Multisectoral models in a Stock – Flow Consistent Framework.
Towards modelling a monetary economy of production.

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1. Introduction

In the present paper, we develop a dynamic economic model where elements from the real side of a disaggregate economic system interact with financial and monetary variables, as we are interested in what Keynes called a monetary economy of production. To do so, we try to put together three theoretical elements and then explore the outcome. The first element is the linear theory of production and the Classical theory of prices, as revived by Sraffa, 1960. The second one is the Theory of the Monetary Circuit (TMC onwards), as developed, amongst others, by Graziani, 2003, combined with the idea, suggested by Nell, 1998, that money circulates through channels in order to monetize transactions required by the economic system to expand. And third, the Stock-Flow-Coherent Accounting and Modelling (SFC hereafter), as developed, amongst others, by Godley and Lavoie, 2007. All of these elements are put under the umbrella of the Keynesian principle of effective demand (Keynes, 1936).

On the one hand, Sraffa's contribution provides a sound basis for analizing production, distribution and prices. But it omits references to money and financial variables. Graziani explains correctly, in our view, how money puts in motion the wheels of production. However he deals with production and prices in a very simplified manner. Combining these strands, we can monetize all transactions and distributive categories within an economy growing *à la* von Neumann, 1945 (see for instance Febrero, 2008). However, this case is not very general.

On the other hand, SFC provides a sound account of the interaction between production, distribution, spending and financial and monetary variables. However, this approach usually deals with a highly aggregated economy (an exception is Kim, 2006) and, as in TMC, production, prices and distribution are, we believe, not dealt with in a completely satisfactory manner.

SFC has two elements: first, balance sheet and transaction matrices, and, second, definitional and behavioral equations. The first two elements can be combined with input-output tables, thus providing the researcher with very interesting information which gives an account of modes of production, distribution, output, prices (implicitly), on the one hand, and also about where the money comes from and where it goes whether it is spent or saved, on the other hand. The second element, the set of equations, give us relevant information on the evolution of elements of the vector of final deliveries.

When we put all these elements together, a new and fruitful field of research is presented to economists.
The structure of this paper is as follows. In Section 2, we provide an account of a highly simplified economic system. It is described by means of input-output relations which, later on, are encapsulated in an input-output table. Also, we provide a price system for this economy. Next, in Section 3, we assume that this system grows *á la* von Neumann and all transactions are monetized, following the main tenets of the TMC. Section 4 provides information about the accounting of transactions and stocks following the basic insights of SFC Accounts. In Section 5, we combine balance sheets and transaction matrices within input-output tables. In Section 6, we depart from the von Neumann growth path. There, we inform about what happens when the interest rate falls below the rate of growth of output, in a literary manner. Then, Section 7 contains the definitional and behavioral equations informing about the evolution of the different components of the vector of final deliveries (amongst other relevant variables). In Section 8, we bring the main conclusions of the previous section into an input-output model (a quantity system). Section 9 offers our finalconclusions.

2. The system of production

We shall assume a multisectoral economy whose structure of production is described in the following figure:

Figure 1

	Inputs						Outputs				
	T	NM	OM	Kc	L		NM	OM	Kc	C	H
SI				Kc_1	L_1		NM^1				
SIIa		NM_{2a}		Kc_{2a}	L_{2a}			OM^{2a}	Kc^{2a}		
SIIb			OM_{2b}	Kc_{2b}	L_{2b}	→			Kc^{2b}		
SIII				Kc_3	L_3					C^3	
SIV	T_4			Kc_4	L_4						H^4

There are four sectors. Sector I, SI produces new machinery, NM^1 by means of circulating capital, Kc_1, and labour, L_1. In sector II, SII, we distinguish two sets of firms; SIIa produces circulating capital by means of labour, circulating capital and new machinery, NM_{2a}, and SIIb which produces circulating capital using old machinery, OM_{2b}. We implicitly assume that the useful lifetime of machinery is two periods of production. Sector III, SIII, produces consumer goods, C^3, by means of circulating capital and labour. The last, sector, four, SIV, produces dwellings, H^4, by means of circulating capital, labour and land, T_4.
In the list of inputs, columns for C and H have been omitted because these commodities are not required as inputs in any production process (nevertheless, see Appendix 1).
As usual, we assume that the system is viable, that all production processes start and finish on the same dates and, hence, that they last the same period of time. There are non-produced inputs (land), basic commodities (fixed capital, circulating capital), and other commodities which may or may not be basic depending on the definition of the wage basket (consumer goods, houses).
Labour is homogeneous and it is given in an amount so that it does not constrain output. Also, we assume that the level of output (and employment) is ruled by the Keynesian principle of effective demand. Part of this demand can be attended to with inventories of (already) produced output.

The quantity system

We define a matrix **A** of inputs and a matrix **B** of outputs. Matrix **A** encapsulates all material inputs except land:

$$\mathbf{A} = \begin{array}{|c|c|c|c|c|}\hline & & Kc_1 & & \\ \hline NM_{2a} & & Kc_{2a} & & \\ \hline & OM_{2b} & Kc_{2b} & & \\ \hline & & Kc_3 & & \\ \hline & & Kc_4 & & \\ \hline \end{array}$$

$$\mathbf{B} = \begin{array}{|c|c|c|c|c|}\hline NM^1 & & & & \\ \hline & OM^{2a} & Kc^{2a} & & \\ \hline & & Kc^{2b} & & \\ \hline & & & C^3 & \\ \hline & & & & H^4 \\ \hline \end{array}$$

Hence, as usual, the quantity system becomes:

(2.1) $\quad \mathbf{qA} + \mathbf{y} = \mathbf{qB}$

With **q** being a (row) vector of commodities denoting the level of intensity (the amount of output produced by each trade).

In terms of ‘conventional’ input-output tables, Figure 1 becomes:

Figure 2

		Intermediate inputs				(Gross) Final deliveries		Total output
		SI	SIIa+b	SIII	SIV	C	I	
SI							NM^1	NM^1
SIIa+b		Kc_1	Kc_{2a+2b}	Kc_3	Kc_4		Kc^2-$\sum Kc_i$	Kc
SIII						C^3		C^3
SIV							H^4	H^4
VA	W	L_1	L_{2a+2b}	L_3	L_4			
	GOS	B_1	B_{2a+2b} + $(NM_{2a}–OM^{2a})$	B_3	B_4 + T			
Total output		NM^1	Kc	C^3	H^4			

Some comments:

- All elements in the table above are computed in monetary units. Thus, although we have used the same symbols as in Figure 1, here they are multiplied by their corresponding prices (profits, as will be shown below, are calculated multiplying the value of advances by a profit rate).
- Fixed capital (in our example, new and old machinery) as an input is not included in the block of intermediate inputs: this sub-matrix accounts for circulating capital inputs

only. Investment in fixed capital is shifted to final deliveries, particularly to the column of investment, I.
- Depreciation (given by NM_{2a}–OM^{2a}) is accounted for in GOS (gross operating surplus) of the sector using fixed capital, SIIa+b.
- Land, T in sector SIV, should be read as a rent, which is included in GOS.
- Dwellings have the same treatment as productive investment.
- As above, blanks stand for zeros.[1]

Alternatively, we can represent the above system in another input-output table, closer to the Classical-Keynesian approach:

Figure 3

		Intermediate inputs				(Net) Final deliveries		Total output
		SI	SIIa+b	SIII	SIV	C	I	
SI			$(NM_{2a}–OM^{2a})$				NM^1 - $(NM_{2a}–OM^{2a})$	NM^1
SIIa+b		Kc_1	Kc_{2a+2b}	Kc_3	Kc_4		Kc^2-$\sum Kc_i$	Kc
SIII		C_1	C_{2a+2b}	C_3	C_4	C^3		$C^3+\sum C_i$
SIV							H^4	H^4
(Net surplus)	L_{saving}	S_{L1}	S_{L2a+2b}	S_{L3}	SL_{L4}			
	NOS	B_1	B_{2a+2b}	B_3	B_4 + T			
Total output		NM^1	Kc	C^3	H^4			

Here we have shifted workers' consumption to the intermediate inputs block. And the same holds regarding depreciation. Then, VA (value added) becomes net surplus which, in turn may be decomposed into workers' savings (S_{Li}), that is wages (incomes) minus consumption spending, and NOS (net operating surplus).
Regarding final deliveries, they now appear in net terms (contrary to the IO Table in Figure 2 where we had final deliveries in gross terms).

The price system
We proceed in two steps because we have to solve a price system for reproducible commodities first, and then we can get the *rental* price of land in the building (SIV) trade.
As usual we assume competitive conditions (in the Classical-Marxian sense) in all trades, so that there is a uniform profit rate. Wages are paid ex-ante.
First, we have to face a price system as follows:

$$
\begin{aligned}
&(p_{Kc} Kc_1 + L_1 w)(1+r) = p_{NM} NM^1 \\
&(p_{NM} NM_{2a} + p_{Kc} Kc_{2a} + L_{2a} w)(1+r) = p_{kc} Kc^{2a} + p_{OM} OM^{2a} \\
(2.2)\quad &(p_{NM} OM_{2b} + p_{Kc} Kc_{2b} + L_{2b} w)(1+r) = p_{kc} Kc^{2b} \\
&(p_{Kc} Kc_3 + L_3 w)(1+r) = p_c C^3 \\
&(p_{Kc} Kc_4 + L_4 w)(1+r) = p_H{}^* H^4
\end{aligned}
$$

[1] Alternatively, some authors prefer to include depreciation and workers' consumption in the intermediate inputs block.

This is a system with (in our example) 5 equations and 7 unknowns (p_{NM}, p_{OM},, p_{Kc}, p_C, and p_H^*, as prices, and additionally, w and r).[2] As usual, there are two degrees of freedom which have to be set from the outside. We choose the nominal wage and the profit rate. Also, it should be noted that once w and r are fixed from the outside, the three upper equations are enough to get the first three prices (p_{NM}, p_{OM},, p_{Kc}). Then, we can get the remaining prices in a recursive manner. In this price system, p_H^* stands for the 'building cost', that is, the cost of building a house leaving aside the price of the land where it is allocated. According to the recent Spanish experience, we shall assume that the price of one house, p_H, which includes the price of the portion of land where it is built, is ruled by the amount of money that a bank agrees to lend (in the form of a mortgage) to the purchaser (for simplicity's sake we shall assume a 100% loan-to-value ratio). And the size of the loan that the bank is willing to grant is ruled by the present value of the stream of future payments from the borrower. Further, payments are limited by the percentage income that banks consider that borrowers can set aside to deal with loan instalments.
Then:

$$(2.3) \qquad p_H = \sum_{t=1}^{T} \frac{z \cdot w}{(1+i)^t}$$

Where z is the percentage income of the borrower (e.g. 30%) which can be (with some effort) saved to cancel mortgage loan instalments, w is the borrower income (the wage per unit of labour and period unit of time) i stands for the interest rate on mortgage loans and T denotes the maturity of the loan (for instance, 5 periods of time). The price p_H holds when it is larger than p_H^*, of course.
Then the rental price of one unit of land, p_T, where h (e.g. 20) houses can be built amounts to:

$$(2.4) \qquad p_T = h\left(p_H - p_H^*\right)$$

And, finally, the corresponding price equation for the building industry (SIV) becomes:[3]

$$(2.5) \qquad p_T T_4 + \left(p_{Kc} Kc_4 + L_4 w\right)(1+r) = p_H H^4$$

3. Monetary circulation

We shall follow the basic tenets of the Theory of the Monetary Circuit –TMC onwards (Graziani, 2003, Lavoie, 1992, chapter 4, Nell, 1998, Parguez and Secareccia, 2000, Realfonzo, 1998, Rochon, 1999, amongst others). Banks create money *at the stroke of a pen* when they grant credit to creditworthy borrowers, to fund the expenses corresponding to particular production processes. However, contrary to the conventional TMC view (see Febrero, 2008) we shall assume that the circulation of money takes place once one production process has already finished and before the next one starts. In formal terms:

[2] The first three equations can be solved simultaneously, once we fix from the outside w and r. In doing so, we obtain p_{NM}, p_{OM} and p_{Kc}. Then, in a recursive manner, we can obtain p_c, and p_H^*.
[3] It should be noted that within this assumption, the purchase of a house by a worker means a re-distribution of wages towards rents (from land ownership).

(3.1) $\mathbf{q_t B} \Rightarrow \mathbf{q_{t+1} A}$

Thus, output at the end of period *t* is given by $\mathbf{q_t B}$. This denotes output at the firms' gates. Firms, now have to decide how much output they are going to produce during period *t*+1. This decision is based on expectations about how much they are going to sell in their corresponding markets at *normal* prices (i.e. according to the Keynesian principle of effective demand). Then, once they have decided on $\mathbf{q_{t+1}}$, they know how much inputs they have to purchase and how much labour they have to hire.

Back to the quantity system.
Although it is not necessary, for the sake of simplicity, we shall assume that the system grows *à la* von Neumann (von Neumann, 1945). This will make the exposition of our argument easier and, also, we shall remove it later. When the system expands *à la* von Neumann all trades grow at the same rate and the vector of sectoral output adopts a particular composition. Returning to the quantity system, once the techniques have been defined, it is rather convenient to define a wage basket in terms of commodities. For simplicity, we can assume that all wages are spent on consumer goods and also, a portion of the labour force purchases a house each period of time. For instance,

(3.2) $w = \left(p_C + 0.05 \cdot p_H\right)$

That is, we assume that each worker purchases one unit of consumer goods with their wage per unit of time (period of production). Additionally, we have assumed that 5% of the labour force purchases a house every year. This is tantamount to assuming that one worker purchases 5% of a house.
Next we define a matrix $\mathbf{A}^+$ (a socio-technical matrix) as:

(3.3) $\mathbf{A}^+ = \mathbf{A} + \mathbf{a_n} \cdot \mathbf{d}$

Where $\mathbf{a_n}$ stands for a (column) vector of direct labour requirements per unit of (total) output and $\mathbf{d}$ stands for a (row) vector corresponding to the wage basket of a unit wage.
In our assumed steady growth path:

(3.4)
$$\mathbf{q_t A}^+ (1+G) = \mathbf{q_t B}$$
$$\mathbf{q_t A}^+ \mathbf{B}^{-1} = \frac{1}{(1+G)} \mathbf{q_t}$$

Where matrix $\mathbf{A}^+$ is:

$\mathbf{A}^+ =$

		Kc_1	L_1	$0.05 \cdot L_1$
NM_{2a}		Kc_{2a}	L_{2a}	$0.05 \cdot L_{2a}$
	OM_{2b}	Kc_{2b}	L_{2b}	$0.05 \cdot L_{2b}$
		Kc_3	L_3	$0.05 \cdot L_3$
		Kc_4	L_4	$0.05 \cdot L_4$

Therefore, the rate of growth of the system, *G*, is given (as is widely known) by the inverse of the maximum eigenvalue of matrix $\mathbf{A}^+\mathbf{B}^{-1}$. Further, the composition of the vector of output $\mathbf{q_t}$ is given by the right hand eigenvector associated to the maximum eigenvalue of $\mathbf{A^{+}B}^{-1}$.

Finally, we shall assume the second part of the Classical hypothesis of expenditure: all profits are saved and ploughed back into investment capacity.

And revisiting the price system

Although the price system has already been presented, at least in its basic form, we now provide a particular way to solve the system when the wage basket is defined *a priori* and the system grows *à la* von Neumann. Once we have assumed an expenditure pattern for wages, and taking the unit price of consumer good as a *numeraire*, the price system above becomes:

$$\text{(3.5)} \qquad \begin{aligned} &\mathbf{A}^{+}\mathbf{p}(1+R)=\mathbf{Bp} \\ &\mathbf{B}^{-1}\mathbf{A}^{+}\mathbf{p}=\frac{1}{(1+R)}\mathbf{p} \end{aligned}$$

As it is widely known, R is now ruled by the inverse of the maximum eigenvalue of $\mathbf{B}^{-1-}$ $\mathbf{A}^{+}$ and the price vector is given by the left hand eivenvector of matrix $\mathbf{B}^{-1}\mathbf{A}^{+}$ (further, $G = R$).

Of course, in this system we have the price of one house that does not include the price of land. Its full price is calculated by the (above stated) expression for the present value of the stream of mortgage instalments that the bank is going to receive after they grant a mortgage loan.

Cnannels of monetary circulation

We assume that we have output at the firms' gates, given by vector $\mathbf{q_t B<p>}$ at the end of period t. Next, firms expect that the demand for their output at the end of period t+1 is going to grow at a rate G. Therefore, we might expect the following facts to take place immediately (almost simultaneously): (1) firms have to purchase inputs in order to increase their output at the desired / expected rate, and (2) they have to sell their already produced output.

In formal terms we have:

$$\text{(3.6)} \qquad \begin{aligned} &\mathbf{q_{t+1}}=\mathbf{q_t}(1+G) \\ &\mathbf{q_t B}=\mathbf{A q_{t+1}} \\ &\mathbf{q_t B}=\mathbf{A q_t}(1+G) \end{aligned}$$

Thus, output at the end of period t equals inputs at the beginning of period t+1.

In order to monetize transactions, we shall assume a pure credit economy (as in Wicksell, 1898, particularly chapter 9, section B). Hence, we can assume that there is one bank, which creates deposits when it grants a loan to a creditworthy borrower. That is, money is endogenous. This bank may make short term loans to fund the purchase of working capital (i.e. circulating capital and wages) and long term loans to fund the purchase of fixed capital (new machinery) and houses.

Also, we shall assume that the interest rate, i, equals the rate of profit R and the rate of growth of output, G.

Although money may circulate through several channels, the following ones lead us to an interesting result: all money which is created at the beginning of the circuit is destroyed once all commodities have circulated, making it possible for a new production process to start with pending debts remaining. Let us see it.

1. SI, SIIa and SIIb and SIV ask for short term loans to pay wages.
2. Workers in these sectors spend a part of their wages on consumer goods. The rest is used to settle debt service payments corresponding to the purchase of houses in the past.
3. SIII has sold part of its output and uses the proceeds to pay wages to its workers which, in turn, set aside a part of their incomes to pay debt services (houses purchased previously) and the rest is spent on consumer goods.
4. SIII needs circulating capital, apart from labour, to produce more consumer goods. It uses proceeds to purchase this input from firms in sector SII.
5. Firms SII use liquidity from the sale of Kc to SIII to cancel part of their short term debt. Now, SIIb also needs new machinery to start a new process of production. It will have to borrow long term (the maturity will match the useful life time of machinery: in our simplified economy, 2 periods of production).
6. SI sells already produced new machinery to SIIb. SI can now cancel part of its short term debt (for the payment of wages), and the rest is used to purchase Kc from SIIa and SIIb.
7. A portion of workers (5%, as assumed) borrow long term to purchase new houses.
8. SIV cancels its short term debt and the remaining proceeds are used to purchase circulating capital from SII and land.
9. Land owners spend their proceeds on consumer goods.

At the end of the circulation we find the following relevant outcomes:

- Sectors SI, SIII and SIV have cancelled all their short term debt.
- Firms in sector SIIa (those who have previously used new machinery, and that are going to use old machinery in the current production process) will show a monetary surplus which equals in absolute terms the monetary deficit of firms in SIIb (those who have purchased new machinery).
- The monetary efflux corresponding to funding the purchase of new houses is offset with the monetary reflux for the debt service payments (due to the purchase of houses in the past).

4. Stock-Flow Consistent Accounting.

The balance sheet matrix corresponding to a situation where $i = G = R$ is:

Table 1: Balance sheet matrix

	Households	Land owners	Firms	Banks	Σ
Money					
Equities / Bonds					
Capital			$+ K_{(OM)}$		$+ K_{(OM)}$
Outstanding debt	$- L_H$		$- L_{dSII}$	$+ L_S$	0
Houses	$+ p_H \cdot H$				$+ p_H \cdot H$
Land					
Equity	$+ e_{Bd} \cdot p_{eB}$			$- e_{Bd} \cdot p_{eB}$	
Balance (- net worth)	$- V_H$		0	0	$- K_{(OM)} - p_H \cdot H$
Σ	0		0	0	

Regarding households, we see that their only asset here is houses, $p_H \cdot H$, whilst their liability is outstanding debt. Households' net worth is given by the difference between both items. With respect to firms, the value of capital equals outstanding debt. This means that all funds to purchase productive capacity come from banks and that there are no capital gains (this means that firms' net worth is nil). Finally, banks assets equal loans (outstanding debts from firms and household). Contrary to usual practice, in the liability side of the bank balancesheet we do not find bank deposits, since no agent holds deposits in this model. Then, this side is occupied by bank capital which, in turn, is owned by households.
The net worth of the whole system is the sum of the stock of capital plus the stock of houses.

And the corresponding transaction matrix is:

Table 2: Transaction matrix

	Households	Land owners	Firms		Banks		
			Current	Capital	Current	Capital	∑
Consumption	$- C_{dH}$	$- C_{dL}$	$+ C_S$				0
Investment (productive)			$+ I^P_S$	$- I_D$			0
Investment (residential)	$- p_H \cdot \Delta H$		I^R_S				
Wages	$+ W_S$		$- W_d$				0
Firms (net) Profits			$- F_U$	$+ F_U$			0
Banks profits					$- F_B$	$+ F_B$	
Rents		$+ T$	$- T$				0
Interest	$- i_{-1} \cdot L_{dH-1}$		$- i_{-1} \cdot L_{dF-1}$		$+ i \cdot L_{S-1}$		0
Δ loans	$+ \Delta L_{dH}$			$+ \Delta L_{dF}$		$- \Delta L_S$	0
Δ money							
Equity Issues							
∑	0	0	0	0	0	0	

As we see here, GDP can be defined in two forms:

$$
(4.1)\qquad
\begin{aligned}
Y &= C + I^{P} + I^{R} \\
Y &= W + F_{U} + R + i_{-1} \cdot L_{dF-1}
\end{aligned}
$$

That is, GDP can be computed as the sum of final deliveries for consumption, productive investment and residential investment. Alternatively, GDP is the adding up of wages, non distributed firms profits, F_U, rents, R, and interest payments made by firms, $i_{-1} \cdot L_{dF-1}$.

Regarding firms and banks, we distinguish current and capital accounts. Firms' capital account informs about the change of the stock of capital (I_D) and its funding: undistributed profits (F_U) and (change of) bank loans (ΔL_{dF}). Banks current account informs about the origin of its profits and how they are used. Its capital account tells us how much its loans grow (ΔL_S) and what its consequences are. One could be tempted to think that bank profits make it possible for them to increase lending but, in our view, the causality goes the other way around: it is new loans which make it possible to pay interest on outstanding debt and, therefore, to monetize bank profits which lead to increasing bank –financial– capital.

Land owners receive proceeds from the sale of land to the building trade and they spend all proceeds on consumer goods.

And with respect to households, reading by columns again, we see that their spending goes to the purchase of consumer goods and residential investment. Additionally, their funds come from wages and bank loans. And, stating the obvious, it should be taken into account that borrowing today leads to debt service settlements in the future, for a long period of time. The part of debt service corresponding to interest payments is made explicit in the row 'Interest', and the remaining part (principal payments) is combined with new borrowing in the row 'Δ loans'.

When the economy grows *à la* von Neumann, *i* equals *G* and *R*, and all savings are spent and all profits saved and invested, households do not save any money and firms get funds from bank borrowing, and not from financial markets.

5. Integrating IO Tables and SFC Accounting models. First round.

Let us now proceed to integrate both information systems. An alternative may be the following one.

Figure 4

		Intermediate inputs				(Gross) Final deliveries		Total output	Banks
		SI	SIIa+b	SIII	SIV	H + L	F		
SI							NM^1	NM^1	
SIIa+b		Kc_1	Kc_{2a+2b}	Kc_3	Kc_4		$Kc^2 - \sum Kc_i$	Kc	
SIII						$+C^3$		C^3	
SIV						$+H^4$		H^4	
VA	W	W_1	W_{2a+2b}	W_3	W_4	$-W$			
	NOS (profits)	B_1	$NB_{2a+2b} + (NM_{2a} – OM^{2a})$	B_3	B_4		$-F_U$		
	(rents)				T	-T			
	interest		$+i_{-1} \cdot L_{dF-1}$			$+i_{-1} \cdot L_{dH-1}$			$-i \cdot L_{S-1}$
Total output		NM^1	Kc	C^3	H^4				
Δ loans						$-\Delta L_{dH}$	$-\Delta L_{dF}$		$+\Delta L_S$
Loans Households		L_1^H	L_{2a+2b}^H	L_3^H	L_4^H	$- L^H$			$+L^H$
Houses						+ H			
Loans Firms		L_1^F	L_{2a+2b}^F	L_3^F	L_4^F		$-L^F$		$+L^F$
Equity						$+e_{Bd} \cdot p_{eB}$			$-e_{Bd} \cdot p_{eB}$
Capital Stock							$+K_{(OM)}$		
Net worth						V_H	V_F		

Some comments on Figure 4:

- Regarding the block corresponding to an input-output table (as in Figure 2) we have made two main changes. In the final deliveries block, instead of distinguishing by uses (i.e. consumption, investment) we have distinguished by institutional agents: households and firms. And in the VA block, we have divided GOS into three parts: net operating surplus, rents from land and interests paid by firms on bank loans previously granted.
- The block in green joined to the final deliveries block, aims at representing the transaction matrix, as in Table 2). The main difference can be found in the fact that here we do not distinguish between current and capital accounts for firms and banks.
- The block in orange accounts for the balance matrix (as in Table 1).
- Finally, we can see in Figure 4 above that banks, in our simplified economy, do not produce anything.

6. Integrating IO Tables and SFC Accounting models. Second round. Leaving the steady state.

Now we leave the von Neumann path. We shall assume that the interest rate *i* falls below the rate of growth of output. Let us proceed step by step.

First, we comment on the reaction of different agents to this shock and then formalize this behavior in the form of equations, following the tradition of SFC models (the standard reference for this is Godley and Lavoie, 2007. See for instance chapter 7).

<u>Households</u>:

- Those who have purchased a house in the last n-years (with n being the mortgage loan maturity –for simplicity we make n = 5), will have to pay a lower debt service since the same outstanding debt generates a lower interest rate when *i* is lower. This will lead to larger consumption and some savings (because disposable income, after debt service payments, increases).

- The amount of people purchasing a flat when interest rates fall will rise. Also those purchasing a house will borrow larger amounts of money.
- With some delay, the price of houses will rise, because banks will give larger mortgages.
- Those owning a house will spend more on consumer goods because of a wealth effect (the latter measured by the change in the price of the stock of houses, deposits and equity stock).

<u>Land owners</u>:

- The rise in the price of houses makes the rent on land rise as well. We may assume that rentiers consumption rises but in a lower proportion. Therefore, they save a part of the larger rent.

<u>Firms</u>:

- As the demand for houses and consumer goods increases, firms react, in the short run, increasing production raising the degree of use of productive capacity (and with some existing inventories).
- When existing productive capacity *generates* more output, the profit rate rises even with constant prices. This, notwithstanding, more intensive use of productive capacity leads to the shorter useful lifetime of capital stock.
- Firms' outstanding debt generates a lower debt service payment when the interest rate falls.
- Increasing profits and decreasing debt service payments make it easier to either distribute some profits and / or cancel some outstanding debt.
- The value of firms equity stock will rise, reflecting larger profits.
- The funding of fixed capacity becomes less dependent on bank loans, since now firms can collect households and (especially) rentiers savings by issuing assets in financial markets.

The new situation can be reflected in a balance sheet and transaction matrices as follows:

Table 3: Balance sheet matrix when i < g

	Households	Land owners	Firms	Banks	Σ
Money	$+ M_H$	$+ M_L$		$- M_S$	
Equities / Bonds	$+ p_e \cdot E_{dH}$ $+ e_{Bd} \cdot p_{eB}$	$+ p_e \cdot E_{dL}$ $+ e_{Bd} \cdot p_{eB}$	$- p_e \cdot E_S$	$- e_{Bd} \cdot p_{eB}$	
Capital			$+ K_{(OM)}$		$+ K_{(OM)}$
Outstanding debt	$- L_H$		$- L_{dSII}$	$+ L_S$	0
Houses	$+ p_H \cdot H$				$+ p_H \cdot H$
Land					
Balance (- net worth)	$- V_H$	$- V_L$	0	0	$- K_{(OM)} - p_H \cdot H$
Σ	0		0	0	

Table 4: Transaction matrix when i < g

	Households	Land owners	Firms Current	Firms Capital	Banks Current	Banks Capital	$\sum$
Consumption	$- C_{dH}$	$- C_{dL}$	$+ C_S$				0
Investment (productive)			$+ I^P_S$	$- I_D$			0
Investment (residential)	$- p_H \cdot \Delta H$		$+ I^R_S$				
Wages	$+ W_S$		$- W_d$				0
Firms (net) Profits	$+ F_{DH}$	$+ F_{DL}$	$- F$	$+ F_U$			0
Banks profits					$- F_B$	$+ F_B$	
Rents		$+ T$	$- T$				0
Interest	$- i_{-1} \cdot L_{dH-1}$		$- i_{-1} \cdot L_{dF-1}$		$+ i \cdot L_{S-1}$		0
Δ loans	$+ \Delta L_{dH}$			$+ \Delta L_{dF}$		$- \Delta L_S$	0
Δ money	$- \Delta M_{dH}$	$- \Delta M_{dL}$				$+ \Delta M_S$	0
Equity Issues	$- p_e \cdot \Delta E_{dH}$	$- p_e \cdot \Delta E_{dL}$		$+p_e \cdot \Delta E_S$			0
$\sum$	0	0	0	0	0	0	

Regarding Table 3, the main changes with respect to Table 1 are that now:
- Households hold money (bank deposits) and equities issued by firms. The same for land owners.
- Firms can collect funds through the issue of equity or bonds in financial markets.
- Banks have deposits on the liability side of their balances.

And with respect to Table 4, the novelties are:
- Households receive some distributed profits from firms (F_{DH}); they may save so that they hold these savings in the form of money (ΔM_{dH}) and equities ($\Delta p_e \cdot E_{dH}$).
- Land owners receive some distributed profits (F_{DL}); when they do not spend all their proceeds on consumer goods, they save and hold these savings in the form of bank deposits and equities.
- Firms distribute part of their profits. Now, part of investment may be funded through the issue of securities in financial markets.[4]
- Banks have deposits on the liability side of their balance sheet. This means that their capital adequacy ratio (the quotient between bank capital –V_B– and –risky– assets) falls. This happens because bank capital grows at a rate ruled by the interest rate whilst assets grow at a rate ruled by the rate of growth of output.

Next we shall shift this situation to a table like the one represented in Figure 4.

[4] Actually, and for simplicity's sake, we shall assume firms ask banks for long term loans to fund the purchase of fixed capital. The difference between profits and debt service payments is distributed to shareholders.

Figure 5

		Intermediate inputs				(Gross) Final deliveries		Total output	Banks
		SI	SIIa+b	SIII	SIV	H + L	F		
SI							NM^1	NM^1	
SIIa+b		Kc_1	Kc_{2a+2b}	Kc_3	Kc_4		$Kc^2 - \sum Kc_i$	Kc	
SIII						$+C^3$		C^3	
SIV						$+H^4$		H^4	
VA	W	W_1	W_{2a+2b}	W_3	W_4	$-W$			
	NOS (profits)	B_1	$NB_{2a+2b} + (NM_{2a} - OM^{2a})$	B_3	B_4	$-F_{DH+L}$	$-F_U$		
	(rents)				T	$-T$			
	interest		$+i_{-1} \cdot L_{dF-1}$			$+i_{-1} \cdot L_{dH-1}$			$-i \cdot L_{S-1}$
Total output		NM^1	Kc	C^3	H^4				
Δ loans						$-\Delta L_{dH}$	$-\Delta L_{dF}$		$+\Delta L_S$
Δ money						$-\Delta M_{dH+L}$			$-\Delta M_S$
Equity issues						$-\Delta p_e \cdot e_{dH+L}$	$+\Delta p_e \cdot e_S$		
Loans Households		L_1^H	L_{2a+2b}^H	L_3^H	L_4^H	$-L^H$			$+L^H$
Houses						$+H$			
Loans Firms		L_1^F	L_{2a+2b}^F	L_3^F	L_4^F		$-L^F$		$+L^F$
Money						$+M_{H+L}$			$-M_S$
Equities						$+p_F \cdot e_{dH+L}$ $+e_{Bd} \cdot p_B$	$-p_F \cdot e_{FS}$		$-e_{BS} \cdot p_B$
Capital Stock							$+K_{(OM)}$		
Net worth						V_{H+L}	V_F		$-V_B$

7. The equations of the model

Following the usual practice in SFC models, we now provide a set of behavioral equations, explaining the evolution of components of aggregate demand.

For the sake of simplicity, we shall assume a situation where $i < g$ (see appendix 2) and where financial markets are irrelevant. The former assumption means that depreciation funds are larger than loan instalments (corresponding to the purchase of fixed capital) which firms have to settle at due dates. Hence, firms are able to distribute part of their profits. The second assumption involves assuming that firms do not collect funds in financial markets in order to finance the purchase of new fixed capital. In order to avoid unnecessary problems (regarding the central aim of this paper, of course) we shall assume that all funds required to monetize long term productive investment are obtained from banks, and the profits above debt service payments are distributed to shareholders. The irrelevance of financial markets means that shares are not exchanged and, therefore, their price does not change. The ownership of shares gives the right to obtain distributed profits, but when the price of shares does not change there is no capital gain on these assets.

Further, we shall assume that the propensity to consume of households and land owners is, by and large, lower than one. However, this does not mean that sometimes they cannot spend more than their proceeds: they (particularly households) can obtain funds from banks, as firms do.

Also, we have assumed that prices are determined *à la* Sraffa, as stated above. Contrary to the usual practice with SFC models, we do not believe that the interest rate rules the profit rate. For simplicity, we shall take the latter from the outside and keep it constant.

Households and Land owners. Consumption.
Households spend on consumer goods:[5]
(7.1) $C^H = C^H{}_0 + \alpha_0 (W_{-1} - DSP_{-1}) + \alpha_1 \Delta V_{H-1}$

That is, household consumption depends on two factors. First, current income (wages, with one lag), minus debt service payments, generated by borrowing to fund the purchase of houses in the past. Second, changes in wealth (with one lag as well), represented here by net value. We include an intercept $C^H{}_0$ as well (see Godley and Lavoie, *op.cit*. p. 225).
Wages are given by the expression:
(7.2.A) $W = w{\cdot}l{\cdot}Y$

Or:
(7.2) $W = Y - L - F - i_{-1} L_{F-1}$

Where w is the nominal wage (given from the outside), l is the inverse of productivity (and can be understood as the total labour requirements per unit of final output), and Y is final output.
Regarding debt service payments, as stated above, we assume that the price of one house is ruled by the amount of money that a bank is willing to lend. And, in turn, this depends on the present value of the amount of income that the purchaser can (in the opinion of the bank) set aside to settle debt service payments. Hence, for any given nominal income w if the bank believes that the purchaser of a house can set apart a percentage income, z (for instance, 30%) then the price of a house becomes:
(7.3) $p_H = z{\cdot}w{\cdot}\dfrac{(1+i)^n - 1}{i(1+i)^n}$

Where n is the maturity of the mortgage loan. In this paper we shall assume 5 periods of time. Of course then:
(7.4) $DSP = \sum_{i=1}^{5} z_{-i} {\cdot} w_{-i} {\cdot} \lambda_{-i} L_{-i}$

Where λ_{-i} stands for the fraction of workers who purchased a new house in period $-i$.
Household net worth is:
(7.5) $V_H = p_H H + M_H - L_H$

Net value is given by the amount of houses, plus money minus outstanding debt.
We are implicitly assuming that household savings are held as money (i.e. bank deposits), and they do not hold any equities, so that they do not receive any distributed profits.
Regarding land owners:

[5] Note that, in this expression, all factors are in real terms. This means that, in each period, all factors are deflated by a consumer price index.

(7.6) $C^L = \beta_0(T_{-1} + F_{D-1}) + \beta_1 \Delta V_{L-1}$

Land owners consumption is a fraction β_0 of rent T one period earlier plus a wealth effect.
Rent can be stated as the difference between the price of a house minus its building cost, c, times the number of houses built in the period of reference.
(7.7) $T = (p_H - c)\Delta H$

Land owners wealth (net worth) is:
(7.8) $V_L = M_L + e_{FL} \cdot p_F + e_{BL} \cdot p_B$

Under the (restrictive, but simplifying) assumption $\alpha_1 = \beta_1$, we get:

(7.9)
$$
\begin{aligned}
C &= C^H + C^L = \alpha_0(W_{-1} - DSP_{-1}) + \alpha_1 \Delta V_{H-1} + \beta_0(T_{-1} + F_{D-1}) + \beta_1 \Delta V_{L-1} \\
C &= \alpha_0(W_{-1} - DSP_{-1}) + \beta_0(T_{-1} + F_{D-1}) + \alpha_1 \Delta V_{H+L-1}
\end{aligned}
$$

Also, net value of households and land owners is:
(7.10) $V_{H+L} = p_H H - L_H + M_{H+L} + (e_B \cdot p_B + e_F \cdot p_F)$

Regarding the change of wealth held by household and land owners:

(7.11)
$$
\begin{aligned}
\Delta V_{H+L} &= (W - DSP - C^H) + (T + F_D - C^L) + CG; \\
\Delta V_{H+L} &= \Delta M + \Delta(H \cdot p_H)
\end{aligned}
$$

In other words, wealth V increases because of savings out of current income plus capital gains, GC. This increase in wealth, in turn materializes into an increase in money held and the rising value of houses.[6] (Note that the price of equities does not change because there is no stock market).

Households. Residential investment.
The demand for houses, residential investment, fits the following simple expression:

(7.12)
$$
\begin{aligned}
I^H &= \Delta H = L_{-1}\lambda \\
\lambda &= \gamma_0\left(1 - \gamma_1 \Delta i_{-1} - \gamma_2 i_{-1} + \gamma_3\left(1 - \frac{OD^H{}_{-1}}{W_{-1}}\right)\right)
\end{aligned}
$$

The demand for houses is a fraction (λ) of current employment, but this fraction depends negatively on levels and first differences of the interest rate with one lag, and on the amount of outstanding debt, OD^H over wages..
Next, we shall assume that the purchase of new houses is completely funded by mortgage loans.
The change of loans to households equals the increase in the number of houses (valued at current prices) minus the payment of principal on outstanding debts (φL_{H-1}).
The total for existing houses is:

[6] For the sake of simplicity, we shall assume that house purchasers do not cancel outstanding debt before due dates. Hence, all savings are held in the form of money. So the only capital gains come from the rising price of houses.

$$(7.13)\quad H = H_{-1} + I^{H}$$

Firms. Productive investment.
Investment encapsulates the demand for circulating capital plus machinery (fixed capital).
We shall assume an investment function for firms:[7]

$$(7.14)\quad I^{F} = \sigma_0 Y_{-1} + \rho\left(K^{T} - K_{-1}\right) + D$$

The first term on the right hand side of the equation above, $\sigma_0 Y_{-1}$ informs about the investment demand of circulating capital. The second one, $\rho\ (K_{-1} - K_{-1}^{T})$ accounts for the demand for new machinery, where ρ is a parameter informing about the speed with which firms shift from the existing stock of capital towards target one (K_{-1} and K_{-1}^{T}, respectively). D is depreciation of capital.
And regarding this expression, we add:[8]

$$K^{T} = \upsilon Y_{-1}$$
$$(7.15)\quad D = \delta K_{-1}$$
$$DA = D\left(1 + g_{-1}\right)$$

The target stock of capital is given by the level of output Y_{-1} times the accelerator, υ (the capital output ratio). Next, DA stands for depreciation allowances, or amortization funds, which give us the amount of money which users of fixed capital have accumulated to settle debt services (principal plus interest on outstanding debt, incurred within the funding of the purchase of productive capacity).
How is firms' investment funded?
Here we face two questions. First, what portion of investment is financed out of profits and second, which part of external funds is collected from banks? In our highly simplified economy, the purchase of new fixed capital is funded with retained profits and long term bank borrowing. The former covers the value corresponding to the first year of useful lifetime, UL, of the new machinery NM. The rest is funded with bank borrowing. In our example, where the lifetime of machinery is two periods, firms borrow long term an amount of value given by the value of used machinery. Distributed profits, F_D, equal total profits minus depreciation times one plus the interest rate, minus the fraction of profits which goes to fund current investment:

$$(7.16)\quad \Delta L_F = \left(\frac{UL-1}{UL}\right)NM = \left(\frac{UL-1}{UL}\right)\rho\left(K^{T} - K_{-1}\right)$$

$$(7.17)\quad F_D = F - D\left(1 + i_{-1}\right) - \left(\frac{1}{UL}\right)\rho\left(K^{T} - K_{-1}\right)$$

And profits are calculated in a simple manner as follows:[9]

$$(7.18)\quad F = r\left(K + D\right)$$

Amortization funds are, of course, generated along with the lifetime of the stock of capital. They have to grow in order to replace scrapped capital growing at a rate g_K. But,

[7] See Godley and Lavoie, 2007, chapter 7.
[8] In our case, where the useful lifetime of the stock of capital is 2 periods, we can assume $\delta = 1/(3+2g_{K-1})$, with g_K being the rate of growth of the stock of fixed capital.
[9] Here we have assumed that profits F include interest on debt as well.

in our model they are used to pay debt services, generated by indebtedness for the funding of the purchase of productive capacity. And that debt which has to be cancelled (in instalments) at due dates, generates an interest *i*.

The stock of productive capital is:

(7.19) $K = K_{-1} + I^F - AD$

And output is:

$$Y = C^H + C^L + I^R + I^P$$

(7.20) $$pY = p_C\left(C^H + C^L\right) + p_H I^R + p_K I^P$$

$$pY = W + T + F$$

8. Back to the input-output model

The equations of the SFC model provide us with useful information about the evolution of components of GDP.

Now, we can write a quantity system close to input-output usual practice, i.e. as in Figure 3 as follows:

$$\mathbf{A}^*\mathbf{x_t} + \mathbf{y_t} = \mathbf{x_t}$$

(8.1) $$\mathbf{y_t} = \begin{pmatrix} NM \\ Kc \\ C \\ H \end{pmatrix}$$

And then, we have:

(8.2) $$\mathbf{x_t} = \left(\mathbf{I} - \mathbf{A}^*\right)^{-1} \begin{pmatrix} NM_0\left(1 + g_{NM}\right) \\ Kc_0\left(1 + g_{Kc}\right) \\ C_0\left(1 + g_C\right) \\ H_0\left(1 + g_H\right) \end{pmatrix}$$

Where matrix $\mathbf{A}^*$ includes depreciation of fixed capital and socially necessary consumption. This we shall call a socio-technical matrix. Vector $\mathbf{y_t}$ encapsulates net output and it changes in time as determined in the set of equations stated in the section above.

We have made a simulation (see Appendix 2 below) with the model developed in the above section. The following pictures show the rates of growth of the different commodities in the vector of final deliveries:

Figure 6

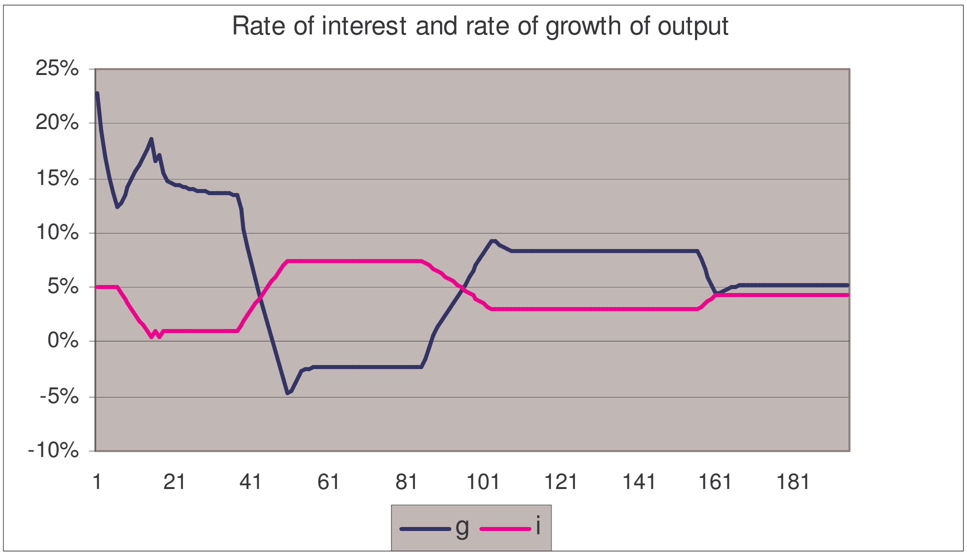


Figure 7

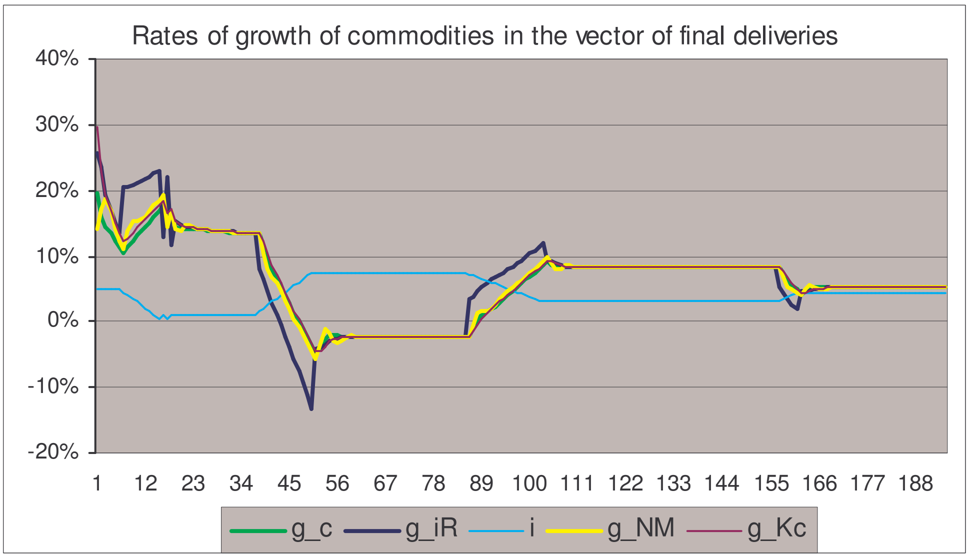


Figure 8

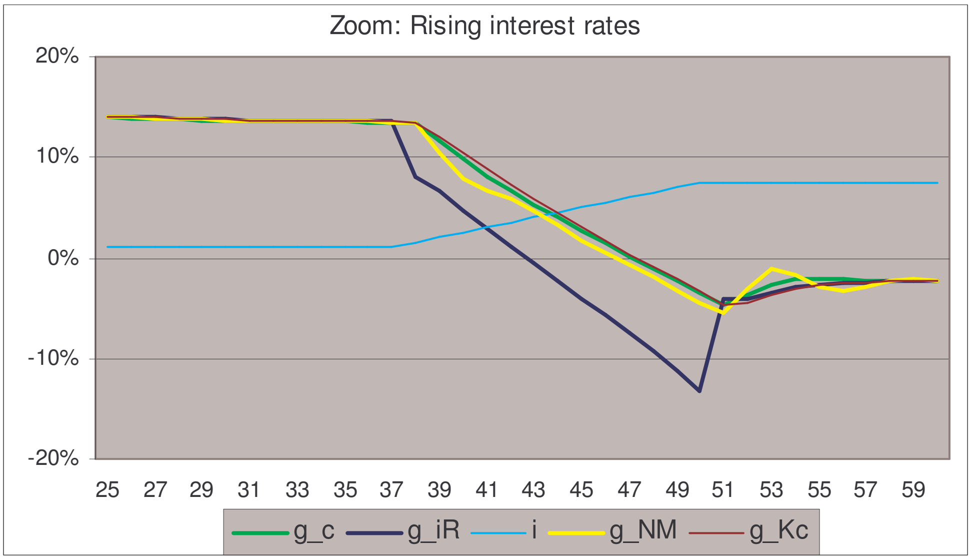
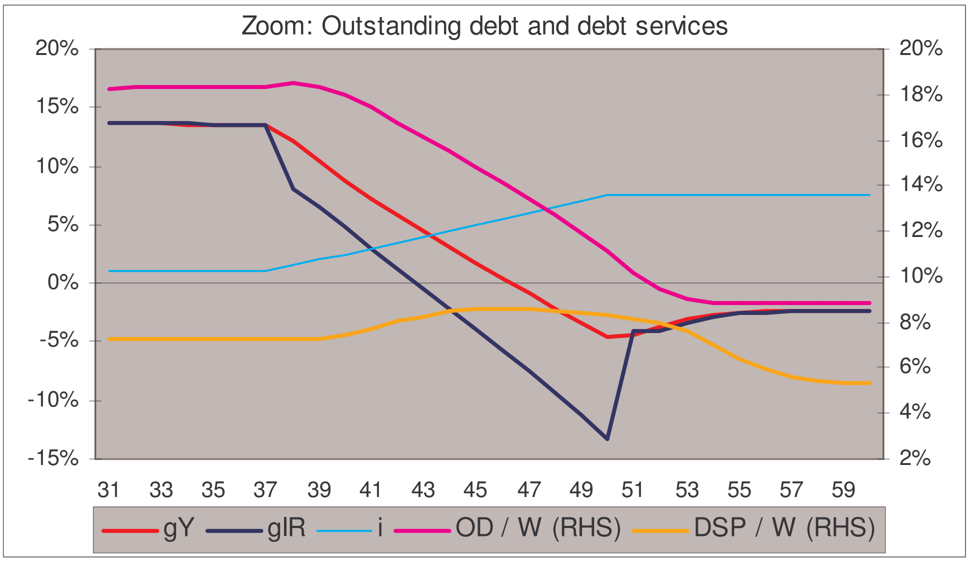


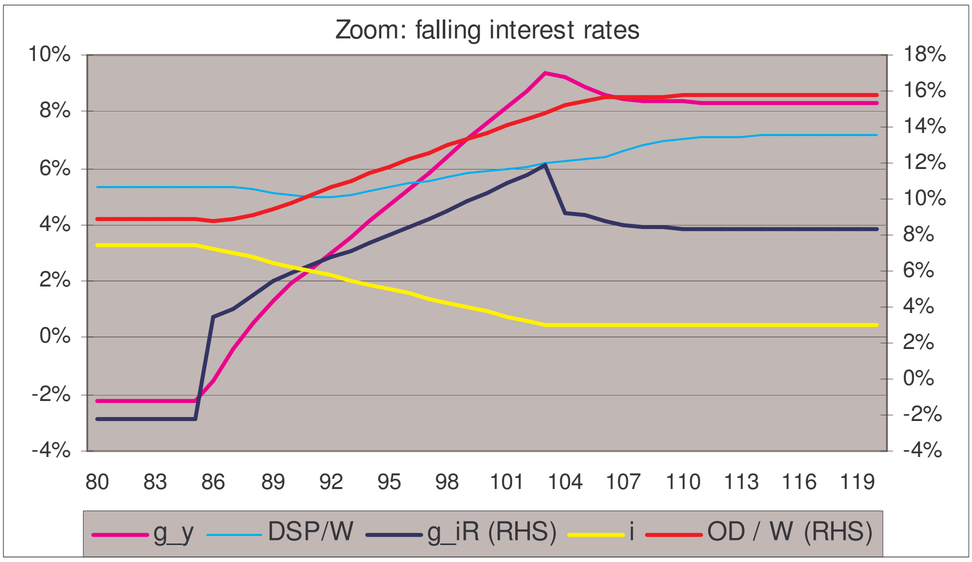
Figure 9



What we show here is that, when the interest rate rises (see figures 8 and 9 which are a zoom-in of what is shown in figures 6 and 7), residential investment falls and this pulls down the rate of growth of output, thus making consumption and investment fall as well. In figure 9, we see that the rise in the interest rate also makes debt service payments relative to wages (DSP / W), rise temporarily as well thus making consumption fall further. Also, the fall in residential investment makes the outstanding debt of households relative to wages (OD / W) fall. When it falls, although the interest rate is now at a relatively higher level, (DSP / W) falls as well, in the long term.

Note also that, when the interest rate remains at too high a level, the rate of growth of output stabilizes around a lower value (see figure 6) and it is when the former falls that the rate of growth increases (figure 10 below). Again, recovery comes from the residential investment sector. Note also that when residential investment rises, outstanding debt rises as well, but, temporarily, debt service payments over wages fall, despite the rising amount of debt, because of the effect of falling interest rates.

Figure 10



Regarding matrix $\mathbf{A}^*$, we find at least two difficulties. First, how to distinguish between *normal* (i.e. a socially necessary consumption, notion used by the English Classics and Marx) and *above normal* consumption. Second, how to define the coefficient of depreciation. With respect to the first question, some doubts arise: should dwellings be considered as part of the matrix of inputs? What happens with forced savings used to pay debt services by households? And with regard to the second question, by and large, the depreciation coefficient is calculated as the deflated value (number) of the fixed capital input (say, machinery), divided by the number of useful lifetime periods under normal utilization conditions and finally, the previous outcome divided by the deflated value (number) of output which it contributes to produce. However, when the degree of utilization of productive capacity rises, then it is reasonable to assume that the lifetime of fixed capital shortens, thus making the depreciation coefficient rise.

9. Conclusion

We have developed a dynamic multisectoral model, combining elements from several theoretical strands. Input-output models provide a sound account of the modes of production, the composition of output, distribution and, implicitly, prices. The Theory of the Monetary Circuit gives us a correct treatment of how (endogenous) money puts in motion the wheels of production. Finally, Stock Flow Consistent modelling explains where the money comes from and where it goes after transactions have been monetized. In this paper we have aimed at exploring a synthesis of rather fruitful elements,when they are taken in isolation but, which prove, in our view, far more fruitful when put altogether.

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Appendix 1. A numerical illustration of the simplest case

Let us give figures to symbols in Figure 1

Table 5

	Inputs							Outputs				
	T	NM	OM	Kc	C	L		NM	OM	X_2	X_3	H
SI				18	10	8		1				
SIIa		1		18		6			1	35		
SIIb			1	18		6	→			35		
SIII				10	5	10					60	
SIV	1			10	10	5						25

Note that here we assume that the consumer good is also used as a material input.
And once we define the wage basket, as in expression (3.2):

Table 6

	Inputs								Outputs				
	T	NM	OM	X_2	X_3	X_3^C	H		NM	OM	X_2	X_3	H
SI				18	10	8	0,4		1				
SIIa		1		18		6	0,3			1	35		
SIIb			1	18		6	0,3	→			35		
SIII				10	5	10	0,5					60	
SIV	1			10	10	5	0,25						25

Then matrices **A** and **B** are:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 18 & 18 & 18 & 10 & 10 \\ 18 & 6 & 6 & 15 & 15 \\ 0.4 & 0.3 & 0.3 & 0.5 & 0.25 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 35 & 35 & 0 & 0 \\ 0 & 0 & 0 & 60 & 0 \\ 0 & 0 & 0 & 0 & 25 \end{pmatrix}$$

The rate of profit, r, the rate of growth, g, and the rate of interest, i, are (according to expressions (1) and (4)):

$$r = g = i = 0.0783 = (1/\lambda_{\max(AB^{-1})} - 1)$$

$$\lambda_{\max(AB^{-1})} = 0.927386$$

And the vectors **p** and **q** are, according to expressions (1), (4):

$$\mathbf{p} = \begin{pmatrix} 97{,}0152 & 50{,}3351 & 3{,}9456 & 1 & 2{,}3744 \end{pmatrix}$$

$$\mathbf{q} = \begin{pmatrix} 1 \\ 0{,}9274 \\ 62{,}5601 \\ 43{,}6603 \\ 1{,}4172 \end{pmatrix}$$

The last component of vector **p** accounts for $p_H^{\ *}$, that is, the *production price* of one house, excluding the price of land required to build this house. According to (3) the nominal wage is:

$$w = 1{,}1187$$

And, therefore, the maximum amount which can be devoted to paying each mortgage instalment (30% w,):

$$mi = 0{,}3356$$

Next, once i is determined and the mortgage maturity amounts, *ex hipotesi*, to 30 periods of time, the price of one house p_{HT} (which includes the price of land required to produce it) and the price of one unit of land (where 25 houses can be built) are, determined by expressions (4)-(5):

$$p_{HT} = 3{,}8397$$
$$p_T = 36.6330$$

Now, if we multiply each element in Table 6 its corresponding price, and make the system expand according to von Neumann's proportions, we have for periods t and t+1:

Table 5

					period t							
	Inputs							Outputs				
	T	NM	OM	X_2	X_3	L		NM	OM	X_2	X_3	H
SI				71,021	10	8,9498		97,0152				
SIIa		89,9705		65,864		6,2249			46,6801	128,068		
SIIb			43,2905	61,081		5,7729	→			118,769		
SIII				28,711	3,6384	8,1406					43,6603	
SIV	2.0766			2,2366	0,5669	0,3178						5,4415

					period t+1							
	Inputs							Outputs				
	T	NM	OM	X_2	X_3	L		NM	OM	X_2	X_3	H
SI	0	0	0	76,582	10,783	9,6506		104,6115	0	0	0	0
SIIb	0	97,0152	0	71,021	0	6,7123		0	50,3351	138,096	0	0
SIIa	0	0	46,6801	65,864	0	6,2249	→	0	0	128,068	0	0
SIII	0	0	0	30,959	3,9232	8,7780		0	0	0	47,0789	0
SIV	2, 2392	0	0	2,4118	0,6113	0,3419		0	0	0	0	5,8676

As stated above, our analysis is placed just after output corresponding to period t has already been produced and before the next production process starts. The circulation of money makes the *transformation* of outputs into inputs in period t +1 possible. Simultaneously, surplus distribution occurs (both wages and profits are monetized).

Before money enters the system, sector SIIa has old machinery which can be used during t+1. This implies that it has to pay a second instalment back to the bank corresponding to the purchase of machinery one period before. Additionally, workers / households are indebted to banks because of the purchase of houses in the past. We shall assume that banks agree to grant mortgages with a maturity no longer than 30 periods of time (say, years) resulting in instalments no larger than 30% of household income (the wage corresponding to one unit of labour). Hence, workers will have to pay mortgage instalments for the purchase of houses during the next 30 years.

Workers / households and land owners have a nil propensity to save, whilst for capitalists it equals unity.

1. Firms borrow short term 22.9297 monetary units (m.u. onwards) to pay wages for workers in sectors SI, SIIb, SIIa and SIV. They use part of these proceeds to pay back mortgage instalments and the rest goes to the purchase of consumer goods. With proceeds, sector SIII pays wages (amounting to 8.7780 m.u.) which, in turn, are used to pay mortgage instalments and to purchase more consumer goods. The amount which workers set aside for mortgage instalments can be calculated as follows. We assume that each period of time, 5% of the labour force purchases a new house. Current purchasers will have to pay instalments during the next 30 periods of time. Therefore in the present period, the amount of workers / households income which is used to pay back households bank debt is:

   $$DSP_{H,t} = mi \cdot z \cdot \sum_{i=1}^{30} L_{t-i} = mi \cdot 0.05 \cdot L_t \cdot \left(\frac{(1+g)^{30} - 1}{g(1+g)^{30}} \right) = 5.4414 \qquad (5)$$

   Where *mi* accounts for the mortgage instalment that the purchaser of a house has to pay during 30 years and L_t is the amount of labour hired in period t.[10] We assume that the rate of growth of the economic system remains constant and the same holds for the interest rate, each mortgage instalment and the proportion of labour force purchasing a house each period of time.

   Once we account for the amount of workers's income going to pay bank debt, what remains for the purchase of consumer goods is 26.2655 m.u.

   Sector IIa can start its production process: it has old machinery, X_2 (produced by itself) and labour.
2. Sector SIII (the producer of consumer goods) has obtained 26.2655 and spent 8.7780. Now it requires 30.959 m.u. of circulating capital (commodity X_2). Therefore SIII has to ask for a short term credit amounting to 13.4716 m.u. We assume that SIIa sells X_2 to SIII. Now, sector SIII can start its production process.
3. SIIb needs, apart from labour, new machinery. It borrows 97.0152 m.u. for its purchase. This credit matches maturity with the lifetime of the machine (in our example, two periods of time). Now it can start its production process.
4. SI sells machinery to SIIb. The proceeds are used to purchase 76.582 m.u. of commodity X_2, 10.783 m.u. of commodity X_3, and to pay back the short term

[10] In period t total labour amounts to 28.3428.

debt corresponding to the payment of wages. SIIa sells 31.2455 m.u. of X_2 to SI; the rest, 45.336 m.u. are sold by SIIb. SI can start its production process.

5. The corresponding 5% of the labour force has to purchase new houses. They ask for mortgages whose maturity amounts to 30 periods of time. They ask for long term debt 5.4423 m.u. It should be noted that this amount equals mortgage instalments paid during this period of time (see point 1 above).
6. SIV sells houses to workers / households. Then it pays 2.0766 m.u. to land owners who spend their proceeds on consumer goods, purchases 2.4118 m.u. of X_2 from SIIb, 0.6113 m.u. of X_3 from SIII and pays back short term bank debt amounting to 0.3419 m.u. (corresponding to the payment of wages for the new production process). SIII obtains liquidity to pay back its short term loan (point 2).

Once money circulation stops, we can see that the balance of bank accounts for sectors SI, SIII and SIV becomes nil. Land owners' savings *ex hipotesi* amount to zero as well. Workers / households remain indebted for the next 30 years. SIIb has a debt amounting to 57.9797 m.u. Simultaneously, SIIa has a surplus of 57.9797 m.u.: this equals, in absolute value, the deficit of SIIb. We interpret this surplus as what SIIa has to pay the bank for the second instalment corresponding to the purchase of new machinery one period before.
In this example the column for banks in Table 1 becomes:

Table 6

Private banking sector			
Assets		Liabilities	
55.9797	Long term credit to SIIb	Equity	111.939
55.9593	Mortgages to households		

The account 'Mortgages to households' accounts for total debt due for payment and which generates an interest. And the account 'Long term credit to SIIb' encapsulates outstanding debt that SIIb has to pay back, plus interest on it, the next year.

Appendix 2. The equations of the SFC Model

The behavioral equations used in the simulation can be written in a summarized form:

(A.2.1) $Y = C + I^H + I^P = C^H + C^L + I^H + I^P$

(A.2.2) $$\begin{aligned} C &= C_0 + \alpha_0 (W_{-1} - DSP_{-1}) + \beta_0 \left((p_{H-1} - c)\Delta H_{-1} + F_{D-1} \right) + \\ &+ \alpha_1 \left((W_{-1} - C_{H-1} - DSP_{-1}) + \left((p_{H-1} - c)\Delta H_{-1} + F_{D-1} - C^L{}_{-1} \right) + \Delta p_{H-1} H_{-1} \right) \end{aligned}$$

(A.2.3) $I^H = L_{-1}\gamma_0 \left(1 - \gamma_1 i_{-1} + \gamma_2 \left(1 - \frac{OD^H{}_{-1}}{W_{-1}} \right) \right)$

(A.2.4) $I^P = \tau Y - 1 + \rho\left(K^T - K_{-1}\right) + D$

(A.2.5) $K^T = vY_{-1}$

(A.2.6) $K = K_{-1} + I - D$

We define wages as:

(A.2.5) $W = pY - T - F = wL = w\frac{Y}{pr}$

And profits are:

(A.2.6) $F = r\left(K_{-1} + D_{-1}\right)$

(A.2.7) $D = \delta K_{-1}$

(A.2.8) $F_D = F - D\left(1 + i_{-1}\right) - \frac{1}{2} NM$

Rents:

(A.2.9) $p_H = \sum_{t=1}^{T} \frac{z \cdot w}{(1+i)^t}$

The parameters used for simulations are:

$C_0 = 10$

$\alpha_0 = 0.725$

$\beta_0 = 0.2$

$\alpha_1 = 0.025$

r = 0.5

pr = 1 / 0.275

v = 0.6

$\rho = 0.5$

$\delta = 0.31$

$\tau = 0.1$

z = 0.3

T = 5

c = 0.187

$\gamma_0 = 0.075$

$\gamma_1 = 10$

$\gamma_2 = 0.2$