left(C^{-1}\right)' \cdot \left(C^{-1} \cdot \overset{\wedge}{A^{r-1} \cdot f}\right)^{-1}$ and $C_r = \left(\overset{\wedge}{A^r \cdot f}\right) \cdot \left(C^{-1}\right)' \cdot \left(C^{-1} \cdot \overset{\wedge}{A^r \cdot f}\right)^{-1}$ will differ from B and C in every step of the circuit. Is this not contrary to de Mesnard's adherence to the assumption of constant absorption and product-mix coefficients as the definition product-technology? The make tables $M_r = \left(\overset{\wedge}{A^r \cdot f}\right) \cdot \left(C^{-1}\right)'$ will always contain negative elements.

Moreover the series of the make tables M_r in such a version of the economic circuit does not converge to the make table of the national accounts:

$$\sum_{r=0}^{\infty}\left(A^{r}\cdot\hat{f}\right)\cdot\left(C^{-1}\right)' = \hat{q}\cdot\left(C^{-1}\right)' \neq M = \sum_{r=0}^{\infty}\left(A^{r}\cdot\hat{f}\right)\cdot D' = \hat{q}\cdot D' \tag{123}$$

But we really do not see what the negative elements of M_r in the example above have to do with the negatives that are currently mentioned in association with product technology. I think I demonstrated a correct version of the economic circuit under product technology where the make table is non-negative in every step since D is non-negative. The negatives that possibly arise, when applying product technology to derive product x product tables from make and absorption tables, are not those in C^{-1}, which are inevitable, but the ones in $A = B\cdot C^{-1}$, which are in theory evitable, since the absence or presence of negatives in $B\cdot C^{-1}$ is conditional (Konijn 1994, Steenge, 1989 and 1990).

These are negative inputs that arise during the transformation of B into A when an industry does not register certain inputs in such a measure as it should do according the product technology assumption. If it does register these inputs (as in our example) these negatives simply never appear. Moreover I have the impression that the matrices M_r in the example above exhibit negative unconditional outputs. Is this not a complete different matter?

What surprises us the most is that in the example given by the De Mesnard the product-mix is obviously not constant while he initially associates product technology with a constant product-mix.

As a conclusion we would like to say that we have serious considerations on de Mesnard’s critique of product technology:

- he defines product technology as the exhibition of a constant product-mix. Is his critique only valid against this special case of product (or industry!) technology combined with a constant product-mix? What remains of his critique if one accepts the more general definition of product technology?

- if one considers his critique as only directed against the assumption of an invariable product-mix why is this one not constant in the example he gives to illustrate his statement? Is he here not in contradiction with his own starting-point?

4. Practical Aspects

4.1. Integration in National Accounts

We know now that input-output tables are never entirely integrated in national accounts. They are always explicitly (product by product tables) or implicitly (industry by industry) based on simple but far-reaching modelling assumptions (strong technology assumptions). They are still closely related to the underlying supply and use tables and basic data but there is a "modelling cut".

When looking superficially at the product by product tables this "modelling cut" is only more apparent:

$$\begin{array}{c c|c} U \cdot (D')^{-1} & f & q \\ va' \cdot (D')^{-1} & & \\ \hline q' & & \end{array}$$

When looking at the value-added quadrant the link with national accounts is clearly cut. But the in the final demand part the link with national accounts is more closely maintained, although it is not perfect. It is divided by product and valued at basic prices. In national accounts two variables of final demand are published:

- fixed capital formation by product (but at a very aggregated level, the Pi7 level of the NACE) at purchaser prices
- final consumption by households is published divided by function (the COICOP classification which is related to the product classification) and valued at purchaser prices

If one takes a quick glance at the industry by industry tables:

$$\begin{array}{c c|c} D \cdot U & D \cdot f & g \\ va' & & \\ \hline g' & & \end{array}$$

Value added (va'), total output (g) and consequently total intermediate consumption are published in the national accounts by industry. Here the link with the national accounts is perfectly maintained. But final demand is now given by industry ($D \cdot f$) while in the national accounts it is published by product or by function (which is related to products not to industries). Here the link with national accounts is clearly cut.

4.2. The statistical unit and the degree of secondary production

Different types of statistical units are considered in official guidelines: the enterprise, the local unit, the kind-of-activity unit (KAU) and the local kind-of-activity unit (local KAU) or establishment as it is called in the SNA.

This last one is the recommended unit. The present Belgian statistical apparatus has no experience with this concept because the enterprise is the statistical unit[6] with the consequence that the supply and use tables are very heterogeneous.

The definition of the local KAU or establishment given in the SNA or ESA is very abstract. The ESA defines it as “the part of a KAU which corresponds to a local unit. The KAU groups all parts of an institutional unit in its capacity as producer contributing to the performance of an activity at class level (4 digits) of the NACE rev. 1 and corresponds to one or more operational subdivisions of the institutional unit. The institutional unit’s information system must be capable of indicating or calculating for each local KAU at least the value of production, intermediate consumption, compensation of employees, the operating surplus and employment and gross fixed capital formation” (Eurostat, 1995)

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[6] Before the introduction of the ESA 95 the local unit was the statistical unit of the industrial branches only. There was no systematical statistical interrogation of the service industries (Avonds and Gilot, 2002)

According to me (by intuition) this means that when an enterprise surpasses a certain size and its secondary production(s) a certain threshold it has to be split up into several KAU’s or local KAU’s (when it has several local units).

The SNA adds the types of employees and hours worked and the stock of capital and land used to the minimum data requirements. Notice that intermediate consumption is not required by product. If only total intermediate consumption is known at the level of the (local) KAU when intermediate consumption by product is known at the level of the enterprise how are the latter attributed the to (local) KAU’s? We suppose that in this case clearly some modelling at the micro-level should be done. Is this done according to the industry technology assumption (purely proportional according to the outputs of the local KAU’s) or closer to the product technology principle when inputs are attributed according to the generally known input structure of the products corresponding with the (nearly homogeneous) outputs of the (local) KAU’s?

Some caution is certainly recommended when comparing input-output tables that are compiled on the basis of different statistical units.

Product by product tables remain in theory perfectly comparable because they are directed towards the same result: the input structure and output of homogeneous branches. The type of statistical unit will off course in practice affect to a certain degree the outcome.

When comparing industry by industry tables based on different statistical units this is not the case because these tables reproduce transactions between enterprises, local units, KAU’s or local KAU’s according to the type of statistical unit.

Regarding secondary production B. Thage declares “The observed extent of secondary production does therefore not posses any observable characteristics of its own. The elusive character of the concept of secondary production makes it difficult to justify that it should be of particular interest statistically just because it is produced in two or more industries at a certain level of industry or product aggregation" (Thage, 2005).

We have to reply by repeating that input-output analysis is based on homogeneous branches (homogeneous within the dimensions of the input-output table). Secondary production remains the essential difference between supply and use tables on the one hand and homogeneous input-output tables on the other. The degree of secondary production should be considered at the level of the working format of the supply and use tables and the input-output table should be compiled at the maximum (square) dimension because in this case the relation with the statistical data is diminishes less and more applications are feasible and accurate.

When the supply and use tables are compiled on the basis of (local) KAU's they are already very homogeneous and the use table approaches the product by product table (for a very good reason). B. Thage mentions the registration of secondary production only for manufacturing industry and the assumption of the service industries as being homogeneous (even at the level of the enterprises) by lack of statistical data. In this case the use table resembles even more to a product by product table but now for a bad reason. Further he makes mention of agriculture, trade and construction as already being constructed as homogeneous branches in the supply and use tables. Honestly, I really ask myself if one can still speak of industry by industry tables in such a case. If the industries in the supply and use tables are already rather (or in many cases already completely) homogeneous the "industry by industry" table can be more or less identified with a Leontief table (Konijn, 1994) but can it still be considered as an industry by industry table?

In Belgium we do not have the (local) KAU as statistical unit and trade and construction are registered as heterogeneous industries in the supply and use tables in order to maintain the link with the business register. But we do have data on the output composition of the service industries (at the enterprise level). If we look at the degree of secondary production of the service industries, the assumption of their homogeneity (at the enterprise level) does not seem acceptable, at least for the so-called business-services (NACE 65 up to and including 74).

Table 1: The degree of secondary production presented at the level of the P6 and A6 classifications for Belgium

These are not the "official" Belgian input-output tables but constant price tables calculated for the EUKLEMS project with the disaggregations of certain industries and one product, all corresponding with the national accounts version 2005 (Avonds et al., 2007). The official tables correspond with different versions of the national accounts and do only exist at current prices. Moreover, the "official" tables for 1996 and 1998 have never been calculated[7].

The table does not reflect the heterogeneity of 6x6 tables but of the square tables at maximum level[8]. The ratio of total secondary output to total output (the ratio of total off-diagonal elements to total elements in the make tables) is used as a criterion. This ratio is very high and fluctuates between 16% and 18%. Regarding the degree of secondary production at the level of the (mega) industries we observe a break between 1999 and 2000.We do fear that the (official) tables for 1997 and 1999 are (partially) calculated as an extrapolation of the 1995 table notwithstanding the fact that the statistical data are yearly available.

[7] We have to admit that the tables for 1996 and 1998 are a retro- and extrapolation of the tables for 1997.
[8] 1995: 159 x 159, 2000: 152 x 152, other years: 137 x 137.

From 2000 on the degree of heterogeneity of the business services is equal to 15% or 16%. Compared to 1995 the pattern of secondary production does not really change qualitatively (no disappearance of certain secondary production and the appearance of new ones), the share of the existent secondary production has increased.

- Banks do have a large secondary production of supporting services of financial institutions. The insurance companies have large secondary activities in real estate services (CPA 70) and business services (CPA 74)
- The real estate services do have a considerable secondary output of construction activities (CPA 45)[9]
- The set of industries belonging to the NACE Division 74 "Other Business Services" do form a cluster. The largest part of the secondary output is always secondary output characteristic of other industries belonging to Division 74. Further there are also secondary activities of computer services (CPA 72)[10] and waste collection and treatment (CPA 90). At last there is an industry of the "other "category within this division that has a whole series of rather small secondary productions of manufacturing goods which all counted together are significant, comparable to the wholesale trade industry (if the statistical unit is the local KAU this activity is normally moved the manufacturing industries, the PRODCOM statistic has a low threshold)
- The whole of other services (NACE 75 up to and including 95) has a small but gradually increasing degree of secondary production. The significant secondary outputs are the incidental sales of government (an ESA 79 concept: Eurostat,

[9] Here is taken care of when construction is already a homogeneous branch in the supply and use tables.
[10] I am talking here of market computer services and not output for own final use.

1979) and production of market services by the non-market service industries that are the market variant of their characteristic production.

4.3. The distinction between the use of domestic output and imports

In general the use table for imports is calculated by assuming constant import ratio's (a simple proportional distribution of imports over each row of the use table), which is similar to the assumption of a fixed product sales structure. According to B. Thage this is contradictory with the separation of the input-output table into input-output tables for domestic output and imports when assuming product technology.

We think this is only the case when one assumes that the input structure of a product is the same in every industry where it is produced but also that the ratio of domestic/imported for all the intermediary used goods and services for the making of this product is the same in all these industries.

The non-published input-output manual of Eurostat (Eurostat, 2002) proposed a method to avoid this. This method is based on the product technology model, linked to the hypothesis that inside one industry the ratio use of imports/total use of intermediate products is the same for the principal and secondary activities. This means that the technical coefficients are unique for a product no matter in which industry it is produced as principal or secondary output, but the composition in the technical coefficients of domestic output and imports can vary according to the industry.

In Belgium about 70% of the imports of goods are exogenously attributed on the basis of foreign trade data (Van den Cruyce, 2002, 2003 and 2004). This information is maintained when calculating the input-output table for imports.

4.4. Negatives and redefinitions

A serious difficulty encountered during the application of product technology are negatives in the matrix product $U \cdot M^{-1}$ or $B \cdot C^{-1}$. This means that an industry does not use enough (or at all) the inputs it is supposed to use for its secondary production. These negatives can have different reasons: errors in basic data or in the compilation of make and use tables (they can be traced and corrected), heterogeneity of the industry classification, ...

This last phenomenon is often indicated as a possible cause of negatives when applying the product technology model (Gigantes, 1970, Konijn 1994, Rainer and Richter, 1992, Stone et al., 1963): when calculating the input structures of products (homogeneous branches) these are aggregated to the level of the industry classification these aggregated to the level of the industry classification. At this level the principal production of an industry is an aggregation of different original products for which the production processes (inputs) may differ in reality. The input structure of a homogeneous branch is largely determined by the input structure of the primary producer. This means that the input structure of a homogeneous branch is more or less a weighted average of the input structures of the products made by the primary producer. Another industry can produce, as a secondary activity, only some of these original products or in another composition than the primary producer. But this is not taken into account in the transformation matrix of the product technology model. It is assumed here that secondary producers have the same composition as the main one. If this is not the case negatives can be created in the input-output model. A disaggregation of the primary industry can be useful to handle this kind of problem.

If the product technology assumption is not (entirely) valid or if it is not possible to realize the aimed disaggregations in practice (if the industries would have to broken up into an impossible number of activities) "redefinitions" can be carried out. A redefinition is the manual transfer of the inputs of a part of the secondary production of an industry whereby these inputs are estimated exogenously so that that no negatives appear after the transformation. This does not necessarily mean that one rejects entirely the product technology model in favour of the industry technology model. If the input-structure of the manual transformation is similar to the one of the primary producer one remains much closer to product technology than industry technology. If one takes the use table after redefinition as the new use table the mathematical properties of the product technology model remain valid.

During the compilation of the Belgian input-output tables for 1995 and 2000 redefinitions were made under the denomination of "analytical disaggregations" (Avonds and Gilot, 2002, Avonds, 2002 and 2005). The "activity technology model" developed by Konijn does start as a redefinition. The inputs of activities for which no primary producer (industries) exists are estimated exogenously. After that the activity technology model goes one step further: the outputs of these activities are distinguished as separate products. This means that extra products are introduced in the input-output system of which the uses should be distinguished in the use table. In this way, the conditions of the Leontief input-output model (homogeneous branches) are once again met (Konijn, 1994). The US Bureau of Economic Analysis (BEA) applied a "redefinition process" of which they declared that it is similar to product technology as a first step in the transformation of supply and use tables into the "benchmark" input-output tables. The second step consists of the application of the industry technology principle (Guo et al., 2002). A more overall type of redefinition is the application of specific transformation matrices to each row (product) of the absorption table, avoiding in this way the emergence of negatives, instead of one general transformation matrix $(D')^{-1}$ (Reich et al., 1989, Stahmer, 1985).

Small negatives can be corrected by putting them equal to zero and rebalancing the input-output table with the RAS method or applying the Almon algorithm to the absorption table (a mathematical iterative method not entirely equal to but still based on the product technology assumption, Almon, 1970, 1998 and 2000). After applying such mechanical procedures the mathematical properties of the product technology model are not entirely valid anymore and the input-output table does slightly disobeys the 4 assumptions of input-output analysis. In this case one has made an approach of an input-output table meeting these 4 assumptions.

B. Thage also mentions redefinitions in the last part of his paper. He considers manual splitting up of KAU's when no information is available and they span more than one heading at the first level of the industrial classification (for example the A6 version of the NACE)[11]. This is a redefinition that remains within the framework of the supply and use tables. It is also modelling at he micro-level, where have to make the same remark as on page 46. He limits himself to the statement that these redefinitions are "made at hand based on the best available information and judgement of the national accountants". I think one can hardly disagree that when an industry combines productions belonging to the first level of break-down of an industry classification (for example manufacturing and wholesale) the implausibility of the industry technology assumption is obvious. Applying industry technology explicitly or implicitly does not make sense in such a case. These redefinitions can remain within the framework of the supply and use tables or executed only during the transformation into the input-output

[11] The SNA 93 recommends this for vertically integrated enterprises (United Nations et al., 1993). The vertical integration of enterprises is a typical cause of negatives during the application of product technology.

tables. According to him these redefinitions are quit different from the product technology model because:

- only part of secondary production is redefined
- they are not (pure) mathematical procedures
- no negatives appear

Of course we agree that redefinitions are different from the pure mathematical version of the product technology model but we think that they are still related to it. Large secondary productions causing large negatives are in general not mathematically transferred and subsequently mechanically corrected but treated with redefinition type transfers (where the product technology principle is mitigated) as explained above.

When input-output tables are calculated by a combination of redefinitions and explicit or implicit application of industry technology (industry by industry tables based on a fixed product sales structure) one applies in reality a mixed technology model: a combination of (a mitigated version of) product technology and industry technology as was declared in the BEA paper.

Moreover if industry by industry tables would be independent of technology why bother to make them more homogeneous by means of these redefinitions (and even to introduce less heterogeneous statistical units than the enterprise)? Would it still matter? If it matters does this not mean that one wants these tables to resemble more to a Leontief table (a table based on homogeneous branches obeying the axioms of input-output analysis) than a industry by industry table without these interventions and that one is (maybe implicitly or unconsciously) considering technology assumptions?

Conclusions

Input-output tables are never fully integrated into national accounts in the sense that they are always based on far-reaching modelling assumptions, even when these assumptions are very simple. Product by product tables are based on a combination of technology assumptions (the input structure of products and consequently of the industries producing them) and output assumptions (the output composition of industries in terms of products). The industry by industry tables are not only based on market assumptions (the share of deliveries by industries in the final and intermediate use of each product) but also on technology assumptions. The only tables that are really integrated in the national accounts are the supply and use tables from which the input-output tables are derived as models.

Products by product tables based on the product technology assumption have stable coefficients by definition. A certain flexibility in the choice of the output assumption exists here. One simply has to make an output assumption that does not lead to unconditional negatives in the make table and guarantees convergence in terms of the economic circuit. It is on the base of these findings that we have serious considerations with the critique formulated against the product technology model by de Mesnard. Firstly his critique does not seem directed against the product technology model in general but against the output assumption of a constant product-mix (which he identifies with product technology). Secondly in the example in which he illustrates his statement the product-mix is variable (and impossible). I cannot dispose myself of the impression that he is contradicting himself here.

Because all of this I continue to be inclined to think that product by product tables based on product technology remain the only tables that obey the axioms on which input-output analysis is based.

Considering B. Thage's arguments in favour of the industry by industry table based on the market assumption of a fixed product sales structure there is only one that is completely indisputable. It is indeed (much) easier to derive such a table from the supply and use tables than to compile homogeneous tables based on product technology. How much easier this is depends on the degree of secondary production in the supply and use tables, which in turn depends on the type of the statistical unit and the degree in which (simplifying) modelling assumptions are already made when compiling these supply and use tables. If, for a variety of reasons the degree of secondary production is low (in general and in all the industries), are the differences between the different types of input-output tables derived from the supply of use tables not that small that the discussion about which type of table to select in practice resembles a little bit to a "Byzantine" theological dispute? But we cannot agree with his main theoretical argument, namely that this version of industry by industry tables is invariant of the technology assumption. We think it is (implicitly) also based on the industry technology assumption. Considering his other, more practical arguments, they can be toned down by counter-arguments.

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Table 1 Degree of secondary production presented at the level of the P6 and A6 classifications

		1995	1996	1997	1998	1999	2000	2001	2002
1	Agriculture	1.0%	0.5%	0.5%	0.4%	0.6%	0.6%	0.5%	0.5%
2	Industry, including energy	19.7%	20.3%	21.1%	20.7%	21.3%	18.7%	19.1%	19.3%
3	Construction	25.6%	20.6%	20.4%	20.9%	21.1%	16.8%	16.6%	16.8%
4	Trade, transport and communications	24.3%	26.3%	24.6%	27.7%	27.9%	17.5%	18.3%	19.7%
5	Business services	8.7%	8.8%	8.8%	8.7%	9.9%	16.4%	15.2%	15.6%
6	Other services	2.8%	4.7%	5.0%	5.3%	4.9%	6.0%	5.4%	5.7%
	Total	16.1%	16.7%	16.6%	17.4%	17.9%	15.8%	15.9%	16.3%