The use of supply-use tables for the identification of key sectors using unbiased input-output multipliers

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Abstract

From an axiomatic point of view, Kop Jansen and ten Raa (1990) and Rueda-Cantuche and ten Raa (2009) singled out the product technology and the fixed industry sales structure assumptions as the best two models for the construction of either product or industry input-output tables, respectively. However, there is one hard to neglect criticism that has prevented them for a more widespread use in input-output analysis, i.e. the resulting negative coefficients. At this point, this paper proves that under these two assumptions, unbiased and consistent backward and forward input-output multipliers can be respectively estimated econometrically from supply and use tables instead of from input-output tables. The advantages of our econometric approach are twofold, i.e.: not only it circumvents the problem of negatives but also provides unbiased multipliers. We hope this paper allows for a more general use of the two axiomatically best methods in input-output analysis. As an example, this paper analyses the repercussions of the estimated bias in the determination of the key sectors of an economy, as postulated by Rasmussen (1956). As shown in the paper, the estimated bias may induce to wrongly identify key sectors in the Turkish economy for the year 1998.

Keywords: Stochastic input-output analysis, input-output multipliers, key sectors, input-output linkages, supply and use tables.

Note: The views expressed in this paper belong to the authors and should not be attributed to the European Commission or its services.

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1. Introduction

Recent contributions in input-output analysis (e.g. Rueda-Cantuche and Amores, 2010) have shown that unbiased and consistent backward input-output multipliers can alternatively be calculated using supply-use tables and econometric techniques; in contrast to the standard Leontief inverse based method. The elements of the Leontief inverse have proven to be biased (e.g. Simonovits (1975), Kop Jansen (1994), Dietzenbacher (1995, 2006) and Roland-Holst (1989), among others) and therefore, the output multipliers (ten Raa and Rueda-Cantuche, 2007), too.

In the input-output literature, backward and forward multipliers are well known and mainly used together for the identification of key sectors in an economy (Dietzenbacher (2002) and Oosterhaven (1988), among others). Although there is an unsolved controversy (Oosterhaven, 1988, 1989; Miller and Lahr, 2001; and Mesnard, 2009) on the methods proposed so far in the literature with that purpose, this paper will not discuss this issue but rather the consequences of the bias introduced by the use of the Leontief inverse and the Ghosh inverse in the Rasmussen's (1956) approach for the identification of key sectors. We do not aim to single out the best method to make such identification but rather to illustrate the errors arising from the use of the standard Leontief and Ghosh inverses to do the job.

Unfortunately, the use of supply-use tables and a similar econometric approach to that of Rueda-Cantuche and Amores (2010) in order to estimate unbiased and consistent forward input-output multipliers has not been developed yet to allow us completing the Rasmussen's approach for the identification of key sectors. Therefore, this paper fills the gap and presents this as its first major contribution. Subsequent findings will raise the issue of the potential use of the product technology assumption and the fixed industry sales structure assumption not only for the construction of input-output tables but for input-output based impact studies. Secondly, we will explore how the estimated bias of backward and forward multipliers will affect the identification of key sectors for the Turkish economy in 1998. In addition, this paper also generalizes the calculation of backward and forward multipliers from supply-use tables and using econometric techniques.

The main purpose of the next section is to present briefly the theoretical background of the two most commonly and broadly used models in input-output analysis, i.e. the Leontief quantity model and the Ghosh price model, which are the basis for the standard calculation of backward and forward multipliers, respectively. It will follow a summary discussion on the use of them to identify key sectors in an economy brought about by a brief review of the main related literature. Then, we will introduce the econometric approach initially proposed by ten Raa and Rueda-Cantuche (2007) to estimate unbiased and consistent backward and, as a new contribution, forward multipliers. We will discuss briefly the main implications of our findings in relation to the use in practice of the product technology assumption and the fixed industry sales structure model for input-output impact analysis. A discussion on the effect of the estimated bias over the identification of key sectors will follow. Next, the last section concludes.

2. The Leontief quantity model and the Ghosh price model

Dietzenbacher (1997) considered the following input-output table (IOT) in monetary terms for period 0:

X_0	f_0	x_0
v_0^{T}	-	$v_0^{\mathrm{T}}e$
x_0^{T}	ef_0^{T}	

X_0 is the n x n matrix of intermediate uses; its typical element x_{ij}^0 denotes the value (in euros) of the deliveries from industry i to industry j, assuming an industry by industry input-output table. The column vector f_0 can be interpreted as final demand of industry's outputs including private and government consumption, investments and net exports. The row vector v_0^{T} provides the value added in each industry, containing, for instance, payments for the labour and capital primary factors. The value of each industry output is given by the elements of the vector x_0 while e denotes the n-dimension column vector of ones. Column-wise, an IOT depicts input structures and row-wise, output structures. Since the total value of outputs is equal to the total value of inputs, for each industry, the following sets of accounting equations can be defined:

$$x_0 = X_0 e + f_0, \tag{1}$$

$$x_0^{\mathrm{T}} = e^{\mathrm{T}} X_0 + v_0^{\mathrm{T}}. \tag{2}$$

It follows that the input coefficients are defined as the industry *i*'s input into industry *j* as a fraction of the purchaser's output (x_j^0). They are derived as $a_{ij}^0 = x_{ij}^0 / x_j^0$, or in matrix terms, as $A_0 = X_0 \hat{x}_0^{-1}$ where $\hat{x}_0$ stands for a diagonal matrix. Therefore, equation (1) may be written as:

$$x_0 = A_0 x_0 + f_0. \tag{3}$$

Similarly, the output coefficients denote the industry *i*'s delivery to industry *j* as a proportion of the seller's output (x_i^0). These are calculated as $b_{ij}^0 = x_{ij}^0 / x_i^0$ or, in matrix terms, as $B_0 = \hat{x}_0^{-1} X_0$. As a result, equation (2) may be re-specified as

$$x_0^{\mathrm{T}} = x_0^{\mathrm{T}} B_0 + v_0^{\mathrm{T}}. \tag{4}$$

From the accounting equations (3) and (4), it is usual to obtain the so called **Leontief quantity model** and the **Ghosh price model**, respectively. Other models are the Leontief price model and the Ghosh quantity model that are not frequently discussed in the input-output literature except for Dietzenbacher (1997) where the reader can find a more comprehensive description.

Leontief Quantity Model

Equation (3) relies on the assumption of fixed technical coefficients being the new industry output vector (x_1) required for an exogenously specified new final demand vector (f_1) such that,

$$x_1 = (I - A)^{-1} f_1. \tag{5}$$

Provided a shock in the physical amount consumed of a bundle of products produced by a certain industry, both primarily and secondarily produced, by final users, then the effect on the total output value of the industry output is given by x_1. Notice that in the **Leontief quantity model** prices do not change.

Ghosh Price Model

Equation (4) rests on the assumption of fixed output coefficients. For a new value added vector (v_{2}^{T}), the new total output values are obtained by,

$$x_{2}^{\mathrm{T}} = v_{2}^{\mathrm{T}} \left(I - B_0\right)^{-1}. \tag{6}$$

Given a price change in any of the primary factors used (generally speaking, capital and labour), then the effect on the output value of the industry output is x_2. Notice that in the **Ghosh price model** there is neither change in the amounts of primary factors used nor on the goods and services produced.

From now on, we will refer to backward output multipliers to those obtained from Equation (5). For a unitary change in any of the final demand components ($f_1 = I$), the backward multiplier will be denoted as the row sums of each column of the Leontief inverse. However, for Equation (6), for changes in the value added coefficients due to unitary changes in the price of primary factor inputs, the forward output multipliers will be given by the column sums of each row of the Ghosh inverse.

3. Key sector analysis

3.1 What is a key sector? Why is it useful?

Hirschman (1958) introduced the concept of the key sector for the very first time. This is a transposition of the logic of Schumpeter's (1912) concept of economic evolution to the sectoral level. It has been shown that economies are driven by innovative and adaptable firms, whose interaction explains the process of entry and exit of firms. This has been a source of long-term increases of productivity (Eliasson, 1991). At the sectorial level, such a scheme is represented by 'propulsive', 'leading' or 'key' sectors driving the economy to increases in interdependence and income levels (Cuello and Mansouri, 1992).

Obviously, the essence of the key sector concept relates itself to the concept of unbalanced development. Hirschman (1958) argues that the unbalanced development of main final demand sectors will drive the entire economy on the path of efficient growth like that of a competitive economy. The countries that have followed Hirschman's strategy have been the most successful in their development policies (these include Japan, Taiwan, and South Korea). Unfortunately, the countries for which the approach was first proposed (Latin American economies) enacted plans based on other concepts, such as the import substitution of basic industries and infrastructure projects.

However, the key sectors are an issue not only for developing countries. At a time of crisis, budgeting for regional development may also play an increasingly important role. During a crisis, efficient budgeting for Keynesian policies may benefit from input-output information through the identification of narrow key sectors. Moreover, EU regional development plans, essential for territorial cohesion policies, may be more efficient if the concept of key sectors is taken into account. Even Porter's (1990) concept of competitive advantage is closely related to the strategy of unbalanced development. Essential concepts for industrial policy, such as the cluster or value chain, are also closely related to the ideas of key sectors and linkages.

3.2 Multiplier Analysis

The so-called central input-output equation system offers multiple approaches for analysis (Eurostat, 2008):

$$b = c(I - A)^{-1}. \tag{7}$$

where A is the technological matrix, the row vector $\boldsymbol{c}$ includes the input coefficients (per unit of output) of the selected variables for the analysis (*e.g.*, intermediates, labor, capital, energy, emissions) and vector b (backward linkages) shows the direct and indirect requirements (*e.g.*, energy, labor, capital) or joint products (emissions) needed (or generated) to produce goods and services (Eurostat, 2008) that would satisfy one unit increase in final demand of commodities or industry outputs ($f_1 = I$ in Equation (5)) . Within this framework, the use of input-output systems is generally and often applied in the literature to evaluate environmental and employment policies, to productivity analysis, to energy issues, and so on. Notice also that when $c = e$, where $\boldsymbol{e}$ is a unitary vector of suitable dimension, b refers to output multipliers. Eventually, we will denote $L = (I–A)^{-1}$. As noted before, output b may be computed by summing over the rows of L.

Contrary to backward multipliers, the row sums of the Leontief inverse are a traditional but somewhat controversial forward linkage (FL) measure. They are interpreted as the impact on sector i's output of simultaneous unit changes in each and every sector's final demands. This is objected by Jones (1976) for the unrealistic 'simultaneous unit changes' assumption and by Beyers (1976, p. 231) for having 'calculated forward linkages on the basis of the strength of backward linkages'. Despite the controversies, this FL measure is widely supported by many authors (Alauddin, 1986; Haji, 1987; Hewings *et al.* 1989; Sonis *et al.* 2000, etc.). On the contrary, the row sums of the Ghosh inverse (Ghosh, 1958) are suggested to replace the Leontief's approach in estimating FL (Augustinovics, 1970; Beyers, 1976; and Jones, 1976). Despite being endorsed by many authors either conceptually or empirically (Bulmer-Thomas, 1982; Dhawan and Saxena, 1992; Dietzenbacher, 2002; Miller and Blair, 2009; Oosterhaven, 1988; Poot, 1991; among others), the Ghosh inverse row sums (as a forward linkage measure) are criticized by a few hard to neglect authors (e.g. Cella, 1984), who are mainly concerned about the 'implausibility' of the Ghosh model (Oosterhaven, 1988, 1989; Mesnard, 2009).

Nevertheless, the row sums of the Ghosh inverse are widely used as a standard forward linkage measure to capture both direct and indirect effects and this paper will not address this issue. Although we are aware that the Ghosh inverse is not free of controversy, it is perhaps the least controversial forward linkage measure. In an experimental work (Iráizoz, 2006), the Ghoshian measures have proven to provide similar results to those obtained by hypothetical extraction methods or Cai and Leung (2004). Therefore, we will use in this paper the Rasmussen (1956)'s coefficients under a Ghoshian transformation.

Similarly, we will define another input-output equation system for addressing forward multipliers (see the Ghosh price model), i.e.:

$$f = (I–B)^{-1}c^{\mathrm{T}}. \tag{8}$$

That is, an increase in the value added coefficients coming from unitary changes in factor input prices will generate *f* amounts of e.g., energy, labor, capital, output or emissions. This model is mostly known as the "supply-driven" model since the initial shock is located on the value added component of industries (see a more detailed description in Dietzenbacher, 1997). In Equation (8), *B* is the Ghosh matrix, the column vector c^T denotes the input coefficients (per unit of output) of the selected variables for the analysis (*e.g.*, intermediates, labour, capital, energy, emissions), and the vector *f* (forward linkages) shows the direct and indirect supply-driven effects (*e.g.* energy, labour, capital) or joint products (emissions) for the newly produced goods and services. Within this framework, the use of input-output systems is generally applied to evaluate several kinds of policy impacts due to changes in factor prices of inputs and/or taxes (*e.g.*, environmental taxes). Notice also that when $c = e$, then, *f* refers to output multipliers. We will denote $G = (I–B)^{-1}$. Similarly, output *f* may be calculated by summing over the columns of *G*.

By normalising sector *j*'s backward and forward linkages (elements of *b* and *f*) by dividing them over the (simple) average of all backward and forward linkages, respectively, it can be defined that

$$\overline{b_j} = \frac{b_j}{1/n \sum_j b_j}, \qquad (9)$$

$$\overline{f_i} = \frac{f_i}{1/n \sum_i f_i}. \qquad (10)$$

where *n* is the number of sectors of the economy (also the dimensions of *A*, *B* and the input-output table, b_j is the backward linkage of sector *j* (the *j*-th element of vector *b*), and f_i stands for the forward linkage of sector *i* (the *i*-th element of vector *f*). Note that we are deliberately assuming industry by industry input-output tables in order to continue the analysis in terms of key sectors rather than on key commodities. The average value of the normalised measures is equal to one such that sectors "above average" (greater than one) will have stronger total backward and/or forward linkages while sectors "below average" will have exactly the opposite meaning. These indices are known as the "powers of dispersion" according to Rasmussen's (1956) terminology. Table 1 schematically shows the method proposed by Rasmussen (1956).

Table 1. Traditional Multiplier-Based Key Sector Analysis

	$\bar{f} > 1$	$\bar{f} < 1$
$\bar{b} > 1$	Key sector	Backward-oriented sector
$\bar{b} < 1$	Forward-oriented sector	Weakly-linked sector

Notwithstanding the simplicity and intuitive approach of the Rasmussen's methodology, we are fully aware that this is not free of criticism. To mention two examples, the multiplier/linkage-based key sector analysis provides a rigid classification under which only a small distance would separate some sectors (*e.g.*, key sectors) from others (not close enough to the threshold) and the final identification of key sectors may even be very sensitive to the selected threshold (*e.g.*, simple mean, corrected mean, median, etc.). Ultimately, the discussion of these issues is definitely interesting but it clearly falls beyond the scope of this paper.

4. Econometric Approach to Input-Output Multipliers

4.1 A brief review on stochastic input-output analysis

The existing literature on stochastic input–output analysis revolves around the analysis of transmission of errors under the Leontief inverse. Following Dietzenbacher (2006), a stochastic technical coefficients matrix, A, leads to the crucial result that the Leontief inverse, L, is positively biased, with input coefficients that are totally independent (Simonovits, 1975), biproportionally stochastic (Lahiri, 1983) or moment-associated (Flam and Thorlund-Petersen, 1985). That is, denote the true value of a stochastic IO matrix A as A_0 and that of its Leontief inverse L as L_0. Assume the expected value of A be equal to A_0 as $\mathrm{E}(A) = A_0$ (unbiased technical coefficients). Then,

$$\mathrm{E}(L) = \mathrm{E}\left[(I - A)^{-1}\right] > (I - \mathrm{E}[A])^{-1} = (I - A_0)^{-1} = L_0 . \quad (11)$$

Notice that even if the A matrix is unbiased the derived multiplier matrix $(I - \mathbf{A_0})^{-1}$ –see right hand side of the inequality– will not match the true value of L. That would be something like: $E(L) = L_0 + \text{bias}$ (with positive bias). The reader must be aware that L_0 does not correspond to $(I - A_0)^{-1}$ but to the true value of L. Hence, the difficulty to address the unknown true value of $E[(I - A)^{-1}]$ because of its stochastic nature leads IO practitioners to use $[I-E(A)]^{-1}$ instead for a particular case. For instance, Simonovits (1975) assumed: $E(A) = \mathbf{A_0} = A_t$, being A_t the current available matrix of technical coefficients. This incidentally explains why he reported in his work under-estimation of the Leontief inverse while Dietzenbacher (2006) reported over-estimation instead. The interested reader should notice that even though Dietzenbacher (2006) stated over-estimation and Simonovits (1975) under-estimation, they were speaking about the same inequality but from two different viewpoints. This should not get the reader confused.

Denote an econometric (unbiased) estimator of the Leontief inverse as L_e so that $E[L_e] = L_0$. Then, it follows that the bias of the standard Leontief inverse can be estimated by $L_t - L_e$, provided that it is just an estimation of the true value of the bias, i.e.: $E(L_t)-E(L_e)$ (ten Raa and Rueda-Cantuche, 2007).

Alternatively to other stochastic approaches that placed stochastics either directly on the input coefficients or on the symmetric input-output tables (not so often), ten Raa and Rueda-Cantuche (2007) estimated unbiased estimates of the column row sums of the Leontief inverse (backward multiplier estimates) directly from supply and use firms' data. Their work placed stochastics neither on the input coefficients nor on the symmetric input-output tables but rather on the total firms' output and total firms' employment data used for the compilation of the officially published supply and use tables. These authors proposed a single-equation econometric model in which the regression coefficients result in the output (and employment) backward multipliers obtained through the Leontief inverse, assuming a product technology model for the construction of the matrix of technical coefficients, A. The results confirmed the positive expected bias on almost all of significant multipliers.

One of the advantages provided by the econometric approach proposed by ten Raa and Rueda-Cantuche (2007) was the quantification of the unbiased estimate of the backward input-output multiplier of any kind. Moreover, this approach would allow for the estimation of confidence intervals for the backward multipliers and the run of standard hypotheses tests.

However, this method is greatly limited by data restrictions at the firm level. Firms' information needs to be carefully used in terms of filling data gaps, imputing values to non-observed establishments, and having domestic values and transactions at basic prices (once deducting net commodity taxes, non-deductible value added tax (VAT) and distribution margins). On top of it, there is an increasing availability of *U* and *V* matrices over the Internet for free and ready-to-be-used. Therefore, following the approach initiated by ten Raa and Rueda-Cantuche (2007), Rueda-Cantuche and Amores (2010) made a new contribution to the literature by providing unbiased estimations of backward multipliers neither from stochastic assumptions imposed on the input–output coefficients nor from the variability of the underlying input and output statistics across establishments. Instead, they used aggregated products and industries data (given by official use and supply tables). A similar exercise was carried out for the European Union (Rueda-Cantuche, 2011). This paper will continue this research line and in addition, it will extend the analysis to forward input-output multipliers. The reader should also notice that when speaking about FL it is the Ghosh inverse the one that provides biased estimates and not the Leontief inverse.

4.2 Unbiased and consistent output backward multipliers

In Equation (7), each value of b measures the total (direct and indirect) effect of one-unit increase in final demand commodities over socio-economic and environmental variables such as employment, output, emissions, capital consumption, water consumption, etc. Following Rueda-Cantuche and Amores (2010), if we assume proportional coefficients over commodity outputs for any of the mentioned variables, we can specify that:

$$\begin{aligned} C &= cV^T \\ c &= CV^{-T}, \end{aligned} \tag{12}$$

where *C* is a row vector of direct product levels of variables such as employment, emissions, etc.; and V^T (the transpose of the intermediate matrix of a make table) is a production matrix of the supply table at basic prices. Similarly, for the intermediate inputs:

$$\begin{aligned} U &= AV^T \\ A &= UV^{-T}, \end{aligned} \tag{13}$$

where A represents the technical coefficient matrix (product by product) and U represents the intermediate part of a use table at basic prices (product by industry). Subsequently, by assuming the product technology assumption in Equation (5), it yields:

$$b = CV^{-T}\left(I - UV^{-T}\right)^{-1} = C\left[\left(I - UV^{-T}\right)V^{T}\right]^{-1} = C\left(V^{T} - U\right)^{-1}, \tag{14}$$

which can be expressed as

$$C = b(V^{T} - U). \tag{15}$$

If there were the same number of industries and of products, this equation will become into a system of equations with one single solution for the b coefficients. Nevertheless, rectangular systems with different number of industries and of products will allow for the introduction of a random disturbance error ε, where eventually the dependent variable will be explained not only by the net output of commodities but by other uncontrolled variables included in this error term. This error can then be defined as a row vector of m independent and normally distributed errors with zero mean and constant variance:

$$C = b(V^{\mathrm{T}} - U) + \varepsilon. \tag{16}$$

In the last equation, C is an m-order row vector (m industries) of different levels of employment, emissions, capital consumption, income, etc.; b corresponds to an n-order row vector (n products) of backward input-output multipliers; V is the make matrix of order m x n, and U is the use matrix of order n x m (products by industry).

Note that m is not only the number of observations but also the number of industries. Furthermore, the net output of commodities (n) will constitute the independent variables of the regression model. Note that in order to obtain enough degrees of freedom (m-n), the model will require more industries than products ($m>n$). As long as this is fulfilled, the equations system is over-determined and the regression model is computable by Ordinary Least Squares (OLS).

Provided that the dependent variable is total industry output, the backward output multiplier will be calculated using the following econometric regression:

$$eV^T = b\left(V^T - U\right) + \varepsilon \, . \tag{17}$$

4.3 Unbiased and consistent output forward multipliers

In Equation (8), each value of f measures the total (direct and indirect) effect of an increase in the value added coefficients of industries (originated by unitary changes in factor input prices) over socio-economic and environmental variables such as employment, output, emissions, capital consumption, water consumption, etc. Analogously to backward multipliers, now we will assume that the variables to be analysed (emissions, capital consumption, water use, etc.) are to be proportional to industry outputs. Then, it is verified that:

$$\begin{aligned} C &= cV \\ c &= CV^{-1}, \end{aligned} \tag{18}$$

Next, by replacing B in Equation (8) with the corresponding mathematical expression of the fixed industry sales structure assumption, $U = V^{\mathrm{T}}B$ (see Rueda-Cantuche and ten Raa, 2009), the forward multipliers f can be alternatively expressed as:

$$\begin{aligned} f &= [I - B]^{-1} c^{\mathrm{T}} = \left[I - V^{-T}U\right]^{-1} c^{\mathrm{T}} = \left[I - V^{-\mathrm{T}}U\right]^{-1} V^{-\mathrm{T}} V^{\mathrm{T}} c^{\mathrm{T}} = \\ &= \left[V^{\mathrm{T}}\left(I - V^{-\mathrm{T}}U\right)\right]^{-1} V^{\mathrm{T}} c^{\mathrm{T}} = \left(V^{\mathrm{T}} - U\right)^{-1} V^{\mathrm{T}} c^{\mathrm{T}} = \left(V^{\mathrm{T}} - U\right)^{-1} C^{\mathrm{T}}, \end{aligned} \tag{19}$$

and consequently,

$$C^T = \left(V^T - U\right) f \, . \tag{20}$$

Similarly to backward multipliers, this system of equations will have unique solution for the f coefficients only if the number of industries equals the number of products (square supply-use tables) Otherwise, rectangular supply-use systems with different number of industries and of products would allow for the introduction of a random disturbance error ν. Moreover, this error term would include those independent variables that are not explicitly shown in the regression model. This error can then be defined as a column vector of n independent and normally distributed errors with zero mean and constant variance. In contrast to the case of backward multipliers, the number of observations refers to the number of commodities (n) while the number of industries corresponds to the number of independent variables (m). The econometric expression would be:

$$C^T = \left(V^T - U\right) f + v\,. \qquad (21)$$

Note that in order to get enough degrees of freedom, we will need to have more commodities than industries in our equation system ($m<n$), which is completely the opposite to the case of backward multipliers. For instance, provided that the dependent variable is total commodity output, the forward output multiplier will be computed using the following econometric regression:

$$V^T e = \left(V^T - U\right) f + v\,. \qquad (22)$$

4.4 Some further considerations

Even though the product technology assumption and the fixed industry sales structure assumptions have been traditionally conceived for the construction of input-output tables (see ten Raa and Rueda-Cantuche, 2009 and Kop Jansen and ten Raa, 1990), our findings show that they can easily be used in addition for impact analysis with two manifest advantages, i.e.: the econometric approach skips the problem of negatives that arise from the use of the two assumptions and provides unbiased and consistent backward multipliers (for product by product tables) and forward multipliers (for industry by industry tables).

It may have happened in the past that the extremely implausible negative coefficients provided by both the product technology model and the fixed industry sales structure assumption have led to very few applications in practice even though they were proved to be axiomatically the two best methods in each case (Kop Jansen and ten Raa, 1990 and ten Raa and Rueda-Cantuche, 2009). Actually, almost all statistical offices compiling industry by industry tables use the fixed product sales structure instead (with no possible negative outcomes), e.g. Norway, Netherlands and Finland, among others. Furthermore, input-output practitioners generally use the industry technology model for the same reasons.

However, this paper may have found enough reasons to turn this situation over. Indeed, the econometric regressions are built upon the two axiomatically best assumptions and in addition they provide unbiased backward and forward multipliers. What else can we ask for? We definitely have found another important use for the product technology and the fixed industry sales structure models that was so far unknown.

5. Data and results

5.1 Data

The pioneering work initiated by ten Raa and Rueda-Cantuche (2007) used firms' supply and use micro data with econometric techniques. The problem arises when micro-data (data at establishments' level) is not available easily or even inaccurate. Firstly, most of the times gathering a reliable database needs sufficient economic resources, also provided that it is not likely to be publicly available. Secondly, this sort of information contains a sizeable amount of handwork prior to the construction of supply and use matrices (data filling of surveyed establishments that did not report all the information; the extrapolation of imputed values to non-observed establishments; balancing procedures, etc.) Therefore, if data are not carefully prepared, multiple problems may arise. With the purpose of circumventing these problems, we will use supply and use tables instead of micro data. Our econometric approach is rectangular oriented and it fully reflects the Leontief inverse standard approach when using square supply-use tables (see Rueda-Cantuche and Amores, 2010).

The empirical work was carried out using the Turkish economy as a test case with official supply and use tables for the year 1998 (97 industries/commodities) at basic prices expressed in millions of current Turkish Lire.

We have made estimations of linkages according to the A60 CPA (EC, 2002). However, the Turkish economy does not have some of the activities included in the A60 CPA: activities 12 (Mining of uranium and thorium ores), 37 (Recycling) and 99 (Extra-territorial organizations and bodies). On the other hand activities 67 (Activities auxiliary to financial intermediation) and 90 (Sewage and refuse disposal, sanitation and similar services) had to be aggregated with activities 65 (Financial intermediation, except insurance and pension funding) and 85 (Health and social work), respectively.

Table 2. Products Classification

A60 Code	CPA Description*
01	Products of agriculture, hunting and related services
02	Products of forestry, logging and related services
05	Fish and other fishing products; services incidental of fishing
10	Coal and lignite; peat
11	Crude petroleum and natural gas; serv. incidental to oil & gas extraction excluding surveying
13	Metal ores
14	Other mining and quarrying products
15	Food products and beverages
16	Tobacco products
17	Textiles
18	Wearing apparel; furs
19	Leather and leather products
20	Wood & products of wood & cork (except furniture); articles of straw & plaiting materials
21	Pulp, paper and paper products
22	Printed matter and recorded media
23	Coke, refined petroleum products and nuclear fuels
24	Chemicals, chemical products and man-made fibres
25	Rubber and plastic products
26	Other non-metallic mineral products
27	Basic metals
28	Fabricated metal products, except machinery and equipment
29	Machinery and equipment n.e.c.
30	Office machinery and computers
31	Electrical machinery and apparatus n.e.c.
32	Radio, television and communication equipment and apparatus
33	Medical, precision and optical instruments, watches and clocks
34	Motor vehicles, trailers and semi-trailers
35	Other transport equipment
36	Furniture; other manufactured goods n.e.c.
40	Electrical energy, gas, steam and hot water
41	Collected and purified water, distribution services of water
45	Construction work
50	Trade, maintenance & repair serv.motor vehicles & motorcycles; retail sale of automotive fuel
51	Wholesale trade and commission trade services, except of motor vehicles and motorcycles
52	Retail trade serv.except motor vehicles & motorcycles; repair serv.personal & household goods
55	Hotel and restaurant services
60	Land transport; transport via pipeline services
61	Water transport services
62	Air transport services
63	Supporting and auxiliary transport services; travel agency services
64	Post and telecommunication services
65+67	Financial intermed.serv.except insurance & pension funding serv.+Serv.aux.financial intermed.
66	Insurance and pension funding services, except compulsory social security services
70	Real estate services
71	Renting services of machinery & equipment without operator & of personal & household goods
72	Computer and related services
73	Research and development services
74	Other business services
75	Public administration and defence services; compulsory social security services
80	Education services
85+90	Health & social work serv. + Sewage & refuse disposal serv. sanitation & similar serv.
91	Membership organisation services n.e.c.
92	Recreational, cultural and sporting services
93	Other services
95	Private households with employed persons

Note: * This classification is equivalent to A60 NACE Rev 1.1 (EC, 2001).
We refer to sectors or commodities without distinction.

We eventually set up the analysis with 55 groups of commodities/industries (see Table 2) in order to obtain sufficient degrees of freedom (97 industries/commodities - 3 missing industries/commodities – 55 commodities/industries = 39 degrees of freedom) to estimate output backward effects. On the other hand, 55 groups of industries were aggregated in order to obtain sufficient degrees of freedom (97 commodities – 55 industries = 42 degrees of freedom) to estimate forward output effects.

Nevertheless, for comparison purposes, the Leontief inverse based calculations (Table 2) were not constructed on the basis of the official A_{97x97} matrix published by TURKSTAT, but on a pure commodity technology basis for our aggregation to 55 sector/product, A_{55x55} (Table 2). This means that equations (7) and (8) were computed using an aggregated version of published use and make tables, an aggregated version of published SUT_{97x97} and $A = UV^{-T}$. The same applies to *f* with respect to the *B* matrix, computed as $B = V^{-T}U$.

5.2. Results of the unbiased and consistent output backward linkages

The *b* estimates are presented in the first two columns on the left of Table 3. The first one depicts the traditional or Leontief inverse based output backward multipliers under the commodity technology assumption and the second one the econometric estimates of *b*. The model has been estimated for 55 commodities by means of ordinary least squares. The resulting R-squared is 0.9961, which is quite satisfactory. Due to the presence of certain forms of unknown heteroskedasticity, the White estimate (White, 1980) of the covariance matrix of estimated coefficients was used to provide consistent and robust standard errors. No problems of autocorrelation (as expected in cross-sectional data) or multicollinearity were detected. Only 7 out of the 1,485 (0.47%) possible off-diagonal elements of the matrix of correlations between the 55 different explanatory variables were significant with a 5% significance level being only two of them higher than 0.75. Finally, 51 estimated backward output coefficients were significant at the 95% confidence level. All the remaining estimators are assumed to be zero (no impact) due to the acceptance of the null hypothesis. Negative values of multipliers were not significant either.

A careful look at Table 3 will provide us with some further considerations:

Table 3: Backward and Forward Linkages of Output

A 60			b_E bounds		b_E	Est.	Ranking				f_E bounds		f_E	Est.	Ranking	
Code	b_T	b_E	Lower	Upper	p-value	Bias	b_T	b_E	f_T	f_E	Lower	Upper	p-value	Bias	f_T	f_E
01	1.52	1.51 **	1.25	1.76	0.00	0.01	27	27	1.63	1.49 **	0.95	2.02	0.00	-0.14	11	11
02	1.19	1.15 **	1.02	1.29	0.00	0.04	19	19	2.46	2.17 **	1.73	2.62	0.00	-0.29	30	30
05	1.31	1.30 **	1.23	1.36	0.00	0.01	31	31	1.30	1.24 **	1.04	1.43	0.00	-0.06	73	73
10	1.54	1.50 **	1.31	1.69	0.00	0.04	25	25	4.33	4.00 **	3.77	4.23	0.00	-0.33	13	13
11	1.25	0.20	-3.18	3.18	0.90	1.04	28	28	26.23	24.80 **	23.77	25.83	0.00	-1.43	71	70
13	1.58	1.54 **	1.18	1.90	0.00	0.04	18	36	5.68	4.95 **	4.45	5.45	0.00	-0.73	70	71
14	1.35	1.30 **	1.13	1.48	0.00	0.05	36	18	2.75	2.54 **	2.28	2.81	0.00	-0.21	27	10
15	2.18	2.23 **	1.91	2.55	0.00	-0.05	34	34	1.42	0.91 *	0.24	1.59	0.01	-0.51	10	27
16	2.10	2.09 **	1.96	2.22	0.00	0.02	24	29	1.09	0.07	-3.29	3.29	0.97	-1.02	21	21
17	2.16	2.09 **	1.97	2.22	0.00	0.07	20	15	1.99	2.01 **	1.05	2.98	0.00	0.02	24	40
18	2.36	2.32 **	2.24	2.40	0.00	0.04	29	33	1.23	1.12 **	0.84	1.40	0.00	-0.11	33	14
19	2.54	2.52 **	2.38	2.67	0.00	0.02	33	32	1.66	0.78	-2.09	2.09	0.47	-0.88	14	23
20	2.28	2.16 **	1.92	2.41	0.00	0.12	15	20	2.42	1.94 **	1.00	2.88	0.00	-0.49	40	33
21	2.09	2.06 **	1.91	2.20	0.00	0.04	17	24	3.93	3.48 **	2.84	4.13	0.00	-0.44	23	61
22	1.96	1.94 **	1.64	2.25	0.00	0.01	32	17	2.00	1.74 *	0.25	3.23	0.03	-0.26	61	24
23	1.42	1.14 *	0.30	1.98	0.01	0.28	16	63	2.62	2.51 **	2.42	2.59	0.00	-0.12	2	92
24	2.29	2.15 **	1.65	2.66	0.00	0.13	63	16	3.45	2.31 **	1.25	3.38	0.00	-1.14	20	2
25	2.39	2.37 **	2.04	2.70	0.00	0.03	21	21	1.90	1.13	-1.91	1.91	0.25	-0.77	92	64
26	1.83	1.76 **	1.61	1.91	0.00	0.07	45	45	2.06	2.06 **	1.69	2.43	0.00	-0.01	41	66
27	2.71	2.74 **	2.62	2.87	0.00	-0.04	22	22	4.40	3.79 **	3.39	4.19	0.00	-0.62	64	41
28	2.37	2.35 **	2.15	2.56	0.00	0.02	92	92	2.21	2.04 **	1.79	2.29	0.00	-0.17	66	74
29	2.25	2.31 **	2.18	2.43	0.00	-0.06	30	55	1.56	1.49 **	0.75	2.24	0.00	-0.07	28	26
30	1.91	-0.87	-3.25	3.25	0.60	2.77	35	61	9.41	9.25 **	7.99	10.52	0.00	-0.16	74	28
31	2.42	2.42 **	2.34	2.50	0.00	0.01	55	62	1.53	1.37 **	0.85	1.89	0.00	-0.17	26	17
32	2.13	2.17 **	2.04	2.30	0.00	-0.04	61	35	1.76	1.73 **	1.55	1.90	0.00	-0.04	65	65
33	2.21	2.18 **	2.01	2.36	0.00	0.03	26	26	2.85	2.43 *	0.63	4.22	0.01	-0.42	22	20
34	2.33	2.31 **	2.23	2.39	0.00	0.02	62	70	1.48	1.45 **	1.37	1.54	0.00	-0.03	17	72
35	1.87	1.77 **	1.25	2.30	0.00	0.09	72	91	1.92	1.35	-1.62	1.62	0.10	-0.57	72	50
36	2.36	2.33 **	2.18	2.48	0.00	0.02	80	80	1.08	0.89 **	0.37	1.41	0.00	-0.19	35	22
40	1.59	1.44 **	1.03	1.85	0.00	0.15	91	72	2.69	2.55 **	2.40	2.69	0.00	-0.14	25	32
41	1.22	1.23 **	1.09	1.36	0.00	-0.01	40	74	2.24	2.10 **	1.95	2.25	0.00	-0.14	50	62
45	2.00	1.98 **	1.92	2.04	0.00	0.02	70	93	1.03	1.02 **	1.02	1.03	0.00	0.00	32	29
50	1.35	1.34 **	1.29	1.38	0.00	0.01	93	13	1.89	1.82 **	1.78	1.87	0.00	-0.06	19	1
51	1.30	1.29 **	1.27	1.31	0.00	0.01	13	1	1.48	1.42 **	1.37	1.46	0.00	-0.06	1	60
52	1.30	1.28 **	1.26	1.31	0.00	0.01	74	66	1.40	1.34 **	1.29	1.39	0.00	-0.06	29	34
55	1.86	1.86 **	1.77	1.95	0.00	0.01	71	65	1.28	1.28 **	1.10	1.46	0.00	0.00	31	51
60	1.50	1.44 **	1.29	1.60	0.00	0.06	65	10	1.53	1.47 **	1.43	1.52	0.00	-0.06	60	31
61	1.86	1.81 **	1.67	1.95	0.00	0.05	66	60	2.56	2.37 **	2.23	2.51	0.00	-0.19	62	52
62	1.82	1.79 **	1.66	1.93	0.00	0.03	10	40	1.52	1.51 **	1.41	1.60	0.00	-0.01	34	55
63	2.10	2.09 **	2.02	2.16	0.00	0.02	1	85	0.95	0.90 **	0.72	1.09	0.00	-0.05	51	5
64	1.19	1.19 **	1.16	1.21	0.00	0.01	60	50	2.23	2.17 **	2.12	2.22	0.00	-0.06	15	80
65+67	1.55	1.51 **	1.45	1.56	0.00	0.04	85	95	2.03	1.96 **	1.88	2.04	0.00	-0.07	52	18
66	1.54	1.51 **	1.43	1.59	0.00	0.04	23	14	2.21	2.12 **	2.03	2.21	0.00	-0.09	5	93
70	1.59	1.64 **	1.09	2.18	0.00	-0.05	14	5	4.91	4.61 **	4.01	5.20	0.00	-0.31	55	91
71	1.55	1.82	-2.47	2.47	0.15	-0.27	50	51	5.63	4.57 **	2.00	7.15	0.00	-1.06	18	85
72	1.74	1.59 **	1.37	1.81	0.00	0.15	95	52	1.93	1.83 **	1.62	2.04	0.00	-0.10	80	45
73	1.30	2.26	-6.03	6.03	0.46	-0.95	5	41	7.55	7.52 **	6.50	8.54	0.00	-0.03	93	75
74	1.58	1.56 **	1.51	1.61	0.00	0.02	73	64	2.13	2.07 **	1.90	2.23	0.00	-0.07	16	95
75	1.00	1.00	1.00	1.00	-	0.00	51	2	1.00	1.00	1.00	1.00	-	0.00	91	15
80	1.64	1.60 **	1.53	1.67	0.00	0.04	52	23	1.19	1.14 **	0.99	1.28	0.00	-0.05	36	63
85+90	1.43	1.41 **	1.33	1.48	0.00	0.02	11	75	1.04	1.04 **	1.03	1.05	0.00	0.00	85	36
91	1.62	1.60 **	1.56	1.64	0.00	0.02	41	11	1.09	1.08 **	1.05	1.11	0.00	-0.01	45	16
92	1.91	1.89 **	1.81	1.98	0.00	0.02	2	30	2.42	2.28 **	2.10	2.47	0.00	-0.14	75	19
93	1.58	1.55 **	1.46	1.65	0.00	0.03	64	71	1.17	1.10 **	0.86	1.34	0.00	-0.07	95	25
95	1.31	1.30 **	1.29	1.32	0.00	0.01	75	73	1.00	1.00 **	0.99	1.00	0.00	0.00	63	35
Average	1.79	1.65							2.81	2.49						
Corr.Av	1.79	1.78							1.89	1.66						
Median	1.74	1.60							1.93	1.82						
St.Dev	0.42	0.62							3.58	3.45						

Notes: p-values = 0.00: p-values lower than 10^{-3} are rounded to zero.

b_T: Leontief inverse based or traditional backward output multiplier

b_E: Econometrically estimated backward output multiplier

f_T : Leontief inverse based or traditional forward output multiplier

f_E : Econometrically estimated forward output multiplier

Significance level of 5% (*) and 1% (**)

Bounds: Confidence interval bounds for econometric estimations at a 95% confidence level

Est. Bias: Estimated bias: Traditional estimation - Econometric estimation

a) As in ten Raa and Rueda-Cantuche (2007) and Rueda-Cantuche and Amores (2010), in most cases, the Leontief inverse based multipliers overestimate the true values. Indeed, 48 out of 55 (87.27%) commodities have lower estimated values for b than those calculated with the traditional approach, while only 6 (10.91%) have higher values. Furthermore, the estimated average bias is higher for overestimated (5.16%) coefficients than for underestimated coefficients (0.12%). These results firm up the conclusions of Dietzenbacher (1995), Roland-Holst (1989) and ten Raa and Rueda-Cantuche (2007).

b) Input–output estimates are unbiased and consistent, providing confidence intervals for b. These intervals might be seen as a measure of the accuracy of the true estimates of multipliers. Notice also that all multipliers derived from the traditional approach fell within the confidence intervals.

c) The estimated bias of b is generally positively related with secondary production (Pearson correlation coefficient = 0.7). Commodities of which a large share is produced as secondary output have backward multipliers with larger estimated bias, as it was also found in ten Raa and Rueda-Cantuche (2007).

The Pearson and Spearman correlation coefficients between econometric and traditionally estimated multipliers are 0.782 and 0.935, respectively (both significant at a confidence level of 99%). This means implicitly that the econometric approach arrives at coherent results when comparing with the traditional approach. In addition, the top five commodities remains unchanged, i.e. basic metals, leather and leather products, electrical machinery and other apparatuses, rubber and plastic products, and fabricated metal products, except machinery and equipment. With the aim to test the robustness and coherence of the results obtained through equation (17) we used the econometric estimates of the output backward multipliers and the published net outputs matrix for the calculation of the total output of the economy. Consequently, the estimated total output, which yielded 90,883 thousand billions of Turkish Lires, was just 0.04% lower than the published total production (90,923 thousand billions).

5.3. Results of the unbiased and consistent output forward linkages:

With the same number of observations as in the last subsection, the values for f are presented in Table 3. The proposed model has been estimated for 55 industries by means of ordinary least squares and, with quite satisfactory goodness of fit, too (R-squared equals 0.968). The White (1980) estimated covariance matrix of estimated regression coefficients was used to obtain consistent standard errors. The model is again free from serial correlation and multicollinearity problems. None of the 1,485 possible correlations was neither significant nor higher than 0.5 (in absolute value).This time, 51 estimated multipliers were significant at a 5% significance level (which does not mean that they were exactly the same industries as in the case of backward linkages). There are not any negative values.

The analysis of the results of forward linkages provides similar conclusions as backward linkages:

a) Mostly, traditionally computed output multipliers overestimate rather than underestimate the true values of input-output multipliers. It is remarkable that 52 out of 55 (94.55%) of the estimated values of f are lower than those computed under the traditional approach, whilst only 2 (3.64%) commodities present higher f values. Again the estimated average bias is greater for overestimated coefficients (7.63%) than for underestimated figures (0.01%). Most of the Ghosh inverse based values of f were overestimated rather than underestimated, confirming the ideas developed by Dietzenbacher (1995), Roland-Holst (1989) and ten Raa and Rueda-Cantuche (2007), although for the Leontief inverse.

b) The econometrically estimated values of f are unbiased and consistent with confidence intervals that cover more than three quarters of the traditional estimated multipliers.

The correlation coefficients of Pearson and Spearman between the estimated and the traditionally computed multipliers are 0.991 and 0.919, respectively, (both significant at a confidence level of 99%). Moreover, the top four industries remains unchanged, i.e.: crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying, office machinery and computers, research and development services and metal ores.

The robustness and coherence of the results were tested by using the econometrically estimated input–output multipliers and the published supply-use data. This time, the estimated total output, which yield 83,884 thousand billions of Turkish Lires, was around 7.7% lower than published total production (90,923 thousand billions).

Generally, the forward multipliers are more accurate than backward multipliers in the sense that 61.82% of the p-values for forward multipliers in Table 3 are smaller than those of the backward multipliers. However, forward multipliers are more dispersed than backward multipliers (their Pearson coefficients of variation equal 0.376 and 1.384, respectively). Moreover, the econometrically computed multipliers have slightly higher dispersion than those traditionally computed (with coefficients of variation 0.233 and 1.275, respectively). This might be mainly because non significant multipliers are assumed to be zero in the econometric approach, which may increase observed dispersions.

In addition, the Pearson and Spearman correlation coefficients between backward multipliers and forward multipliers (both estimated through the econometric approach) yield -0.576 and -0.361, respectively (both significant at a 99% confidence level), which means that the dispersion power and the absorption capacity are slightly related.

6. Misleading identification of key sectors

The Leontief inverse based or traditional key sector analysis focuses on the comparison of the different linkages with respect to their average (see equations 9 and 10). Its main advantage is derived from the straightforward comparison with respect to unity. Sectors are classified depending on whether they have normalized backward and forward multipliers ($\bar{b}_j$ and $\bar{f}_i$) higher or lower than one. In this sense, Figure 1 depicts graphically the eventual identification of activities according to the econometric approach while Figure 2 shows its counterpart regarding the traditional approach. Table 4 shows the results obtained under both econometric and traditional approaches. One of the main difficulties in the traditional key sector analysis is its average dependence to determine the character of the sectors of an economy. This is due to the fact that most activities are concentrated around their average linkage, which is highly affected by outliers.

We will provide several alternatives to the average as a threshold to identify key sectors. In this sense, we propose to make comparisons with respect to: the average (option *a*, in continuous line in Figures 1 and 2); the average excluding outliers[2] (option *b*, in striped line in Figures 1 and 2); and the median (option *c*, in dotted line in Figures 1 and 2), a rather simple descriptive statistic, which is not affected by outliers. Table 4 shows the different identifications reported by each threshold.

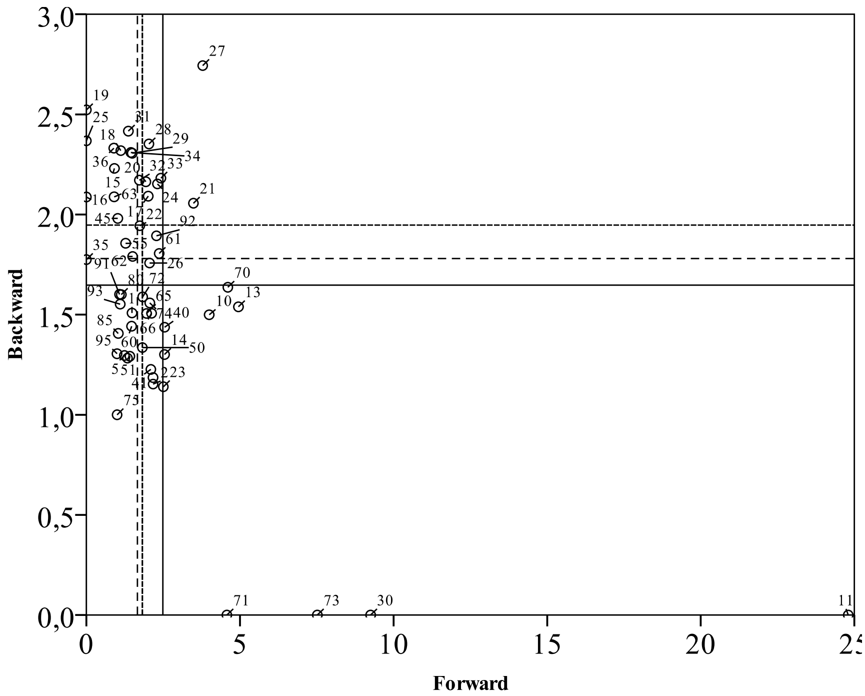

Figure 1: Key-Activities according to econometrically estimated multipliers
Key: Average (solid line); Average without outliers (striped line); Median (dotted line)

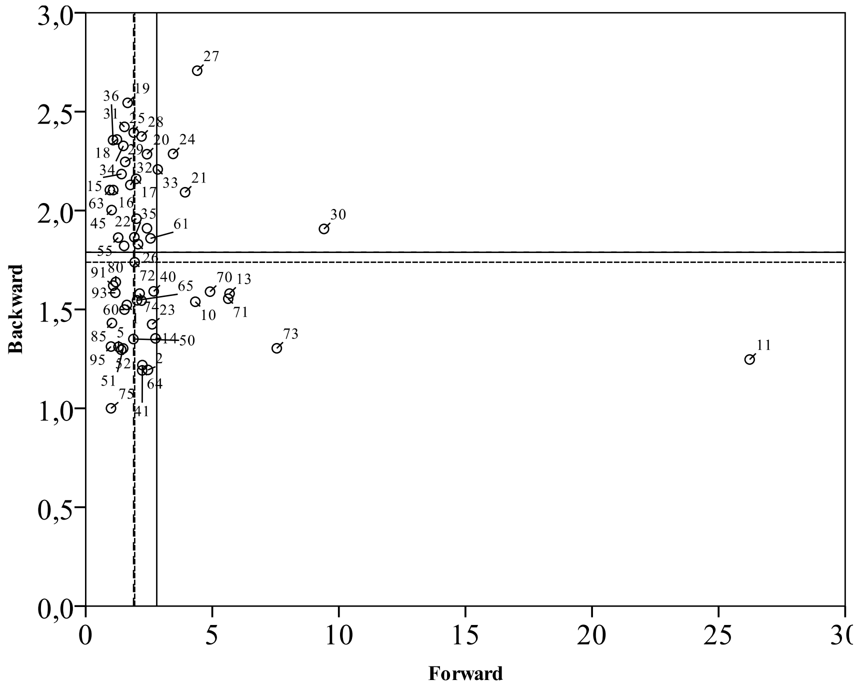

Figure 2: Key-Activities according to traditionally estimated multipliers
Key: Average (solid line); Average without outliers (striped line); Median (dotted line)

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[2] Consider an outlier to be a value outside one and a half times the interquartile range.

Table 4: Secondary productions & Classification of Activities

A 60 Code	% Secondary production per Sector	% Secondary production per Product	Type of Activity: Traditional Approach a) Average	Traditional Approach b) NO outlier	Traditional Approach c) Median	Econometric Approach a) Average	Econometric Approach b) NO outlier	Econometric Approach c) Median
01	9.76%	0.01%	W	W	W	W	W	W
02	0.00%	0.02%	W	F	F	W	F	F
05	1.39%	0.00%	W	W	W	W	W	W
10	4.81%	0.00%	F	F	F	F	F	F
11	3.63%	5.92%	F	F	F	F	F	F
13	2.13%	0.00%	F	F	F	F	F	F
14	1.22%	0.06%	W	F	F	F	F	F
15	1.74%	15.25%	B	B	B	B	B	B
16	1.61%	0.00%	B	B	B	B	B	B
17	5.34%	2.42%	B	K	K	B	K	K
18	4.46%	4.83%	B	B	B	B	B	B
19	3.24%	1.92%	B	B	B	B	B	B
20	2.55%	0.68%	B	K	K	B	K	K
21	6.52%	3.65%	K	K	K	K	K	K
22	22.40%	1.19%	B	K	K	B	K	B
23	0.74%	2.14%	W	F	F	F	F	F
24	3.11%	3.18%	K	K	K	B	K	K
25	2.22%	2.98%	B	K	B	B	B	B
26	1.87%	0.37%	B	K	K	B	F	K
27	5.47%	1.19%	K	K	K	K	K	K
28	6.73%	9.72%	B	K	K	B	K	K
29	9.04%	9.08%	B	B	B	B	B	B
30	28.86%	56.36%	K	K	K	F	F	F
31	10.01%	2.80%	B	B	B	B	B	B
32	7.25%	9.60%	B	B	B	B	K	B
33	6.61%	10.36%	K	K	K	B	K	K
34	8.63%	1.51%	B	B	B	B	B	B
35	4.80%	1.13%	B	K	B	B	W	B
36	1.99%	1.28%	B	B	B	B	B	B
40	0.95%	0.39%	W	F	F	F	F	F
41	3.91%	3.74%	W	F	F	W	F	F
45	1.40%	0.06%	B	B	B	B	B	B
50	3.20%	0.93%	W	W	W	W	F	W
51	3.92%	5.80%	W	W	W	W	W	W
52	3.68%	1.96%	W	W	W	W	W	W
55	0.79%	0.62%	B	B	B	B	B	B
60	1.02%	3.37%	W	W	W	W	W	W
61	0.02%	4.04%	B	K	K	B	K	K
62	15.79%	0.00%	B	B	B	B	B	B
63	14.09%	15.58%	B	B	B	B	B	B
64	0.20%	0.00%	W	F	F	W	F	F
65+67	0.00%	0.00%	W	F	F	W	F	F
66	0.00%	0.97%	W	F	F	W	F	F
70	0.02%	71.09%	F	F	F	F	F	K
71	0.12%	85.20%	F	F	F	F	F	F
72	0.00%	2.64%	W	F	W	W	F	F
73	0.00%	83.72%	F	F	F	F	F	F
74	0.83%	12.15%	W	F	F	W	F	F
75	0.00%	0.00%	W	W	W	W	W	W
80	1.04%	0.37%	W	W	W	W	W	W
85+90	0.35%	0.08%	W	W	W	W	W	W
91	0.00%	0.00%	W	W	W	W	W	W
92	0.40%	1.01%	B	K	K	B	K	K
93	0.11%	0.00%	W	W	W	W	W	W
95	0.00%	0.00%	W	W	W	W	W	W

Notes: Type of Activity (*bj* : backward linkage; *fi* : forward linkage)
K: Key Activity (*bj* >1, *fi* >1) W: Weakly Linked Activity (*bj* <1, *fi* <1)
B: Backward Oriented Activity (*bj* >1, *fi* <1) F: Forward Oriented Activity (*bj* <1, *fi* >1)

Table 5 summarizes the number of key activities, backward and forward oriented activities together with those with weak linkages depending on the different thresholds and classifications (Table 4). Panel I counts the activities that have been identified as key, forward oriented, backward oriented or weakly linked using the three different thresholds (*a*, *b* or *c*), which were calculated from the Leontief inverse based multipliers. Analogously, Panel II counts the same but for the econometrically calculated multipliers. Panel III counts the number of sectors for which the traditional and econometric approaches provide different outcomes in each option (*a*, *b* and *c*). Finally, Panel IV accounts for the different outcomes when comparing the three thresholds (*a* vs. *b*, *b* vs. *c* or *a* vs. *c*) in every type of multiplier (Trad. or Econom.).

Table 5: Count of Classifications and its differences

I. Traditional	Average	No outliers	Median
K	5	14	12
F	6	16	15
B	22	13	15
W	22	12	13

II. Econometric	Average	No outliers	Median
K	2	11	11
F	10	19	16
B	24	13	16
W	19	12	12

III. Trad. vs. Ec.	Average	No outliers	Median
	6	6	4

IV. Criteria	a) vs. b)	b) vs. c)	a) vs. c)
Traditional	19	3	16
Econometric	19	6	16

Notes: Type of Activity (*bj* : backward linkage; *fi* : forward linkage)
K: Key Activity (*bj* >1, *fi* >1)
B: Backward Oriented Activity (*bj* >1, *fi* <1)
W: Weakly Linked Activity (*bj* <1, *fi* <1)
F: Forward Oriented Activity (*bj* <1, *fi* >1)

Classification Criteria
a) Average
b) Average without outliers
c) Median

As a whole (see Table 5.III), there are 6 activities (*ca.* 10%) where both approaches (traditional vs. econometric) differ in classification according to option *a* or *b* (these are other mining and quarrying products; coke, refined petroleum products and nuclear fuels; chemicals, chemical products and man-made fibres; office machinery and computers; medical, precision and optical instruments, watches and clocks; and electrical energy, gas, steam and hot water).

As expected, comparing Panels I and II of Table 5 one can observe that as far as the econometric approach corrects the overestimation of linkages, it identifies less number of key sectors ($\bar{b}_j > 1$ and $\bar{f}_i > 1$) than the traditional approach (under criterion *a*). Not surprisingly, as far as reducing overestimation 'moves down' the average, it also identifies less number of weakly linked sectors ($\bar{b}_j < 1$ and $\bar{f}_i < 1$). Both differences are not so evident when comparing results under criteria *b* or *c*. However, as mentioned before, this way of identifying key sectors relies heavily on an arbitrary threshold (call it average, corrected average or median) and in one way or another, it might not be quite satisfactory in terms of providing an accurate identification of industries.

Activities identified differently depending on the selected threshold can also be seen graphically in Figures 1 and 2 (between lines). The differences are similar for both types of multipliers (see Table 5.IV).

As regard the positive bias inherent to traditionally estimated multipliers, all multipliers will be affected to a different extent, particularly the highest multipliers which are good candidates to be outliers maybe due to bias. Bias will affect not only to the simple average (to a lesser extent to the corrected average or median) but also to the identification of each activity either with respect to the average, the corrected average or the median. Therefore, the estimated bias is not only an issue for impact analysis but also for key-sector analysis since traditional Leontief multipliers tend to over-identify more key and weakly linked sectors than the unbiased and consistent econometric multipliers proposed in this paper.

7. Conclusions

Kop Jansen and ten Raa (1990) and Rueda-Cantuche and ten Raa (2009) singled out axiomatically the product technology and the fixed industry sales structure models as the two best ways for the construction of either product or industry input-output tables, respectively. However, these models have generated a lot of criticism due to their possible negative resulting elements that may appear in the intermediate matrix. At this point, we have proven in this paper that under these two assumptions, unbiased and consistent backward and, particularly, forward input-output multipliers can be respectively estimated from supply and use tables instead of from input-output tables. The advantages of the proposed econometric approach are twofold, i.e.: not only it avoids the problem of negatives but also provides unbiased multipliers.

This paper has extended the pioneering work of ten Raa and Rueda-Cantuche (2007) for backward input-output multipliers in two ways, i.e.: it replaces firms' data by aggregated industry and product supply and use tables and provides the theoretical framework for the econometric estimation of forward input-output multipliers. In addition, the mathematical expressions for backward and forward output multipliers are formalized. Other kinds of econometric based multipliers are mentioned already in ten Raa and Rueda-Cantuche (2007) for employment and Rueda-Cantuche and Amores (2010) and Rueda-Cantuche (2011), for air emissions.

The conclusions related to the bias for the forward output multipliers are very similar to that of the backward output multipliers. There seems to be indeed a positive bias inherent to the Ghosh and Leontief inverses that affect the correct identification of key sectors when using the Rasmussen's (1956) approach. It is remarkable that around 10% of the economic activities are differently classified independently of the threshold used. It has also been proved that the corrected average does not reduce significantly the number of different classified sectors as far as bias affects not only to extreme values (good candidates for being outliers) but to all multipliers. All in all, the Rasmussen's (1956) procedure to identify key-sectors might not be either completely satisfactory itself since it does not provide an accurate identification of sectors due to the somewhat arbitrary and rigid thresholds used. Even so, the overestimation of the multipliers computed using the Leontief and Ghosh inverses tend to over-estimate the number of key sectors and the number of weakly linked sectors. At least in our test case, we have found that the estimated bias may induce to wrongly identify key sectors in the Turkish economy for the year 1998.

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