Trade Coefficients and the Role of Elasticity in a Spatial CGE Model Based on Armington Assumption

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Abstract

Armington Assumption in the context of multi-regional CGE models is commonly interpreted that the same commodity with different origins is an imperfect substitute of each other. A static spatial CGE model that is compatible with this assumption and explicitly considers the transport sector and regional price differentials is formulated. It is shown that the trade coefficients, which are derived endogenously from the optimization behaviors of firms and households, take the form of a potential function. To investigate how the elasticity of substitutions affects the equilibrium solutions, a simpler version of the model, which incorporates 3-regions and 2-commodities besides the transport sector, is introduced. It is found that (1) if the commodities produced in different regions are perfect substitutes, the regional economies will either be autarkic or completely symmetric, and (2) if they are imperfect substitutes, the impacts of the elasticity on trade coefficients as well as the closure errors of model solutions are not necessarily monotonic and sometimes very sensitive.

Keyword: Armington assumption, spatial CGE, elasticity of substitution

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1 Introduction

According to the traditional trade theory (e.g. Samuelson (1953)), the phenomenon known as "cross-hauling" or "two-way trade" may not arise under perfect competition. On the contrary, it is quite common that a pair of countries trade the same commodities with each other. Brander (1981) explains existence of cross-hauling through introducing imperfect competition (strategic interaction among firms) into traditional trade theory. In addition to the theoretical explanations, cross-hauling can also be interpreted from the following statistical viewpoints; 1) every practical classification of a commodity involves great diversity in quality, 2) a country often represents a highly aggregated area, and 3) trade statistics captures transactions in a finite period, during which a country would seek supply of a commodity to various countries due to seasonality and other reasons. It must be noted that the first point explains "intra-industry trade" of half-products that belongs to the same category as the final products.

In many multi-regional models, the potential type interregional trade coefficients are formulated to accommodate the observation of cross-hauling. For example, the most popular formulation is to assume that the quantities of interregional trade are positively related to production (supply) capacities and negatively related to CIF prices. This kind of formulation can be derived from Wilson's entropy model (see Wilson (1970)). However, the problem is that such kind of formulations is based upon analogies in physics or on statistical principles, they do not provide a theoretical explanation from the view of a firm's or individual's rational and deterministic decision making. Therefore, when such formulation is used in economic models, some kind of inconsistency may take place.[1] On the other hand, in the CGE literatures, the widely used method to justify the phenomenon of cross-hauling under perfect competition assumption is to employ the Armington assumption, which assumes that the same commodity produced in different origins is an imperfect substitute of each other. The assumption of perfect competition, to some extent, is out of touch of economic reality we observed in the real world. In this regard, an imperfect competition approach would be more preferable in CGE model to justify the existing of cross-hauling. However, this requires additional information on industry agglomeration (number of firms) as well as scale economies (data about fixed cost) for the model calibration, which is generally, sometimes extremely difficult to be obtained. This is particularly true when developing economies or relatively small regions are studied. In addition, the statistical reasons of cross-hauling described above seem difficult to be explained by imperfect competition. Therefore, in so many spatial CGE (SCGE) models, perfect competition and Armington assumption are still the most popular and standard assumptions for CGE modelers.

The Armington assumption is easy to use and also can justify the existing of cross-hauling under perfect competition. However, it seems that the relationships among the Armington elasticity, trade coefficient, spatial price equilibrium (SPE) and model solutions have not been carefully clarified yet in the existing SCGE literatures. One of the considerable reasons is that the existing studies tend to regard the transport sector as an ordinary service sector or imaginary transport agency that requires no resource for producing transport services (see Miyagi and

[1] Meng and Ando (2005) shows that very similar potential type interregional trade coefficients can be logically derived from the economic principle of firms' (individuals') deterministic decision making under the framework of multiregional input-output framework, rather than from the vague and irrelevant concepts of social physics.

Hongbu (1993) and GTAP [2] (1997)). However, the problem is that transport conditions, fares in particular, are a source of regional price differentials, and should be consistent with the SPE system. Therefore, without an explicit consideration on the unique characteristics of transport sector, it is difficult to explain how transport conditions affect trade pattern and SPE system under given Armington elasticity, also can not show how Armington elasticity affect trade pattern and SPE system under given transport conditions. For the justification of considering the behavior of transport firm explicitly in SCGE model, one can refer to Harker (1987)[3], Haddad and Hewings (2001), and Macann (2005) .

In addition, using the Armington assumption requires information about elasticity of substitution between goods from different regions, which is normally difficult to estimate when the numbers of regions and sectors are large. In many existing SCGE models, such information is always based on some other existing literatures or exogenously given by the modelers without enough verification on its accuracy. Without significant information on the elasticity of substitution, the use of Armington assumption may make the model simulation results very arbitrary. This is another reason why the detailed estimation on the property of Armington assumption in SCGE model is important.[4]

This paper proceeds as follows: In Section 2, based on the Armington assumption and the assumption of perfect competitive market, a three-region, three-sector SCGE model is formulated. The main feature of the model is that behaviors of transport sector and transportation networks are explicitly considered. In Section 3, the computation algorithm of our SCGE model is first discussed, and then the detailed evaluation on the impacts of the Armington assumption is summarized according to the simulation results which are based on three different benchmark calibrations. Section 4 gives the conclusion remarks.

2 The SCGE Model

In this section, we first introduce the basic assumptions of the model followed by detailed description on the behavior of individual economic agents (general industries, households, and transport sector). Then we show that (1) trade coefficient can be endogenously derived from the deterministic decision making of firms or households under the Armington assumption; (2) the spatial price equilibrium condition can be obtained from the cost-minimization behavior of transport firm. Finally, the general equilibrium conditions of the entire system are summarized. Definitions of all notations used in the formulations are given in Appendix B.

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[2]Developed by the World Trade Analysis Center in 1992. See http://www.gtap.agecon.purdue.edu/ for details.

[3]Harker (1987) introduces transport firms and networks into Takayama and Judge's (1971) framework. This made the SPE model a specific antecedent to development of the SCGE model.

[4]Due to the similar reason, Lofgren and Robinson (2002), Florenz (2005) and Ando and Meng (2009) use perfect substitution assumption to avoid using the Armington assumption in their CGE models. This paper will also discuss the validity of using perfect substitution assumption in SCGE model under perfect competition assumption.

2.1 Basic Assumptions

(1) Numbers of sectors and regions: Three industrial sectors, and three regions.

(2) Two factors of production: Two production factors of labor and physical capital are considered, and both of them are immobile across regions and sectors[5].

(3) Three types of economic agents: General industrial sectors (non-transport firms), transport firms and households.

(4) Transport demand: We assume that the demand for the transport services consists solely of derived demand that accompanies purchases of other commodities[6], transport services are supplied by the region of origin, and all transport costs are paid at origin.

(5) One final demand item: The final demand is only from households' expenditure, and households' disposable income is equal to their consumption expenditure.

(6) Imperfect substitutes: Commodities produced in different regions are imperfect substitutes of each other (Armington assumption).

2.2 Behavior of Economic Agents

2.2.1 General Industries (Non-transport Firms)

The (aggregate) production function of sector j in region s combines the two factor inputs, labor L_j^s and capital stock K_j^s of sector j in region s, with the intermediate inputs x_{ij}^{rs} of commodity i produced in region r.

$$X_j^s = A_j^s \prod_{i \neq 3} (\sum_r (x_{ij}^{rs})^{-\rho_{ij}^{\ s}})^{\frac{\alpha_{ij}^{\ s}}{-\rho_{ij}^{\ s}}} (L_j^s)^{\alpha_{Lj}^{\ s}} (K_j^s)^{\alpha_{Kj}^{\ s}} \tag{1}$$

The upper level of the production function uses a Cobb-Douglas type technology, and the lower level for intermediate inputs employs a CES type technology. X_j^s denotes the amount of output produced by industry j in region i, $\rho_{ij}^{\ s}$ is the substitution parameter[7], A_j^s is the scale parameter. Notation 3 represents the transport sector. For the parameters $\alpha_{ij}^{\ s}$, $\alpha_{Kj}^{\ s}$ and $\alpha_{Lj}^{\ s}$, the following is assumed:

Assumption 1 *The production function is linearly homogeneous for each region, i.e.,* $\sum_{i \neq 3} \alpha_{ij}^{\ s} + \alpha_{Lj}^{\ s} + \alpha_{Kj}^{\ s} = 1$[8].

[5] This assumption can easily be modified to facilitate mobile capital and (or) labor.

[6] For the sake of simplicity, transport services are just considered as freight transport, passenger transport is combined with other services sector.

[7] The elasticity of substitution: $\sigma_{ij}^{\ s} = \frac{1}{1+\rho_{ij}^{\ s}}$, where $\rho_{ij}^s \geq -1$.

[8] According to the Basic Assumption (4), transport services ($i = 3$) are not considered as one of the intermediate inputs.

As a whole, non-transport firms face the problem of choosing a combination of $\{x_{ij}^{rs}, K_j^s, L_j^s\}$ to maximize their profits described as follows:

$$\pi_j^s = p_j^s X_j^s - \sum_{i \neq 3} \sum_r (p_i^r + c_i^{rs}) x_{ij}^{rs} - \omega_j^s L_j^s - \gamma_j^s K_j^s \tag{2}$$

where, p_j^s is the producer's (FOB) price of commodity j in region s, and c_i^{rs} is the transport cost of a unit commodity i from region r to s, $p_i^r + c_i^{rs}$ is the purchasing (CIF) price of region s for the intermediate commodity i produced in region r. γ_j^s and ω_j^s are the capital rent and wage rate respectively.

One of the first-order conditions for equation (2) can be written as:

$$\frac{\partial \pi_j^s}{\partial x_{ij}^{rs}} = \frac{p_j^s \alpha_{ij}{}^s X_j^s (x_{ij}^{rs})^{-\rho_{ij}^s}}{x_{ij}^{rs} \sum_r (x_{ij}^{rs})^{-\rho_{ij}{}^s}} - (p_i^r + c_i^{rs}) = 0 \tag{3}$$

According to the Chenery-Moses's assumption, the intermediate input in physical term is given as follows:

$$x_{ij}^{rs} = a_{ij}^{rs} X_j^s = t_i^{rs} a_{ij}{}^s X_j^s \tag{4}$$

where a_{ij}^{rs} is the interregional input coefficient in physical term. t_i^{rs} and $a_{ij}{}^s$ are respectively the regional trade coefficient and the regional input coefficient. Based on the above equation, equation (3) can be then simplified as the following form:

$$\alpha_{ij}{}^s = \frac{\sum_r (t_i^{rs})^{-\rho_{ij}{}^s}}{p_j^s} \cdot \frac{t_i^{rs}(p_i^r + c_i^{rs})}{(t_i^{rs})^{-\rho_{ij}^s}} \cdot a_{ij}{}^s \tag{5}$$

The above solution for x_{ij}^{rs} is available to any region r'. Thus a similar result for $x_{ij}^{r's}$ is obtained:

$$\alpha_{ij}{}^s = \frac{\sum_r (t_i^{rs})^{-\rho_{ij}{}^s}}{p_j^s} \cdot \frac{t_i^{r's}(p_i^{r'} + c_i^{r's})}{(t_i^{r's})^{-\rho_{ij}^s}} \cdot a_{ij}{}^s \tag{6}$$

Dividing (5) by (6),

$$\frac{t_i^{rs}}{t_i^{r's}} = \left(\frac{p_i^r + c_i^{rs}}{p_i^{r'} + c_i^{r's}}\right)^{\frac{-1}{1+\rho_{ij}{}^s}}. \tag{7}$$

Summarizing both sides with r', and using the condition $\sum_{r'} t_i^{r's} = 1$, then trade coefficients can be derived as follows:

$$t_i^{rs} = \frac{(p_i^r + c_i^{rs})^{\frac{-1}{1+\rho_{ij}{}^s}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{-1}{1+\rho_{ij}{}^s}}}. \tag{8}$$

The above form implies that trade coefficients depend on producer's prices p_i^r and transport costs c_i^{rs}.

Here, defining the composite price (market price) q_i^s as the weighted average CIF prices of commodity i supplied from various regions (see Appendix A), namely,

$$q_i^s = \frac{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_{ij}{}^s}{1+\rho_{ij}{}^s}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{-1}{1+\rho_{ij}{}^s}}} = \sum_r (p_i^r + c_i^{rs}) t_i^{rs} \tag{9}$$

then, the profit function (2) can be rewritten as follows:

$$\pi_j^s = p_j^s X_j^s - \sum_{i\neq 3} q_i^s \sum_r x_{ij}^{rs} - \omega_j^s L_j^s - \gamma_j^s K_j^s \tag{10}$$

First-order conditions to the profit-maximization problem of equation (10), can be written as follows:

$$\alpha_{ij}^{\ s} = \frac{q_i^s \sum_r x_{ij}^{rs}}{p_j^s X_j^s} \quad , \quad \alpha_{Lj}^{\ s} = \frac{\omega_j^s L_j^s}{p_j^s X_j^s} \quad , \ and \quad \alpha_{Kj}^{\ s} = \frac{\gamma_j^s K_j^s}{p_j^s X_j^s} \tag{11}$$

The above parameters are nothing but the regional input coefficients measured in monetary terms. Since the regional input coefficients in physical terms can be given as:

$$a_{ij}^{\ s} = \frac{\sum_r x_{ij}^{rs}}{X_j^s} \quad , \quad a_{Lj}^{\ s} = \frac{L_j^s}{X_j^s} \quad , \ and \ , \quad a_{Kj}^{\ s} = \frac{K_j^s}{X_j^s}, \tag{12}$$

then the relationship between the monetary and the physical regional input coefficients can be written as follows:

$$a_{ij}^{\ s} = \frac{p_j^s}{q_i^s} \alpha_{ij}^{\ s} \quad , \quad a_{Lj}^{\ s} = \frac{p_j^s}{\omega_j^s} \alpha_{Lj}^{\ s} \quad , \ and \ , \quad a_{Kj}^{\ s} = \frac{p_j^s}{\gamma_j^s} \alpha_{Kj}^{\ s}. \tag{13}$$

2.2.2 Households

The source of income for households is the gross regional domestic product V^s comprising rent and wage payments:

$$V^s = \sum_j \omega_j^s L_j^s + \sum_j \gamma_j^s K_j^s, \tag{14}$$

where regions are assumed to be closed in terms of factor income. For simplicity, we consider that firms and their capital are owned by the households of the region where they are located. In addition, since tax and income transfer are ignored, the household disposable income W^s should equal V^s in our model.

The aggregate utility function of households in region s is considered to depend only on y_i^{rs}, the amount of commodity i produced in region r consumed in region s. Then the problem of households is to choose $\{y_i^{rs}\}$ that maximize their utility

$$\underset{y_i^{rs}}{\text{Max}} \qquad U^s = \prod_{i\neq 3} (\sum_r (y_i^{rs})^{-\delta_i^s})^{\frac{\beta_i^s}{-\delta_i^s}}, \tag{15}$$

under the budget constraint

$$s\ .\ t. \qquad \sum_{i\neq 3} \sum_r (p_i^r + c_i^{rs}) y_i^{rs} = W^s, \tag{16}$$

where, $W^s$is the disposable income of households, $\delta_i^s \geq -1$ is the substitution parameter, and β_i^s is final demand parameter.

Parallel to the production function, linear homogeneity of the utility function is assumed:

Assumption 2 *The utility function is linearly homogeneous, viz.* , $\sum_{i\neq 3}\beta_i^s = 1$.

From the first-order condition of the above problem, we can get the final demand parameter for y_i^{rs} as follows:

$$\beta_i^s = \frac{\lambda^s(p_i^r + c_i^{rs})\sum_r(y_i^{rs})^{-\delta_i^s}}{U^s(y_i^{rs})^{-\delta_i^s-1}}, \tag{17}$$

and the same parameter for $y_i^{r's}$ can be given as:

$$\beta_i^s = \frac{\lambda^s(p_i^{r'} + c_i^{r's})\sum_r(y_i^{rs})^{-\delta_i^s}}{U^s(y_i^{r's})^{-\delta_i^s-1}}. \tag{18}$$

Dividing (17) by (18), the trade coefficient of final demand goods can obtained as follows:

$$t_i^{rs} = \frac{(p_i^r + c_i^{rs})^{\frac{-1}{1+\delta_i^s}}}{\sum_r(p_i^r + c_i^{rs})^{\frac{-1}{1+\delta_i^s}}} \tag{19}$$

The form of the above trade coefficient is very similar to the form in equation (8). For the sake of simplicity, the following *Assumption* 3 is used in the model.

Assumption 3 *Substitution parameters of general industries and households are dependent only on its destination and commodity, and both of them are equal to each other. i.e.* $\rho_{ij}^s = \rho_i^s$ *, and* $\delta_i^s = \rho_i^s$.

Under the above assumption, we can get a general form of trade coefficient which includes both intermediate inputs and final demands:

$$t_i^{rs} \equiv \frac{T_i^{rs}}{\sum_r T_i^{rs}} = \frac{\sum_j x_{ij}^{rs} + y_i^{rs}}{\sum_r(\sum_j x_{ij}^{rs} + y_i^{rs})} = \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r(p_i^r + c_i^{rs})^{-\sigma_i^s}}. \tag{20}$$

Defining q_i^s as the composite price of composite consumption $\sum_r y_i^{rs}$, equation (16) can be rewritten as follows:

$$\sum_{i\neq 3}\sum_r(p_i^r + c_i^{rs})t_i^{rs}\sum_r y_i^{rs} = \sum_{i\neq 3} q_i^s \sum_r y_i^{rs}, \tag{21}$$

then the composite price can be given as the following form:

$$q_i^s = \sum_r(p_i^r + c_i^{rs})t_i^{rs}. \tag{22}$$

This is consistent with the similar equation shown in Appendix A. In addition, from the above equation and the first-order condition, the composite consumption of commodity i by households in region s ($y_i^s = \sum_r y_i^{rs}$) can be then written as follows:

$$y_i^s = \frac{\beta_i^s W^s}{q_i^s} \tag{23}$$

2.2.3 Transport Sector

Under Basic Assumption (4), all demands of this sector are derived from purchases of other commodities. Non-transport firms can determine output levels to maximize their profits, but transport firms are required to provide transport services that are needed to fulfill demands of other commodities and services. Thus they seek to minimize costs given the level of services.

For convenience, the following assumption concerning transport cost payments is introduced:

Assumption 4 *The transport costs are paid at the origin. This scheme also applies to the purchases by the transport sector itself. However, they do not recognize the imputed costs that accompany their own purchases from the regions they are located*[9].

The total transport demands originating in region s, in monetary terms, would be given by the LHS of the following formula:

$$c_i^{sr}(\sum_j x_{ij}^{sr} + y_i^{sr}) = p_3^s X_3^s \tag{24}$$

Under Assumption 4, these demands would be fulfilled by transport firms in region s, whose monetary output $p_3^s X_3^s$ must exceed these demands. The cost to provide services required may then be written as follows:

$$C_3^s = \sum_{i\neq 3}\sum_{r\neq s}(p_i^r + c_i^{rs})x_{i3}^{rs} + \sum_{i\neq 3} p_i^s x_{i3}^{ss} + \omega_3^s L_3^s + \gamma_3^s K_3^s \tag{25}$$

The production function of transport firms is also given by equation (1). The problem is to choose $\{x_{i3}^{rs},\ K_3^s,\ L_3^s\}$ that minimize the total cost (25) while satisfying the transport demands (24).

The first-order condition of intermediate inputs can be written with the Lagrange multiplier μ^s associated with (24) as follows:

$$a_{i3}^{\ s} = \frac{\mu^s p_3^s}{q_i^s}\alpha_{i3}^s = \frac{\mu^s p_3^s}{p_i^s + \mu^s c_i^{ss}}\alpha_{i3}^s. \tag{26}$$

The first expression is for the purchases from other regions, x_{i3}^{rs} $(r \neq s)$, and the second one is for the intra-regional purchases. From the above equation, the relation between FOB and CIF prices is given as follows:

$$q_i^s = p_i^s + \mu^s c_i^{ss}. \tag{27}$$

Finally, conditions for factor inputs can be written as follows:

$$a_{K3}^{\ s} = \frac{\mu^s p_3^s}{\gamma_3^s}\alpha_{K3}^{\ s} \quad and \quad a_{L3}^{\ s} = \frac{\mu^s p_3^s}{\omega_3^s}\alpha_{L3}^{\ s} \tag{28}$$

[9] The transport costs that accompany the intra-regional purchases of transport sectors are to be paid to the transport sectors themselves. Thus they can be deducted from the total cost of producing the transport services required.

2.3 Equilibrium Conditions

In this section, equilibrium conditions will be summarized for the model. Many of them are obtained by incorporating the first-order conditions of individual agents into the price and output equations of the interregional input-output system.

2.3.1 Price Equations

Price equations correspond to column sums of the input-output table. Three different patterns of equations must be prepared for non-transport and transport sectors as well as for final demands. The equation for non-transport sectors may be written as follows:

$$p_j^s X_j^s = \sum_{i \neq 3} \sum_r p_i^r t_i^{rs} a_{ij}{}^s X_j^s + \sum_{i \neq 3} \sum_r c_i^{rs} t_i^{rs} a_{ij}{}^s X_j^s + \omega_j^s a_{Lj}^s X_j^s + \gamma_j^s a_{Kj}{}^s X_j^s. \tag{29}$$

Using (13) to eliminated $a_{ij}{}^s$, and dividing both sides by $p_j^s X_j^s$,

$$1 = \sum_{i \neq 3} \frac{\alpha_{ij}{}^s}{q_i^s} \sum_r (p_i^r + c_i^{rs}) t_i^{rs} + \alpha_{Lj}{}^s + \alpha_{Kj}{}^s. \tag{30}$$

According to the definition of market price q_i^s, it is easy to see that the above equation is nothing but *Assumption* 1 of linear homogeneity in general sectors.

A similar argument can be applied to the final demand:

$$W^s = \sum_{i \neq 3} \frac{\beta_i^s W^s}{q_i^s} \sum_r (p_i^r + c_i^{rs}) t_i^{rs}. \tag{31}$$

It is easy to see that the above equation is consistent with the *Assumption* 2 , namely $\sum_{i \neq 3} \beta_i = 1$. In the similar way the price equation of transport sectors can be written as follows:

$$\frac{1}{\mu^s} = \sum_{i \neq 3} \sum_r \frac{(p_i^r + c_i^{rs}) t_i^{rs}}{q_i^s} \cdot \alpha_{i3}{}^s + \alpha_{L3}{}^s + \alpha_{K3}{}^s, \tag{32}$$

where costs accompanying the intra-regional purchases of its own are taken into account. Under *Assumption* 1, $\mu^s = 1$ must hold in order to comply with equation (22). Then equation (27) can be rewritten as follows:

$$q_i^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs} = p_i^s + c_i^{ss}. \tag{33}$$

This is the only one meaningful condition derived from the price equations.

2.3.2 Output Equations

Output equations correspond to the row sums of the input-output table. Output levels for non-transport sectors can be measured in physical units. Hence,

$$X_i^r = \sum_s \frac{t_i^{rs}}{p_i^s + c_i^{ss}} \left(\sum_j \alpha_{ij}{}^s p_j^s X_j^s + \beta_i^s W^s \right). \tag{34}$$

For the special property of transport sectors defined in the *Basic Assumption* 4, output level in transport sector can only be written in monetary terms:

$$p_3^r X_3^r = \sum_{i \neq 3} \sum_s \frac{c_i^{rs} t_i^{rs}}{p_i^s + c_i^{ss}} \left(\sum_j \alpha_{ij}{}^s p_j^s X_j^s + \beta_i^s W^s \right). \tag{35}$$

2.3.3 Factor Market and Final Demand

According to *Basic Assumption* 2 the capital rent and the wage rate are determined as the following forms:

$$\omega_j^s = \alpha_{Lj}^s p_j^s \frac{X_j^s}{L_j^s} \tag{36}$$

and

$$\gamma_j^s = \alpha_{Kj}^s p_j^s \frac{X_j^s}{K_j^s}. \tag{37}$$

Meanwhile the formula for the expenditure item can be summarized as follows:

$$W^s = \sum_j \omega_j^s L_j^s + \sum_j \gamma_j^s K_j^s. \tag{38}$$

3 Simulation Analysis

In this section, we first summarize the equilibrium conditions and variables used in the model, and then explain the computational procedure applied for the calculation of benchmark equilibriums. Finally, the benchmark equilibrium solutions will be used to evaluate the relationship among the Armington elasticity, transport conditions and endogenous solutions of the model in details.

3.1 Equations, Variables and Calculation Procedure

Equations describing the equilibrium are summarized in table 1. Since price in transport sectors cannot be distinguished from their quantities, their product, $p_3^r X_3^r$ is considered independent variable. Variables and parameters of the system are summarized in table 2. The number of endogenous variables is 54, which is equal to the number of equilibrium conditions.

The model composes a system of nonlinear simultaneous equations. However, each equation is not uniformly interconnected with other equations. Several blocks of equations can be identified that are relatively independent from other blocks. Considering this structural features of the equation system, we divide the entire system into three blocks (see Table 1). These include trade coefficient block (T), price block (P) and the block (X,W, ω, γ) for other endogenous variables. Each block takes the form of nonlinear programming to minimize the sum of squared errors from relevant equilibrium conditions. The system solution constitutes a series of convergence calculations by using iterative procedure based on quasi-Newton algorithm.

It should be noted that interregional transport costs are considered exogenous to the model. Actual transport costs c_i^{rs}, which are different among sectors, can be assumed to be proportional to interregional time-distances d^{rs}: $c_i^{rs} = \xi_i d^{rs}$. d^{rs} can be given based on the shortest time paths between pairs of regional geographical centers or capitals.

Table 1: Equilibrium conditions

Equations	Numbers	Blocks
general sectors:	3×2	
$X_i^r = \sum_s \frac{t_i^{rs}}{p_i^s + c_i^{ss}} (\sum_j \alpha_{ij}^{\ s} p_j^s X_j^s + \beta_i^s W^s)$	eq.(3.43)	
transport sectors:	3	
$p_3^r X_3^r = \sum_{i \neq 3} \sum_s \frac{c_i^{rs} t_i^{rs}}{p_i^s + c_i^{ss}} (\sum_j \alpha_{ij}^{\ s} p_j^s X_j^s + \beta_i^s W^s)$	eq.(3.44)	X
wage rate:	$3 \times 2 + 3$	W
$\omega_j^s = \alpha_{Lj}^{\ s} p_j^s X_j^s / L_j^s$	eq.(3.45)	ω
capital rent:	$3 \times 2 + 3$	γ
$\gamma_j^s = \alpha_{Kj}^{\ s} p_j^s X_j^s / K_j^s$	eq.(3.46)	
households:	3	
$W^s = \sum_j \omega_j^s L_j^s + \sum_j \gamma_j^s K_j^s + TR^s$	eq.(3.47)	
price system:	3×2	
$q_i^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs} = p_i^s + c_i^{ss}$	eq.(3.42)	P
trade coefficient:	$3 \times 2 \times 3$	
$t_i^{rs} = \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}}$	eq.(3.24)	T
	subtotal: 54	

Table 2: Variables and parameters

endogenous variables and subtotal	X_i^r (3×2) , $p_3^s X_3^s$ (3) , p_i^r (3×2) , ω_j^s $(3 \times 2 + 3)$, γ_j^s $(3 \times 2 + 3)$, W^s (3) , t_i^{rs} $(3 \times 2 \times 3)$ subtotal: 54.
exogenous variables	K_j^s , L_j^s , $c_i^{rs} (c_i^{rs} = \xi_i d^{rs})$.
parameters	$\alpha_{ij}^{\ s}$, $\alpha_{Kj}^{\ s}$, $\alpha_{Lj}^{\ s}$, β_i^s , σ_i^s , ξ_i .
	$\sum_{i \neq 3} \alpha_{ij}^{\ s} + \alpha_{Lj}^{\ s} + \alpha_{Kj}^{\ s} = 1$, $\sum_{i \neq 3} \beta_i^s = 1$.

3.2 Simulation Results

Three different benchmark equilibriums were calculated for testing the impacts of Armington elasticity on trade coefficients, price system and other endogenous solutions in details under given transport conditions. Benchmark 1 represents an economic system, in which the distribution pattern of interregional transport costs is completely uniform. Benchmark 2 provides an economy, in which the distribution pattern of transport costs is completely symmetric with relatively low intra-regional transport costs. Benchmark 3 shows an non-symmetric economic system, in which transport cost between two selected regions is lower than the other regions. These three benchmark situations are compared under the following two scenarios: (1) the Armington elasticities are perfect substitutes, (2) the Armington elasticities are imperfect substitutes.

3.2.1 Benchmark 1

The parameters and exogenous variables used in benchmark 1 are shown below:
$\alpha_{ij}^s = 0.25, \quad \forall\ i, j, s$
$\alpha_{Lj}^s = \alpha_{Kj}^s = 0.25 \quad \forall j, s$
$\beta_i^s = 0.50 \quad \forall\ i, s$
$L_j = K_j = 100.00 \quad \text{for } j = 1, 2,\ L_3 = K_3 = 40$
$c_i^{rs} = 0.20 \quad \forall\ r, s, i$ (see Figure 1) .

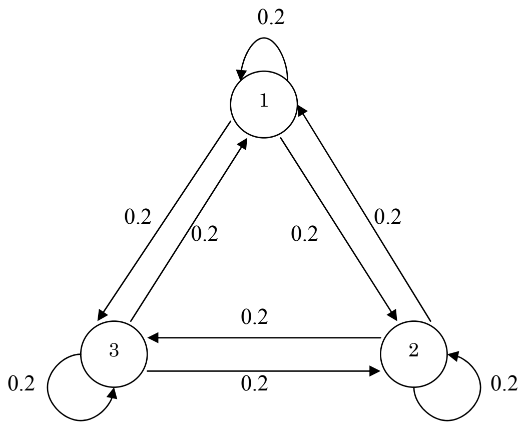

Figure 1: c_i^{rs} in benchmark 1

The endogenous variables for convergence calculation are initialized to the following values:
$p_i^s = 1.00, \quad \forall i, s$
$X_j^s = 100.00, \quad \forall j, s$

From the above conditions, it is easy to see that the economy given is a completely uniform system. Here, we solve the system under the following two scenarios:
scenario 1: $\sigma_i^s = 10, \quad \forall i, s$
scenario 2: $\sigma_i^s = \infty, \quad \forall i, s$, where $\sigma_i^s = 1/(1 + \rho_i^s)$

The calculation results of Benchmark 1 are summarized in Table 3. Obviously, The endogenous solutions are also completely uniform under both scenarios. This means the relationship between the Armington elasticity and model solutions is very robust when the distribution pattern of interregional transport costs is completely uniform.

Table 3: Solutions of endogenous variables in Benchmark 1, ($\sigma_i^s = 10$ *or* ∞)

	p_i^r		X_i^r		$p_3^s X_3^s$	ω_j^s			W^s	t_i^{rs}		
	goods 1	2	1	2		1	2	3		region 1	2	3
*region*1	1.00	1.00	100	100	40	0.25	0.25	0.25	120	1/3	1/3	1/3
*region*2	1.00	1.00	100	100	40	0.25	0.25	0.25	120	1/3	1/3	1/3
*region*3	1.00	1.00	100	100	40	0.25	0.25	0.25	120	1/3	1/3	1/3

3.2.2 Benchmark 2

Benchmark 2 presents a relatively real situation, in which the interregional transport costs are changed as follows:
$c_i^{rs} = 0.20 \quad \forall\ i$, when $r \neq s$,
$c_i^{rr} = 0.10 \quad \forall\ r, i$ (see Figure 1) .
The other initialization conditions are the same as benchmark 1.

The calculation results of Benchmark 2 under scenario 1 and 2 are summarized in Table 4 and 5 respectively. Both of them give completely symmetric solutions. Comparing with those for benchmark 1, under scenario 1, namely, the elasticities of substitution are 10, the outputs of non-transport sectors, the disposable incomes for each region, and factor prices for each region go up. This means the reduction of intraregional transport costs give an positive effect on outputs of non-transport sectors, regional incomes and factor prices. However, this will directly decrease the revenue of transport sector, therefore the output and the labor wage in transport sector seems to be damaged. On the other hand, the relatively lower intraregional transport costs give an positive impact on the intraregional trade coefficients. This can easily be understood, since the reduction of intraregional transport cost will cut down the production cost inside region, then the firms and households can enjoy relatively low CIF prices inside region. As an extreme case, when the Armington elasticity approximates to an infinite value, the economic system becomes autarkic pattern for each region. This means when the intermediate inputs can be perfectly substituted each other among regions, every region will just import goods and services from its own region, since the CIF prices inside region are the smallest one.

Table 4: Solutions of endogenous variables in Benchmark 2, ($\sigma_i^s = 10$)

	p_i^r	X_i^r	$p_3^s X_3^s$	ω_j^s		W^s	t_i^{rs}		
	1 / 2	1 / 2		1 / 2	3		1	2	3
1	1.00	101.57 ↑	27.37 ↓	0.25↑	0.17 ↓	115.25 ↓	0.54↑	0.23↓	0.23↓
2	1.00	101.57 ↑	27.37 ↓	0.25↑	0.17 ↓	115.25 ↓	0.23↓	0.54↑	0.23↓
3	1.00	101.57 ↑	27.37 ↓	0.25↑	0.17 ↓	115.25 ↓	0.23↓	0.23↓	0.54↑

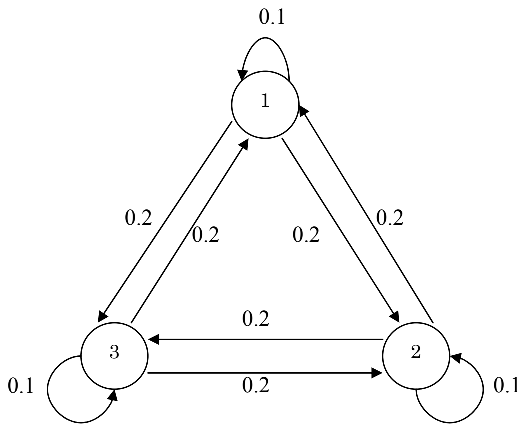

Figure 2: c_i^{rs} in benchmark 2

Table 5: Solutions of endogenous variables in Benchmark 2, $(\sigma_i^s = \infty)$

	p_i^r	X_i^r	$p_3^s X_3^s$	ω_j^s		W^s	t_i^{rs}		
	1 / 2	1 / 2		1 / 2	3		1	2	3
1	1.00	101.95 ↑	20.47 ↓	0.26↑	0.13 ↓	112.18 ↓	1.00↑	0 ↓	0↓
2	1.00	101.95 ↑	20.47 ↓	0.26↑	0.13 ↓	112.18 ↓	0↓	1.00↑	0↓
3	1.00	101.95 ↑	20.47 ↓	0.26↑	0.13 ↓	112.18 ↓	0↓	0 ↓	1.00↑

3.2.3 Benchmark 3

Comparing with Benchmarks 1 and 2, Benchmark 3 presents a non-uniform and non-symmetric economic initial conditions, in which the transport cost of commodity 1 from region 1 to region 2 decreases from 0.2 to 0.15. This setup not only helps us to check the impacts of the Armington elasticity on model solutions, but also helps us to simulate the impacts of transport cost reduction for selected pair of regions on the whole economy. The interregional transport costs for Benchmark 3 are given as follows:
$c_i^{rs} = 0.20 \;\; \forall\, i$, when $r \neq s$,
$c_i^{rr} = 0.10 \;\; \forall\, r, i$
$c_1^{12} = 0.15$ (see Figure 3).
The other initialization conditions are the same as benchmark 2.

The calculation results of Benchmark 3 under scenarios 1 and 2 are shown in Tables 6 and 7 respectively. Obviously, under the non-symmetric distribution pattern of interregional transport costs, the equilibrium solutions also give an non-symmetric image. Under scenario 1, the prices of commodity 1 for region 1 and 2 reduced, this is because the interregional transport cost for commodity 1 from region 1 to region 2 cut down the CIF prices in both region 1 and region 2. Comparing with the decreasing of CIF prices in these two regions, the CIF price in region 3 will become relatively high inevitably. This high price will give an negative impact on the output of commodity 1 in region 3, since high price results in low demands. It should be noted that the output of commodity 2 in region 1 and 2 also accept negative effects. This

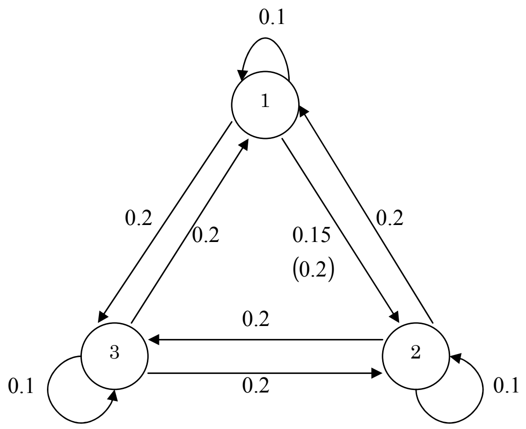

Figure 3: c_i^{rs} in benchmark 3

is because the relatively lower CIF price of commodity 1 in region 1 and 2 will boost up the demand for commodity 1 produced in these regions, for satisfying the increased demand, the non-transport firms in these region have to transfer the resources originally used for the production of commodity 2. As a result, the outputs of commodity 2 in both regions fall down. The pattern of trade coefficients also becomes non-symmetric. Comparing with Benchmark 2, every region tends to import relatively more commodity 1 from region 1 and 2. This also results from the reduction of transport costs between region 1 and region 2. However, when the Armington elasticity approaches to an infinite value, the economic system becomes autarkic for each region as well as the solution of Benchmark 2. This means when the intermediate inputs can be perfectly substituted each other among regions, every region will just import goods and services from its own region, even if the initial condition is non-symmetric.

Table 6: Solutions of endogenous variables in Benchmark 3, $(\sigma_i^s = 10)$

	p_i^r		X_i^r		$p_3^s X_3^s$	ω_j^s			W^s	t_1^{rs}		
	1	2	1	2		1	2	3		1	2	3
1	0.99	1.00	112.13	97.91	23.26	0.28	0.26	0.15	116.15	0.56	0.29	0.27
2	0.97	1.00	115.00	96.36	24.18	0.28	0.24	0.15	115.85	0.29	0.58	0.33
3	1.04	1.00	71.27	103.72	22.61	0.17	0.26	0.14	100.31	0.15	0.13	0.41

Summarizing the above discussion, it can be concluded that, if the intermediate inputs from different regions are aggregated by Cobb-Douglas technique, namely, the elasticity of substitution is set as an infinite value, just two solution patterns can be obtained: (1) completely symmetric regional economic structure, (2) complete autarkic regional economies. This implies that if Cobb-Douglas technique is used to aggregate interregional intermediate inputs, it is impossible to calibrate the spatial CGE model by using real data without any closure errors, unless the real data originally reflects either one of the above two solution patterns. For other calculation results in details, one can refer to Appendix D.

Table 7: Solutions of endogenous variables in Benchmark 3, $(\sigma_i^s = \infty)$

	p_i^r		X_i^r		$p_3^s X_3^s$	ω_j^s			W^s	t_1^{rs}		
	1	2	1	2		1	2	3		1	2	3
1	1.00	1.00	101.95	101.95	20.46	0.26	0.26	0.13	112.18	1.00	0	0
2	1.00	1.00	101.95	101.95	20.46	0.26	0.26	0.13	112.18	0	1.00	0
3	1.00	1.00	101.95	101.95	20.46	0.26	0.26	0.13	112.18	0	0	1.00

4 Conclusion

In this paper, we formulated a SCGE model based on Armington assumption with multi-regional framework (3 regions, 3 sectors). To complete the formulation, we haven given a separate presentment to the transport sectors so that the CIF price can be distinguished from FOB price. And then we simplified the model according to the Chenery-Moses assumption, which defines intermediate inputs and final demands through a set of trade coefficients in MRIO analysis. As a result of the simplification, we got a unified form of trade coefficients for all sectors and regions. The equilibrium conditions are summarized, and the parameters are calibrated then. At last, the benchmark status is simulated under each benchmark.

To evaluate the performance of the model, we divided the equilibrium conditions into several blocks(shown in flow chart of Computation procedure), then, we set three benchmark equilibriums to complete the simulation. Comparing the simulation results with each other, we can see that, if the intermediate inputs from different regions are aggregated by Cobb-Douglas technique, namely, the elasticity of substitution is set as an infinite value, just two solution patterns can be obtained: (1) completely symmetric regional economic structure, (2) complete autarkic regional economies. This implies that if Cobb-Douglas technique is used to aggregate interregional intermediate inputs, it is impossible to calibrate the spatial CGE model by using real data without any closure errors, unless the real data originally reflects either one of the above two solution patterns.

Several limitations of this paper should be mentioned here. First, the model is formulated under the framework of three regions and three sectors, in consideration of regional differentials it can be enlarged more concretely. Second, as a MRIO model, this paper falls short of providing a reasonable explanation of many other economic agents, such as government and foreign investment. Third, the model has not been simulated based on actual dates of I-O table. Finally, the model is formulated as a system of non-linear equations. Hence, existence and uniqueness of equilibrium as well as development of formulas to reduce Walras error should be investigated. So there is room for improvement of the model.

The Armington assumption takes the products with the same name but coming from different country of origin to be imperfect substitutes for each other. This assumption has been widely used in the existing CGE models. For investigating the impacts of this assumption on trade coefficients and model solutions in details, we formula a 3-region, 2-commodity simple spatial CGE model with explicit consideration on transport sector and regional price differentials. Under the Armington assumption, the model shows that trade coefficient can be endogenously derived from the rational and deterministic decision making of firm or household. Using this trade coefficient, the model solutions show that (1) if commodities produced in different regions are perfect substitutes, the regional economies will become autarkic or complete symmetric pattern; (2) if the commodities are imperfect substitutes, the impacts of the Armington elasticity on trade coefficient and model solution will be non-monotone and sometimes very sensitive.

Appendix A: Composited price

Derivation of composited price.
From the first-order condition equation (3.3) the composite price can be derived. First, rewriting equation (3.5) as follows:

$$\alpha_{ij}^{\ s} = \frac{x_{ij}^{rs}(p_i^r + c_i^{rs})\sum_r (x_{ij}^{rs})^{-\rho_{ij}^{\ s}}}{p_j^s X_j^s (x_{ij}^{rs})^{-\rho_{ij}^{\ s}}} \tag{A.1}$$

And making an identical transformation on the above equation,

$$\alpha_{ij}^{\ s} = \frac{\sum_r x_{ij}^{rs}}{p_j^s X_j^s} \cdot \underline{\frac{x_{ij}^{rs}\sum_r (x_{ij}^{rs})^{-\rho_{ij}^{\ s}}}{(x_{ij}^{rs})^{-\rho_{ij}^{\ s}}\sum_r x_{ij}^{rs}} \cdot (p_i^r + c_i^{rs})} \tag{A.2}$$

Then according to the relations between monetary and physical regional input coefficients, we find that, the terms with an underline in equation (A.2) are equal to the prices of composited goods. Arranging it by equation (3.4) as follows:

$$\frac{x_{ij}^{rs}\sum_r (x_{ij}^{rs})^{-\rho_{ij}^{\ s}}}{(x_{ij}^{rs})^{-\rho_{ij}^{\ s}}\sum_r x_{ij}^{rs}} \cdot (p_i^r + c_i^{rs}) = \frac{t_i^{rs}\sum_r (t_i^{rs})^{-\rho_{ij}^{\ s}}}{(t_i^{rs})^{-\rho_{ij}^{\ s}}\sum_r t_i^{rs}} \cdot (p_i^r + c_i^{rs}) \tag{A.3}$$

Finally, substituting equation (3.8) for t_i^{rs} in the RHS of the above equation, and the definition of composited price can be obtained.

$$q_i^s \quad = \quad \frac{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_{ij}^{\ s}}{1+\rho_{ij}^{\ s}}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{-1}{1+\rho_{ij}^{\ s}}}} \tag{A.4}$$

The above equation just stands for the same meaning with price index of the Krugman model.

Appendix B: Symbols in this paper

Table B.1: Symbols in this paper

Symbols	Definitions
X_j^s	production amount of industry j in region s
x_{ij}^{rs}	amount of good j from region s, used in production of good i in region r
L_j^s	input amount of labor, used in production of industry j, in region s
K_j^s	input amount of capital, used in production of industry j, in region s
π_j^s	profit of industry j in region s
p_j^s	supply price of good j in region s
q_i^s	demand price of good i in region s
c_i^{rs}	transport cost of goods i carried from region r to region s
U^s	households utility in region s
y_i^{rs}	households consumption in region s of good i produced in region r
W^s	income of households in region s
A_j^s	scale coefficient
$\rho_{ij}^{\ s}$	substitution parameter about elasticity
$\alpha_{ij}^{\ s}$	regional input coefficient of intermediate goods, measured in monetary term
$\alpha_{Lj}^{\ s}$	regional input coefficient of labor, in monetary term
$\alpha_{Lj}^{\ s}$	regional input coefficient of capital, in monetary term
ω_j^s	wage rate of industry j in region s
γ_j^s	rental rate of capital of industry j in region s
$a_{ij}^{\ s}$	regional input coefficient of intermediate goods, measured in physical term
t_i^{rs}	trade coefficient (physical term)
δ_i^s	substitution parameter about elasticity of households in region s
β_i^s	final demand parameter (monetary term)
TR^s	net income transfer to region s
TFM^s	regional net export of region s

Appendix C: Alternate derivation of trade coefficients

It is well known in modern microeconomics that a duality exists between the production function and the cost function. According to the so-called Shephard's duality, the unit cost function can be represented as follows:

$$p_j^s = \prod_i \left[\frac{1}{\alpha_{ij}} \Big(\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}} \Big)^{\frac{1+\rho_i}{\rho_i}} \right]^{\alpha_{ij}} \left[\frac{\gamma_j^s}{\alpha_{Kj}^s} \right]^{\alpha_{Kj}^s} \left[\frac{\omega_j^s}{\alpha_{Lj}^s} \right]^{\alpha_{Lj}^s}. \tag{C.1}$$

We further employ the Shephard's lemma, and then obtain the unit demand function for input x_{ij}^{rs} shown below, which theoretically equals interregional input-output coefficient.

$$\frac{\partial p_j^s}{\partial (p_i^r + c_i^{rs})} = \frac{\alpha_{ij} p_j^s}{p_i^r + c_i^{rs}} \frac{(p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}} = a_{ij}^{rs} \tag{C.2}$$

Using equations (8) and (13) to arrange the above equation, we have

$$t_i^{rs} = \frac{q_i^s}{p_i^r + c_i^{rs}} \frac{(p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}. \tag{C.3}$$

And then move the term $p_i^r + c_i^{rs}$ to the left side and compute their $\sum_r$ for both sides, we can obtain

$$q_i^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs} \tag{C.4}$$

which implies that the purchasing price index can be considered as an average value of the purchasing prices weighted by the trade coefficients. Since $\sum_r t_i^{rs} = 1$, then directly summarize both sides of (C.3) by r, we have a different expression of the purchasing prices indices as follows:

$$q_i^s = \frac{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}{\sum_r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}. \tag{C.5}$$

We further substitute the above equation into equation (C.3) to calculate t_i^{rs}. The results are as follows

$$t_i^{rs} = \frac{(p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}{\sum_r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}$$

which is the same as the earlier equation (8).

Appendix D: Results of computation

Table D.1: x_{ij}^{rs} in physical terms for Benchmark 2,($\sigma_i^s = 10$)

x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	12.56	12.56	3.36	5.26	5.26	1.42	5.26	5.26	1.42
1	2	12.56	12.56	3.36	5.26	5.26	1.42	5.26	5.26	1.42
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	5.26	5.26	1.42	12.56	12.56	3.39	5.26	5.26	1.42
2	2	5.26	5.26	1.42	12.56	12.56	3.39	5.26	5.26	1.42
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	5.26	5.26	1.42	5.26	5.26	1.42	12.56	12.56	3.86
3	2	5.26	5.26	1.42	5.26	5.26	1.42	12.56	12.56	3.86
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table D.2: y_i^{rs} in physical terms for Benchmark 2,($\sigma_i^s = 10$)

y_i^{rs}		1	2	3
	1	28.51	11.94	11.94
1	2	28.51	11.94	11.94
	3	0.00	0.00	0.00
	1	11.94	28.51	11.94
2	2	11.94	28.51	11.94
	3	0.00	0.00	0.00
	1	11.94	11.94	28.51
3	2	11.94	11.94	28.51
	3	0.00	0.00	0.00

Table D.3: x_{ij}^{rs} in FOB price for Benchmark 2,($\sigma_i^s = 10$)

x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	12.56	12.56	3.39	5.26	5.26	1.42	5.26	5.26	1.42
1	2	12.56	12.56	3.39	5.26	5.26	1.42	5.26	5.26	1.42
	3	5.29	5.29	1.79	4.88	4.88	1.68	4.88	4.88	1.68
	1	5.26	5.26	1.42	12.56	12.56	3.39	5.26	5.26	1.42
2	2	5.26	5.26	1.42	12.56	12.56	3.39	5.26	5.26	1.42
	3	4.88	4.88	1.69	5.29	5.29	1.79	4.88	4.88	1.69
	1	5.26	5.26	5.26	1.42	5.26	5.26	1.42	12.56	12.56
3	2	5.26	5.26	5.26	1.42	5.26	5.26	1.42	12.56	12.56
	3	4.88	4.88	1.69	4.88	4.88	1.68	5.29	5.29	1.79

Table D.4: y_i^{rs} in FOB price for Benchmark 2,($\sigma_i^s = 10$)

y_i^{rs}		1	2	3
	1	28.51	11.94	11.94
1	2	28.51	11.94	11.94
	3	12.39	11.44	11.44
	1	11.94	28.51	11.94
2	2	11.94	28.51	11.94
	3	11.44	12.39	11.44
	1	11.94	11.94	28.51
3	2	11.94	11.94	28.51
	3	11.44	11.44	12.39

Table D.5: x_{ij}^{rs} in physical terms for Benchmark 2,($\sigma_i^s = \infty$)

x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
1	2	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	23.17	23.17	4.65	0.00	0.00	0.00
2	2	0.00	0.00	0.00	23.17	23.17	4.65	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
3	2	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table D.6: y_i^{rs} in physical terms for Benchmark 2,($\sigma_i^s = \infty$)

y_i^{rs}		1	2	3
	1	50.99	0.00	0.00
1	2	50.99	0.00	0.00
	3	0.00	0.00	0.00
	1	0.00	50.99	0.00
2	2	0.00	50.99	0.00
	3	0.00	0.00	0.00
	1	0.00	0.00	50.99
3	2	0.00	0.00	50.99
	3	0.00	0.00	0.00

Table D.7: x_{ij}^{rs} in FOB price for Benchmark 2,($\sigma_i^s = \infty$)

x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
1	2	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
	3	9.27	9.27	1.86	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	23.17	23.17	23.17	0.00	0.00	0.00
2	2	0.00	0.00	0.00	23.17	23.17	23.17	0.00	0.00	0.00
	3	0.00	0.00	0.00	9.27	9.27	9.27	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
3	2	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
	3	0.00	0.00	0.00	0.00	0.00	0.00	9.27	9.27	1.86

Table D.8: y_i^{rs} in FOB price for Benchmark 2,($\sigma_i^s = \infty$)

y_i^{rs}		1	2	3
	1	50.99	0.00	0.00
1	2	50.99	0.00	0.00
	3	20.40	0.00	0.00
	1	0.00	50.99	0.00
2	2	0.00	50.99	0.00
	3	0.00	20.40	0.00
	1	0.00	0.00	50.99
3	2	0.00	0.00	50.99
	3	0.00	0.00	20.40

Table D.9: x_{ij}^{rs} in physical terms for Benchmark 3,($\sigma_i^s = 10$)

x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	14.42	12.71	3.02	7.85	6.80	1.71	4.40	6.14	1.34
1	2	13.74	12.11	2.88	5.76	4.99	1.25	3.85	5.37	1.71
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	7.14	6.29	1.50	14.89	12.90	3.24	5.24	7.32	1.60
2	2	5.76	5.07	1.21	13.75	11.92	3.00	3.85	5.37	1.17
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	3.90	3.44	0.82	3.32	2.88	0.72	6.62	9.25	2.02
3	2	5.76	5.07	1.21	5.76	5.00	1.25	9.19	12.83	2.80
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table D.10: y_i^{rs} in physical terms for Benchmark 3,($\sigma_i^s = 10$)

y_i^{rs}		1	2	3
	1	30.15	16.36	11.87
1	2	28.73	12.00	10.39
	3	0.00	0.00	0.00
	1	14.93	31.03	14.15
2	2	12.03	28.65	10.40
	3	0.00	0.00	0.00
	1	8.15	6.93	17.89
3	2	12.03	12.00	24.81
	3	0.00	0.00	0.00

Table D.11: x_{ij}^{rs} in FOB price for Benchmark 3,($\sigma_i^s = 10$)

x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	14.30	12.60	2.99	7.78	6.74	1.70	4.36	6.08	1.33
1	2	13.74	12.11	2.88	5.76	4.99	1.25	3.85	5.37	1.17
	3	5.59	5.26	1.70	5.12	4.80	1.62	4.43	5.08	1.61
	1	6.90	6.08	1.44	14.39	12.47	3.13	5.06	7.07	1.54
2	2	5.76	5.07	1.21	13.75	11.92	2.99	3.85	5.37	1.71
	3	5.36	5.05	1.65	5.64	5.26	1.73	4.60	5.32	1.66
	1	4.06	3.58	0.85	3.46	3.00	0.75	6.90	9.64	2.10
3	2	5.76	5.07	1.21	5.76	4.99	1.25	9.17	12.83	2.80
	3	4.71	4.48	1.52	4.59	4.35	1.51	4.36	4.99	1.59

Table D.12: y_i^{rs} in FOB price for Benchmark 3,($\sigma_i^s = 10$)

y_i^{rs}		1	2	3
	1	29.88	16.21	11.77
1	2	28.73	12.00	10.39
	3	12.55	11.52	11.12
	1	14.43	29.99	13.68
2	2	12.03	28.65	10.39
	3	12.06	12.64	11.58
	1	8.49	7.22	18.64
3	2	12.03	12.00	24.81
	3	10.70	10.45	10.94

Table D.13: x_{ij}^{rs} in physical terms for Benchmark 3,($\sigma_i^s = \infty$)

x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
1	2	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	23.17	23.17	4.65	0.00	0.00	0.00
2	2	0.00	0.00	0.00	23.17	23.17	4.65	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
3	2	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table D.14: y_i^{rs} in physical terms for Benchmark 3,($\sigma_i^s = \infty$)

y_i^{rs}		1	2	3
	1	50.99	0.00	0.00
1	2	50.99	0.00	0.00
	3	0.00	0.00	0.00
	1	0.00	50.99	0.00
2	2	0.00	50.99	0.00
	3	0.00	0.00	0.00
	1	0.00	0.00	50.99
3	2	0.00	0.00	50.99
	3	0.00	0.00	0.00

Table D.15: x_{ij}^{rs} in FOB price for Benchmark 3,($\sigma_i^s = \infty$)

x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
1	2	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
	3	9.27	9.27	1.86	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	23.17	23.17	23.17	0.00	0.00	0.00
2	2	0.00	0.00	0.00	23.17	23.17	23.17	0.00	0.00	0.00
	3	0.00	0.00	0.00	9.27	9.27	9.27	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
3	2	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
	3	0.00	0.00	0.00	0.00	0.00	0.00	9.27	9.27	1.86

Table D.16: y_i^{rs} in FOB price for Benchmark 3,($\sigma_i^s = \infty$)

y_i^{rs}		1	2	3
	1	50.99	0.00	0.00
1	2	50.99	0.00	0.00
	3	20.40	0.00	0.00
	1	0.00	50.99	0.00
2	2	0.00	50.99	0.00
	3	0.00	20.40	0.00
	1	0.00	0.00	50.99
3	2	0.00	0.00	50.99
	3	0.00	0.00	20.40

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