The evolution of the theory of value from Dmitriev and Bortkiewicz to Charasoff

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Abstract

This work presents the theories of prices of some authors that belong to the Russian-German school of political economy. In particular, we consider the contributions of Dmitriev, Bortkiewicz and Charasoff. They developed and reformulate many fundamental concepts of classical authors such as Smith, Ricardo and Marx. Besides, they anticipated several results that later would appear in the works of authors of classical inspiration as Leontief, Von Neumann and Sraffa. This works aims to present in a simplified form the main results and evolution of these contributions giving special attention to Georg Charasoff´s, since he remains forgotten and for our concern he presents the best version of classical theory of prices before the publication of Piero Sraffa´s mature works.

I. Introduction

The authors studied in this article belong to the so called Russian-German school of political economy. This school kept a strong bond to the classical-marxist tradition of thought and participated in the debates that followed the upsurge of the marginalist school. Among these authors the most significant ones are Dmitriev, Bortkiewicz, Charasoff, Leontief, Von Neumann, Adolph Lowe, Tugan-Baranovsky, Kalecki, Spiethoff, Aftalion and Fel'dman. This tradition has lost strength since the persecution suffered after Hitler´s ascension to power in Germany. However, some of these authors remained working in exile.

The majority of these theoretical contributions by these authors remained forgotten by economists and historians of economic history of thought. Dmitriev and Bortkiewicz are moderately known as a result of Paul Sweezy and Piero Sraffa´s researches. Though Charasoff had only being rediscovered after the 1980´s after the publication of an Italian article by Egidi and Gilibert (1984). The scope of this article is to go deep inside Charasoff´s ideas starting with a quick introduction based on the previous contributions made by his predecessors on the problem of value.

In the works of these three authors, it appears some fundamental concepts developed later by Leontief (1928, 1953), Von Neumann (1945) and Sraffa (1951, 1960). It means that the study of their works is relevant to investigate the classical-marxist roots of thought.

In sections II and III, we briefly show the most relevant theoretical contributions of Dmitriev and Bortkiewicz in relation to the classical theory of value. In section IV, which is the core of this article, we present Charasoff´s work on the problems faced by Marx, Dmitriev and Bortkiewicz in his respective theories of value. Section V shows a short conclusion.

II. Dmitriev: a reduction of prices in dated quantities of labour

Dmitriev (1974) discusses Ricardo's theory of value, what is called the labour theory of value, which establishes a relation between relative prices and the relative quantities of labour required to produce commodities. Dmitriev deduces the total quantity of labour required to produce each commodity starting from the following identity:

$$v = l + Av \qquad (1)$$

where v is the vector of total quantities of labour (direct and indirect) necessary to produce different commodities; l is the vector of direct quantities of labour; "A" is the technical coefficients matrix. By means of successively substituting v in equation (1), the author achieves this reduction:

$$v = l + AI + A^2v$$

$$v = l + Al + A^2I + A^3 I + ... + A^n l \qquad (2)$$

We can demonstrate that this succession is compatible with Leontief´s inverse matrix. Multiplying (2) by matrix A we have:

$$Av = Al + A^2l + A^3l + ... + A^{n+1}l \qquad (3)$$

Subtracting equation (3) from (2), we obtain:

$$v – Av = l - A^{n+1}l$$

If the system is viable in Sraffa´s sense, i.e., if the sum of technical coefficients of matrix A (a_{ij}) is superior than 0 and inferior than 1, the elements of matrix A^{n+1} converge to null values when n tends to infinite. So, we again have equation (1):

$$v – Av = l$$

$$(I – A)v = l$$

$$v = (I – A)^{-1}l$$

This way, it´s proven that Dmitriev´s method is equivalent to using Leontief´s inverse matrix $[(I – A)^{-1}]$. This reduction to quantities o labour deduced from technical conditions is not and must not be confused with a historical regression. In other words, the total quantity of labour inferred from equation (2) is not the effective quantity used in the past, but the required one by the *present* conditions of production represented by the technique in use. If some inputs were produced with past technologies, this would not be taken into consideration in this reduction which depends only on the dominant technique defined by the current ones. This is clearly stated in Dmitriev:

> We can always find the total sum of the labour directly and indirectly expended of the production of any product *under present day production conditions*… the fact that all capital under present day conditions is itself produced with the assistance of other capital in no way hinders a precise solution of the problem (DMITRIEV, 1974; p.44)

An element that distinguishes Dmitriev´s position to Charasoff′s is, as will be seen below, the fact that his series of dated quantities of labour are finite. Because of this, Dmitriev´s procedure is similar to the Austrian method formulated by Von Bawerk and his followers that present capital as a finite series of labour done in the past. However, this method implies that there are not basic commodities, i.e., commodities that participate in the production of all the other commodities (Sraffa, 1960), because the effective reduction ends in a certain point and, as a consequence, is finite. Then, the Austrian method makes a reduction to "original factors", i.e., factors not produced and, by this way, considered exogenous. Then, capital is considered a quantity of labour done in the past reducible to an original endowment, of labour and land. Since the reduction is finite, there is production of commodities by means of commodities only after a certain point, i.e., until the reduction reaches the original factors. On the contrary, if the reduction is infinite, we face a true circular flux and infinite of wealth where capital goods can never be eliminated. Consequently, as capital participates in every phase of production, which means, it can not disappear from the analysis, it is guaranteed the existence of at least one basic commodity. In this case, it is possible deduce a maximum rate of profit since the labour always and in every stage of production is assisted by capital.

This last point distinguishes Dmitriev, Ricardo and Bortkiewicz´s contributions from those of Marx, Charasoff, Von Neumann and Sraffa. If Dmitriev´s procedure were valid, the rate of profit would be infinite. However, if in every phase of production there is capital, as is supposed by Marx, by distinguishing constant from variable capital, the rate of profit reaches a finite maximum value (Gehrke e Kurz, 2006).

Nevertheless, Dmitriev made various contributions to the classical theory of prices and to the determination of the rate of profit. In order to analyse these aspects, we present Dmitriev´s price equation:

$$p^T = w[(1+r)l^T + (1+r)^2 l^T A + (1+r)^3 l^T A^2 + ...] \qquad (4)$$

Where p^T is the transverse vector of prices; w the nominal wage; r the normal rate of profit. Consider still that the workers consumption basket consists of only one basic commodity, as in Ricardo´s example. Then, we have:

$$w = p_c c$$

Where p_c is the price of the basic commodity and c is the quantity received by the worker. So, we can substitute the previous equation in equation (4) to obtain:

$$p_c = p_c c[(1+r)l^T + (1+r)^2 l^T A + (1+r)^3 l^T A^2 + ...]$$

$$1/c = [(1+r)l^T + (1+r)^2 l^T A + (1+r)^3 l^T A^2 + ...]$$

As we can attest from the equation above, the normal rate of profit (r) depends only on technology (l, A) and the quantity of basic commodity (c). So, we have:

$r = f\,(l, A, c)$ *a la* Ricardo

With this reasoning, Dmitriev obtains a consistent system of prices determined together with the normal rate of profit starting from the same independent variables as Ricardo. By this way, he manages to refuse Walras´s critique that proposes the Ricardian system would present logical inconsistencies, such as the determination of prices by means of prices and the existence of more unknowns than equations. In Gerhrke´s words:

> Dmitriev deserves the credit for having demonstrated that starting from the data of Ricardo's approach, relative prices and the rate of profits can be determined simultaneously. The system is complete and all objections of the kind put forward by Walras among others, that Ricardo's cost of production explanation of prices is circular since it defined prices from prices, are untenable (GEHRKE, 1998, p. 225).

Dmitriev also demonstrated that even if one inserts in the previous example, based on Ricardo, various wage goods[1], the rate of profit is affected by changes in condictions of production of non-basic activities, i.e., in this example, commodities which are not used directly or indirectly to produce wage goods. Then, it would still be valid that the determination of the rate of profit depends only on technology and wage goods.

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[1] Remember that the problem of heterogeneity of wage goods, raised by Malthus, instigated Ricardo to formulate the theory of prices. However, He left unfinished this theory, because, among other things, of the difficulty to determine a system of prices together with a uniform rate of profit.

Though Dmitriev searched for a synthesis between the classical and the marginal utility theories, he retained the fundamental asymmetry in the treatment of the distributive variables that characterize the classical tradition, proposing that the conditions which affect the real wage level are out of the scope of the classical political economy (Dmitriev, 1974; p. 74; cited by Gehrke, 1998).

III. Bortkiewicz: a critic to the Marxian theory of value

Marx shows his transformation of labour value into prices of production, which, formally speaking, can be interpreted in the form presented below. Assume that production is realized by three departments, and there is one technique in use which only uses circulating capital and that does not exist joint production. Then, we have:

Department I $(c_1 + v_1)(1+r) = \lambda_1 p_1$

Department II $(c_2 + v_2)(1+r) = \lambda_2 p_2$

Department III $(c_3 + v_3)(1+r) = \lambda_3 p_3$

Department I involves the production of constant capital (c_i; i = 1, 2, 3), department II of variable capital (v_i; i = 1, 2, 3) and department III of luxury goods. λ_i (i = 1, 2, 3) are the labour values; s_i (i = 1, 2, 3) are the sectoral surplus value; p_i the "conversors" of labour value into prices of production. Marx determines the rate of profit as the quotient between aggregate surplus value and the aggregate capital.

$$r = \sum_i s_i / \sum_i (c_i + v_i)$$

According to Marx, even if the theory of labour value does not explain relative prices, labour values should coincide to the prices of production measured in labour *in the aggregate*[2]. In other words, to Marx the following identities are valid (Howard e King, 1998):

[2] "It is then only an accident if the surplus-value, and thus the profit, actually produced in any particular sphere of production, coincides with the profit contained in the selling price of a commodity." (Marx, 1894, cap.9).

i. $\sum_i \lambda_i = \sum_i \lambda_i\, p_i$

ii. $\sum_i s_i = r\sum_i (c_i + v_i)$

iii. $r = \sum_i s_i / \sum_i (c_i + v_i)$

Marx´s reasoning can be illustrated with the following numerical example:

	CC	**CV**	**MV**	**V**	**L**	**Pp**
I	10	10	10	30	7.5	27.5
II	20	12	12	44	12	44
III	15	5	5	25	7.5	27.5
Total	45	27	27	99	27	99

r = 0.375

This example is based on the previous system of equations. Column CC represents constant capitals in departments I, II and III; CV the variable capital; MV the surplus value; V labour values; Pp the prices of production. As it can be observed the sum of labour values is equal to the sum of the prices of production and the sum of profits is equal to the sum of surplus value. In department II, the organic composition, i.e., the relation between constant capital and variable (CC/CV) coincides to the medium organic composition. By this reason, only on this department the price of production matches the respective labour value.

However, the transformation proposed by Marx, as he recognised, to be complete needed also to tranform the labour values into prices of production of constant and variable capitals, i.e., in his scheme capital is not transformed. Nevertheless, to complete the transformation, *first* it is necessary to know the prices of production of constant and variable capitals. By this way, Marx´s transformation seemed to have a problem of logical circularity.

This was the problem studied and solved by Bortkiewicz that starts from the previous scheme, but manages to determine the rate of profit and the system of production *simultaneously*. The procedure proposed by Marx is denominated “successive method” by Bortkiewicz, since it determines the rate of profit and the prices separately and successively. Bortkiewicz´s method is simultaneous, since the author determines both variables together. In order to develop this method, he supposes stacionary conditions (simple reproduction in Marx´s terms). The author´s scheme is presented in the following manner:

a. $(c_1\, p_1 + v_1\, p_2)\,(1+r) = \lambda_1 p_1$
b. $(c_2\, p_1 + v_2\, p_2)\,(1+r) = \lambda_2 p_2$
c. $(c_3\, p_1 + v_3\, p_2)\,(1+r) = \lambda_3 p_3$
d. $p_3 = 1$

Equation iii of Marx´s scheme, which determines the rate of profit starting from labour values in a direct way, is substituted by a numerarie (d) by Bortkiewicz. Then, the rate of profit is determined together with prices and the prices of production of constant and variable capital.

We highlight two of Bortkiewicz´critics on Marx. First, in Bortkiewicz´s analysis the three conditions that are valid in the aggregate in Marx´s analysis are not valid here:

$$\sum_i \lambda_i \neq \sum_i \lambda_i\, p_i \;; \qquad \sum_i s_i \neq r \sum_i (c_i + v_i) \;; \qquad r \neq \sum_i s_i / \sum_i (c_i + v_i)$$

Second, in Bortkiewicz´s analysis it is possible to deduce that Marx´s critic on Ricardo in the sense that the rate of profit is independent to the conditions of production of luxury goods (department III in this example) is also not valid. In other terms, from the equations, one can deduct that the determination of the rate of profit depends only on the conditions of production of those goods that are directly and indirectly used in the production of wage goods (departments I and II).

To Bortkiewicz, the labour values are exogenous when different techiques are available and the choice among those is an object of the theory on itself. In other words, if the choice of technique is endogenous, the labour values are explained not explicative variables.

IV. Charasoff: the most developed (and forgotten) version of the classical theory of value

Charasoff was a Russian author who published two texts (Charasoff 1909 and 1910) i Germany which deal with the classical-marxist theory of prices. His works are relatively less known that those of Dmitriev and Bortkiewicz. It possibly is because he was an independent researcher (Kurz and Salvadori, 1995) whose ideas were only revived by Egidi and Gilibert (1984). However, we understand that his masterpieces show an analysis much more advanced than his predecessors as we will try to show later. Charasoff has antecipated many results later developed by Von Neumann, Leontief and Sraffa.

Charasoff defines a group of simplified hypothesis that allow an economic system to self-reproduce itself. He assumes the existence of constant returns of scale and simple production, i.e., it does not consider joint production nor fixed capital. The capitalists´s consumption is given as a fixed basket of goods included in the technical coeficients of production in the same manner that any other productive input, which means, is part of the intermediary consumption and integrates the advanced capital by capitalists.

Commodities are produced by means of commodities. Starting from this premisse, Charasoff formalises his concept of capital proposing that an economy is characterized by a circular flux of production. Let us take a look at a brief presentation of the author´s theoretical proposal.

IV.1 The determination of the corresponding quantities of ‘original capital’

Starting from the final production (Q), he deduces the necessary inputs to produce it (Q_1), called the ‘first generation’, and defined by the technical in use. Then, the author deduces the inputs necessary to produce other inputs, i.e., infers the inputs of the ‘second generation’ (Q_2). By doing the same procedure, it is possible to deduce the 'third generation' (Q_3) and so on. Each reduction of inputs by means of inputs is called a new 'generation' of capital. In order to formaly deduce this method, assume a vector of final commodities (Q). This vector uses $Q^T A$ inputs, where A is the matrix of technical coeficients[3]. By this way, we have:

$$Q^T A = Q_1 \rightarrow Q$$
$$Q_1{}^T A = Q_2 \rightarrow Q_1$$
$$Q_2{}^T A = Q_3 \rightarrow Q_2$$
$$\vdots$$
$$Q_n{}^T A = Q_{n+1} \rightarrow Q_n$$

For this reduction to be possible, it is necessary to suppose that the productive system, in Staffa's (1960) sense, i.e., one in which is possible to produce of any commodity at least the same amount which is required of it as an input[4]. Then, it is possible to estabilish the following inequations:

$$Q > Q_1 > Q_2 > Q_3 > ..... > Q_n \qquad (Q_i > 0;\ i = 1, 2, ..., n)$$

This reduction can also be presented in this way:

$$Q^T A = Q_1$$
$$Q^T A^2 = Q_2$$
$$Q^T A^3 = Q_3$$
$$\vdots$$
$$Q^T A^n = Q_n$$

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[3] Actually, Charasoff does not use a matrix of technical coeficients, but an stilized matrix (Egidi, Gilibert, 1984). Nevertheless, one can tranform the latter matrix into the former by a simple scalar transformation. Since the matrix of technical coeficients is more used in the comtemporary literature, we use it in our presentaion. For more details, see Stamatis (1999).

[4] This condition is also known as the Hawking-Simon condition.

As ‘i ’ grows, Matrix A^i goes changing. Though the elements of matrix tend to zero as i grows ($\lim A^i = 0$), Charasoff shows that, under some restrictions, the vectors that compose the successive matrix tend to keep among them determined fixed proportions. In other words, as matrix A^i is multiplied, the absolute values of the technical coeficients drop to zero. What is important in this analysis is the relation between these multipied matrixes and to know if they keep or not a determined relation. In other terms, Not only the lines but aalso the columns of matrix A^i become colinears or ones are multiplies of the others. By this way, in some moment vectors of matrix A^i reach proportions very close to constant values, i.e., even as the inputs diminish from reduction to reduction, the proportion among them are the same. Charasoff calls this constant composition among inputs ‘original capital’. A condition to reach this composition is to exclude from the matrix as the reduction goes the non-basic commodities in Charasoff´s and Marx´s terms[5].

The process by which the reduction excludes the non basic commodities can be summarized in the following manner: suppose the n^{th} component of vector Q corresponds to a commodity not used as an input in any productive process. As a result, the n^{th} component that corresponds to $Q_1 = Q^T A$ will be equal to zero. Then, the n^{th} commodity is called “luxury commodity of first order”. The same applies to the commodities of second order, i.e., those commodities that integrate Q_1 and do not participate the production of no other commodity and so on.

IV.2 The determination of Normal Prices

Charasoff uses the same method to deduce the set of normal prices. Beginning from the “original capital”, the author obtains the normal rate of profit as a relation between the net product and the generation of previous capital without any reference to the price system. He makes use of a similar procedure to that utilized to deduce the maximum rate of profit obtained by Sraffa with the standard commodity. By one way, the normal rate of profit of Charasoff coincides with the rate of profit that corresponds to Von Neumann’s System (1945). For both, Charasoff and Von Neumann, the rate of profit could be interpreted as the maximum rate of Growth.

[5] This restriction is reached when the technical coeficients of matrix A are irreducible and primitive.

In Charasoff's approach, as in Von Neumann, there is a perfect duality between prices and quantities[6].

In a similar way to the procedure used to deduce quantities, the author determines the prices of the first (p_1), second (p_2) and third (p_3) orders and so on by means of a recursive method. Assuming a vector of final prices (p), we can represent this reduction in this way:

$Ap = p_1 \rightarrow p$

$Ap_1 = p_2 \rightarrow p_1$

$Ap_2 = p_3 \rightarrow p_2$

$p > p_1 > p_2 > p_3 > ..... > p_n \qquad (p_n > 0)$

Observe that in this case the only formal difference with the deduction of the vector of quantities that correspond to the original capital is the fact that the price vector is multiplied by the matrix 'A' from the left. In the quantities case the vector multiplies the same matrix 'A' from the right. This fact shows the price-quantity duality present in Charasoff analysis.

In his analysis, we can start from any arbitrary price vector belonging to whatever generation of capital. Later, with the reduction analysed before, the successive vector of prices tend to converge to the normal prices. In this way, this reduction could be realised starting from products to costs, or, inversely, from costs to products.

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[6] This duality is only valid for simplified systems as that of Von Neumann (1945) and Charasoff. In these models, the dominant techniques determine the prices as well as the quantities. But, if obsolete capital goods are discharge for the estimation of potential employment and production, it will imply an overestimation of productive capacity and a underestimation the potential level of employment. This is because the dominant techniques are usually more productive than the others. On the contrary, if a hierarchy of techniques is absent in the determination of normal prices, as occurs in Leontief's model (1928, 1953), the influence of competition on price's determination would be dismissed. Usually, prices obtained in this way would be higher than prices obtained by dominant techniques in competitive conditions. On the other hand, this duality requires unlimited the natural resources. If this requirement is absent, the duality disappears, even in the long-run, because prices would be determined by inferior methods ('marginal' lands), and quantities by all methods in use.

In the second case (from costs to prices), Charasoff starts from the labour-value of means of production. By the reduction he arrives to a price system very approximate to the prices of production of some generation of capital. In this point, Charasoff seams to follow the procedure adopted by Marx in his transformation of labour-values to prices of production (Marx, 1894, chapter XIII).

The difference between them appears in the conclusions of their reasoning. Charasoff demonstrates that the reduction converges to price of production starting from labor-values and starting from whatever vector of prices arbitrarily chosen. In Charasoff words: "*Marx wished... to start from the values of the commodities: but this is absolutely inessential for the theory of prices as such. The starting prices can be arbitrary*". (CHARASOFF, 1910, p.138 quoted by Egidi, 1998). Consequently, one important conclusion of his assessment of Marx's work is that labour-values are not necessary to obtain prices of production and the normal rate of profit.

In Charasoff analysis, it is possible to deduce normal prices without the rate of profit. Or, symmetrically, it is possible to deduce the rate of profit without normal prices. In an advance stage of reduction, this rate appears as the net part of the scalar that equalizes the input matrix with the products matrix. In formal terms:

$$A^{i} = A^{i-1} (1+ r)$$

Where r is a very approximate value of the normal rate of profit for a 'i' sufficiently high. Thus, even in a multisectoral economy suffices any ration between the same technical coefficient in two successive generation of capital to determine the normal rate of profit. This result is similar to the deduction made by Ricardo (1815), when he determines the rate of profit as the ratio of two magnitudes of corn in a corn's economy, i.e., when the economy only uses corn to produce corn.

Charasoff is able to determine normal prices without the rate of profit. It is because relative prices appear as the scalar that equalize the columns of Matrix A^{i} when 'i' is sufficiently high. Formally:

$A^i = (A^i_1, A^i_2,..., A^i_n)$

Where $A^i_1, A^i_2,..., A^i_n$ are the column-vectors of matrix A^i. Thus, we have:

$A^i_1 = \alpha A^i_2$

.

.

$A^i_1 = \beta A^i_n$

$A^i_2 = \gamma A^i_n$

Where α, β, γ are the relative prices of goods 1, 2 and n (p_1/p_2, p_1/p_n, p_2/p_n) respectively. It is possible to obtain prices and quantities when the reduction is sufficiently high. Thus, we obtain a matrix A^i where the inputs participate in the production of all products in the same proportion. In this way, we obtain a capital, even composed by heterogeneous commodities, which could be considered as a unique commodity reproduced by itself. Charasoff called this capital as ‘original capital’. He proposes that the heterogeneity of capitals would be falling while the reduction persists, i.e., the relation between the capital of ‘n’ order (A^n) and the capital of ‘n’ plus one order (A^{n+1}) is necessarily equal or less heterogeneous that the relation of capitals of inferior orders. When this succession is very high, capital tends to by composed by the same and homogenous structure. Using Marx’s terminology, we could say that when the reduction go on from one generation of capital to the next one, the successive capitals have an organic composition more uniform that the previous one. The original capital has a strictly uniform composition of capital. For example, imagine an economy which produces corn and iron. In order to determine the rate of profit without knowing the relative price corn-iron, both commodities must appear as inputs in the same proportions. Charasoff obtains this result by means of the reduction beginning by any productive system.

In deduction of prices’ case, we can start from an arbitrary price vector. These prices appear as cost of production in the successive period. However, this vector not necessarily assures the uniformity of profits rates. After that, competition tends to equalize them.

If competition tends to establish the equalization between the different rates of profits in the case of initial arbitrary costs, the prices of second generation will be nearer the prices of production than the initial costs and so on. In this way, Charasoff regression could be interpreted as a process of adjustment of market prices to production or natural prices. So, if we have $P^i = A^i P = AP^{i-1}$, when 'i' rise, actual prices tend to converge to $A^n P$, i.e., to production prices. Why an arbitrary price vector should converge to the vector of normal prices? The answer is that each new vector of prices is obtained multiplying the previous vector by matrix 'A'. Thus, considering that the multiplication tends to A^n, the result also tends to the set of prices defined by matrix A^n, i.e., normal prices.

The columns are also proportional to prices, i.e., from its ratios we derive relative prices. Symmetrically, the files are proportional to the quantities that correspond to the 'original capital'. In other words, the relative quantities that form the 'original capital' are the scalars that equalize the files of different stages. Formally:

$$A^i = (A_{i1}, A_{i2}, \ldots, A_{in})$$

Where $A_{i1}, A_{i2}, \ldots, A_{in}$ are the file-vectors of matrix A^i. Symmetrically, we have:

$$A_{i1} = \delta\, A_{i2}$$

.

.

$$A_{i1} = \varepsilon\, A_{in}$$

$$A_{i2} = \theta\, A_{in}$$

Where δ, ε, θ are scalars which represent the relative proportions of products 1, 2 and n (q_1/q_2, q_1/q_n, q_2/q_n) respectively that appear in the 'original capital'. However, the iterated or recursive method of Charasoff does not imply a separate or non-simultaneous determination of prices and rate of profit. On the contrary, both are determined simultaneously as occurs in the analysis of Dmitriev, Bortkiewicz, Von Neumann, Leontief and Sraffa. In other words, Charasoff´s method does not entail that these variables are determined in successive stages as, according to Bortkiewicz, happens in

Marx's method. Undoubtly, Charasoff method is different from the others. But it is compatible with them.

The equivalence between Charasoff´s procedure and the other authors is explained by the fact that his reduction is an alternative way to represent Leontief's inverse matrix. Therefore, in the reduction the non-basic commodities are eliminated, i.e., the final matrix is indecomposable. Charasoff reduction is the succession of matrixes showed in Dmitriev section that is also the identical to Leontief's inverse matrix.

Starting from a viable production system, Charasoff´s reduction ends as an unreduced matrix where all vectors that represent luxury or non-basic products are eliminated. Besides, the remained components are all positive. This method demonstrates that the rate of profit and the vectors of production are necessarily positive. In other words, he offers an indirect demonstration of the so called Perron's theorem of 1907. As it was showed above, without considering prices, the relation between the capital of one stage and the previous one (or the next one) gives the system rate of profit that correspond to the inverse of the maximum eigenvalue of Perron's theorem.

Therefore, in the same way as Sraffa, Charasoff demonstrates that any economic system contains a system (or 'original capital') in which the organic compositions are uniform between all productive sectors. This fact provides the rate of profit without any reference to prices, and, symmetrically, gives prices without any reference to the rate of profit. Consequently, Charasoff´s original capital is an alternative version of Sraffa standard commodity[7].

As a result, matrix A^n contains all the system relevant properties. On the other hand, the original capital (A^n) does not represent a concrete productive configuration nor is an intellectual devise. On the contrary, this capital is contained in any productive system defined by the prevalent technique and the normal level of wages, which in Charasoff system appears implicit in the prevalent technique.

[7] Strictly speaking, Sraffa standard commodity is a more developed version than Charasoff´ original capital because it admits fix capital and joint production.

The goal of Charasoff in representing an economy in which all profits are saved (and reinvested) is to compare it with the real capitalist economies in which these facts usually do not happen. Therefore, he describes, as Von Neumann, a balanced trajectory of growth that corresponds to the maximum or potential rate of growth. But this result does not pretend to explain the actual capitalist system but compare it with an ideal situation.

V. Conclusion

In this work, we presented a brief description of Dmitriev´s, Bortkiewicz´s and Charasoff's contributions in relation to the theory of prices. These authors belong to the Russian-German school of political economy and develop fundamental concepts later developed by Leontief, Von Neumann and Sraffa.

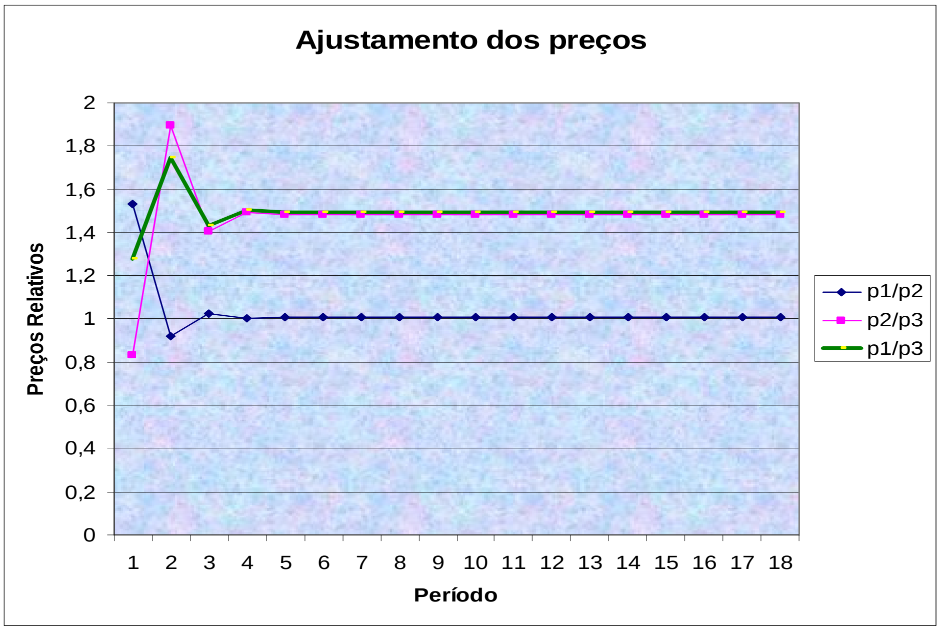
We gave special enphases to Charasoff because he has the most unknown and, in our concern, the one that most developed the classical theory of prices untill the apearance of Piero Sraffa´s masterpiece. In this sense, the study of Charasoff is fundamental to everyone who wishes to understand the classical-marxist thought in the twentith century.

Apendices I: Adjustment of relative quantities and prices

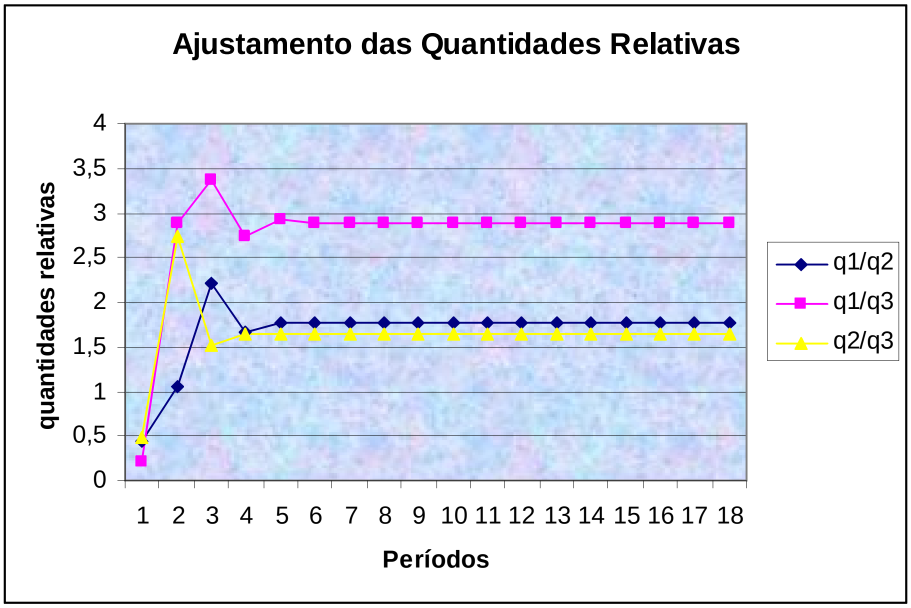
Starting from a 3x3 matrix chosen by chance, we present the adjustment process of an arbitrary set of prices to the normal prices proposed by Charasoff

departments	I	II	III
I	0.4	0.3	0.2
II	0.7	0.06	0.1
III	0.1	0.4	0.09

For example, let us start from the following arbitrary column-vector [23, 15, 18]. Graphically, this is the sequence of relative prices in departments I, II and III.



Now we do the same for the relative quantities that appear in the 'original capital' using the same matrix. We use the following arbitrary line-vector of quantities: [10, 23, 48]. Graphically, this is the sequence of relative quantities between departments I, II and III.



Apendices II: Matemathical

According to Perron-Frobenious's theorem, the non-negative matrix have non-negative autovalues. To the maxim autovalue is associated autovector x, which satifies the following equation:

1) $Ax = \lambda x$

From Sraffa (1960), we can derive the following identities:

2) $Ax\,(1+R) = x$

Where R is the maximum rate of profit or surplus rate. From equations 1 and 2, we have:

3) $\lambda = 1/(1+R)$

In economic terms, the maximum autovalue of a matrix of technical coeficients is the value which makes zero the difference between inputs and commodities. In other terms, it is the inverse of the surplus rate. If the salary is considered part of the surplus, as in Sraffa (1960), the maximum autovalue coincides to the maximum rate of profit of the economy. If the salary is considered part of the intermediare consumption as in Charasoff and Von Neumann, the maximum autovactor coincides to the normal rate of profit.

A matrix A is irreducible if there is not the possibility of making a partition neither in their lines nor in their columns in two (or more) mutually disjunct groups. If a matrix is reductible, the elements of one group are not used in the production of one group, i.e., the coeficients that relate the product of one group to the product of another group are equal to zero ($a_{ij} = 0$).

In economic terms, a matrix of technical coeficients will be irreducible when the non basic commodities are ecluded. It is worth mention that Charasoff, by doing the reduction to the original matrix (original capital), uses only basic commodities. Every irreducible matrix has a maximum positive characteristic root or autovalue, known as the Frobenius root, which are associated to the characteristic column and line.

A positive and irreducible matrix A is primitive if it has only one positive Frobenius autovalue (Meyer, 2000; pág. 688). Being primitive and existing the $\lim_{t\to\infty} (A/\lambda)^t$, every column (and line) of the G limit matrix is a column (line)-vector characteristic associated to its autovalue (λ). Besides that:

$$\lim_{t\to\infty} (A/\lambda)^t = G = p\, q^T / q^T p$$

Where p and q are respectively the Perron-vectors of A and A^T (Nikaido, 1962 and Meyer, 2000).

In economic terms, the limit matrix G is the one that represents the ‘original capital’. This matrix presents colinears lines and columns from which one deduces the group of normal prices and quantities correspondent to the original capital.

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