Balance, Manhattan norm and Euclidean distance of industrial policies for the US

Maurizio Ciaschini, Rosita Pretaroli, Claudio Socci[a*]

[a] *University of Macerata, Dipartimento di Scienze della Comunicazione, Via Armaroli 9, 62100 Macerata-Italy; Phone +39 0733 258 2560, Fax +39 0733 258 2553*;

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The present day economic circumstances require the design of policy controls strongly oriented to specific industrial sectors suitable for supporting output and mitigate the effect of recession. Within the scientific debate the implementation of such policies highlights a set of problems of choice of the macro variables that make up the policy control - final demand components, disposable income or other - of the determination of its amount, structure and balance. In a multi-sectoral framework these issues require a careful identification of the relationship among amount and structure of the policy control - which normally is brought back to a demand control - since the aggregation criterion influences the achievable results and their interpretability. The Macro Multiplier approach, which extends the Leontief multipliers analysis, identifies the complete representation of the structures of the macro variables, components of final demand, consistent with the technologies in use. The set of MM gives then a set of scalars in which each MM operates, in an aggregated fashion, on an associated structure of the macro variable in a multiplicative way. The potentialities of the reply in the objectives will rise out, as well as the compatibility of the final demand structures with the technologies characterizing the producing processes. Within the contributions on the attempt to attenuate a recession phase through demand oriented policies, the work proposed tries to identify the "convenient" composition of the policy control variable and its impact on production. The application is performed on an Input-Output table for the year 2007.

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1 Economic policy and multisectoral analysis

The progressive opening of macroeconomic analysis to the multisectoral viewpoint, encouraged and confirmed also by the progressive integration of aggregate national accounts of the final demand with the multisectoral accounts of industry flows, cannot set aside the macroeconomic implications of multisectoral results, perceived nowadays as critical.

On the other side multisectoral analysis is developing also in terms of broadening the model towards other aspect of the socio economic contest in terms of social accounting. The attempt in this paper is that of a second thought on the multisectoral reference scheme, that is the Leontief model, in terms of economic policy, of relationship between the aggregated level and the disaggregated one, on the need to produce results for the complete structure of the macroeconomic variables, overcoming the inconvenient of looking to sectoral one-to-one effect, in the awareness that these considerations and results will be extended to multisectoral model with more complex reduced forms and disaggregation criteria as those that emerge from the social accounting.

In this respect, section 2 shows the Macro Multipliers and the key structures of the Leontief reduced form. In section 3 the policy isoquants as reachable policy-objective sets are showed. Section 4 discusses of the Leontief multipliers and Macro Multipliers at work. Section 5 shows

*Corresponding author. email: Maurizio Ciaschini (ciasco@unimc.it), Pretaroli Rosita (pretaroli@unimc.it), Claudio Socci (socci_claudio@unimc.it).

Leontief Multipliers for the US economy and section 6 describes the Macro Multipliers approach. Section 7 focuses on policy design as an alchemy of policy key-structures.

2 Policy isocosts and the Leontief reduced form

In order to allocate the policy problem in a multisectoral frame work we refer to the Leontief model(Leontief 1965). This is the simplest multisectoral model, compared, for example, to that emerging from the SAM, that gives an evaluation of the macro variables both in terms of sectoral composition and aggregate value.

The macro variables implied are total output value ($\mathbf{x}$), industry demand ($\mathbf{m}$), final demand ($\mathbf{f}$) and, in an ancillary role of accounting check, value added ($\mathbf{y}$). The equilibrium relationships that connect intermediate expenditure, final demand expenditure to total output value, verified at the industry level can be written, according the well known formula, as:

$$\mathbf{x} = \mathbf{m} + \mathbf{f} \tag{1}$$

Given the technic coefficient matrix $\mathbf{A}$ the structural formo $\mathbf{f}$ the Leontief model is written as:

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{x} + \mathbf{f}$$

That implies a value added of the type:

$$y = \sum_j (1 - \sum_i a_{ij} x_j)$$

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which connects to the concept of GDP, the most relevant macrovariable in aggregate modelling.

To this structural form of the model corresponds the reduced form which is given by the specification of the model where the variables that play a strategic role in defining the policy problem are put in evidence. These are the endogenously determined *policy objectives*, on which the effects of the policy control are observed, and the exogenously given *policy controls*, which the policy maker has to determine to attained the desired or convenient configuration of the policy objectives(Ciaschini et al. 2009). The structural model is then manipulated in order to put the policy objectives in direct relationship with the policy controls determining the reduced form of the model:

$$\mathbf{x} = \mathbf{R} \cdot \mathbf{f} \tag{2}$$

where $\mathbf{R} = [\mathbf{I} - \mathbf{A}]^{-1}$.
For the Leontief model the policy objectives are linked to the policy controls through the well known Leontief inverse $[\mathbf{I} - \mathbf{A}]^{-1}$. Though the study of the parametric structure of the reduced form we determine both the scale and the structure of the policy control – a change in the final demand expenditure $\Delta \mathbf{f}_o$ – necessary top obtain a convenient change in the policy objectives given by the change in the scale and the structure of $\Delta \mathbf{x}_o$. Once determined the solutions of the policy problem, the remaining endogenous variables are then consistently determined:

$$\Delta \mathbf{m}_o = \Delta \mathbf{x}_o - \Delta \mathbf{f}_o \tag{3}$$

$$y_o = \sum_j [\Delta x_{oi} - \sum_i a_{ij} \Delta x_{oj}] \tag{4}$$

Given a vector that shows the value of the sectoral components of a macro variable, defining the structure of such macro variable, the delicate question of how to define its scale emerges i.e. we need to define in a scalar taken out from the sectoral components its aggregate value. The question is critical since it involves the possibility itself of obtaining consistent results at the different levels of aggregation.

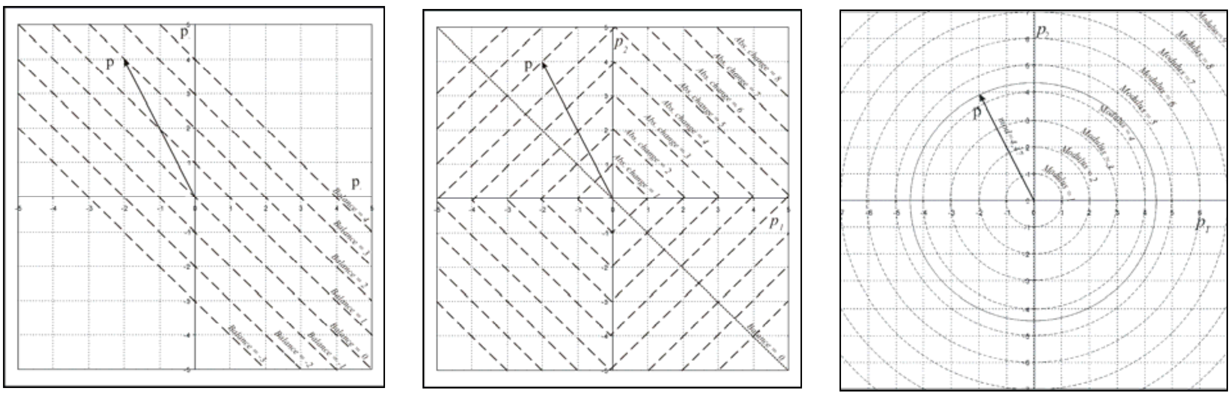
The most immediate aggregation criterion is given by the sum of sectoral elements. Given that the sectoral components of a vector can assume both positive and negative values, since in general they represent changes form an initial condition, we define this procedure synthetically as *balance*.

Given then the policy control vector $\mathbf{p}$ a first definition of its scale will be given by:

$$bal(\mathbf{p}) = \sum p_i$$

The vectors that show the same balance will be allocated along the same line as shown in figure 1(a). Grouping all the possible policy controls $\mathbf{p}$ according their balance, a set of lines will be determined that we define isocosts or, more precisely, balance-isocosts of the policy control. In the policy application is of sure relevance the zero-balance manoeuvre which is the policy control which is performed without making the original level of the sectoral variables change through variations that compensate within the same policy control.

Figure 1. Isocosts of the policies



(a) Map of the balance-isocosts Policy control $p = (-2 \quad 4)^T$ belongs to the isocost 2

(b) Map of the balance-isocosts Policy control $p = (-2 \quad 4)^T$ belongs to the isocost 6

(c) Map of the balance-isocosts Policy control $p = (-2 \quad 4)^T$ belongs to the isocost 4.472

It is however apparent that balance is not insufficient to define the scale of a macro variable (change), since the balance can easily hide variations of very diverse relevance. Together with balance we need to dispose of a quantification of the scale that gives information on the real amount of resources that have been activated. This information can be provided by the sum of the absolute values of the vector components, that we will indicate with the abridged expression absolute *change*:

$$abs(\mathbf{p}) = \sum |pi|$$

The absolute change of vector $\mathbf{p}$ quantifies the amount of the policy manoeuvre in terms both of the expansion realized and the restraints imposed to sectors. Vectors that show tha same absolute change will locate along the same square with diagonal equal twice the absolute change.

Grouping all the possible policy control $\mathbf{p}$ according the scale defined as absolute value, a set of squares will be generated, as shown in figure 1(b), that we define as absolute change-isocosts of the policy control, or more simply isocosts of the policy control when the context allows no ambiguity.

The absolute change of a vector, according our definition, is a vector norm and defines a type of distance between two points in terms of the sum of the (absolute) differences of their coordinates. Sometimes is known as Manhattan norm[1]. The notion of norm of a vector in mathematics is essential for defining the concept of "distance " or "length" in a linear vector space as that in which the policy control isocosts map is defined. The norm of a vector $\mathbf{p}\varepsilon\ \mathbf{P}^n$ is a mapping that associates to each element in R^n a real number. Moreover the norm verifies three conditions:

l) the norm of a vector different from zero is positive. [2]

2) The norm of the sum of two vectors is not greater then the sum of the norms of the two vectors.

3) Scaling a vector by a constant the norm of the vector is scaled by the same constant.

We note that while the absolute change, which gives a measure of the scale of the vector satisfies the requisites to be a norm, the balance even providing a measure of the scale of the vector doesn't satisfy them and then is not a norm, in this confirming its scarce reliability in aggregating in a scalar the entire vector. Of course it remains the relevance of its economic meaning.

Since we want to operate with multidimensional macrovariables, in particular worth the aim of operating on the multidimensional policy objectives through the use of the multidimensional policy control, we need to answer to the question whether the aggregation criteria we have chosen, that generate the isoquants map we have seen, are suitable and convenient to be transformed into policy objective sets operating on them with the Leontief reduced form.

As it will be shown further on in this paper, a matrix transformation of the vector space, the map of isocosts of the policy control, takes place through a process that implies three phases: rotation, scaling and counter rotation. The main question is now whether the morphology of the absolute change isocosts of the policy control is neutral with respect to the three phases of the matrix transformation. In this case we could attribute all the effect, rotation scaling and counter rotation to the Leontief reduced form. Of course each rotation of the axes transforms the coordinates of the vectors. However in the case of the absolute change isocost the scale – the norm – of vector $\mathbf{p}$, determined as absolute change transforms the isocosts in a non uniform manner and then is not neutral with the respect to an axes rotation.

We then conclude that even if the two aggregation criteria described, balance and absolute change, are sufficient under the economic profile to synthetize the characteristics of the vectors scale, we need a further attempt to identify an aggregation criterium that can generate an isocost map neutral with respect to an axes rotation with the aim to isolate uniquely the effects of the transformation generated by the Leontief reduced form.

[1]This name alludes to the grid layout of most streets on the island of Manhattan (Lorenz et al. 1952), which causes the shortest path a car could take between two points in the city to have length equal to the sum of the (absolute) differences of their coordinates rather than the euclidean distance.

[2]The norm is a definite positive function: $\|\mathbf{p}\| = \mathbf{0}\ \forall\ \mathbf{p}\ \varepsilon\ \mathbf{P}\ \|\mathbf{p}\| = \mathbf{0}$ if and only if p_i=0
Satisfies the triangular inequality: $\|\mathbf{p}_1 + \mathbf{p}_2\| = \|\mathbf{p}_1\| + \|\mathbf{p}_2\|\ \mathbf{p}_1,\ \mathbf{p}_2\ \varepsilon\ \mathbf{P}$
Is homogeneous $\|\lambda_p\| = |\lambda| \cdot \|p\|$ for each scalar λ

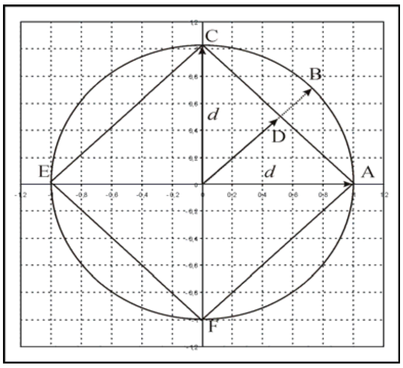
An aggregation criterion that overcomes these drawbacks is that of assigning to the vectors scale the value of its Euclidean norm, that for aim of simplicity we will refer as modulus:

$$mod(\mathbf{p}) = \sqrt[2]{p_1^2 + p_2^2}$$

A ll the policy vectors that have the same modulus, describing a circle with radius equal to the modulus are invariant with respect to rotations of the axes as shown in figure 3. This aggregation criterion, also if less immediate in its economic interpretation, when used together with the two already described, allows for an interesting development of the analysis allowing for overcoming some of the difficulties posed by the aggregation process and able to shed light on the Leontief reduced form in terms of the evaluation of the effects of structure and scale of the policy variables.

In particular is of interest the relationship that exists between absolute change and modulus as defined previously. As shown in figure 2 distance d, which determines the abscissa of point A, reduces along the isocost of absolute change for rotations until 45 degrees, for expanding again towards d when the axes rotation has reached 90 degrees, point C in figure 2. Then the absolute

Figure 2. Rotation effects on absolute change-isocost



change of a vector is an aggregate evaluation that changes its value from $\overline{OA}$ in point A to $\overline{OD}$ in point D, while it should be equal to$\overline{OB}$ if it had to be kept constant in length, and again to $\overline{OC}$= $\overline{OA}$ in point C. the segment $\overline{DB}$ quantifies the difference between the unit modulus and the unit absolute change for varying structures of vector $\mathbf{p}$.

3 Macro multipliers and key structures of the Leontief reduced form

The policy control $\mathbf{p}_o$, consisting in a vector of $n$elements according the number of sectors composing the macrovariable is then premultiplied by the reduced form of the model, that in the case of the Leontief model is an $(n\text{x}n)$ matrix.

In general however the reduced form of a multisectoral model can be given by an $(m\text{x}n)$ matrix. We will then refer to this case since the other case is easily derived from it. In the reduced form all the endogenous macroeconomic variables, other than the objectives, have disappeared. The policy control will then have an effect on the macrovariable considered as objective which is given by a vector $\mathbf{z}_o$ with m elements as the sectors that compose the macrovariable.

$$\mathbf{z}_o = (\tilde{\mathbf{z}}_1\tilde{\mathbf{p}}_1\mathbf{m}_1 + \tilde{\mathbf{z}}_2\tilde{\mathbf{p}}_2\mathbf{m}_2 + \ldots + \tilde{\mathbf{z}}_k\tilde{\mathbf{p}}_k\mathbf{m}_k)\mathbf{p}_o$$

The reduced form can be rewrittten as: $\mathbf{R} = (\tilde{\mathbf{z}}_1 m_1 + \tilde{\mathbf{z}}_2\tilde{\mathbf{p}}_2 m_2 + \ldots + \tilde{\mathbf{z}}_k\tilde{\mathbf{p}}_k m_k)$ so that

$$\mathbf{z}_o = (\tilde{\mathbf{z}}_1 m_1 + \tilde{\mathbf{z}}_2\tilde{\mathbf{p}}_2 m_2 + \ldots + \tilde{\mathbf{z}}_k\tilde{\mathbf{p}}_k m_k)\mathbf{p}_o \tag{5}$$

The k scalars m_i are the Macro Multipliers (Ciaschini and Socci 2007) which are present in matrix $\mathbf{R}$, that can be activated singularly through the use of convenient "keys". These keys are the k policy control structures $\tilde{\mathbf{p}}_i$, that we define as key structures of the policy control, which are determined in matrix $\mathbf{R}$, which are reciprocally perpendicular,and form a basis for defining whatever policy control one may design. In particular each policy key structure $\tilde{\mathbf{p}}_i$, multiplied by the policy control vector $\tilde{\mathbf{p}}_i p_o$ generates a scalar α_i that quantifies the degree of activation of the associated multiplier m_i. Given the two multipliers m_1 and m_2, we can determine the measure in which each policy objective key structure undergoes the impact of the two multipliers according the respective degree of activation. Such an effect quantified as ω_i will be given by $\omega_1 = m_1\alpha_1$ and $\omega_2 = m_2\alpha_2$.

In matrix terms the macro multipliers can be determined as singular values of the reduced form. Given an mxn[3] $\mathbf{R}$ matrix always exists the decomposition $\mathbf{R}$=$\tilde{\mathbf{Z}}\mathbf{M}\tilde{\mathbf{P}}^T$.

Matrix $\tilde{\mathbf{Z}} = [\tilde{\mathbf{z}}_1 \ldots \tilde{\mathbf{z}}_m]$ is an mxm unitary matrix, i.e a matrix composed by a set of orthonormal vectors (Lancaster and Tiesmenetsky 1985), that represents what we define as "the policy-objectives key-structures", i.e. structures of the objective variable through which all the results for the objectives are observed and evaluated. Matrix $\tilde{\mathbf{P}}^T = [\tilde{\mathbf{p}}_2 \ldots \tilde{\mathbf{p}}_k]$ is a nxn unitary matrix, given by the set of the policy-control key-structures, i.e. those policy structures determined by $\mathbf{R}$ which in fact measure and establish the multipliers impact of all the possible policy controls.

Matrix $\mathbf{M}$ is an mxm diagonal matrix with all elements equal to zero outside the diagonal. The elements along the diagonal are all real and ordered according their magnitude as:: $m_1 \geq m_2 \geq \ldots \geq m_k \geq 0$ and $k = min[m, n]$.

In general the decomposition may be compacted as:

$$\mathbf{R} = \begin{bmatrix} \tilde{\mathbf{Z}}_1 \tilde{\mathbf{Z}}_2 \end{bmatrix} \begin{bmatrix} \mathbf{M}_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{P}}_1^T \\ \tilde{\mathbf{P}}_2^T \end{bmatrix}$$

Or

$$\mathbf{R} = \tilde{\mathbf{Z}}_1 \mathbf{M}_1 \tilde{\mathbf{P}}_1^T$$

Where $\mathbf{M}_1$ is a kxk diagonal matrix where k are the non zero macro multipliers. $\tilde{\mathbf{Z}}_1$ mxk represents the first k columns of $\tilde{\mathbf{Z}}$ and is a base in the policy objective space $\Im$ ($\mathbf{R}$). $\tilde{\mathbf{P}}_1$ is an nxk matrix and represents the first k columns of $\tilde{P}^T$ and is a base in the policy control space $\wp(\mathbf{R})$.

From these consideration we can get suggestions useful for the decomposition of the Leontief reduced form given in the second paragraph. Taking the square of that matrix, and remembering that it is not symmetric in the mathematical sense, we get two matrices: $\mathbf{R}^T\mathbf{R}$ and $\mathbf{R}\mathbf{R}^T$.

The symmetric matrix $\mathbf{R}^T\mathbf{R}$ will be given by:

$$\mathbf{R}^T\mathbf{R} = \tilde{\mathbf{P}}\mathbf{M}^2\tilde{\mathbf{P}}^T$$

Since $\mathbf{R}^T\mathbf{R}$=$(\tilde{\mathbf{Z}}\mathbf{M}\tilde{\mathbf{P}}^T)^T(\tilde{\mathbf{Z}}\mathbf{M}\tilde{\mathbf{P}}^T)$. From this result we get that the macro multipliers are the square root of the eigenvalues of matrix $\mathbf{R}^T\mathbf{R}$, that is:

$$m_i = \sqrt{\lambda_i(\mathbf{R}^T\mathbf{R})}$$

Moreover the policy-control key-structures $\tilde{\mathbf{p}}_i$, which evaluate i.e. give a weight in terms of activation of the multipliers to all possible policy controls, are obtained as eigenvectors of $\mathbf{R}^T\mathbf{R}$.

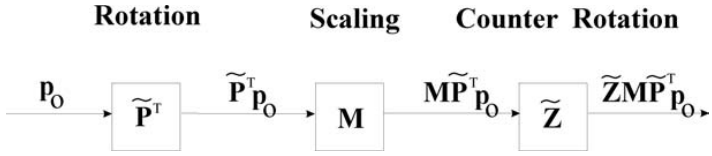
[3]We will treat the general case of a mxn matrix of the reduced form. In the case of the Leontief reduced form, R will be in general a square matrix that can be dealt with along the same lines.

Similarly if we consider $\mathbf{RR}^T$ we obtain:

$$\mathbf{RR}^T = \tilde{\mathbf{Z}}\mathbf{M}^2\tilde{\mathbf{Z}}^T$$

since $\mathbf{RR}^T = (\tilde{\mathbf{Z}}\mathbf{M}\tilde{\mathbf{P}}^T)(\tilde{\mathbf{Z}}\mathbf{M}\tilde{\mathbf{P}}^T)^T$, this means that the macro multipliers can also be calculated as square root of the eigenvalues of matrix $\mathbf{RR}^T$, that is:

Figure 3. Matrix of the Leontief reduced form



$$m_i = \sqrt{\lambda_i(\mathbf{RR}^T)}$$

Moreover the vectors that represent the policy-objectives key structures $\tilde{\mathbf{z}}_i$, are obtained as eigenvectors of $\mathbf{RR}^T$. Its worthwhile mentioning that the policy objective key structures are different from the policy control key structures since the Leontief reduced formi is not symmetrical.

$$\mathbf{z}_o = \mathbf{R}\mathbf{p}_o$$

$$\mathbf{z}_o = \tilde{\mathbf{Z}}\mathbf{M}\tilde{\mathbf{P}}^T\mathbf{p}_o$$

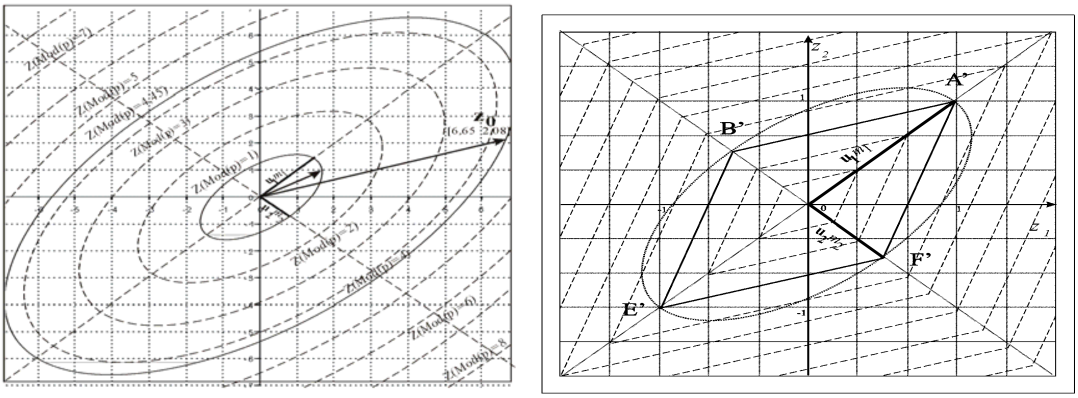
The Leontief reduced form operates then, as shown in figure 3, in three steps:

(1) Rotation. The actual policy control, $\mathbf{p}_o$, is evaluated in terms of the policy key-structures in order to determine the weight at which each policy key-structure is present in the actual policy control, these weights are quantified by vector $\tilde{\mathbf{P}}^T\mathbf{p}_o$;
(2) Scaling. A scale effect is attributed to each policy key-structure, according its presence in the actual policy control. Vector $\mathbf{M}\tilde{\mathbf{P}}^T\mathbf{p}_o$ quantifies this effect.
(3) Counter Rotation. The effect of the actual policy control on the policy objective, $\mathbf{z}_o$, through a counter rotation that transforms the macro-multipliers scaled effect of actual policy control, $\mathbf{M}\tilde{\mathbf{P}}^T\mathbf{p}_o$, into the vector of actual objective, $\tilde{\mathbf{Z}}\mathbf{M}\tilde{\mathbf{P}}^T\mathbf{p}_o = \mathbf{z}_o$.

4 Policy isoquants as reachable policy-objective sets

In determining the set of the policy-objectives which are reachable with a predetermined policy control we need to establish the aggregation criterion of the policy-objective sectoral components. Keeping in mind that the modulus-isocosts are invariant with respect to axes rotations, we will firstly concentrate on the determination of the reachable set of the policy objectives with the same policy-control isocost. In other words we reply to the question: "which is the policy-objective set reachable though the use of a policy for which we keep constant the aggregate value but we make the policy control assume all the possible structures" In the policy objective space, that will be determined by the transformation produced by the matrix R of the reduced form, the unit ball will undergo a deformation and become a ellipsoid where the length of the axes represents the macro multipliers.

Figure 4. Isoquants of the policies

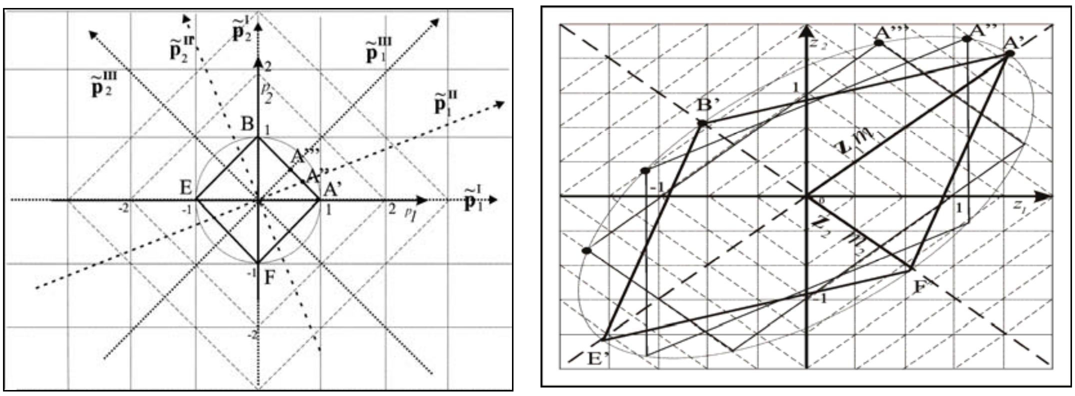


(a) Modulus- Isoquants of the policies (b) Absolute change- Isoquants of the policies

When vector po takes the structure, the first policy-control key-structure, the corresponding vector $\mathbf{z}_o$ takes the structure $\tilde{\mathbf{z}}_1$, the first policy-objective key-structure, and its modulus results multiplied by m1 with respect to the policy control modulus.

Conversely when vector $\mathbf{p}$ takes the structure $\tilde{\mathbf{p}}_2$ the corresponding vector $\mathbf{z}$ assumes structure $\tilde{\mathbf{z}}_2$ and its modulus is multiplied by m_2. Outside these two structures the policy objective reply to the unit policy control will be given by the quadratic combination of the two structures along the surface of the ellipsoid of the objectives. The reply in the policy objective space is not limited to the unit isocost, changing the isocost will result in a change in the isoquant determining a map of isoquants of the objective according to the scale of the policy control.

Figure 5. Isocosts and Isoquants of the policies



(a) Absolute change- Isocosts of the policies (b) Absolute change- Isoquants of the policies

Let us now compare the two types of isocosts - the modulus isocost absolute change isocost of the policy control – with the two types of isoquants - the modulus isocost absolute change isocost of the policy control and take into consideration the unit absolute change isocost of the policy control and the unit modulus isocost of the policy control. In this case as shown in figure 10 it exists only four structures $\overline{OA}$, $\overline{OC}$, $\overline{OE}$ and $\overline{OF}$, for which modulus - isocost and absolute value - isocost coincide and this happens exclusively in the four intersections of the policy control isocost with the axes where $Mod(\mathbf{p}_o) = Abs(\mathbf{p}_o) = 1$.

Outside these cases the coordinates of the vectors in the unit absolute change- isocost will be given by a convex linear combination of couples of orthogonal vectors to the reference system of coordinates as shown in figure 10. In the policy-objective space, along the corresponding absolute-change-isoquant of the policy control, we note only four vectors: $\overline{OA}'$, $\overline{OC}'$, $\overline{OE}'$ and $\overline{OF}'$ from which the absolute-change-isoquant – given by the parallelogram A'B'E'F' – meets the modulus- isoquant- given by all vector on the ellipsoid.

Outside these cases le vectors coordinates will follow an behaviour indicates by the parallelogram as shown in figure 11. However the actual form of the parallelogram – the absolute change isoquant- will depend on the parameters of the Leontief reduced form. Since only in the case where the policy-control key-structures would coincide with the Cartesian axes – $\tilde{\mathbf{p}}_1^I$ and $\tilde{\mathbf{p}}_2^I$ in figure 5(a): no rotation in the policy key structures – the parallelogram vertices would coincide with the vertices of the ellipsoid (distances from the centre $\mathbf{z}_1 m_1 \quad \mathbf{z}_2 m_2$).

Where the policy-control key-structures implicit in matrix $\mathbf{R}$ are different, for example located in $\tilde{\mathbf{p}}_1^{II}$ e $\tilde{\mathbf{p}}_2^{II}$ or in $\tilde{\mathbf{p}}_1^{III}$ e $\tilde{\mathbf{p}}_2^{III}$, then, since the absolute change is not invariant to axes rotations, the set of reachable policy objectives with a unit absolute change (policy control isocost =1) modifies.

Figure 5(b) shows the transformation of the policy objective isoquant for rotations in the reference axes of the policy-control key-structures. When the rotation takes place the point A' moves towards A" and then to A"' in the [absolute-change] isocost of the policy-control, see figure 5(a), correspondingly point A' moves towards A" and then to A"', in figure 5(a), and the entire absolute change-isoquant changes his configuration. It has to be noted, however, that only the vertices of the [absolute change -] isoquant remain tied to the [modulus-] isoquant.

If we then decide to measure the aggregate value of a multisectoral change with the sum of the absolute values of its components the determinations of its effects on the objective variable is rather complex, since this measure is not invariant with respect to axes rotations.

Nonetheless this aggregation criterion is immediately interpretable in economic terms with respect for example to the Euclidean norm. On the other hand the Euclidean norm, being invariant with respect to axes rotations, allows for the consistent separation of the magnitude effect, scale effect, from the composition effect, the structural effect in the Leontief reduced form. In the application that follows we will perform the analysis exploiting the idea of key structures and macro multipliers consistently defined, in the idea that both the aggregated and disaggregated feature of the macroeconomic variable has to play a role, especially when the macro variable can assume the role of either the policy objective or policy control.

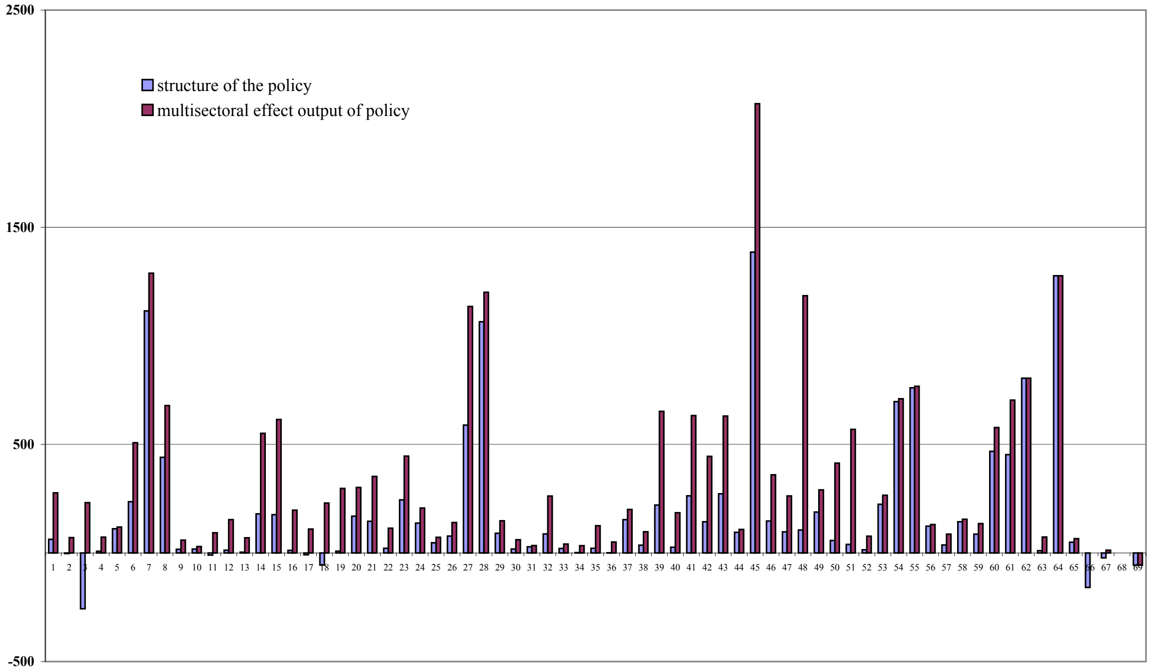
5 Leontief multipliers and macro multipliers at work

The empirical analysis aims at quantifying the short run impact of a government strategy that acts on the negative effects of the current economic crisis. Principally the policymaker appears to have considered how to restore the aggregate demand with a set of sector-specific subsidies like those designed for the as automobile sector. The policy measure is usually evaluated in terms of its direct impact on the sector's performance, but it is crucial to asses both its indirect and induced effects on the production system as a whole.

The traditional approach, based on the Leontief inverse allows for the quantification of this type of aggregate effects, however it is limited by the an unrealistic composition of the exogenous final demand shock traditionally used for the impact analysis. As mentioned above, the Macro Multipliers approach overcomes this limit, and reveals the key structures of the policy control that activate different effects on the policy objective, starting from the observed composition of the final demand.

The empirical application focuses on the government strategies for the automobile sector of the U.S. that allocate and reallocate resources in order to keep up the demand for automobiles. In our experiment we keep the same amount of the strategy announced for the industry but redesign the policy on final demand in order to obtain the higher effect on total output and the automobile output sector. For this purpose we use the symmetric Input-Output (IO) table based on the Make (*industry* by *commodity*) and Use (*commodity* by *industry*) tables of the U.S.

Figure 6. Policy structure and multisectoral effects through the Leontief Multiplier



economy in year 2007. The symmetric IO table allows to determine the 69x69[4] matrix of direct and indirect requirements that makes up the parametric set of the Leontief reduced form. We, then, analyse the policy that faces a decrease of the 6% in the final demand, with an impulse on the activity of "Motor vehicles, bodies and trailers, and parts" of an amount of 3.5 million of US$ dollars.

6 Leontief Multipliers for the US

The scenario that synthesises the policy illustrated is such that final demand has decreased in all sectors according the percentage forecasted, while sector 23 is adjusted with the subsidy of the federal government. The results can be seen on figure 6.

The main aggregated results are shown in table 1. The policy on final demand has a balance of 12,983 million US$ but a change of 14,124 million US$. The difference is caused by the negative demand of the goods 2 "Forestry, fishing, and related activities", 3 "Oil and gas extraction", 17 "Non metallic mineral products", 18 "Primary metals", 66 "Non comparable imports", 67 "Scrap, used and second hand goods" and 69 "Inventory valuation adjustment".

Table 1. Main aggregate results through the Leontief Multiplier

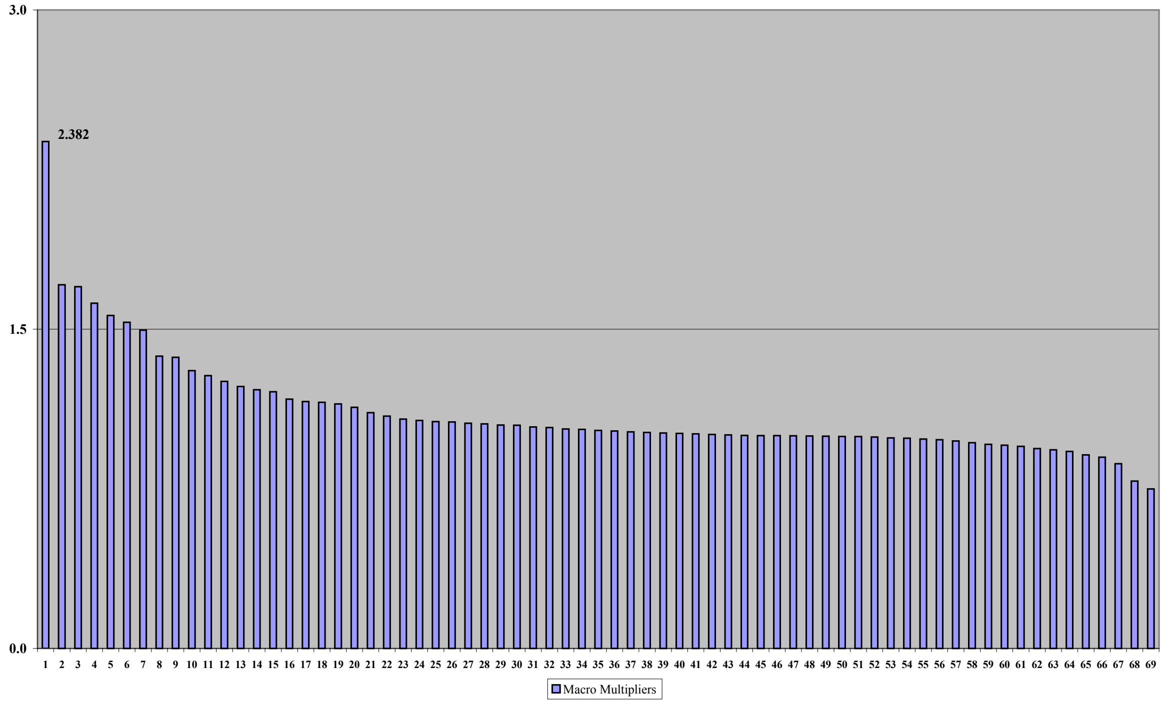
Policy Control balance*	Policy Control Change*	Total Output Balance*	Total Output Change*	Effect on Output 23*	Multiplier Balance	Multiplier Change	Multiplier Modulo
12983	14124	24270	24383	446	1.869	1.726	1.425

*All values are in million of Dollars.

If we focus on the output variable we can observe an increase on industrial output of US$ 24,270 million (balance), a change of US$ 24,383 million for the system as a whole and a positive

[4]The symmetric IO table is built by the Bureau of Economic Analysis and is available at www.bea.gov/industry/iotables/table_list.cfm?anon=95800 (BEA 2007). The commodity classification is shown in table A1 in appendix.

Figure 7. The Macro Multipliers for the U.S. economy



effect of US$ 446 million for the industry 23 "Motor vehicles, bodies and trailers, and parts". There is a difference between the change and the balance of the policy objective caused by the sector 69 "Inventory valuation adjustment". In table 1 the Macro Multipliers are shown in terms of balance and change.

The policy we analysed highlights a multiplier effect both in term of change and of balance. In particular, the goods that are privileged are: 45 "Real estate"; 7 "Construction"; 64 "State and local general government"; 28 "Retail trade"; 48 " Miscellaneous professional, scientific and technical services"; 27 "Wholesale trade". Last, the good 23 "Motor vehicles, bodies and trailers, and parts" obtain an increase of production of US$ 441.58 million of dollars.

7 Key structures and Macro Multipliers

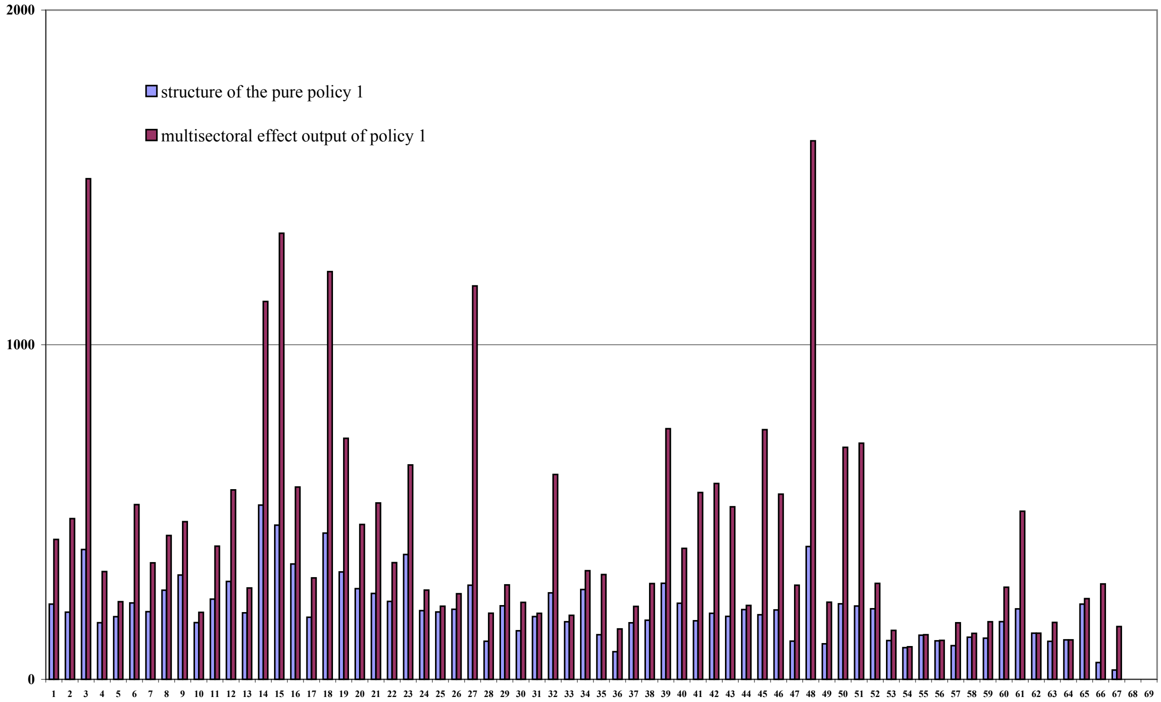
The Macro Multiplier (MM) approach allows to find key-structures of the policy control each activating singularly a multiplying effect on the policy objective that is total output. In this way we identify 69 MM, as shown in figure 7, 69 key-structures of the policy control and 69 key-structures of the policy objectives that direct the impact of each activated multiplier towards the specific sectoral component of the policy objective.

The dominating, i.e. highest, multiplier m_1 shown in figure 7 is equal to 2.38. Each of the 69 MM is associated with a structure of a policy control $\tilde{\mathbf{p}}_i$ that activates each multiplier effect. This multiplier effect is directed towards specific sectoral components of the policy objective according the objective key-structures $\tilde{\mathbf{z}}_i$[5].

If we focus on the dominant multiplier, m_1 , we are looking for a general positive effect on the system as a whole. The associated key-structures (control and objective) are all positive thus the policy control increases both the scale of total output and each industry output. In order to asses the impact of an policy alternative to the leontevian one, we search for a different composition of the total amount used before, US$ 14.124, i.e. we search for a different structure of the policy

[5]Each of the 69 policy-objective key structures multiplied by the correspondent MM is shown in appendix at figure A1, the key-structures of policy control are to be found in figure A2.

Figure 8. Policy structure 1 (dominating policy) and Multisectoral effects on output



control keeping constant its scale at US$ 14.124. We fix a predetermined change for the policy control and, referring to the structure of the policy control associated to the dominant macro multiplier, we obtain a new vector of final demand, $((\mathbf{f}_1^*)$, which has the same composition of the key-vector $\tilde{\mathbf{p}}_1$ and the given predetermined change. Through the use of this new vector of final demand we obtain an impact on the production system that has the structure of $(m_1 \cdot \mathbf{z}_1)$. The impact is different on level and multiplier power.

Table 2. Main aggregate results through the MM approach: the dominant policy

Policy Control balance*	Policy Control Change*	Total Output Balance*	Total Output Change*	Effect on Output 23*	Multiplier Balance	Multiplier Change	Multiplier Modulo
14124	14124	29319	29319	641	2.076	2.076	2.382

*All values are in million of US$.

In particular, as it can be seen in table 2, while the resources are US$ 14,124. million of dollars the whole impact on the production system is US$ 29.319 million of dollars, in terms of change and balance. The multiplier effect is 2.076 both in terms of absolute change and balance[6]. The modulus-multiplier is equivalent to the dominating MM, $m_1 = 2.382$. The disaggregated effects are illustrated in figure 15. The commodities that are favoured by this policy are: 48"Miscellaneous professional, scientific and technical services"; 3"Oil and gas extraction"; 15"Chemical products"; 18"Primary metals"; 27"Wholesale trade"; 14"Petroleum and coal products". Commodity 23"Motor vehicles, bodies and trailers, and parts" gets an increase of production of US$ 641 millions of dollars.

The new structure of final demand allows to redistribute the amount of the government policy improving noticeably the positive impact on the production system. As we stressed above the

[6]The structure of the policy control and objective have all positive elements thus the sum of the absolute changes and the balance do not differ as to final demand and total output.

analysis focuses on commodity 23"Motor vehicles, bodies and trailers, and parts" on which the federal government aims to counteract the decrease of final demand and thus employment level. Focusing on commodity 23, the "most convenient" composition for commodity 23 should be found among the policy key-structures. In the set of the policy objectives we can choose structure 9 because is the most favourable structure for commodity 23, see figure A1A.1. We construct a new vector of final demand, whose structure is suggested by the policy-control key-structure 9, $\tilde{\mathbf{p}}_9$, as shown in figure A2.

Table 3. Main aggregates results for policy oriented to "Motor vehicles, bodies and trailers, and parts" (MM approach)

Policy Control balance*	Policy Control Change*	Total Output Balance*	Total Output Change*	Effect on Output 23*	Multiplier Balance	Multiplier Change	Multiplier Modulo
-385	14124	1023	17701	4633	-2.657	1.253	1.368

*The values are million of Dollars.

The new vector of final demand ($\mathbf{f}_9^*$) has the a specific structure but the same absolute change. The aggregate results of this application are shown in table 3. As it can be seen, the policy has a negative balance ($ -385 million of dollars) but the absolute change is the same as the dominating policy ($ 14.124 million of dollars). The output vector associated with this final demand structure has a positive balance. 1.023 million of dollars, and an absolute change of 17.701 million of dollars. The production of commodity "Motor vehicles, bodies and trailers, and parts" has increased of 4.633 million of dollars. The disaggregated results shown in figure 9 stresses the fact that the composition of the policy control requires positive and negative variations of the level of final demand. This is also observed in the structure of the policy objective where commodity 23."Motor vehicles, bodies and trailers, and parts"; 8."Food and beverage and tobacco products"; 1."Farms"; 61."Other services, except government"; 60."Food services and drinking places" are show a positive change while others as 18"Primary metals"; 22"Electrical equipment, appliances, and components"; 15"Chemical products"; 24"Other transportation equipment" present a negative variation. Even if this policy increases the automobile output it depresses the majority of the production activities.

8 Policy design as an alchemy of policy key-structures

There is a trade-off between the increase of the output of the automobile industry and the output of a set of other goods, as stressed in figure 9. The MM approach allows for a policy design that defines a policy control whose aim is to reduce the negative effects on the policy objective that have been shown above. This aim can be attained through a policy that combines the positive effect on output of the good 23 without neglecting the performance of the system as a whole. It is possible to define a "non key" policy combining the key structures, in particular the dominating policy control, $\tilde{\mathbf{p}}_1$, that achieve the best aggregate performance and the policy control that achieve the greater effect for the automobile output, $\tilde{\mathbf{p}}_\mathbf{9}$. Using a coefficient[7] β we can provide a policy control structure which is a combination of two policy control key-structures. The new policy control will be given by: $\mathbf{p}_{1,9}\ (\beta)=\beta\tilde{\mathbf{p}}_1 + (1-\beta)\tilde{\mathbf{p}}_9$. All the feasible combinations of the new policy vector have an absolute change maximum and equal to the original amount. The results of combination in term of output, and multiplier power are presented in table 4.

When $\beta = 1$ we activate and discuss the results of policy 1, characterized by MM m_1, policy key structure, and objective key-structure $\tilde{\mathbf{z}}_1$. On the other hand when $\beta = 0$ we activate the results of policy 9, characterized by MM m_9, policy key structure $\tilde{\mathbf{p}}_9$, and objective key-structure

[7]With $0 \leq \beta \leq 1$.

Figure 9. Policy structure 9 and Multisectoral effect on output

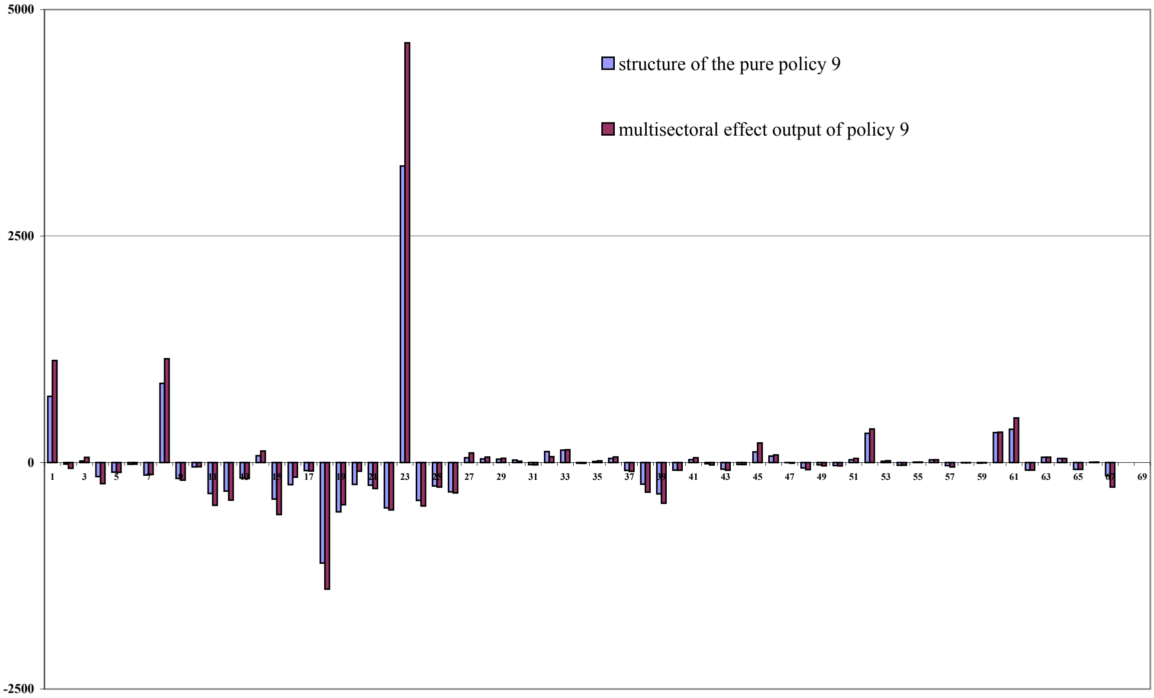


Table 4. Combination between structures 1-9

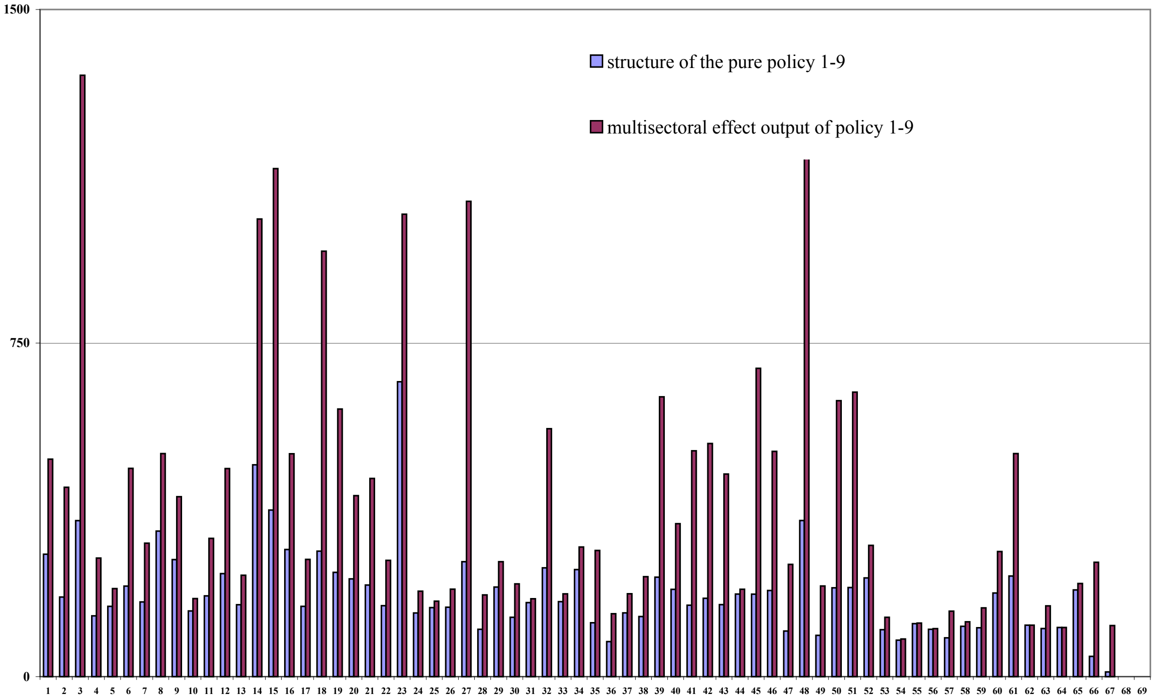
Weight for policy the combination		Policy Control Balance* $\sum p_i^\beta$	Policy Control Change* $\sum \lvert p_i^\beta \rvert$	Total Output Balance* $\mathbf{x}_i^\beta$	Total Output Change* $\lvert \mathbf{x}_i^\beta \rvert$	Effect Output 23*	Multiplier Balance	Multiplier Change	Multiplier Modulo
1	0	14124	14124	29319	29319	641	2.076	2.076	2.382
0.9	0.1	12673	12673	26490	26490	1040	2.090	2.090	2.341
0.8	0.2	11222	11235	23660	23660	1439	2.108	2.106	2.205
0.7	0.3	9771	9873	20830	20830	1838	2.132	2.110	1.997
0.6	0.4	8320	9032	18001	18092	2237	2.163	2.003	1.781
0.5	0.5	6869	8779	15171	16096	2637	2.209	1.833	1.607
0.4	0.6	5419	9160	12341	14779	3036	2.278	1.613	1.492
0.3	0.7	3968	10004	9512	14323	3435	2.397	1.432	1.423
0.2	0.8	2517	11142	6682	14825	3834	2.655	1.331	1.388
0.1	0.9	1066	12474	3852	15912	4234	3.614	1.276	1.372
0	1	-385	14124	1023	17701	4633	-2.657	1.253	1.368

*All values are in million of dollars.

$\tilde{\mathbf{z}}_9$. Any other value of coefficient β determines results that may be studied as combinations of the two policies. As it can be seen from table 4 choosing a policy combination with an higher effect on industry 23 results in a policy with lower effects on all other industries.

The criteria through which the convenient combination is chosen may be different. If we compare the results with the impact of the Leontief policy we can easily choose policy characterized by $\beta = 0.9$. The final demand and the output impact are shown in figure **??**.

Figure 10. Convenient policy structure 1-9 and multisectoral effect on output ($\beta = 0.9$)



This figure shows that this policy combination has a structure very similar to the dominating one: the favourite commodities are the same[8]. If we focus on the absolute change multiplier, the convenient policy control is the structure given by $\beta = 0.5$ with the value of 1.833 that is greater than 1.726 the previous structure's multiplier. Last, if we aim to avoid the negatives component on sectoral output we can combine the structures using $\beta = 0.7$.

9 Conclusion

The present day economic crisis emphasizes the relevance of economic policy to design and determine, at the disaggregated level and in the short term, the effects on output of a stimulus on private consumption and investment. Observing the strategies of the national governments facing the crisis, it can be claimed that all act to support the aggregate demand of a cyclic good in the short run. In the U.S. economy the activity that suffers more than others of a lack of demand is the automobile sector that used to employ a large part of labour force. Multisectoral analysis is a suitable tool to analyse this type of government strategy because can quantify the indirect effect on the production system.

The traditional multiplier analysis, based on the Leontief inverse, forces to rethink on how much the structure of the predetermined exogenous shock might be unrealistic. If the period can be assumed to be the short term, the most realistic structure of final demand is the structure suggested by the IO table, i.e. the observed composition of final demand. But the Macro Multipliers approach allows to find key policies of final demand that are endogenously suggested by the structure of the economy. It also allows for the identification of map of the isocost and isoquant of policy and evaluate the policy impacts in terms of aggregated macro multipliers.

If the policy maker aims to increase the output of a single commodity, like automobiles, he has to investigate and determine which key-structures are favourable to that particular commodity. Moreover, the policy maker can have complex aims oriented both to favour a single good and

[8]In this case the industry most more stimulated is 3"Oil and gas extraction".

the production system as a whole. In this case it may have trade-offs between the objectives and the solution will be represented by a combination of policies.

In this paper we have shown the dominating policy control for the U.S. economy that is strongly oriented on some particular commodities. If the policy maker wants to increase the output of the automobiles he has to choose a particular endogenous policy that generates negative effects on the system. The policies we construct combining policy key-structures allow to increase the output of automobile industry and reduce the negative performance of some other activities.

References

BEA, (2007), "Annual Input-Output data," http://www.bea.gov/industry/index.htm#annual.

Ciaschini, M., and Socci, C. (2007), "Final demand impact on output: a Macro Multiplier approach.," *Journal of Policy Modeling*, 29, 115–132.

Ciaschini, M., Pretaroli, R., and Socci, C. (2009), "A conveniet multi sectoral policy control for the ICT in the USA economy," *Metroeconomica*, (forthcoming).

Lancaster, P., and Tiesmenetsky, M. (1985) *The Theory of Matricies*, second ed., New York: Academic Press.

Leontief, W. (1965), "The Economic Impact -Industrial and Regional- of the Arms Cut," *Review of Economics and Statistics.*

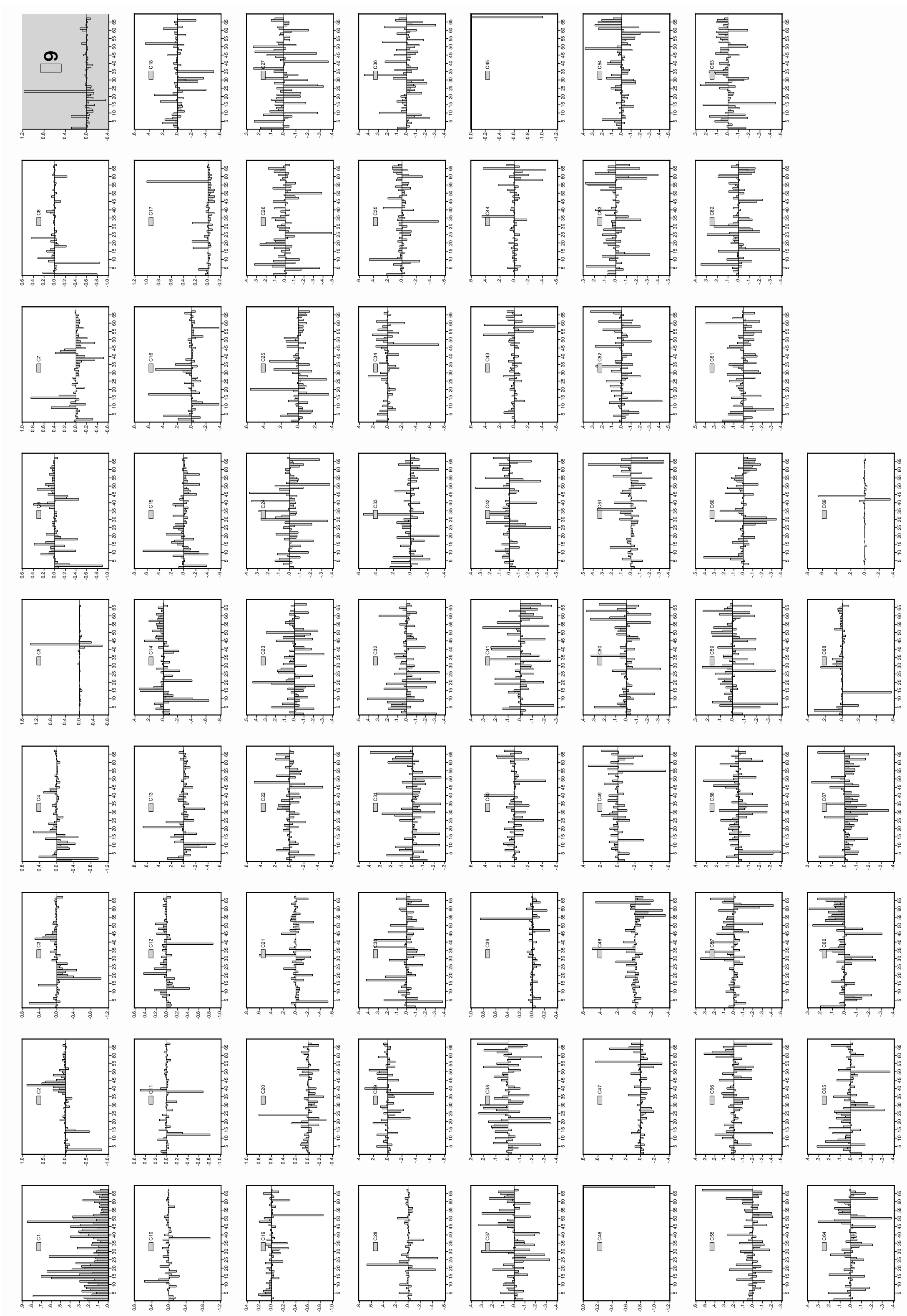
Lorenz, H., Einstein, A., Minkowski, H., and Hermann, W. (1952), "The Principle of Relativity: A Collection of Original Memoirs.," *Dover.*

Appendix A: Tables and figures

Table A1. Classification of commodities (NAISIC)

1	Farms	36	Warehousing and storage
2	Forestry, fishing, and related activities	37	Publishing industries (includes software)
3	Oil and gas extraction	38	Motion picture and sound recording industries
4	Mining, except oil and gas	39	Broadcasting and telecommunications
5	Support activities for mining	40	Information and data processing services
6	Utilities	41	Federal Reserve banks, credit intermediation, and related activities
7	Construction	42	Securities, commodity contracts, and investments
8	Food and beverage and tobacco products	43	Insurance carriers and related activities
9	Textile mills and textile product mills	44	Funds, trusts, and other financial vehicles
10	Apparel and leather and allied products	45	Real estate
11	Wood products	46	Rental and leasing services and lessors of intangible assets
12	Paper products	47	Legal services
13	Printing and related support activities	48	Miscellaneous professional, scientific and technical services
14	Petroleum and coal products	49	Computer systems design and related services
15	Chemical products	50	Management of companies and enterprises
16	Plastics and rubber products	51	Administrative and support services
17	Non metallic mineral products	52	Waste management and remediation services
18	Primary metals	53	Educational services
19	Fabricated metal products	54	Ambulatory health care services
20	Machinery	55	Hospitals and nursing and residential care facilities
21	Computer and electronic products	56	Social assistance
22	Electrical equipment, appliances, and components	57	Performing arts, spectator sports, museums, and related activities
23	Motor vehicles, bodies and trailers, and parts	58	Amusements, gambling, and recreation industries
24	Other transportation equipment	59	Accommodation
25	Furniture and related products	60	Food services and drinking places
26	Miscellaneous manufacturing	61	Other services, except government
27	Wholesale trade	62	Federal general government
28	Retail trade	63	Federal government enterprises
29	Air transportation	64	State and local general government
30	Rail transportation	65	State and local government enterprises
31	Water transportation	66	Non comparable imports
32	Truck transportation	67	Scrap, used and secondhand goods
33	Transit and ground passenger transportation	68	Rest of the world adjustment
34	Pipeline transportation	69	Inventory valuation adjustment
35	Other transportation and support activities		

Figure A1. Pure objective policy ($m_i\mathbf{z}_i$)



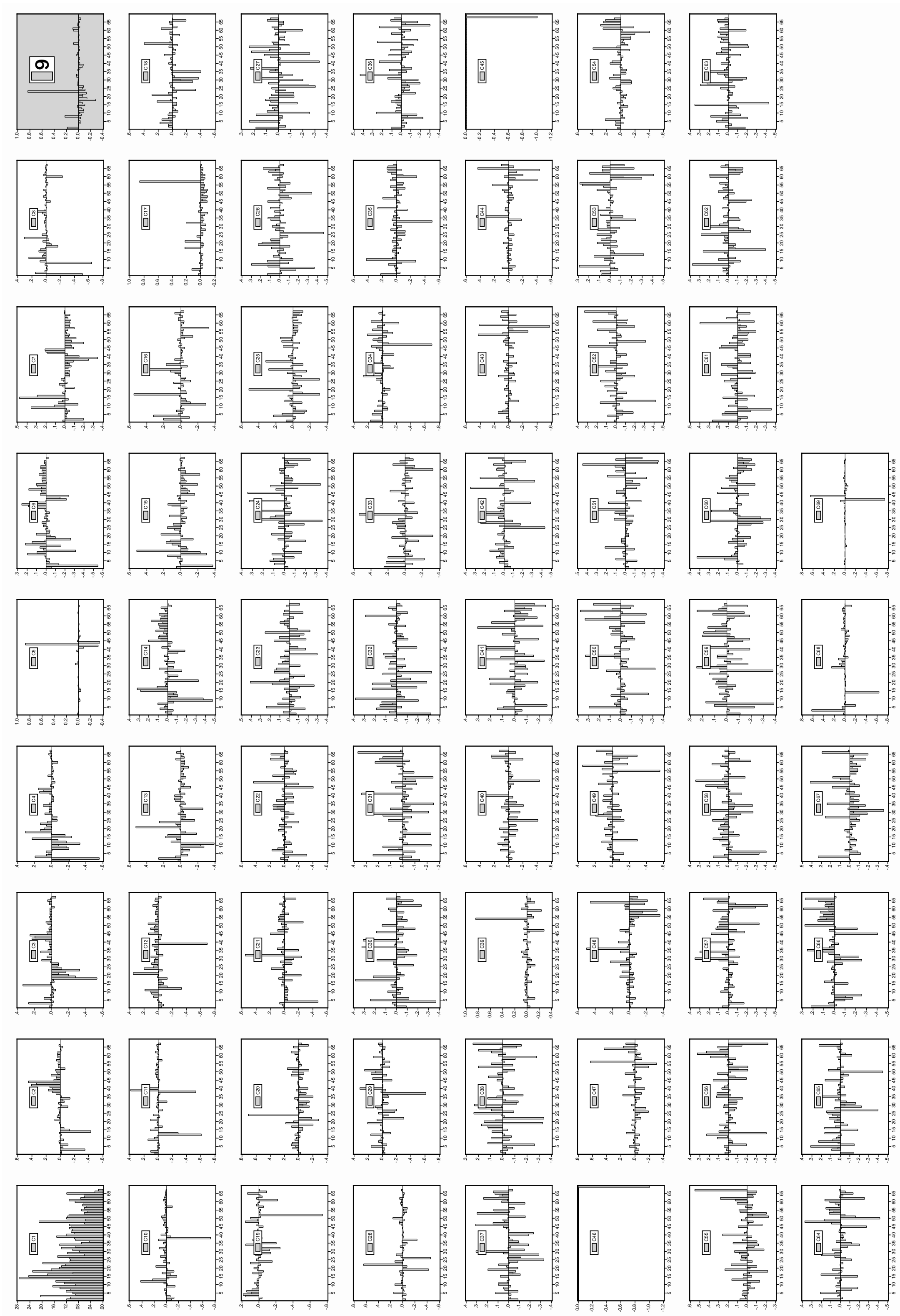


Figure A2. Pure control policy ($\mathbf{p}_i$)