Merging the Hypothetical Extraction Method and the Classical Multiplier Approach: A Hybrid Possibility

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Abstract

The two main alternative methods used to identify key sectors within the input-output approach, the Classical Multiplier method and the Hypothetical Extraction method, are formally compared in this paper. Our findings indicate that the main distinction between the two approaches stems from the role of the internal effects. These internal effects are quantified under the Classical Multiplier Method while under the HEM only external impacts are considered. These conclusions allow us to develop a hybrid proposal that combines these two existing approaches. This hybrid model has the advantage of making it possible to distinguish and disaggregate external effects from those that a purely internal. This proposal has also an additional interest in terms of policy implications. Indeed, the hybrid approach may provide useful information for the design of "second best" stimulus policies that aim at a more balanced perspective between overall economy-wide impacts and their sectoral distribution.

Keywords: Sectoral linkages, Key sectors, Extraction methods, Multiplier methods.

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I. Introduction

The seminal work of Leontief (1941) stressed the fact that in modern economies markets are not isolated. Disregarding this fact and focusing on a partial rather than a general equilibrium perspective downward biases the evaluation of changes on economic variables. The potential derived effects in the economic system should also include indirect and possibly induced interactions. Leontief pointed out that a real market economy is characterized by a complex network of economic interdependencies between sectors that inform about the structure of the economic system as a whole. Consequently, a change in a given market, i.e. a demand shock, works its way throughout the “grid” of sectoral linkages, where mutual interconnections are duly taken into account, and yield endogenous repercussions affecting most or all the interlocking economic pieces of the system.

The input-output model, as first developed by Leontief, is constructed from a table of quantitative information that reflects the recorded bilateral transactions between economic sectors, i.e. the well-known input-output tables. This is the reason why the empirical tool known as the input-output analysis, has been broadly used by analysts in order to shed some light on a large variety of economy-wide issues, taking into account in particular the role and nature of market interdependencies.

Also within the input-output framework, Hirschman (1958) was the first to suggest the relevance of sectoral linkages in economic development. The more developed an economy is, the higher the proportion of inter-sectoral transactions to total output. According to this author, industrial or sectoral linkages constitute a measure of the degree of efficiency in production in an economy (i.e. the higher the degree of industrial integration, the lower the costs of production) but they are also an index of policy effectiveness (i.e. the effects of an increase in one sector investment will be transferred to the rest of the production block thanks to the network of industrial interdependencies). In his pioneer work Hirschman stated that, within these industrial interdependencies, two inducement mechanisms might be considered at work between each pair of industries: the direct backward linkage (or input-provision effects) and the direct forward linkage (or output-utilization effect). The former informs about one sector potential capability to induce the supply of inputs by other sectors while the latter is a measure of the potential effect of this sector over other sectors’ input demand.

Hirschman's approach was therefore the first relevant quantitative attempt for the identification of "key sectors" as a mean of planning better and more effective industrial development policies. "Key sectors" are defined as those that have either an above average backward strength (key pull sector) or an above average forward linkage index (key push sector).

For the empirical identification of "key sectors" under the input-output framework, analysts have been using two methods: the Classical Multiplier Method (CMM) based on Rasmussen (1957) and the Hypothetical Extraction Method (HEM) initially proposed by Strassert (1968) and later reformulated by Cella (1984) and Clements (1990)). The Hypothetical Extraction Method is a technique developed to measure the role of a sector within a network of sectors, typically in multisectoral models, to elicit its 'key' character in terms of its economic relevance or implicit weight. It is an improvement over the Classical Multiplier approach that measures 'keyness' merely in terms of simple averages of technical coefficients (direct and indirect). The HEM, in contrast, weights the 'keyness' of a sector by way of simulating the elimination of all of its external linkages from the economy, in other words, the hypothetical elimination of its sales to and its purchases from all other sectors. The output loss that would follow from this hypothetical cessation of economic activities quantifies the underlying network of linkages and provides a measure of 'keyness'. The empirical literature uses both of these approaches liberally to detect and measure how 'key' a sector is though a consensus is emerging that the HEM may go deeper to the root of the problem (Miller and Lahr, 2001).

The HEM method, then, quantifies the relevance of one sector in terms of its external, i.e. out-block, contribution to the market interdependencies while the CMM omits this distinction since it measures the total contribution originating in a sector over the whole set of sectors. Nevertheless, the two methods share the same theoretical assumptions in the sense that both CMM and HEM have their roots in Leontief's quantity model.

The aforementioned distinctions and similarities of the two approaches suggest then that their combined use is not only feasible from a pure theoretical point of view but it may also be useful for empirical work. In fact, this constitutes the main contribution of this paper. We present a novel "hybrid" methodology that merges the two existing approaches to single out sectors' "keyness" in an economy. Since differently to the CMM the HEM only accounts for the external interdependencies of sectors, the usefulness in combining the two existing approaches stems from isolating these effects from those that are merely internal, i.e. from self-supply. Therefore, the use of our proposed "hybrid" framework allows measuring sectors' forward and backward "keyness" in terms of both economy-wide impacts and the economy-distributive effects. This makes possible to attain a "second best" situation that makes compatible economy-wide policy effectiveness and its sectoral distributive impacts.

In order to illustrate the viability and the usefulness of our "hybrid" proposal, we also present an empirical exercise that aims at identifying "key sectors" for energy efficiency policies in the context of the Spanish economy. As already pointed out by Hirschman (1958), both these indices, backward and forward, provide useful information to design in a more cost-effective way general economic policy as well as more specific policies, as are those related to energy efficiency gains. Taking into account the characteristics of these policies and the seminal ideas of Hirschman, it is relevant to identify those energy and non-energy sectors that play a relevant role in providing production requirements to the remaining production blocks, that is to say, "key" push sectors. The reason behind this statement stems from the existing relationship between technology, production efficiency and cost structure. In an interconnected market economy, energy efficiency gains that occur in a specific sector reduce its overall production costs but also those of other sectors to which it provides intermediate inputs. Thanks to the existence of integrated markets, then, these efficiency gains are transferred from one sector to the other, round by round, favouring overall reduction in the intermediate use of energy in the economy as a whole. Furthermore, our proposed "hybrid" model allows also identifying those sectors with the largest external "push" effect, i.e. the larger the external "push" effect, the stronger the distributive effect of energy efficiency gains. Therefore, the combined used of the two existing methods under our "hybrid" proposal mat enrich both the empirical results and the conclusions drawn for policy guidance.

This paper is organized as follows. After formally describing the characteristics and the main differences between the two existing methodologies for identifying "key-sectors" in Section II, in Section III we formally present the new "hybrid" approach whereby the two frameworks may be complementarily used to disaggregate total effects into internal and external linkages. The empirical exercise of our "hybrid" proposal related to energy efficiency policies in the Spanish context is presented and described in Section IV while Section V concludes this analysis.

II. The Classical Multiplier Method and the HEM: A Review

In this introductory Section, we describe and compare formally both methodologies. On doing so, we have used partitioned matrices following the usual approach in the HEM. This practise eases the comparison the two alternative approaches, the Classical Multiplier method and the HEM and helps in the presentation of the novel "hybrid" approach presented in Section III. Our point of departure is the supply-demand balance system that corresponds to the familiar Leontief's quantity model. This system of equations in matrix notation is given by:

$$X = (I - A)^{-1} \cdot f \qquad (1)$$

where X refers to the column vector of sectoral production levels, and $(I - A)^{-1}$ is the so-called Leontief inverse that relates final demand f with total output X . Provided some technicalities that are associated to the productivity of matrix A are satisfied[1], the system in (1) has a unique and positive solution.

With the aforementioned goal in mind, i.e. comparing formally the CMM with the HEM, and using partitioned matrices expression (1) can be rewritten in the following way:

$$\begin{bmatrix} X_K \\ X_{-K} \end{bmatrix} = \begin{bmatrix} I - A_{K,K} & A_{K,-K} \\ A_{-K,K} & I - A_{-K,-K} \end{bmatrix}^{-1} \begin{bmatrix} f_K \\ f_{-K} \end{bmatrix} \quad \text{with} \quad \underbrace{1,..k..,K}_{K},\underbrace{K+1,..,-k...N}_{-K} \qquad (2)$$

[1] a) Matrix $(I - A)$ is non-singular, $|I - A| \neq 0$, and, b) productivity of matrix A with respect to all column vectors $f \geq 0$: $AX \leq X$

The set of equations in (2) indicates that the production block composed by N sectors is sub-divided into two production blocks: block K and block $-K$. The way sectors are grouped under the "key sectors" analysis usually depends on the nature of problem researchers want to tackle, i.e. for the case of energy efficiency policies, we disaggregate sectors into an energy block and a non-energy block.

Applying the generalized inverse of partitioned matrices, following Moore (1935) and Penrose (1955), the partitioned Leontief inverse that solves the supply-demand balance in matrix and scalar notation is given by:

$$\begin{bmatrix} I_{K,K} - A_{K,K} & A_{K,-K} \\ A_{-K,K} & I_{-K,-K} - A_{-K,-K} \end{bmatrix}^{-1} = \begin{bmatrix} T_{K,K} & U_{K,-K} \\ V_{-K,K} & -Q^{-1}_{-K,-K} \end{bmatrix} \quad (3)$$

where

$$\begin{aligned}
T_{K,K} &= (I_{K,K} - A_{K,K})^{-1}\left[I_{K,K} - A_{K,-K}Q^{-1}_{-K,-K}A_{-K,K}(I_{K,K} - A_{K,K})^{-1}\right]; \text{with} \quad [T]_{k,k} = \alpha_{k,k} \\
U_{K,-K} &= -(I_{K,K} - A_{K,K})^{-1}A_{K,-K}Q^{-1}_{-K,-K}; \text{with} \quad [U]_{k,-k} = \alpha_{k,-k} \\
V_{-K,K} &= -Q^{-1}_{-K,-K}A_{-K,K}(I_{K,K} - A_{K,K})^{-1}; \text{with} \quad [V]_{-k,k} = \alpha_{-k,k} \\
Q^{-1}_{-K,-K} &= \left[(I_{-K,-K} - A_{-K,-K}) - A_{-K,K}(I_{K,K} - A_{K,K})^{-1}A_{K,-K}\right]^{-1}; \text{with} \quad \left[Q^{-1}\right]_{-k,-k} = \alpha_{-k,-k}
\end{aligned}$$

Using the notational conventions in expression (3), we derive the solution of the system in matrix notation as:

$$\begin{aligned}
X_K &= (T_{K,K}\, f_K + U_{K,-K}\, f_{-K}) \\
X_{-K} &= (V_{-K,K}\, f_K - Q^{-1}_{-K,-K}\, f_{-K})
\end{aligned} \quad (4)$$

The system in (4) above implies that each unit of output can be decomposed in two parts: a first one that is required to satisfy self-final demand, i.e. $(T_{K,K}\, f_K)$ for block K and $(-Q^{-1}_{-K,-K}\, f_{-K})$ for block $-K$, and a second one that is needed to fulfil the final consumption requirements of the remaining production block, i.e. $(U_{K,-K}\, f_{-K})$ for block K and $(V_{-K,K}\, f_K)$ for block $-K$. Similarly, if there is an exogenous shock in overall final demand, i.e. $(\Delta f_K, \Delta f_{-K})$ the endogenous increase in the level of output is divided also in two effects, one that relates to the effect of *within-block* final demand, i.e. $(T_{K,K}\Delta f_K)$ for block K and $(-Q^{-1}_{-K,-K}\Delta f_{-K})$ for block $-K$, and another one that relates to the impact of *out-block* final demand, i.e. $(U_{K,-K}\Delta f_{-K})$ for block K and $(\Delta V_{-K,K} f_K)$ for block $-K$. Total derived shock in production levels is then the result of total sectoral linkage effects, both direct and indirect.

We now proceed to present the partitioned formulation of the Classical Multiplier method, also termed in the literature as Rasmussen indices (Rasmussen, 1957), describing its un-weighted version[2]. Under this method total backward linkages for each production block $(TBL_k; TBL_{-k}$ thereafter) are defined by means of the sum of columns of the Leontief inverse. Following the scalar notation in (3), the algebraic expression of these indices reads as:

$$\begin{aligned} TBL_k &= \left[\sum_{i=1}^{K}\alpha_{ik} + \sum_{i=K+1}^{N}\alpha_{ik}\right] \\ & \qquad \forall i = \underbrace{1,....,K}_{K}\ \underbrace{K+1,....,N}_{-K} \qquad (5) \\ TBL_{-k} &= \left[\sum_{i=1}^{K}\alpha_{i-k} + \sum_{i=K+1}^{N}\alpha_{i-k}\right] \end{aligned}$$

The coefficients in expression (5), i.e. TBL_k and TBL_{-k} respectively inform about the sectoral stimuli in sector k and sector $-k$ on its activity level due to a unitary final demand change in overall sectors in the economy.

Proceeding along similar lines, the sectoral total forward linkage of each of the sector in block K and $-K$ ($(TFL_k; TFL_{-k}$ thereafter), in absolute terms is written as:

[2] There are also different weighted versions of the Rasmussen indices. Clements and Rossi (1991) were first in suggesting weighting sectoral linkage indices by output shares. Weighting sectoral stimuli by the relevance of endogenous impacts, allows controlling not only for the size of each sector but also for the distribution of the exogenous demand shock. Laumas (1976) considered that sectoral backward stimulus indices should be rather weighted by the exogenous stimulus i.e. a final demand weighted version. Other authors have considered that linkages should even been expressed in terms of elasticities (Mattas and Shrestha, 1991).

$$TFL_k = \sum_{j=1}^{K} \alpha_{kj} + \sum_{j=K+1}^{N} \alpha_{kj}$$

$$\forall j = \underbrace{1,....,K}_{K}; \underbrace{K+1,....,N}_{-K} \qquad (6)$$

$$TFL_{-k} = \sum_{j=1}^{K} \alpha_{-kj} + \sum_{j=K+1}^{N} \alpha_{-kj}$$

These indices are in turn interpreted as the impact on sector *k*'s output of a simultaneous unit change in each and every sector's final demands. Forward effects indices are then an "output-utilization" measure. In terms of the CMM, a sector is considered to be a "key backward" ("forward") sector if its push or dispersion (pull or absorption) power is above the average sectoral impact.

The second approach to identify key sectors is the HEM. This alternative method aims at measuring the role of a sector or a production block by computing the loss in total output when the external or out-block relations with other sectors hypothetically disappear. A different interpretation of the evaluated output loss through this method has been proposed by Cardenete and Sancho (2006). According to these authors, the HEM also provides an efficiency measure from vertical integration. Thus, the higher the degree of vertical integration is, the greater will be the strength of the production links between sectors and, as a consequence, the stronger will be the forward and backward stimuli.

Table I: Alternative Hypothesis for computing output losses under the HEM

HEM FOR BLOCK *K*	**HYPOTHESIS DESCRIPTION**	**IMPLICATIONS ON THE STRUCTURAL MATRIX *A***	**AUTHORS**
HYPOTHESIS I	*Extract all three matrices in which block K has any influence.*	$A_{K,K} = A_{-K,K} = A_{K,-K} = 0$	*Strassert (1968)* *Schultz (1977)* *Heimler (1991)*
HYPOTHESIS II	*Extract two of the three matrices in which block K has any <u>external</u> influence*	$A_{-K,K} = A_{K,-K} = 0$	*Miller (1966, 1969)* *Miller and Blair (1983)* *Cella (1984)* *Dietzenbacher et al. (1993)*

While in the case of the CMM, the debate is centered on how linkage indices should be weighted, in the case of the HEM a first element of discussion relates to how the extraction of a sector should be simulated. As an illustrative example, we have summarised in Table I above two alternative ways of hypothetically extracting sector *K*. The most widely used "extraction" case is the one proposed by Cella (1984) (Hypothesis II in Table I). Cella proposed this hypothesis in response to the one proposed by Schultz (Hypothesis I in Table I), basically because the HEM aims at measuring the cost of the missing linkages with other sectors and not the internal ones. This is the reason why most of the authors have recently advocated for applying Hypothesis II rather than Hypothesis I (Sánchez-Chóliz and Duarte, 2003 and Cardenete and Sancho, 2006). Since we agree with the view of these authors, this will be the methodology described and used in what follows. The second element of discussion around the HEM deals with how the evaluated impacts should be classified (Cella, 1984 and Clements, 1990). We will turn to this issue later in this section when defining backward and forward impacts under the HEM.

Then, using the HEM under Hypothesis II, we aim at tackling the relevance of the *K* first sectors of the economy extracting the two matrices where this group of sectors has any external influence:

$$A_{-K,K} = A_{K,-K} = 0 \Rightarrow \bar{A}_{(K)} = \begin{bmatrix} A_{KK} & 0 \\ 0 & A_{-K-K} \end{bmatrix}$$

$$(I - \bar{A}_{(K)})^{-1} = \begin{bmatrix} (I_{K,K} - A_{K,K})^{-1} & 0 \\ 0 & \left(I_{-K-K} - A_{-K,-K}\right)^{-1} \end{bmatrix} \tag{7}$$

Applying the formulae of the inverse for partitioned matrices, the vectors of the simulated loss in the output of block *K* and –*K*, Δ_K and Δ_{-K} respectively, are given in absolute terms by:

$$\Delta_K = \left[(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q^{-1}_{-K,-K} A_{-K,K} (I_{K,K} - A_{K,K})^{-1} \right] f_K + (I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q^{-1}_{-K,-K} f_{-K} \tag{8}$$

$$\Delta_{-K} = Q^{-1}_{-K,-K} A_{-K,K} (I_{K,K} - A_{K,K})^{-1} f_K +$$
$$(I - A_{-K-K})^{-1} A_{-KK} \left[I_{KK} - A_{KK} - A_{K-K} (I - A_{-K-K})^{-1} A_{-KK} \right]^{-1} A_{K-K} (I - A_{-K-K})^{-1} f_{-K}$$

Similarly to the Classical Multiplier method, under the HEM, the absolute total linkage losses in expression (8) above can be decomposed in two parts: the one related to the costs of satisfying final demand of the *K*-extracted sectors that refers to the backward linkages of group *K* on the rest of the economy and those costs that are necessary to fulfil the final consumption of the remaining sectors that corresponds to the definition of forward linkages of group *K* on the rest of the economy.

The expression for the backward linkage under the HEM losses of the production block *K* is then given by:

$$\begin{aligned} BL_K = e'\left[(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q^{-1}_{-K,-K} A_{-K,K} (I_{K,K} - A_{K,K})^{-1} \right] f_K + \\ + e' Q^{-1}_{-K,-K} A_{-K,K} (I_{K,K} - A_{K,K})^{-1} f_K \end{aligned} \quad (9)$$

where e' refers to a summation vector of the proper dimension. The first term on the right hand side of expressions (9) can be interpreted as backward linkage costs derived from the “extracted group” self-supply, while the second term constitutes the backward linkage costs due to the intermediate flows from the extracted group to the other sectors. For the case of the forward linkages under the HEM and according with the proposed Cella’s measure:

$$\begin{aligned} FL_K = e'(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q^{-1}_{-K,-K} f_{-K} + \\ + e'(I - A_{-K-K})^{-1} A_{-KK} \left[I_{KK} - A_{KK} - A_{K-K} (I - A_{-K-K})^{-1} A_{-KK} \right]^{-1} A_{K-K} (I - A_{-K-K})^{-1} f_{-K} \end{aligned} \quad (10)$$

The first term in (10) is the output loss on sector *K* required in order to support the final demand of block -*K* while the second term is the feedback loss on sector -*K* coming from self-supply requirements.

In our description of the HEM we have followed Cella's interpretation of forward and backward indicators (Cella, 1984). However, this classification has been subjected to several criticisms and constitutes the second type of debate around the HEM mentioned above. Related to this, Clements (1990) argued that the second component in (10) should rather be interpreted as a backward effect because this external impact is the response of the remaining block when block *K* is purchasing intermediate inputs. Nevertheless, for the ease of the comparison between the classic approach and the HEM, we have chosen Cella's interpretation.

We can conclude from this formal description of the two approaches, the CMM and the HEM, that the main difference between the two methodologies stems from the simulation of output changes once there is a positive stimulus in final demand. Under the CMM, the "keyness" of a sector corresponds to output gains, while under the HEM has to do with output losses. These are, in fact, opposite ways when measuring a sector contribution to economic efficiency. However, the simultaneous use of these two methodologies is feasible leading to a "hybrid" approach that makes possible to disaggregate useful information about the strength of both sectoral forward and backward impacts in applied analysis.

III. The Classical Multiplier Method and the HEM: A "Hybrid" Possibility.

Once we have presented and described the two alternative methodologies, the Classical Multiplier method and the HEM, we move then to formally compare these approaches stressing their main distinctions. In doing so, we use the un-weighted versions of the two approaches assuming that under the two methodologies the exogenous shocks in final demand are unitary, i.e. output gains and losses per unit of final demand. The backward effect measure for block *K* in matrix notation under the Classical Multiplier method, i.e. TBL_K is given by:

$$TBL_K = e'(I_{K,K} - A_{K,K})^{-1}\left[I_{K,K} + A_{K,-K}Q_{-K,-K}^{-1}A_{-K,K}(I_{K,K} - A_{K,K})^{-1}\right] + \\ + e'\left[-Q_{-K,-K}^{-1}A_{-K,K}(I_{K,K} - A_{K,K})^{-1}\right] \qquad (11)$$

In the case of the HEM, i.e. $BL_{K(\Delta f_k = 1)}$ this expression reads as:

$$BL_{K(\Delta f_k=1)} = e'\left[(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q_{-K,-K}^{-1} A_{-K,K} (I_{K,K} - A_{K,K})^{-1}\right] + e' Q_{-K,-K}^{-1} A_{-K,K} (I_{K,K} - A_{K,K})^{-1} \quad (12)$$

Expression (12) exactly corresponds to expression (9) when there is a homogenous unitary change in the final demand of those sectors included in block *K*.

In the comparison of backward linkage measures under the Classical Multiplier method and the HEM using absolute endogenous effects per unit of final demand, we have to interpret the difference between (11) and (12):

$$TBL_K - BL_{K(\Delta f_k=1)} = e'(I_{KK} - A_{KK})^{-1} \quad (13)$$

Expression (13) implies that those transactions purely internal to block *K* are not accounted for under the HEM since they are not considered to "disappear" under Hypothesis II. The output of block *K* produced to satisfy internal input requirements are not lost, only those input requirements that are external to this block. The amount of production coming from block *K* that is hypothetically lost and that is considered under the HEM, is the *out-block* impact of self-supply requirements. Under this method, this is the weight attributed to block *K* in order to measure its economic "keyness". Therefore, when classifying "key sectors" in terms of backward linkage effects, differences between both methods stem from the relevance of the production chains internal to the block. If the degree of block's dependency on the internal linkages, i.e. "horizontal integration" is very strong, block *K* may turn out to be a "key backward sector" under the Classical Multiplier method. This classification of block *K* might be different however, under the HEM method, whereby only the degree of vertical integration is considered for.

Following the same procedure, the corresponding measure for the forward effect under the Classical Multiplier method, i.e. TFL_K , is given by:

$$TFL_K = e'(I_{K,K} - A_{K,K})^{-1} \left[I_{K,K} - A_{K,-K} Q^{-1}_{-K,-K} A_{-K,K} (I_{K,K} - A_{K,K})^{-1} \right] +$$

$$+ e'(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q^{-1}_{-K,-K} \quad (14)$$

and similarly, in the case of the HEM, $FL_{K(\Delta f_{-k}=1)}$:

$$FL_{K(\Delta f_{-k}=1)} = e'(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q^{-1}_{-K,-K} +$$

$$+ e'(I_{-K-K} - A_{-K-K})^{-1} A_{-KK} \left[I_{KK} - A_{KK} - A_{K-K} (I_{-K-K} - A_{-K-K})^{-1} A_{-KK} \right]^{-1} A_{K-K} (I_{-K-K} - A_{-K-K})^{-1} \quad (15)$$

Expression (15) also coincides with expression (10) when there is a homogenous unitary exogenous increase in the final demand of those sectors included in block *-K*

The difference between the traditional forward linkage measure and the one under the *HEM* in absolute terms will be:

$$TFL_K - FL_{K(f_{-k}=1)} = e'(I_{K,K} - A_{K,K})^{-1} - e'(I_{K,K} - A_{K,K})^{-1} \left[A_{K,-K} Q^{-1}_{-K,-K} A_{-K,K} (I_{K,K} - A_{K,K})^{-1} \right] -$$

$$- e'(I_{-K-K} - A_{-K-K})^{-1} A_{-KK} \left[I_{KK} - A_{KK} - A_{K-K} (I_{-K-K} - A_{-K-K})^{-1} A_{-KK} \right]^{-1} A_{K-K} \left(I_{-K-K} - A_{-K-K} \right)^{-1} \quad (16)$$

The interpretation of the difference between these forward measures is considerably more complex than in the case for backward indicators. Comparing expressions (14) and expression (15), under the classical method changes in overall sectors' final demand are considered while the HEM only accounts for "*out-block*" final demand changes i.e. (f_{-k}) . Thus, under the HEM forward and backward indicators do not overlap, they are completely isolated which is not the case under the CMM. Furthermore, under the HEM the forward indicators are measured in overall output terms in the sense that once the complete "out-block" final demand impacts are hypothetically extracted, economy-wide impacts are controlled for. Under the Classical Multiplier method, however, only the own output effect over block *K* is considered.

Interpreting expression (16), the first two elements of the difference (that are included in the Classical method but not under the HEM) refer to the effect on output levels of block *K* coming from self-demand: the "purely internal" stimulus, i.e. $e'(I_{KK} - A_{KK})^{-1}$ and the derived external impact coming from other sectors, i.e.

$e'(I_K - A_{KK})^{-1} A_{K,-K} Q^{-1}_{-K,-K} A_{-K,K} (I_{K,K} - A_{K,K})^{-1}$. The third element in expression (16), however, is accounted for by the HEM but it is disregarded when using the Classical Multiplier approach. This component refers to the impact over the output level of block *–K* as a feedback of the output of the targeted block *K* to support final demand of sector *–K*. This later element is a "pure" external impact coming from sector *K*, i.e. not including the purely within block effect of block *–K*, i.e. $(I_{-K-K} - A_{-K-K})^{-1}$.

Under the Classical Multiplier method, therefore, those sectors that have higher internal inter-industrial linkages coming from self-supply, direct and indirect, might be consider as "key forward sectors" while under the HEM they might get a very different position since only purely external final demand impacts are controlled for.

Summing up, this formal comparison between both methodologies indicates that under the Classical Multiplier Method, the two types of production interdependencies are considered for, both horizontal and vertical integration. When using the HEM, however, only vertical integration is accounted for in weighting sectoral "keyness" in an economy. In fact, the two types of production integration are relevant in terms of economic efficiency. This implies that their complementarity in applied work makes it possible the isolation of within blocks' effects, i.e. internal backward and forward effects from those that are purely external. In other words, the combined use of the two methodologies allows separating the contribution of a specific block in terms of vertical integration from that related to horizontal integration in an economy.

In the description of our proposed hybrid approach, the backward linkage measures under the two methods are first compared for their complementary use in applied analysis. Through the analysis of expression (13) and using the HEM backward measure proposed by Cella (1984), we have reached the conclusion that the differences in the "push" power measure stems from the relevance of "internal" self-supply effects. Differently to the HEM, this effect is considered under the Classical Multiplier method. Therefore, the combined use of the two approaches through our proposed hybrid method allows us to distinguish three backward measures: internal (I^B_K), external (E^B_K) and total (TBL_K) indicators. Following the same notation, these three measures are defined as:

$$I_K^B + E_K^B = TBL_K$$
$$I_K^B = TBL_K - BL_{K(f_k=1)} = e'(I_{KK} - A_{KK})^{-1}$$
$$\begin{aligned} E_K^B = BL_{K(f_k=1)} = e'\Big[(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q_{-K,-K}^{-1} A_{-K,K} (I_{K,K} - A_{K,K})^{-1}\Big] + \\ e' Q_{-K,-K}^{-1} A_{-K,K} (I_{K,K} - A_{K,K})^{-1} \end{aligned} \qquad (17)$$

Similarly to the backward indicators, the application of the hybrid model that combines both approaches we can split the total forward effect TFL_K into “internal” (I_K^F) due to self-supply, “external” (E_K^F) coming from the inter-industrial linkages with other sectors. These three terms are defined as follows:

$$I_K^F + E_K^{F_{cella}} = TFL_K$$
$$I_K^F = e'(I_{K,K} - A_{K,K})^{-1} + e'(I_{K,K} - A_{K,K})^{-1}\Big[A_{K,-K} Q_{-K,-K}^{-1} A_{-K,K} (I_{K,K} - A_{K,K})^{-1}\Big]$$
$$\begin{aligned} E_K^{F_{cella}} = FL_{K(f_{-K}=1)} = e'(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q_{-K,-K}^{-1} + \\ +(I - A_{-K-K})^{-1} A_{-KK}\Big[I_{KK} - A_{KK} - A_{K-K}(I - A_{-K-K})^{-1} A_{-KK}\Big]^{-1} A_{K-K}(I - A_{-K-K})^{-1} \end{aligned} \qquad (18)$$
$$E_K^{F_{clements}} = FL_{K(f_{-K}=1)} = e'(I_{K,K} - A_{K,K})^{-1} A_{K,-K} Q_{-K,-K}^{-1}$$

The definitions of the external forward and backward linkage components have drawn some controversies in the literature. According to Clements’ approach, the correct measure for the external forward effects should be that denoted by $E_K^{F_{Clements}}$ in expression (18) above. However, Clements (1990) considers the second component of this expression as a backward effect because it measures the stimulus generated in supplying sectors of block K by its own intermediate demand. As a result, Clements proposes that only the first component in his definitional expression should be considered as a “pure” forward impact of block K (i.e. $E_K^{F_{Clements}}$). As mentioned in Section II, the question on how these components should be distributed between forward and backward effects constitutes the second source of debate around the HEM and has not yet been cleared up in the literature.

As we mentioned before, the distinction between internal and external effects is relevant for a better understanding of industrial integration. Additionally, this information is also useful for a more complete guidance of specific policies as it is the case of energy efficiency policies. In this sense, the hybrid approach presented in this section is helpful for knowing whether the transmission of the evaluated energy efficiency gains might be potentially concentrated within a specific block or rather transmitted to other production units. Additionally, this allows deciding how to allocate policy inflows over energy and non-energy sectors to maximise not only economy-wide impacts but also the redistribution of efficiency improvements along the whole economic system. In Section IV we present the results of this hybrid model and how the twofold information obtained through this novel approach, i.e. the average and the distribution might be used. The detailed disaggregation of the evaluated endogenous impacts under the hybrid model outlined in this section can be use to guide the degree of effectiveness of energy efficiency policies for the case of the Spanish economy in a more complete way than if the two approaches, the Classical multiplier method and the HEM were used in isolation.

IV. An Empirical Exercise of the Hybrid Model: Identifying Key Sectors for Energy Efficiency Policies in the Spanish Economy.

This section is devoted to implement an empirical exercise applying the hybrid model formally outlined in Section III to Spanish data with the objective of identifying "key sectors" for energy efficiency policies. Our data set refers to a symmetric input-output table. This table has been constructed by the authors from the make and use tables published by the Spanish National Institute of Statistics for the year 2004. On squaring the economic flows coming from tables, we have applied the industry-technology assumption as indicated in ESA-95[3]. Formal details of the application of this assumption can be found in Ten Raa (1995).

As indicated in Section II the main objective of our proposed hybrid approach is to combine in a complementary way the two existing methodologies, the CMM and the HEM, to single out sectors' "keyness". In this empirical exercise we have therefore used expressions (17) and (18) from Section III that combine both methodologies to disaggregate the internal and the external effects from total backward and forward impacts. These six measures were computed for the 17 sectors in the database. The

[3] This acronym refers to the European Systems of Accounts (EUROSTAT, 1995).

sectoral disaggregation applied of the Spanish input-output Table for 2004 is included in Table A.I in the Annex. This sectoral break-down distinguishes 5 energy sectors and 11 non-energy sectors. All the backward and forward empirical indicators presented in this section refer to un-weighted measures.

For each production block, the results for the three measures presented in expression (17) in Section III that refer to backward impacts in absolute terms, i.e. overall impact on output when final demand in an specific sector increases by one unit, are shown down the first three columns of Table II. The last two columns of this table refer, on the other hand, to the backward indices that correspond to the “pure” methodologies for detecting key sectors—the CMM and the HEM. Backward indices under these two alternative approaches have been normalized and they show the distance with respect to the average backward impact. Before applying the hybrid approach to identify key sectors for energy efficiency policies in the Spanish context, we first compare the hybrid model to the “pure” methodologies in terms of our empirical results for the Spanish economy.

As it can be asserted from this Table, in most of the cases those sectors that are “key backward sectors” under the CMM criteria (fourth column of Table II), i.e the *Construction Sector* followed by the *Food, beverage, tobacco, textile and leather products* and the *Electricity Sector* have the same classification under the HEM criteria (fifth column of Table II). However, as mentioned above when interpreting expression (13), the weight of the internal backward effect influences significantly the classification under the indicators that correspond to the Classical Multiplier method indicators. As we can observe, the *Construction Sector,* that has the highest internal effect, i.e. one unit increase in the final demand of the *Construction Sector* potentially increases its own final demand by 1,466 units, has also the first position in the “key backward sectors” whereas under the HEM criteria it is the sector of *Food, beverage, tobacco, textile and leather products* the one that occupies the first place. Even for certain sectors the classification as “key backward sector” is completely different as in the case of the *Manufacturing* sector. This sector was considered to have a backward effect above average under the CMM criteria with 5,4 percent above average but not under the HEM measure with almost 23 percent below average. Again, this result is due to the higher relevance of the internal backward effect over the total backward impact in this sector as a result of its horizontal inter-dependencies.

We now proceed to compare our hybrid model with the “pure” methods in terms of the forward effects. Recall that these forward indicators refer to the absolute effect over each sector’s output if final demand for goods of each block increases by one unit. The results of the three disaggregated forward measures isolated thanks to our hybrid model, i.e. expression (18) in Section III are depicted in the three first columns of Table III. Similarly as when we presented the backward indicators, we show the results of the normalized forward effects under both the Classical Multiplier method and the HEM in the three last columns. Additionally, and to complete our analysis, within the HEM classification, we have distinguished between Cella’s (1984) and Clements’(1990) criteria.

Table II. Hybrid and "Pure" Methods: Backward Indicators.
Symmetric input-output Table for Spain 2004.

INTERNAL, EXTERNAL AND TOTAL BACKWARD EFFECTS IN ABSOLUTE TERMS. HYBRID METHOD.				*NORMALIZED BACKWARD INDICATORS ACCORDING TO THE "PURE" METHODOLOGIES*	
Production Block	$T_K^B = I_K^B + E_K^B = TBL_K$	I_K^B	E_K^B	*CLASSICAL MULTIPLIER METHOD*	*HEM*
Extraction of Anthracite, Coal, Lignite and Peat	*1,376*	*1,001*	*0,375*	**0,771**	**0,58**
Extraction of Crude, Natural Gas, Uranium and Thorium	*1,019*	*1*	*0,019*	**0,571**	**0,029**
Coke, Refinery and Nuclear fuels	*1,718*	*1,067*	*0,651*	**0,962**	**1,008**
Electricity Sector	*2,099*	*1,171*	*0,928*	**1,176**	**1,437**
Gas Sector	*1,764*	*1*	*0,764*	**0,988**	**1,183**
Primary Sector	*1,705*	*1,059*	*0,646*	**0,955**	**1**
Other Extraction Industries	*1,743*	*1,008*	*0,735*	**0,976**	**1,138**
Water Sector	*1,858*	*1,001*	*0,857*	**1,041**	**1,327**
Food, Beverage, Tobacco, Textile and Leather Products	*2,19*	*1,238*	*0,952*	**1,227**	**1,474**
Other Industrial Sectors & Recycling	*2,097*	*1,293*	*0,804*	**1,175**	**1,245**
Chemistry Industry, Rubber and Plastic Industry	*1,777*	*1,237*	*0,54*	**0,995**	**0,836**
Manufacturer Industry	*1,882*	*1,38*	*0,502*	**1,054**	**0,777**
Construction	*2,346*	*1,466*	*0,88*	**1,314**	**1,362**
Commercial & Transport Activities	*1,796*	*1,172*	*0,624*	**1,006**	**0,966**
Market Services	*1,77*	*1,005*	*0,765*	**0,992**	**1,184**
Non Market Services & Public administration	*1,515*	*1,05*	*0,465*	**0,849**	**0,72**
Average Absolute Impact	**1,784**	**1,138**	**0,645**	**1,784**	**0,645**

Table III. Hybrid and "Pure" Methods: Forward Indicators.
Symmetric input-output Table for Spain 2004.

INTERNAL, EXTERNAL AND TOTAL FORWARD INDICATORS IN ABSOLUTE TERMS. HYBRID METHOD.					***NORMALIZED FORWARD INDICATORS ACCORDING TO THE "PURE" METHODOLOGIES***		
Production Block	$T_K^F = I_K^F + E_K^{Fclements}$	I_K^F	$E_K^{Fclements}$	E_K^{Fcella}	CLASSICAL MULTIPLIER METHOD	HEM CLEMENTS	HEM CELLA
Extraction of Anthracite, Coal, Lignite and Peat	*1,114*	*1,004*	*0,11*	*0,151*	***0,624***	***0,174***	***0,170***
Extraction of Crude, Natural Gas, Uranium and Thorium	*2,321*	*1,000*	*1,321*	*1,346*	***1,301***	***2,095***	***1,515***
Coke, Refinery and Nuclear fuels	*1,514*	*1,070*	*0,444*	*0,712*	***0,848***	***0,702***	***0,801***
Electricity Sector	*1,525*	*1,181*	*0,344*	*0,610*	***0,854***	***0,544***	***0,686***
Gas Sector	*1,137*	*1,001*	*0,136*	*0,239*	***0,637***	***0,215***	***0,269***
Primary Sector	*1,503*	*1,091*	*0,412*	*0,642*	***0,842***	***0,651***	***0,723***
Other Extraction Industries	*1,065*	*1,010*	*0,055*	*0,096*	***0,597***	***0,088***	***0,108***
Water Sector	*1,038*	*1,002*	*0,036*	*0,066*	***0,581***	***0,056***	***0,074***
Food, Beverage, Tobacco, Textile and Leather Products	*1,762*	*1,286*	*0,476*	*0,810*	***0,987***	***0,753***	***0,911***
Other Industrial Sectors & Recycling	*1,739*	*1,308*	*0,431*	*0,691*	***0,974***	***0,683***	***0,778***
Chemistry Industry, Rubber and Plastic Industry	*1,979*	*1,248*	*0,731*	*1,041*	***1,109***	***1,158***	***1,172***
Manufacturer Industry	*3,453*	*1,411*	*2,042*	*2,722*	***1,935***	***3,234***	***3,064***
Construction	*1,895*	*1,480*	*0,415*	*0,657*	***1,062***	***0,656***	***0,739***
Commercial & Transport Activities	*2,435*	*1,196*	*1,239*	*1,859*	***1,364***	***1,961***	***2,092***
Market Services	*3,537*	*1,246*	*2,291*	*3,082*	***1,982***	***3,628***	***3,469***
Non Market Services & Public Administration	*1,260*	*1,055*	*0,205*	*0,294*	***0,706***	***0,325***	***0,331***
Average Impact	**1,784**	**1,162**	**0,631**	**0,888**	**1,784**	**0,631**	**0,888**

We already explained when describing expression (16) in Section III that those sectors that have a high forward internal effect take also the first positions under the Classical Multiplier approach. This is the case of the *Construction* Sector followed by the *Manufacturer* Sector and *Market Services*. Consequently, the size of this internal effect explains the reason why under the Classical Multiplier method a sector might turn to be "key forward sector" while not being so under the HEM. The *Construction* sector is a case in point. According to the CMM, this sector presents a forward effect that accounts for 6,2 percent above sectors' average while under the HEM its push impact is almost 35 percent and 25 percent bellow sectors' average according respectively to Clements' and Cella's criteria.

Lastly, we use the illustrative example of resource policies and, more specifically, energy efficiency policies to highlight the usefulness of combining the two "pure" methodologies through our proposed hybrid approach. Policy makers may well consider, in fact, that what matters is not only the total effect of a policy but also the distributive effect of the policy. Thus, when seeking economy-wide growth, the most effective solution might be to stimulate final demand of those sectors with the largest total backward-linkage. However, this policy might not turn to be the more efficient and the more equitable taking into account the relative impact at the sectoral level. If the target of the policy is to spread its impacts throughout the whole economic system, policy makers should be more concerned about those "key backward sectors" that present a higher external backward impact. In relation to energy efficiency policies, according to the results presented in Table III that refer to forward effects measures, those policies targeted at increasing energy efficiency in final demand should be orientated over the final use of *Electricity* not only because the economy-wide impact would be the strongest but also because of its external distributive effect favouring in a more equitable way sectors' output growth.

When dealing with policies that aim at improving technological efficiency similar conclusions can be drawn. It might be more effective to concentrate investment in those sectors that have not only the highest total forward linkage but also the strongest external component, again for distributive reasons. This is in fact how technological change is transferred throughout the economy since this change is reflected in production costs (Rosenberg, 1982). Those sectors that have high external forward effects make possible a more equitable distribution of technological improvements as are, for example, the *Manufacturing* sector and *Chemistry Industry* in the Spanish context. Coming back again to the specific case of energy efficiency policies, those policies that aim at improving energy efficiency in its intermediate use should specially tackle the *Extraction of Crude, Natural Gas, Uranium and Thorium Industry* and the *Electricity Sector*. In that way, efficiency gains would lead to the highest economy-wide impacts distributing also these improvements in a more equitable way among sectors.

V. Conclusions

The work of Hirschman (1958) constituted a milestone for the analysis of "key sectors" within the input-output framework. The relevance of identifying key sectors for specific policies pursues the maximization of their cost-effectiveness. Hence the main motivation behind the approach was to concentrate the policy inflow over those sectors that might potentially maximise the economy-wide impacts of that policy. The related literature on this field has been using alternatively two main approaches, the Classical Multiplier method (Rasmussen, 1957) and the Hypothetical Extraction method (Schultz, 1977, Cella, 1984, and Clements, 1990). There is still debate and not a clear consensus, however, about which procedure is the most appropriate. Furthermore, and to the best of our knowledge the differences between these two existing methods have not been explored. The first main contribution of this paper is to clarify the distinctions between these two aforementioned methods. The second main contribution is to develop a different approach, what we have referred to as the "hybrid" model, which combines these two approaches to help identify "key sectors".

The formal comparison of the two "pure" approaches carried out in this analysis indicates that the most important distinction between the two pure approaches stems from the internal effects that are captured by the Classical Multiplier Method whereas under the HEM only external impacts are considered. Consequently, the interest of our proposed "hybrid" model that combines simultaneously aspects of these two approaches relies on making possible the disaggregation of external (*out block*) and internal (*within block*) backward and forward effects. An additional advantage of the "hybrid" approach outlined in this analysis is that it also makes possible to find a balance between economy-wide impacts and their sectoral distribution as a kind of "second best". When seeking to pursue this kind of mixed objective, policy makers should concentrate policy inflows over those sectors that present not only strong total "push" and "pull" effects but also over those that show external above average impacts.

According to the empirical application for the Spanish economy related to energy efficiency policies, the recommendation is that policies whose target is to improve energy efficiency in intermediate use should be specially focused on the *Extraction of Crude, Natural Gas, Uranium and Thorium* industry and the *Electricity* sector. This is because these two energy sectors present the highest total and external forward effects that are relevant for transferring technological efficiency improvements**.** Following these guidelines, the energy efficiency improvements that initially occur in these production sectors would spread more equitably throughout the whole economic system while leading at the same time to the highest economy-wide impacts.

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Annex.

Table A.I. Sectoral breakdown for the Spanish Input-output Table. 2004

Classification	*Sectors*	*NACE-93 code*
Energy Sectors	*Extraction of Anthracite, Coal, Lignite and Peat*	10
	Extraction of Crude, Natural Gas, Uranium and Thorium	11-12
	Coke, Refinery and Nuclear fuels	23
	Electricity Sector	401
	Gas Sector	402-403
Non Energy Sectors	*Primary Sector*	01, 02, 05
	Other Extraction Industries	13-14
	Water Sector	41
	Food, Beverage, Tobacco, Textile and Leather Products	151-152, 154-155, 156-159, 16-19
	Other Industrial Sectors & Recycling	20-22,37
	Chemistry Industry, Rubber and Plastic Industry	24-25
	Manufacturer Industry	261-268, 27-36
	Construction	45
	Commercial & Transport Activities	50-52, 61-62, 601-603, 63.1-63.2, 63.4
	Market Services	65-67, 70-72, 74, 80, 85, 90, 92, 93, 63.3
	Non Market Services & Public administration	75, 80, 85, 90, 92