# *Does bias really matter in input-output analysis? An almost definite answer*

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**Abstract**

Pioneering works on stochastic input-output analysis usually assumed stochastics on the technical coefficients and proved that under certain circumstances the Leontief inverse is biased (e.g. Simonovits, 1975). More recently, stochastics was alternatively imposed on the intermediate transactions of an input-output table rather than on its technical coefficients (e.g. Dietzenbacher, 2006). The findings of the latter experiments turned out that the bias tends to be rather small and needs a large sample size to get significant relevance. This paper however shifts the attention to supply and use tables, which really constitute the basic units of the elements of an input-output table and therefore, of the technical coefficients. By means of the same kind of experiment as in Dietzenbacher (2006), we prove that the bias might be small indeed but its consequences over the output multipliers (column sums of the Leontief inverse) might not be so tiny but to the contrary, rather more significant. The Leontief inverse estimations of the output multipliers are confronted with the unbiased and consistent econometric estimations of the output multipliers as in ten Raa and Rueda-Cantuche (2007). A similar application on carbon dioxide emission multipliers is also tested in order to quantify the estimated bias of a different kind of multiplier as in Rueda-Cantuche and Amores (2010). The results suggest that the use of supply-use tables and stochastics in the determination of multiplier impact estimates should be increasingly applied in all kind of forthcoming scientific studies that currently (ab)use of the Leontief inverse. We would say therefore that bias does matter in input-output analysis and this paper provides an almost definite solution to circumvent this so far everlasting problem.

**JEL Codes:** C15, C51, C67, Q5.

**Keywords**: input-output multipliers; input-output linkages; supply and use tables; Monte-Carlo; econometrics; bias.

# 1 All the way from establishment data to impact effects

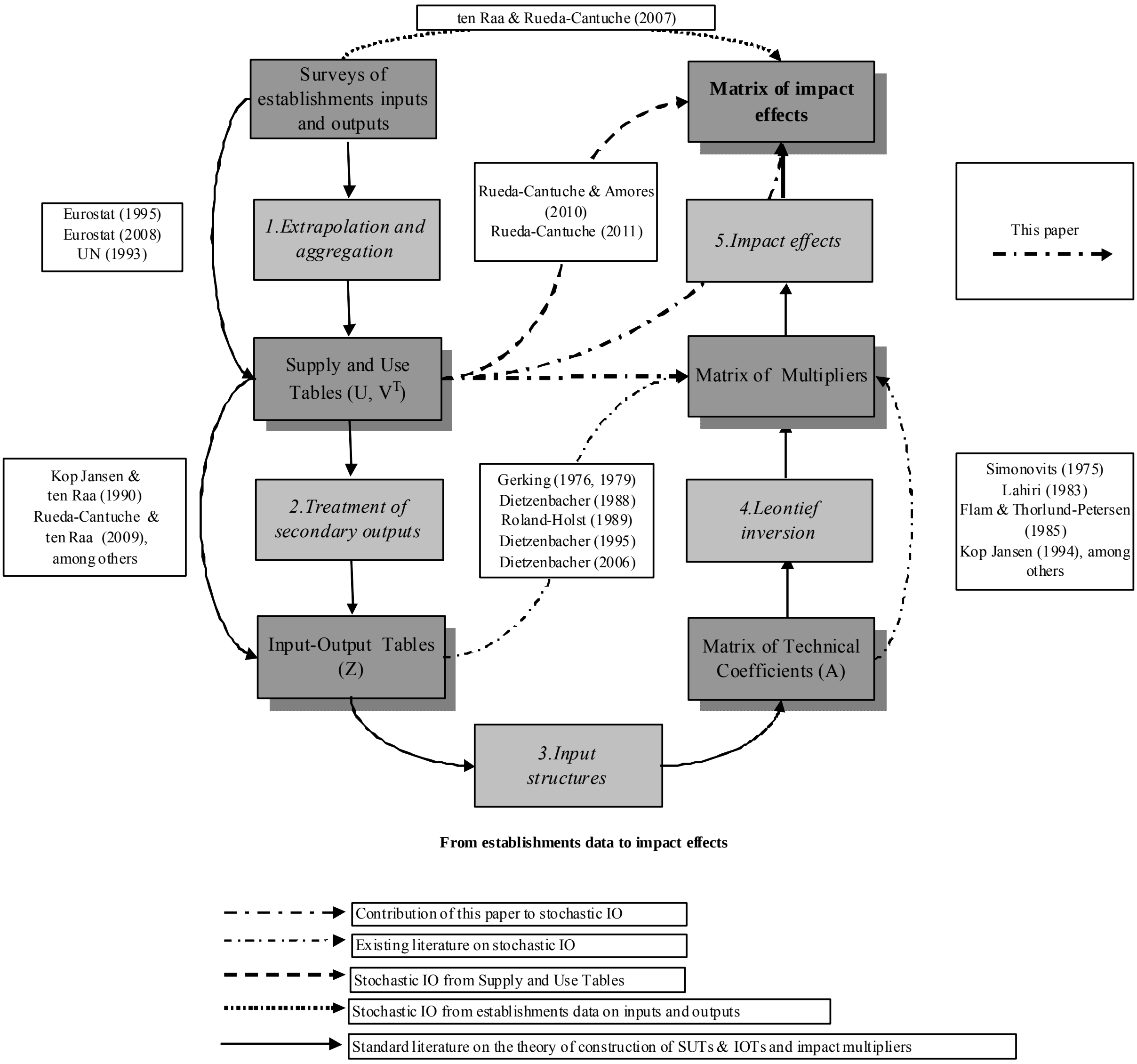
Surprisingly, not many authors are acquainted with the fact that input-output multipliers relate back to establishments data on inputs and outputs. Moreover, they seem to disregard the errors made in the extrapolation and aggregation of surveyed establishment data for the construction of supply and use tables let alone the construction of input-output tables. Doing so, one can find a plethora of published papers that uses the Leontief inversion to calculate input-output multipliers of any kind as if there were no errors at all and also providing point estimates of the impact effects of one-unit changes in final demand rather than their respective confidence intervals. Nevertheless, the existing literature on stochastic input-output analysis provides enough proofs that the elements of the Leontief inverse are biased and can lead to biased impact effects, too; unfortunately, almost nobody seems to be worried about it.

Figure 1 shows an overview of the state-of-the-art of the calculation of impact effects from establishment data, which is actually an extended and improved version of Figure 1 in ten Raa and Rueda-Cantuche (2007). In Step 1, Eurostat (1995, 2008) and United Nations (1993) provide guidelines about how to construct supply and use tables from the surveyed establishment input and output data. Basically, these methods consist of filling data gaps, imputing values to non-observed establishments, gathering of additional information from other non-surveyed data sources and summation over firms within industries. The resulting compiled supply and use tables display the commodity outputs and inputs of the industries.

The off-diagonal elements of the supply table are the secondary outputs of industries, which must be treated one way or another to compile subsequently either an industry by industry input-output table or a commodity by commodity input-output table[1] (Step 2). More precisely, the way secondary outputs are treated is another source of errors in the estimation of impact effects since there are manifold assumptions to be employed with that purpose (Kop Jansen and ten Raa, 1990; Rueda-Cantuche and ten Raa, 2009; among others). Moreover, statistical offices frequently introduce manually specific adjustments that are very difficult to be traced back by modellers and input-output practitioners.

[1] For the sake of simplification, we will always refer to commodity by commodity input-output tables hereafter.

**Figure 1. Overview of the state-of-the-art for the calculation of impact effects from establishment data**




Step 3 consists of a straightforward calculation of the matrix of technical coefficients ($A$) or, in other words, the input shares of the total output value, be either industry or commodity output. Subsequently, Step 4 involves the inversion of the Leontief matrix, i.e.: $(I-A)$, which is widely known by input-output practitioners as the matrix of output multipliers. Column-wise, the elements of the Leontief inverse can be interpreted as the impact effect on the output value of commodities as a result of one-unit change in the amount of final demand consumed of the commodity to which the column refers. Therefore, the sum of the impact effects over all commodities yields the so called backward output multipliers that reflect the total impact effects on the overall output value of an economy as a result of one-unit change in the quantities consumed by final users of a certain commodity (column).

Last but not least, Step 5 extends the analysis of impact effects to other variables different from the output values; namely, employment, carbon dioxide emissions, income, value added, gross domestic product, among others. The calculation of the impact effects of any kind are thus given by the pre-multiplication of the Leontief inverse by a row vector of employment, emissions... per unit of output value.

As mentioned before, to our knowledge, the five steps mentioned above have been so far addressed with very little attention paid to the inherent errors arising mainly from the first two steps, i.e. the compilation of supply and use tables and the construction of input-output tables. Input-output tables are published officially and they seem to be believed as the incommensurable truth, capable of providing us with exact estimates of impact effects of any kind through the corresponding matrix of multipliers. It must be clear though that our intention is not at all to put in doubt the credibility of input-output tables for carrying out input-output analyses but rather to highlight that impact effects derived from their use must come in the form of a range of possible values with a certain confidence level instead of just a single one. Otherwise, the use of the Leontief inverse to calculate input-output multipliers would simply assume that the published supply-use tables and input-output tables match exactly those that one could have constructed using full information on all industries and commodities, without any room for errors to be made. Therefore, we find this situation not very satisfactory and propose to include stochastics progressively in all related input-output studies. Next, we will comment on the existing literature on stochastic input-output analysis and the contribution of this paper at this respect.

The rest of the paper is divided in seven more sections. The second section describes briefly the state-of-the-art on stochastic input-output analysis and the related contributions of this paper. Section 3 introduces the *Supply-Use Based Econometric* (*SUBE*) approach for the estimation of unbiased backward multipliers that will help the reader to understand the next section explaining the details of the Monte Carlo simulation experiment. Section 5 presents and discusses the results for the individual elements of the Leontief inverse. Section 6 discusses the bias in output multipliers and tests the *SUBE* approach against the Leontief inverse based results. Section 7 presents a similar discussion but for $CO_2$ emission multipliers. The last section concludes followed by three appendices that provide detailed results of the calculations.

## 2 A review of the literature on stochastic input-output analysis and related contributions

The literature on stochastic input-output analysis dates back to Simonovits (1975), who firstly assumed stochastics on the matrix of technical coefficients, which were supposed to be totally independent. His main result was that the Leontief inverse was positively biased or, in other words, that the expected value of the Leontief inverse is always greater or equal than the Leontief inverse of the expected value of the matrix of technical coefficients, even in the case that the latter matrix were to be unbiased. Similar results were obtained by Lahiri (1983) assuming bi-proportionally stochastic input coefficients and Flam and Thorlund-Petersen (1985) assuming moment-associated technical coefficients. In addition, Lahiri and Satchell (1985) analyzed the bias of the Leontief inverse by ascribing stochastic errors to the prices at which the technical coefficients were evaluated. Kop Jansen (1994) specified the error distribution that yielded minimal lower bounds for the multipliers, which also yielded minimal lower bounds for the bias, and showed that the bounds were easily calculable by Monte Carlo simulations; his formulas were proven to be good approximations in the presence of interdependencies; see also ten Raa and Kop Jansen (1998).

West (1986) assumed that technical coefficients errors were independently and normally distributed with zero mean and known variance, and derived formulas for the approximation of the mean and variance of input-output multipliers as well as for their confidence intervals. However, ten Raa and Steel (1994) pointed out that normality on input coefficients does not admit finite moments for the elements of the Leontief inverse. Basically, normality attaches positive probability not only to coefficients taking negative values, but also to the spectral radius exceeding unity. Instead, these authors postulated a beta density function. Under the assumption that technical coefficients are independently and symmetrically distributed, Kop Jansen (1994) proved that maximal lower bounds for the first and second order moments of the Leontief inverse can be more widely applied than West's (1986) formulas[2].

[2] The regional input-output literature has also dealt with stochastic input-output analysis. The main interest is errors in regional non-survey input-output models and the accuracy of non-survey techniques vs. survey-based models: Czamanski and Malizia (1969), Schaffer and Chu (1969), Morrison and Smith (1974), McMenamin and Haring (1974), and Round (1978). Several studies analyze the roles of errors in technical coefficients and of errors in regional purchase coefficients (RPCs) in regional output multipliers. Stevens and Trainer (1980) and Park, Mohtadi and Kubursi (1981) concluded that errors in RPCs were of greater significance than those in technical coefficients, assuming a multiplicative structure of errors, but Garhart (1985) and Giarratani (1986) concluded that these results should be interpreted with caution when errors are additive or combined additive-multiplicative.

The reader may notice that so far stochastics has been imposed one way or another on the technical coefficients. However, there is another branch of literature that dates back to Gerking (1976, 1979) and Dietzenbacher (1988), in which stochastics is imposed on the transactions table or input-output table rather than on the matrix of technical coefficients. The reason for the rarity of this approach is likely that the additional step for transforming intermediate uses into input coefficients seriously complicates the analysis, typically inducing a rather complex stochastic nature of the input coefficients. More recently, Dietzenbacher (2006) provides a good overview of the recent contributions at this respect and finds that the bias of the Leontief inverse elements tends to be rather small and needs a large sample size to get significant relevance. As Dietzenbacher (2006) suggested, Roland-Holst (1989) did not gather a big enough sample and concluded misleadingly that the Leontief inverse was not biased. Dietzenbacher (2006) eventually showed that the bias is therefore clearly significant but can be negligible as regard the individual elements of the Leontief inverse so... who must care about bias?

As shown in Figure 1, this paper will give one further step forward and will contribute to the literature on stochastic input-output analysis by providing an almost definite answer to the unbiasedness issue of the elements of the Leontief inverse. We will assume stochastics on the elements of the supply and use tables, which really constitute the basic units of the elements of an input-output table and therefore, of the technical coefficients. By means of the same kind of Monte Carlo experiment as in Dietzenbacher (2006), we will prove that the bias might be small indeed but its consequences over the backward output multipliers (column sums of the Leontief inverse) might not be so tiny but to the contrary, rather more significant. So, the question should rather be... who mustn’t care about bias?

As regard other kind of multipliers (or impact effects), the literature on stochastic input-output analysis is still very limited. Moreover, they only refer to the so called backward input-output multipliers and not to the fully fledged matrix of multipliers. Indeed, ten Raa and Rueda-Cantuche (2007) estimated econometrically unbiased and consistent output and employment backward multipliers for the Andalusian economy in 2000, providing in addition confidence intervals. These authors used establishment data on inputs and outputs and assumed the commodity technology assumption in the formulation of their econometric regression. Following this approach, which we will call hereafter *Supply-Use Based Econometric* (*SUBE*) approach, Rueda-Cantuche and Amores (2010) extended the analysis to carbon dioxide emission multipliers but using this time official supply and use tables from Denmark in 2003, which represented a considerable improvement in terms of data availability. They also gave a more in-depth insight of the stochastic nature of the new approach. Basically, the two approaches consist of an econometrically estimated linear regression where the regression coefficients measure the output backward multipliers or any other kind of multiplier. Doing so, it is relatively easy to estimate unbiased and consistent estimates of the regression coefficients (including also confidence intervals) through standard econometric techniques.

Eventually, the econometric approaches postulated by ten Raa and Rueda-Cantuche (2007) and Rueda-Cantuche and Amores (2010) certainly open the door to test the Leontief inverse estimations of the output multipliers (assuming for the very first time stochastics on the supply-use tables) against the unbiased and consistent econometric estimations of the output multipliers. Using the same data as for Rueda-Cantuche and Amores (2010), our results will suggest that the *SUBE* approach for the determination of unbiased multiplier impact estimates should be increasingly applied in all kind of forthcoming scientific studies that currently (ab)use of the Leontief inverse. A similar application on carbon dioxide emission multipliers is also tested for Denmark in order to quantify the estimated bias of a different kind of multiplier.

To cut a long story short, this paper presents two main contributions in the field of stochastic input-output analysis. Firstly, it provides evidence from stochastic supply-use tables and assuming the commodity technology assumption that the bias in the elements of the Leontief inverse is indeed small (as in Dietzenbacher, 2006 for input-output tables) but to the contrary, the accumulated bias in the output multiplier may not be so negligible. And secondly, it provides empirical evidence that the *SUBE* approach provides unbiased estimates of any kind of backward multiplier while the Leontief inverse may incur in a significant bias.

# 3 The *supply-use based econometric* (*SUBE*) approach for the calculation of backward multipliers of any kind

Following Eurostat (2008, p. 493), the central equation system of input-output analysis is given by:

$$Z = B(I - A)^{-1}Y$$

Matrix $B$ includes row-wise the input coefficients of the variable(s) under investigation (intermediates, labour, capital, energy, emissions, etc.). The diagonal matrix $Y$ denotes exogenous final demand for goods and services. The matrix $Z$ depicts the results for the direct and indirect requirements (intermediates, labour, capital, energy) or joint products (emissions) for the produced goods and services.

Particularly in our analysis, we will consider hereafter impact effects derived from one-unit exogenous changes in final demand (i.e.: $Y = I$) and that the impact effects are summed over all commodities. The latter implies that B becomes into a row vector that we will denote $c$ and so, therefore, $Z$ will do, too (which will be denoted as $\gamma_c$).

Bearing this in mind and with a different notation, let us assume an economy with $m$ different industries that produce $n$ commodities. Let $C$ be the ($1$ $x$ $m$) row vector that contains the levels of the variable of interest per industry and $c$ the respective ($1$ $x$ $n$) row vector of coefficients per unit of commodity output value. Let $U$ be the intermediate matrix of the use table with dimension ($n$ $x$ $m$) and $V^{\mathrm{T}}$ the production matrix of the supply table with dimension ($n$ $x$ $m$). Subsequently, the matrix of technical coefficients is constructed under the product technology assumption (Kop Jansen and ten Raa, 1990), which is: $A = UV^{-\mathrm{T}}$. Similarly, if we assume that direct requirements of the variable $C$ per commodity output value ($c$) are as well independent of the producing industry (namely, the product technology assumption), then $C$ can be defined as: $c = CV^{-\mathrm{T}}$.

Consequently, as a result of the assumption of the product technology model both in input coefficients ($c$) and technical coefficients ($A$), the central equation system turns out to be as follows:

$$\gamma_c = CV^{-\mathrm{T}}\left(I - UV^{-\mathrm{T}}\right)^{-1} = C\left[\left(I - UV^{-\mathrm{T}}\right)V^{\mathrm{T}}\right]^{-1} = C\left(V^{\mathrm{T}} - U\right)^{-1}$$

being each value of $\gamma_c$ the total (direct and indirect) impact effects on the variable $C$ (be they air emissions, employment, etc.) produced as a result of a one-unit increase in the quantities consumed by final users of a certain commodity. Note that, for a row vector of ones denoted as $e$, if $C = eV^{\mathrm{T}}$ (row vector of industry output), then the $\gamma_c$-vector (now differently, $\gamma_p$) precisely stands for the backward output multipliers. That is,

$$\gamma_p = e(I - A)^{-1} = e\left(I - UV^{-\mathrm{T}}\right)^{-1} = eV^{\mathrm{T}}\left(V^{\mathrm{T}} - U\right)^{-1}$$

For both cases, we can formulate the following two systems of equations:

$$C = \gamma_c\left(V^{\mathrm{T}} - U\right) \qquad (1)$$

$$eV^{\mathrm{T}} = \gamma_p \left(V^{\mathrm{T}} - U\right). \tag{2}$$

In both equations, there will only be one single solution if and only if the number of industries equals the number of commodities or, in other words, the number of equations (*m*, industries) equals the number of unknowns (*n*, commodities). Moreover, Rueda-Cantuche and Amores (2010) proved that the solutions to equations (1) and (2) are exactly the same as those provided by the Leontief inverse assuming the commodity technology assumption. However, these results have incidentally been proven to be biased in the literature as mentioned in the previous section.

However, equations (1) and (2) can also be seen as linear econometric regressions with zero degrees of freedom ($m = n$) and thus, with a perfect fit. In other words, solving econometrically the mentioned equations will result in the assumption that there is no error in C or industry output (explained variables) and that the elements of the net output matrix ($V^{\mathrm{T}}$ - $U$) explain fully their behaviour. So the key question would be: have we ever found in Economics that any variable fits perfectly a linear econometric regression without any kind of error? Wouldn't it be much sounder to assume stochastics on the explained variables as it is done for decades in standard econometrics? Maybe the answer to these questions complicates the scheme but nobody told us that life was meant to be easy. With this purpose and following ten Raa and Rueda-Cantuche (2007) and Rueda-Cantuche and Amores (2010), the *SUBE* approach adds a stochastic error term to equations (1) and (2). A further detailed justification of this error term can be found in Rueda-Cantuche and Amores (2010). This error can then be defined as a row vector of *m* independent and normally distributed errors with zero mean and constant variance:

$$C = \gamma_c \left(V^{\mathrm{T}} - U\right) + \varepsilon_c \tag{3}$$

$$P = eV^{\mathrm{T}} = \gamma_p \left(V^{\mathrm{T}} - U\right) + \varepsilon_p . \tag{4}$$

Next, multipliers are assumed as a vector of regression coefficients, $\gamma_c$ and $\gamma_p$. For instance, in (3), $C$ is an $m$-order row vector ($m$ industries) with the values of the variable of interest; $\gamma_c$ corresponds to an $n$-order row vector ($n$ products) of multipliers; $V^{\mathrm{T}}$ is the production matrix of the supply table of order $n$ x $m$ (products by industry), and $U$ is the intermediate use matrix of order $n$ x $m$ (products by industry). Note that although $m$ represents the number of industries, it is also the number of observations. Moreover, the net output of commodities ($n$) would constitute the independent variables of the resulting model.

The *SUBE* approach is particularly a rectangular oriented approach since it needs to have by definition different number of industries (*m* observations) than of commodities (*n* regressors) to get enough degrees of freedom ($m - n$). Otherwise, we would have to come back to the Leontief inverse (biased) approach, which might even be considered in this sense a specific case of the *SUBE* approach. For backward multipliers, more industries than products ($m > n$) are required and as long as we fulfil this requirement (rectangularity) and also have enough degrees of freedom, the equations systems (3) and (4) will be sufficiently over-determined so that the regression coefficients could be estimated by minimizing the residual sum of squares (i.e. ordinary least squares). Finally, the need to have rectangular supply and use tables can easily be accommodated for backward multiplier by splitting up the industries of a square supply-use system or, conversely, aggregate them until a desired number of degrees of freedom is reached.

## 4 Description of the Monte-Carlo experiments and the database

In this section, we describe the numerical experiment carried out to measure the bias of the individual elements of the Leontief inverse; their column sums (output multipliers) and in particular, $CO_2$ emission multipliers; by introducing stochastics on supply-use tables (SUTs).

In each trial *k* of the numerical experiment, we will simulate the presence of random disturbance errors in the elements of the production matrix of a supply table and in the intermediate part of a use matrix. With this purpose, we used as benchmark the official supply and use tables compiled by Statistics Denmark (2007) for 2003 but accommodated to 59 industries and 21 aggregated commodities (namely $V^{of}$ and $U^{of}$), all valued at basic prices and expressed in millions of Danish Crones at current prices. The breakdown of the selected 21 commodities can be found at Appendix 4.

Firstly, in order to test cell-wise the significance of the elements of the Leontief inverse we have randomized every element of the official supply and use matrices around their observed values by adding a different stochastic term in each trial *k*. Specifically, $\upsilon_{tij} = \upsilon_{ij}^{of} + \varpi_{tij}$, where $\varpi_{tij} \sim \mathrm{N}\left(0, \rho\upsilon_{ij}^{of}\right)$; and $u_{tij} = u_{ij}^{of} + \omega_{tij}$, where $\omega_{tij} \sim \mathrm{N}\left(0, \rho u_{ij}^{of}\right)$. In addition, with the aim to test the robustness of our results with respect to the standard deviation $\rho$, we have carried out six different variants of the experiment for six different levels of $\rho$, i.e.: 0.05, 0.10, 0.15, 0.20, 0.25 and 0.30. We run 1,000 trials in every experiment and consequently, generated six times 1,000 stochastic supply and use matrices following this procedure[3]. The results of each trial and of each experiment can be considered independent of each other.

For the sake of accounting consistency, we have also imposed the restriction that the column and row sums of the stochastically generated supply-use matrices must be equal to the corresponding published intermediate totals. Doing so, we keep value added and final demand totals unchanged. Therefore, the different stochastically generated supply-use matrices will only differ one from each other just in their different input structures. This condition is imposed by modifying the last row and the last column of the generated supply-use matrices in such a way that they will capture the gap between the generated column and row sums (evidently excluding the last row and the last column) and the official intermediate totals. Nevertheless, we are aware that this mechanism actually modifies the distribution of the last row and the last column of the corresponding matrices but still this is preferable to allowing for arbitrary (and even non-sense) changes in value added and final demand components. Even so, we explored this argument quantitatively by leaving the last row and the last column free and the results confirmed our intuition.

Secondly, we used the commodity technology assumption to compile a technical coefficient matrix in each trial $k$ such that: $A^k = U^k (V^k)^{-\mathbf{T}}$. For the sake of further comparisons, the use of this assumption is justified because the *SUBE* approach is based on the same assumption (as explained in Section 3). Maybe, it would be interesting as well to study the bias with a different technology assumption but this is clearly beyond the scope of this paper.

As regard the output multipliers, we calculated 1,000 of Leontief inverses, i.e.: $(I - A^k)^{-1}$ in each experiment and computed their corresponding column sums per commodity (i.e. output multipliers). Let us denote them as: $\gamma_p^L$. Note that the Leontief inverse-based "true" vector of output multipliers $\gamma_p$ was not constructed on the basis of the official $SUT_{59x59}$ matrices published by Statistics Denmark but rather on a pure product technology basis for 59 industries and 21 aggregated commodities, i.e. with an input matrix $A_{21x21}$. This means that, accordingly, the Leontief inverses of the simulations were computed as well using aggregated versions of the $SUTs_{59x59}$ (i.e. 59 industries and 21 commodities). Doing so, we would say that if the expected value of the 1,000 vectors of the Leontief-inverse based output multipliers matches its "true" vector of output

[3] We also run 5,000 trials once and the results did not change significantly.

multipliers, denoted as $\gamma_p$, then the Leontief inverse will provide undoubtedly unbiased estimates. However, we can tell in advance that this is not likely what will happen.

Alternatively to the Leontief inverse option, the *SUBE* approach is based on the econometric estimation of equation (4) for output multipliers. Take the same "true" vector of output multipliers $\gamma_p$ and the same 1,000 simulated supply and use matrices. Next, we will assume that the error term $\varepsilon_p$ will follow a Normal distribution with zero mean and a constant standard deviation $\sigma$, which will be given by the standard error of the regression for output multipliers reported by ten Raa and Rueda-Cantuche (2007). Subsequently, equation (4) will generate the "observed" values for industry outputs in each trial $k$, as follows:

$$P^k = e\left(V^k\right)^{\mathrm{T}} = \gamma_p\left[\left(V^k\right)^{\mathrm{T}} - U^k\right] + \varepsilon_p^k . \qquad (5)$$

Note that the "true" output multipliers remain constant throughout all the 1,000 trials. The final step will be to use the generated observations of $P^k$, $V^k$ and $U^k$ in each trial to run 1,000 econometric estimations of equation (4) in order to estimate by ordinary least squares (OLS) 1,000 vector of output multipliers, which will be denoted as: $\gamma_p^{OLS}$. As it will be shown later, the expected value of the 1,000 OLS estimations will result in almost equality with respect to the "true" output multipliers. It is interesting to remark in advance that the Gauss-Markov Theorem (very well known in standard Econometrics) is the main responsible of the latter result since it guarantees that the OLS estimates of any kind of linear regression will be unbiased and therefore, output multipliers, too. As expected, our empirical results will confirm this theory.

The Monte Carlo experiments carried out for emission multipliers are very similar to those already addressed of output multipliers. However, there are several significant differences, namely: (a) the "true" values of the emission multipliers were obtained by randomizing those reported by Rueda-Cantuche and Amores (2010) for Denmark; with that purpose, we used a Uniform distribution that varied between 1 and the maximum value reported by these authors; (b) the constant standard deviation of the error term introduced in equation (3) was taken from the standard error of the regression on carbon dioxide emissions reported by Rueda-Cantuche and Amores (2010); (c) the "observed" values of emissions corresponding to equation (3) were calculated for each trial $k$, as follows:

$$C^k = \gamma_c\left[\left(V^k\right)^{\mathrm{T}} - U^k\right] + \varepsilon_c^k ; \qquad (6)$$

and (d) the "observed" values of $C^k$ were further use to compute emission coefficients $c = CV^{-\mathrm{T}}$ in each trial $k$ of the Leontief inverse based simulations.

## 5 Preparing the discussion of results: unweighted and weighted indicators of bias and a significance *t*-statistic for unbiasedness

Dietzenbacher (2006) concluded that the bias of the elements of the Leontief inverse can be negligible in practice and that the Leontief inverse may remain "unbiased for practical purposes". So, in plain words, his message can be interpreted as: "... who must care about bias?" This section however will develop a complementary test to check whether Dietzenbacher's (2006) conclusions remain unchanged in case that stochastics is imposed on supply and use tables rather than on input-output tables. Furthermore, the experiments will be carried out for different levels of standard deviation $\rho$ in order to determine the robustness of the results in terms of the variability of the stochastically generated supply-use elements. Also, we will provide not only simple but weighted indicators of bias in order to take into account the relevance of the industries and commodities over the whole economy. We will consider two different weights, i.e.: the final demand shares of commodities and the industry output shares over the whole economy.

To sum up, this paper differs from Dietzenbacher (2006) in three aspects: (a) we impose stochastics on the elements of SUTs rather than on the elements of input-output tables; (b) we provide in addition weighted indicators of bias in order to capture the real structure of the base economy; and (c) we compare bias under different levels of $\rho$ to determine if the statistically significant biases found in Dietzenbacher (2006) remain being small irrespective of the variability introduced in the experiment.

Let us define the bias of the (*i,j*)-th element of the Leontief inverse as:

$$b_{ij}^{k} = l_{ij}^{k} - l_{ij}^{of} ,$$

where $l_{ij}^{k}$ is the value of the (*i,j*)-th element in the iteration $k$ of the Monte Carlo experiment and $l_{ij}^{of}$ is the "true" initial value derived from official published SUTs and assuming the commodity technology assumption in the construction of the matrix of technical coefficients.

The *average bias* for each of the elements of the Leontief inverse considering all 1,000 trials will be denoted as:

$$\overline{b}_{ij} = \frac{1}{1,000}\sum_k b_{ij}^k$$

and the average bias of the averaged Leontief inverse matrix (*overall average*) will be given by:

$$\overline{b} = \frac{1}{N^2}\sum_i \sum_j \overline{b}_{ij} \quad , \qquad (7)$$

Similarly, we also considered these indicators in absolute values due to the presence of positive and negative biases. That is,

$$\overline{|b_{ij}|} = \frac{1}{1,000}\sum_k |b_{ij}^k|$$

and,

$$\overline{|b|} = \frac{1}{N^2}\sum_i \sum_j \overline{|b_{ij}|} \quad . \qquad (8)$$

In addition, the overall average biases (equations 7 and 8) were re-calculated using as weights the final demand shares of commodities ($0 \leq w_i \leq 1$) and the industry supply output shares ($0 \leq s_j \leq 1$). Then, equation (7) would become into:

$$\overline{b}_w = \frac{1}{N^2}\sum_i \sum_j \left( \frac{1}{1,000}\sum_k b_{ij}^k w_i \right) , \qquad (9)$$

for final demand weights, and into:

$$\overline{b}_s = \frac{1}{N^2}\sum_i \sum_j \left( \frac{1}{1,000}\sum_k b_{ij}^k s_j \right) . \qquad (10)$$

for output weights.

Similarly, the same expressions can be re-written for the absolute values of the bias,

$$\overline{|b|}_w = \frac{1}{N^2}\sum_i \sum_j \left( \frac{1}{1,000}\sum_k |b_{ij}^k| w_i \right) \qquad (11)$$

$$\overline{|b|}_s = \frac{1}{N^2}\sum_i \sum_j \left( \frac{1}{1,000}\sum_k |b_{ij}^k| s_j \right) \qquad (12)$$

In the following, we will discuss the unbiasedness of the elements of the Leontief inverse using equations (7) to (12) in order to illustrate the Monte Carlo results.

With the purpose of evaluating the statistical significance of the bias in every element of the Leontief inverse, we eventually ran the following $t$-statistic:

$$t_{ij} = \frac{\frac{1}{N}\sum_k l_{ij}^k - l_{ij}^{of}}{r_{ij}/\sqrt{N}},$$

where $r_{ij}$ denotes the standard deviation of the $N$ = 1,000 values (one for each trial $k$) of the $(i,j)$-th element of the Leontief inverse. The null hypothesis refers to unbiasedness, i.e.:

$$H_0 : E\left(l_{ij}^k\right) = l_{ij}^{of}$$

For the evaluation of the bias in the output multipliers, the null hypothesis is defined differently,

$$H_0 : E\left(g_i^k\right) = g_i^{of}$$

where $g_i^k$ is the value of the $i$-th component of the vector $\gamma_{\boldsymbol{p}}$ (output multipliers) in the $k$-th iteration of the Monte Carlo experiment and $g_i^{of}$, the "true" values derived from the official published SUTs and the use of the commodity technology assumption to construct the technical coefficients matrix. This time, we ran the following slightly different $t$-statistic for each component of the vector $\gamma_{\boldsymbol{p}}$:

$$t_i = \frac{\frac{1}{N}\sum_k g_i^k - g_i^{of}}{r_i/\sqrt{N}}$$

The same could be applied to any kind of backward multiplier, e.g. carbon dioxide emission multipliers (see Section 8).

## 6 Does bias really matter in the Leontief Inverse elements?

Table 1 presents the results for the six experiments and also provides a summary of the $t$-statistic tests[4]. The unweighted average bias is generally very small although increasingly growing with the standard deviation of the elements of the supply and use matrices. It reaches only 0.0046 when the values of the elements of the supply-use matrices are allowed to deviate up to 30% of their actual

[4] Detailed results can be found in Appendix 1.

value and 0.0053 if absolute bias is considered instead. The resulting average bias is greater than that of Dietzenbacher (2006), although it remains in the same order of magnitude ($10^{-4}$).

**Table 1. Evaluation of the Leontief Inverse elements in the numerical simulations**

| ***L*** | ρ | | | | | |
|---|---|---|---|---|---|---|
| **Indicators of bias** | **0.05** | **0.10** | **0.15** | **0.20** | **0.25** | **0.30** |
| *Unweighted* | | | | | | |
| Average bias | 0.0001 | 0.0003 | 0.0008 | 0.0015 | 0.0036 | 0.0046 |
| Absolute average bias | 0.0003 | 0.0005 | 0.0010 | 0.0018 | 0.0041 | 0.0053 |
| *Weighted by final demand* | | | | | | |
| Average bias | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Absolute average bias | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0001 |
| *Weighted by output* | | | | | | |
| Average bias | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Absolute average bias | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0002 |
| *Significance tests* | | | | | | |
| % Significant | 16% | 39% | 64% | 75% | 73% | 80% |
| % Positively significant | 14% | 33% | 54% | 64% | 62% | 68% |
| % Negatively significant | 2% | 6% | 10% | 11% | 11% | 12% |

Source: Own elaboration. Significance level: 5%.

As regard weighted indicators, it is interesting to note that the average bias (also in absolute values) becomes absolutely negligible, as Dietzenbacher (2006) concluded. This actually suggests that the greatest bias may be located in commodities and industries that are not very relevant for the overall final demand and output structures of the base economy (e.g. uranium extraction).

Either bigger or smaller, it is very clear in our findings though (see Table 1) that the number of significant biased elements increases as long as the variability of the supply-use elements increases, too. Indeed, if we allowed up to a 30% of variability with respect to the actual elements of the supply-use matrices then around 80% of the elements of the Leontief inverse will present a significant bias.

As shown in Table 1, around 85% of the significant biased elements of the Leontief inverse happen to have positive bias. This result remains almost unchanged irrespective of the variability in the data, which is, by the way, a quite relevant finding to our knowledge bearing in mind that the theory precisely indicates over-estimation (see Section 1 for details).

However, considering the same sample size and variability in the data ($\rho$ = 0.1) as in Dietzebacher (2006), the relative number of significant biased elements observed in our experiment

(39.23%) is considerably smaller than the one reported by Dietzebacher (2006), which varies from 50% to 75%. This may have two possible causes: (a) element-wise deviation s in supply matrices may be compensated by the same type of deviations in use matrices, thus reducing the resulting bias; recall that Dietzebacher (2006) imposed stochastics on the input-output table rather than on SUTs; (b) the assumption of the commodity technology model for the construction of the matrix of technical coefficients has provided negative elements that may have affected the indicators used for comparisons; we are fully convinced that Dietzebacher (2006) did not face such a problem since he actually used non-negative input-output tables. Incidentally, this discussion opens the way to possibly carry out the same experiments but using instead the industry technology assumption to construct the technical coefficients matrix. Its main advantage is that it cannot give negatives by definition. However, this task clearly falls beyond the scope of this paper.

In sum, the results depicted in Table 1 confirmed the conclusions of ten Raa and Rueda-Cantuche (2007) about the likely positive bias of the elements of the Leontief inverse; and the findings of Dietzenbacher (2006) in relation with the negligible bias attributed to the elements of the Leontief inverse. More precisely, this paper confirms the results of Dietzenbacher (2006) irrespective of: (a) where stochastics may be imposed (SUTs or input-output tables); (b) the consideration of unweighted or weighted averages, the latter capturing the real structure of the base economy; and (c) the variability of the supply-use data ($\rho$).

## 7 Does bias really matter in backward output multipliers?

Beyond the value of each element of the Leontief inverse, most of the published input-output studies use the column sums of the Leontief Inverse (backward output multipliers) for impact analyses. Then, the next question to be answered would be: is the bias associated to such column sums also too small to be considered? This section gives an answer to this question by comparing the results obtained through the Leontief inverse based approach with respect to the *SUBE* approach (see Section 3).

Table 2 reports a summary of the results[5] obtained along the 1,000 trials both when the output multipliers are estimated by the column sums of the Leontief inverse and by the *SUBE* approach, respectively. Maybe the most important finding made here was that the *SUBE* approach provides unbiased output multipliers. Indeed, there is no significant bias in the *SUBE* output

[5] Detailed results can be found in Appendix 2.

multipliers while up to a hundred percent of the Leontief inverse based output multipliers are biased with a clear trend to over-estimation (positive bias). Moreover, the bias of output multipliers derived from the Leontief inverse tends to increase with the variability of the data. For instance, if we allow a deviation of 25% of the actual values of the elements of the supply-use matrices ($\rho$ = 0.25) then, the total bias amounts around 1.6% of the total final demand of the base economy and approximately 1.06% of the overall expected output.

**Table 2. Evaluation of output multipliers in the numerical simulations**

| **Leontief** | | | $\rho$ | | | |
|---|---|---|---|---|---|---|
| **Indicators of bias** | **0.05** | **0.10** | **0.15** | **0.20** | **0.25** | **0.30** |
| *Unweighted* | | | | | | |
| Average bias | 0.0011 | 0.0066 | 0.0160 | 0.0314 | 0.0754 | 0.0958 |
| Absolute average bias | 0.0021 | 0.0075 | 0.0181 | 0.0356 | 0.0832 | 0.1081 |
| *Weighted by final demand* | | | | | | |
| Average bias | 0.0001 | 0.0009 | 0.0027 | 0.0051 | 0.0163 | 0.0152 |
| Absolute average bias | 0.0012 | 0.0042 | 0.0108 | 0.0209 | 0.0464 | 0.0629 |
| *Weighted by output* | | | | | | |
| Average bias | 0.0000 | 0.0002 | 0.0005 | 0.0013 | 0.0059 | 0.0054 |
| Absolute average bias | 0.0013 | 0.0051 | 0.0127 | 0.0253 | 0.0516 | 0.0776 |
| *Significance tests* | | | | | | |
| % Significant | 100% | 100% | 90% | 100% | 81% | 86% |
| % Positively significant | 100% | 100% | 86% | 95% | 76% | 81% |
| % Negatively significant | 0% | 0% | 5% | 5% | 5% | 5% |
| *Expected bias as share of* | | | | | | |
| Total final demand | 0.005% | 0.088% | 0.265% | 0.507% | 1.629% | 1.524% |
| Total expected output | 0.004% | 0.057% | 0.173% | 0.331% | 1.064% | 0.995% |

| ***SUBE* (OLS)** | | | $\rho$ | | | |
|---|---|---|---|---|---|---|
| **Indicators of bias** | **0.05** | **0.10** | **0.15** | **0.20** | **0.25** | **0.30** |
| *Unweighted* | | | | | | |
| Average bias | -0.0019 | -0.0033 | 0.0054 | 0.0145 | 0.0138 | 0.0129 |
| Absolute average bias | 0.0041 | 0.0091 | 0.0132 | 0.0214 | 0.0232 | 0.0258 |
| *Weighted by final demand* | | | | | | |
| Average bias | -0.0001 | 0.0000 | -0.0004 | -0.0002 | -0.0002 | -0.0004 |
| Absolute average bias | 0.0010 | 0.0024 | 0.0028 | 0.0036 | 0.0037 | 0.0058 |
| *Weighted by output* | | | | | | |
| Average bias | -0.0003 | -0.0002 | -0.0002 | -0.0004 | -0.0003 | 0.0001 |
| Absolute average bias | 0.0011 | 0.0023 | 0.0026 | 0.0036 | 0.0038 | 0.0063 |
| *Significance tests* | | | | | | |
| % Significant | 0% | 0% | 0% | 0% | 0% | (*) 0% |
| % Positively significant | 0% | 0% | 0% | 0% | 0% | (*) 0% |
| % Negatively significant | 0% | 0% | 0% | 0% | 0% | 0% |
| *Expected bias as share of* | | | | | | |
| Total final demand | -0.009% | -0.004% | -0.045% | -0.025% | -0.025% | -0.037% |
| Total expected output | -0.006% | -0.002% | -0.029% | -0.016% | -0.016% | -0.024% |

Source: Own elaboration. Significance level: 5% except (*): 1%.

As regard the unweighted and weighted (absolute) average bias, the Leontief inverse approach provides greater biased results as long as the variability in the data increases. Indeed, the Leontief approach provides for $\rho$ = 0.3 an average unweighted bias more than seven times greater than that of the *SUBE* approach and more than four times in the case of absolute average biases. The differences get larger for weighted average biases. The weighted average bias expected from the Leontief inverse approach is (for the same variability) 38 times that of the *SUBE* approach if we use final demand weights and 54 times, otherwise. The figures in terms of the absolute average bias are not such high being only around eleven times when the weights correspond to final demand shares and more than twelve times, otherwise.

In our view, these results could be considered an almost definite proof that the *SUBE* approach is clearly superior to the Leontief inverse based estimations of output multipliers irrespective of the variability of the data ($\rho$) and the weights imposed to the selected indicators of bias. The errors to be made in impact analyses may amount to 1.6% of the total final demand if the data suffers from relatively high variability. Therefore, the main conclusions of this section may be summarized in two: (a) although the bias in each element of the Leontief inverse can be very small, they accumulate in the output multipliers vector and could lead to significant deviations; (b) the *SUBE* approach proposed originally by ten Raa and Rueda-Cantuche (2007) and extended further by Rueda-Cantuche and Amores (2010) lead to unbiased backward output multipliers.

## 8 Does bias really matter in $CO_2$ emission multipliers?

It is very common in input-output analysis to work not only with output multipliers but also with multipliers of diverse kind, such as labour, capital, energy, emissions, income, etc. Particularly, the calculation of air emission multipliers is currently a very prominent topic in many studies related to Environmental Extended Input-Output Analysis and Life Cycle Analysis.

The questions addressed in this section are therefore twofold. Firstly, we wonder whether the bias originated from the use of the Leontief inverse applied to other kind of multiplier is of the same magnitude than that of the output multipliers. We take carbon dioxide emissions in 2003 from Statistics Denmark as an exemplary exercise (see more details of the database used in Rueda-Cantuche and Amores, 2010); and secondly, we prove that the *SUBE* approach provides unbiased emission multipliers, too.

For the evaluation of the bias of the $CO_2$ emission multipliers, we performed the same test as in the previous section (see the details at the end of the fifth section) but considering now that $g_i^k$ is the value of the *i*-th component of the vector $\gamma_c$ in the *k*-th iteration of the Monte Carlo experiment and $g_i^{of}$, the "true" values of $CO_2$ emission multipliers derived from a randomly modified version (see section 4 for details) of the same emission multipliers reported by Rueda-Cantuche and Amores (2010).

Table 3 shows a summary of the results[6] obtained through the 1,000 trials carried out by means of the column sums of the Leontief inverse pre-multiplied by the corresponding row vector of emission coefficients and by the *SUBE* approach. Similarly to output multipliers, it is empirically proved that the latter approach provides unbiased emission multipliers while the Leontief inverse based method yields biased emission multipliers, which can deviate around 1.5% of the total expected emissions if the variability of the data is high. The deviation with respect to the total final demand yields 1.7%.

Furthermore, the number of significant biased emission multipliers when using the *SUBE* approach remains being almost zero irrespective of the variability of the data with the exception of one single commodity (i.e.: manufacture of other non-metallic mineral products; incidentally the second most polluting commodity in Denmark). Instead, the Leontief inverse approach may present a number of significant biased emission multipliers that generally increases with the variability of the data, e.g. from nearly half of them ($\rho$=0.15) to around eighty percent ($\rho$=0.25). At this point, it is also proven that the bias tends to be positive (see Table 3). Contrarily to output multipliers, as variability in the supply-use data increases, the share of positively biased emission multipliers increases, too; it varies from two thirds up to a hundred percent.

In relation to the unweighted and weighted absolute average bias, the Leontief inverse based method yields emission multipliers with larger bias as long as the variability in the data increases. Particularly, much less if the average bias is weighted by total output. Indeed, the Leontief approach yields for $\rho = 0.3$ an unweighted absolute average bias that is nearly four times greater than that of the *SUBE* approach. The differences get larger for weighted average biases and very similar to output multipliers in the case of the absolute average bias. The latter figures are not as much higher as in non-absolute values being only around eleven times when the weights

[6] Detailed results can be found in Appendix 3.

correspond to final demand shares and more than fourteen times, otherwise. Note that the weighted average biases are always smaller than their unweighted counterpart, which means that the multipliers with larger bias refer to commodities that are relatively not very relevant in terms of total final demand and total industry output.

**Table 3. Evaluation of CO2 emission multipliers in the numerical simulations**

| **Leontief** | ρ | | | | | |
|---|---|---|---|---|---|---|
| **Indicators of bias** | **0.05** | **0.10** | **0.15** | **0.20** | **0.25** | **0.30** |
| *Unweighted* | | | | | | |
| Average bias | 0.0019 | 0.0061 | 0.0139 | 0.0265 | 0.0528 | -0.0145 |
| Absolute average bias | 0.0024 | 0.0075 | 0.0164 | 0.0307 | 0.0604 | 0.1052 |
| *Weighted by final demand* | | | | | | |
| Average bias | 0.0005 | 0.0012 | 0.0025 | 0.0045 | 0.0076 | 0.0170 |
| Absolute average bias | 0.0012 | 0.0042 | 0.0096 | 0.0178 | 0.0335 | 0.0465 |
| *Weighted by output* | | | | | | |
| Average bias | 0.0003 | 0.0005 | 0.0011 | 0.0020 | 0.0039 | 0.0041 |
| Absolute average bias | 0.0015 | 0.0051 | 0.0117 | 0.0222 | 0.0431 | 0.0530 |
| *Significance tests* | | | | | | |
| % Significant | 14% | 24% | 48% | 67% | 81% | 43% |
| % Positively significant | 10% | 19% | 43% | 62% | 76% | 43% |
| % Negatively significant | 5% | 5% | 5% | 5% | 5% | 0% |
| *Expected bias as share of* | | | | | | |
| Total final demand | 0.048% | 0.116% | 0.249% | 0.450% | 0.764% | 1.698% |
| Total expected emissions | 0.043% | 0.106% | 0.227% | 0.410% | 0.696% | 1.545% |

| ***SUBE* (OLS)** | ρ | | | | | |
|---|---|---|---|---|---|---|
| **Indicators of bias** | **0.05** | **0.10** | **0.15** | **0.20** | **0.25** | **0.30** |
| *Unweighted* | | | | | | |
| Average bias | -0.0030 | -0.0037 | -0.0053 | -0.0060 | -0.0092 | -0.0080 |
| Absolute average bias | 0.0038 | 0.0057 | 0.0094 | 0.0110 | 0.0233 | 0.0276 |
| *Weighted by final demand* | | | | | | |
| Average bias | 0.0002 | 0.0003 | 0.0006 | 0.0004 | 0.0000 | -0.0001 |
| Absolute average bias | 0.0009 | 0.0013 | 0.0020 | 0.0021 | 0.0037 | 0.0040 |
| *Weighted by output* | | | | | | |
| Average bias | -0.0001 | -0.0001 | 0.0000 | -0.0001 | -0.0001 | -0.0002 |
| Absolute average bias | 0.0005 | 0.0009 | 0.0015 | 0.0014 | 0.0031 | 0.0037 |
| *Significance tests* | | | | | | |
| % Significant | (*) 5% | (*) 5% | (*) 0% | (*) 0% | 0% | 5% |
| % Positively significant | 0% | (*) 0% | (*) 0% | (*) 0% | 0% | 5% |
| % Negatively significant | (*) 5% | 5% | 0% | 0% | 0% | 0% |
| *Expected bias as share of* | | | | | | |
| Total final demand | 0.016% | 0.032% | 0.060% | 0.043% | -0.004% | -0.015% |
| Total expected emissions | 0.014% | 0.029% | 0.054% | 0.039% | -0.003% | -0.013% |

Source: Own elaboration. Significance level: 5% except (*): 1%.

Note: The 5% of significant biased emission multipliers refers only to a single commodity in all cases, i.e.: manufacture of other non-metallic mineral products (incidentally, the second most polluting product in Denmark after electricity).

From our viewpoint, the results for emission multipliers also show a considerably definite proof that the *SUBE* approach is evidently superior to the Leontief inverse based approach independently of the variability of the supply-use data ($\rho$) and the weights used in the selected indicators of bias. If data suffers from relatively high variability, then the errors may amount to 1.7% of the total final demand. Hence, the same two main conclusions can also be derived from emission multipliers: (a) even though the element-wise bias of the Leontief inverse can be negligible, they could lead to relatively important deviations in the calculation of emission multipliers; (b) the *SUBE* approach yields unbiased carbon dioxide emission multipliers.

### 8.1 Some further considerations on the comparison between output and $CO_2$ emission multipliers

The comparison of Tables 2 and 3 provide interesting results in relation to the behaviour of the differences in bias between output and any other kind of multiplier, i.e.: carbon dioxide emission multipliers. Our findings can be summarized in the following four main arguments:

a) In both approaches, the output multipliers have greater bias on (weighted) average than the emission multipliers irrespective of the variability of the supply-use data.

b) For unweighted averages, the Leontief inverse based approach provides exactly the opposite results to the *SUBE* approach. That is, while in the latter the output multipliers have larger bias than $CO_2$ emission multipliers for relatively low dispersed data ($\rho$) and just the opposite for relatively high dispersed supply-use data, it is exactly the other way round for the Leontief inverse method.

c) In both approaches, the emission multipliers have bigger bias than output multipliers in terms of total final demand and expected industry output/total $CO_2$ emissions and for low variability in the supply-use data. This pattern is clearer in the *SUBE* approach.

d) From the Leontief inverse approach, the number of significant biased output multipliers is much higher than that of emission multipliers irrespective of the variability of the data. Moreover, the share of positively biased emission multipliers increases as variability in the data grows but remains more or less constant in the case of output multipliers. The SUBE approach provides exactly the opposite results, i.e.: independently of the variability of the supply-use data, the number of significant biased emission multipliers is greater than that of output multipliers.

# 9 Conclusions

Far from concluding just with a summary of the main findings of this paper, we would like to add some reflections on their potential practical use for input-output practitioners. To begin with, this paper complements the results of Dietzenbacher (2006) and both prove empirically that the elements of the Leontief inverse are biased irrespective of: (a) where stochastics is imposed (SUTs or input-output tables); (b) the consideration of weights in the computation of the average bias; and (c) the variability of the supply-use data. Moreover, the bias may be considered negligible and most of the times, positive.

However, input-output studies use more often the column sums of the Leontief inverse (overall output effects by commodity) than the Leontief inverse matrix itself (matrix of output multipliers by commodity and industry). In doing so, the derived output multipliers will also be biased as much as 1.5% of the total final demand and 1% of the total expected industry output when the variability of the supply-use data is relatively high. For carbon dioxide multipliers, the expected average bias amounts to 1.7% of the total final demand and 1.5% of the total expected $CO_2$ emissions for the same high variability in the data.

For further interpreting the main findings of these results, a brief discussion on the variability of the supply-use data is in order. Take, for instance, the purchases of fabricated metal products by the food industry. Firstly, let us assume now that there is only one firm in the economy that belongs to the food industry and purchases such commodities. In this case, the variability in the use data is evidently zero. Secondly, suppose now that there are only two firms in the economy making such kind of transaction and that we only happen to dispose of the information of one of them that amounts 90% of the total of transactions made by the two firms. Independently of the actual value of the transaction of the non-surveyed firm, we can conclude easily that the estimated figure (without the non-surveyed firm) will not vary very much from the situation in which we could have had full information on the two firms. Nevertheless, the variability could be very high in those cases where the surveyed number of firms is too small compared to the large number of firms that populate the corresponding industry. In such cases, the variability in the supply-use data may have easily a high deviation from its estimated value, which may be relevant enough if we think of the trade and construction sectors, which are two very well known examples of limited sample coverage.

Obviously, one could ignore the issue of variability and perfectly use this paper to argue in the opposite direction, i.e.: assuming low variability in the supply-use data will lead us to continue using the Leontief inverse based method for impact analyses. Nevertheless, this paper also proves empirically that the so-called *Supply-Use Based Econometric* (*SUBE*) approach proposed by ten Raa and Rueda-Cantuche (2007) and extended by Rueda-Cantuche and Amores (2010) will provide unbiased backward input-output multipliers (of any kind: output, emissions, etc.) or at least, multipliers with much lower bias than that of the Leontief inverse approach irrespective of the variability of the supply-use data. Therefore, we can conclude that bias does matter in input-output analysis and this paper provides an almost definite solution to circumvent this problem, i.e.: the replacement of the Leontief inverse method by the *SUBE* approach for carrying out impact analyses.

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## References

Czamanski, S. and Malizia, E. E. (1969): Applicability and Limitations in the Use of National Input-Output Tables for Regional Studies, Papers of the Regional Science Association, 23, 65–77.

Dietzenbacher, E. (1988) Estimation of the Leontief Inverse from the Practitioner's Point of View, Mathematical Social Sciences, 16, 181–187.

Dietzenbacher, E. (1995) On the Bias of Multiplier Estimates, Journal of Regional Science, 35, 377–90.

Dietzenbacher, E. (2006) Multiplier Estimates: to Bias or not to Bias?, Journal of Regional Science, 46, 773–786.

Eurostat (1995) European Accounts System – ESA. Luxembourg: Office for Official Publications of the European Communities.

Eurostat (2008) Eurostat Manual of Supply, Use and Input-Output Tables, Methodologies and Working Papers. Luxembourg: Office for Official Publications of the European Communities.

Flam, S. D. and Thorlund-Petersen, L. (1985) Underestimation in the Leontief Model, Economics Letters, 18, 171–174.

Garhart, R. Jr. (1985): The Role of Error Structure in Simulations on Regional Input-Output Analysis, Journal of Regional Science, 25, 353–366.

Giarratani, F. (1986): Evidence on the Structure of Errors, paper presented at the British Section Regional Science Association. Bristol: United Kingdom.

Gerking, S. D. (1976) Input-Output as a Simple Econometric Model, Review of Economics and Statistics, 58, 274–282.

Gerking, S. D. (1979) Input-Output as a Simple Econometric Model: Reply, Review of Economics and Statistics, 61, 623–626.

Kop Jansen, P. S. M. (1994): Analysis of Multipliers in Stochastic Input-Output Models, Regional Science and Urban Economics, 24, 55–74.

Kop Jansen, P. S. M. and ten Raa, T. (1990) The Choice of Model in the Construction of Input–Output Coefficients Matrices, International Economic Review, 31, 213–27.

Lahiri, S. (1983) A Note on the Underestimation and Overestimation in Stochastic Input-Output Models, Economics Letters, 13, 361–365.

Lahiri, S. and Satchell, S. (1985): Underestimation and Overestimation of the Leontief Inverse Revisited, Economic Letters, 18, 181–186.

McMenamin, D. G. and Joseph E. Haring (1974): An Appraisal of Nonsurvey Techniques for Estimating Regional Input-Output Models, Journal of Regional Science, 14, 191–205.

Morrison, W. I. and Smith, P. (1974): Nonsurvey Input-Output Techniques at the Small Area Level: an Evaluation, Journal of Regional Science, 14, 1–14.

Miller, R. E. and Blair, P. (2009) Input-Output Analysis. Foundations and Extensions. 2nd ed. Cambridge University Press, Cambridge.

Park, S. Mohtadi, M. and Kubursi, A. (1981) Errors in Regional Non-Survey Input-Output Models: Analytical and Simulation Results, Journal of Regional Science, 21, 321–340.

Roland-Holst, D. W. (1989) Bias and Stability of Multiplier Estimates, Review of Economics and Statistics, 71, 718–21.

Round, J. I. (1978) An Interregional Input-Output Approach to the Evaluation of Non-Survey Methods, Journal of Regional Science, 18, 179-194.

Rueda-Cantuche, J. M. and ten Raa, T. (2009) The Choice of Model in the Construction of Industry Coefficients Matrices, Economic Systems Research, 21(4), 2009, 363–376.

Rueda-Cantuche, J. M. and Amores, A. F. (2010) Consistent and Unbiased Carbon Dioxide Emission Multipliers: Performance of Danish Emission Reductions via External Trade, Ecological Economics, 69, 988–998.

Schaffer, W. A. and Chu, K. (1969) Nonsurvey Techniques for Constructing Regional Interindustry Models, Papers of the Regional Science Association, 23, 83–101.

Stevens, B. H. and Trainer, G. A. (1980) Error Generation in Regional Input-Output Analysis and its Implication for Non-Survey Models, in Economic Impact Analysis: Methodology and Application ed. by S. Pleeh, 68–84. Boston: Martinus Nijhoff.

Simonovits, A. (1975) A Note on the Underestimation and Overestimation of the Leontief Inverse, Econometrica, 43, 493–498.

Statitistics Denmark. (2007) Danish Input-Output Tables and Analysis 2005. Imports, Employment and Environmet; Copenhagen: Statitistics Denmark.

ten Raa, T. and Steel, M. F. J. (1994) Revised Stochastic Analysis of an Input-Output Model, Regional Science and Urban Economics, 24, 361–371.

ten Raa, T. and Kop Jansen, P. S. M. (1998) Bias and Sensitivity of Multipliers, Economic Systems Research, 10, 275–283.

ten Raa, T. and Rueda-Cantuche, J. M. (2007) Stochastic Analysis of Input-Output Multipliers on the Basis of Use and Make Matrices, Review of Income and Wealth, 53, 1–17.

UN: United Nations (2003) Integrated Environmental and Economic Accounting 2003. New York: United Nations.

West, G. R. (1986): A Stochastic Analysis of an Input-Output Model, Econometrica, 54, 363–374.

## Appendix 1: Leontief Inverse elements results[7].

**Table A1.1. Leontief inverse obtained from an aggregated version of the officially published Supply and Use Tables, Denmark 2003.**

| $l_{ij}^{of}$ | d1 | d2 | d3 | d4 | d5 | d6 | d7 | d8 | d9 | d10 | d11 | d12 | d13 | d14 | d15 | d16 | d17 | d18 | d19 | d20 | d21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 1.1924 | 0.0509 | 0.0439 | 0.0250 | 0.0811 | 0.0110 | 0.0594 | 0.0276 | 0.0103 | 0.0599 | 0.0098 | 0.0022 | 0.0029 | 0.0372 | 0.0116 | 0.0203 | 0.0173 | 0.0015 | 0.0081 | 0.0206 | 0.0035 |
| **d2** | 0.0026 | 1.0122 | 0.0014 | 0.0061 | 0.0040 | 0.0009 | 0.0140 | 0.0008 | 0.0004 | 0.0020 | 0.0011 | 0.0003 | 0.0002 | 0.0012 | 0.0004 | 0.0012 | 0.0010 | 0.0001 | 0.0004 | 0.0010 | 0.0001 |
| **d3** | 0.0001 | 0.0004 | 1.0374 | 0.0014 | 0.0001 | 0.0009 | 0.0033 | 0.0016 | 0.0014 | 0.0009 | 0.0010 | 0.0002 | 0.0001 | 0.0015 | 0.0004 | 0.0006 | 0.0018 | 0.0002 | 0.0005 | 0.0008 | 0.0000 |
| **d4** | 0.0003 | 0.0002 | 0.0020 | 1.0208 | 0.0002 | 0.0013 | 0.0010 | 0.0008 | 0.0030 | 0.0027 | 0.0023 | 0.0006 | 0.0004 | 0.0006 | 0.0036 | 0.0008 | 0.0104 | 0.0017 | 0.0010 | 0.0025 | 0.0001 |
| **d5** | 0.8186 | 0.6662 | 0.0781 | 0.0340 | 1.2147 | 0.0453 | 0.0828 | 0.0657 | 0.0311 | 0.0955 | 0.0225 | 0.0052 | 0.0063 | 0.0723 | 0.0180 | 0.0398 | 0.0584 | 0.0089 | 0.0192 | 0.0395 | 0.0035 |
| **d6** | 0.0146 | 0.0044 | 0.0273 | 0.0069 | 0.0068 | 1.2222 | 0.0186 | 0.0395 | 0.0295 | 0.0353 | 0.0105 | 0.0013 | 0.0070 | 0.0365 | 0.0082 | 0.0256 | 0.0323 | 0.0022 | 0.0131 | 0.0224 | 0.0009 |
| **d7** | 0.0062 | 0.0006 | 0.0672 | 0.4634 | 0.0009 | 0.0036 | 1.0524 | 0.0059 | 0.0086 | 0.0077 | 0.0059 | 0.0013 | 0.0009 | 0.0120 | 0.0108 | 0.0028 | 0.0089 | 0.0011 | 0.0031 | 0.0062 | 0.0004 |
| **d8** | 0.0147 | 0.0096 | 0.0816 | 0.0112 | 0.0175 | 0.0273 | 0.0285 | 1.1853 | 0.0259 | 0.0337 | 0.0129 | 0.0096 | 0.0150 | 0.0393 | 0.0087 | 0.0187 | 0.0292 | 0.0035 | 0.0190 | 0.0257 | 0.0022 |
| **d9** | 0.0011 | 0.0007 | 0.2836 | 0.0084 | 0.0008 | 0.0081 | 0.0201 | 0.0098 | 1.0835 | 0.0139 | 0.0033 | 0.0008 | 0.0040 | 0.0133 | 0.0019 | 0.0064 | 0.0142 | 0.0017 | 0.0024 | 0.0040 | 0.0002 |
| **d10** | 0.0012 | 0.0009 | 0.0299 | 0.0049 | 0.0010 | 0.0104 | 0.0128 | 0.0210 | 0.0177 | 1.1940 | 0.0211 | 0.0048 | 0.0353 | 0.0205 | 0.0034 | 0.0127 | 0.0140 | 0.0013 | 0.0056 | 0.0080 | 0.0004 |
| **d11** | 0.0023 | 0.0023 | 0.0272 | 0.0074 | 0.0022 | 0.0210 | 0.0189 | 0.0359 | 0.0548 | 0.2414 | 1.1963 | 0.0290 | 0.0083 | 0.0284 | 0.0078 | 0.0345 | 0.0291 | 0.0033 | 0.0196 | 0.0199 | 0.0009 |
| **d12** | 0.0014 | 0.0010 | 0.0095 | 0.0029 | 0.0009 | 0.0143 | 0.0076 | 0.0228 | 0.0186 | 0.0731 | 0.0447 | 1.0845 | 0.0065 | 0.0132 | 0.0040 | 0.0144 | 0.0109 | 0.0008 | 0.0084 | 0.0105 | 0.0006 |
| **d13** | 0.0035 | 0.0013 | 0.0067 | 0.0027 | 0.0015 | 0.0407 | 0.0071 | 0.0164 | 0.0103 | 0.0299 | 0.0043 | 0.0006 | 1.0566 | 0.0120 | 0.0028 | 0.0098 | 0.0155 | 0.0008 | 0.0054 | 0.0074 | 0.0003 |
| **d14** | 0.0068 | 0.0006 | 0.0754 | 0.0832 | 0.0009 | 0.0030 | 0.0413 | 0.0018 | 0.0072 | 0.0058 | 0.0037 | 0.0010 | 0.0007 | 1.0065 | 0.0103 | 0.0021 | 0.0033 | 0.0004 | 0.0025 | 0.0050 | 0.0004 |
| **d15** | 0.0241 | 0.0060 | 0.4190 | 0.0493 | 0.0064 | 0.1259 | 0.0672 | 0.0631 | 0.6632 | 0.2509 | 0.0668 | 0.0390 | 0.0210 | 0.0336 | 1.0242 | 0.0757 | 0.0696 | 0.0075 | 0.0342 | 0.0627 | 0.0033 |
| **d16** | 0.0648 | 0.0728 | 0.0567 | 0.0298 | 0.0776 | 0.1173 | 0.0756 | 0.0719 | 0.0487 | 0.0689 | 0.0311 | 0.0340 | 0.0140 | 0.1129 | 0.0429 | 1.0709 | 0.2842 | 0.0212 | 0.0857 | 0.1332 | 0.0066 |
| **d17** | 0.0023 | 0.0017 | 0.0100 | 0.0273 | 0.0021 | 0.0113 | 0.0624 | 0.0061 | 0.0091 | 0.0142 | 0.0097 | 0.0089 | 0.0038 | 0.0109 | 0.0103 | 0.0166 | 1.1478 | 0.0335 | 0.0091 | 0.0180 | 0.0019 |
| **d18** | 0.0013 | 0.0007 | 0.0064 | 0.0058 | 0.0010 | 0.0137 | 0.0132 | 0.0034 | 0.0080 | 0.0066 | 0.0070 | 0.0071 | 0.0011 | 0.0060 | 0.0114 | 0.0040 | 0.0068 | 1.0032 | 0.0193 | 0.0526 | 0.0008 |
| **d19** | 0.0008 | 0.0005 | 0.0049 | 0.0246 | 0.0005 | 0.0039 | 0.0559 | 0.0027 | 0.0028 | 0.0045 | 0.0047 | 0.0037 | 0.0015 | 0.0025 | 0.0021 | 0.0051 | 0.0037 | 0.0006 | 1.0100 | 0.0072 | 0.0011 |
| **d20** | 0.0263 | 0.0141 | 0.1269 | 0.0404 | 0.0214 | 0.3305 | 0.0918 | 0.0622 | 0.1731 | 0.0971 | 0.0566 | 0.0540 | 0.0223 | 0.1335 | 0.2511 | 0.0718 | 0.1391 | 0.0142 | 0.3585 | 1.3230 | 0.0189 |
| **d21** | 0.0376 | 0.0166 | 0.0724 | 0.0196 | 0.0215 | 0.0665 | 0.0500 | 0.0560 | 0.0428 | 0.0433 | 0.0229 | 0.0407 | 0.0762 | 0.0912 | 0.0527 | 0.0348 | 0.1155 | 0.0069 | 0.1030 | 0.0913 | 1.0304 |

[7] Asterisks show the significance level of biases: 10% (*), 5% (**) and 1% (***).

**Table A1.2. Average Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.05**

| $\bar{l}_{ij}$ | d1 | d2 | d3 | d4 | d5 | d6 | d7 | d8 | d9 | d10 | d11 | d12 | d13 | d14 | d15 | d16 | d17 | d18 | d19 | d20 | d21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 1.1930 | 0.0510 | 0.0440 | 0.0250 | 0.0810 | 0.0110 | 0.0600 | 0.0280 | 0.0100 | 0.0600 | 0.0100 | 0.0020 | 0.0030 | 0.0370 | 0.0120 | 0.0200 | 0.0170 | 0.0010 | 0.0080 | 0.0210 | 0.0040 |
| **d2** | 0.0030 | 1.0120 | 0.0010 | 0.0060 | 0.0040 | 0.0009 | 0.0140 | 0.0008 | 0.0004 | 0.0020 | 0.0010 | 0.0003 | 0.0002 | 0.0010 | 0.0004 | 0.0010 | 0.0010 | 0.0001 | 0.0004 | 0.0010 | 0.0001 |
| **d3** | 0.0001 | 0.0004 | 1.0370 | 0.0010 | 0.0001 | 0.0009 | 0.0030 | 0.0020 | 0.0010 | 0.0009 | 0.0010 | 0.0002 | 0.0001 | 0.0010 | 0.0004 | 0.0006 | 0.0020 | 0.0002 | 0.0005 | 0.0008 | 0.0000 |
| **d4** | 0.0003 | 0.0002 | 0.0020 | 1.0210 | 0.0002 | 0.0010 | 0.0010 | 0.0008 | 0.0030 | 0.0030 | 0.0020 | 0.0006 | 0.0004 | 0.0006 | 0.0040 | 0.0008 | 0.0100 | 0.0020 | 0.0010 | 0.0020 | 0.0001 |
| **d5** | 0.8170 | 0.6660 | 0.0780 | 0.0340 | 1.2150 | 0.0450 | 0.0830 | 0.0660 | 0.0310 | 0.0960 | 0.0230 | 0.0050 | 0.0060 | 0.0720 | 0.0180 | 0.0400 | 0.0590 | 0.0090 | 0.0190 | 0.0400 | 0.0030 |
| **d6** | 0.0150 | 0.0040 | 0.0270 | 0.0070 | 0.0070 | 1.2230 | 0.0190 | 0.0400 | 0.0300 | 0.0360 | 0.0100 | 0.0010 | 0.0070 | 0.0370 | 0.0080 | 0.0260 | 0.0320 | 0.0020 | 0.0130 | 0.0220 | 0.0009 |
| **d7** | 0.0060 | 0.0006 | 0.0670 | 0.4650 | 0.0009 | 0.0040 | 1.0530 | 0.0060 | 0.0090 | 0.0080 | 0.0060 | 0.0010 | 0.0009 | 0.0120 | 0.0110 | 0.0030 | 0.0090 | 0.0010 | 0.0030 | 0.0060 | 0.0004 |
| **d8** | 0.0150 | 0.0100 | 0.0820 | 0.0110 | 0.0170 | 0.0270 | 0.0290 | 1.1860 | 0.0260 | 0.0340 | 0.0130 | 0.0100 | 0.0150 | 0.0390 | 0.0090 | 0.0190 | 0.0290 | 0.0040 | 0.0190 | 0.0260 | 0.0020 |
| **d9** | 0.0010 | 0.0007 | 0.2850 | 0.0080 | 0.0008 | 0.0080 | 0.0200 | 0.0100 | 1.0840 | 0.0140 | 0.0030 | 0.0008 | 0.0040 | 0.0130 | 0.0020 | 0.0060 | 0.0140 | 0.0020 | 0.0020 | 0.0040 | 0.0002 |
| **d10** | 0.0010 | 0.0009 | 0.0300 | 0.0050 | 0.0010 | 0.0100 | 0.0130 | 0.0210 | 0.0180 | 1.1950 | 0.0210 | 0.0050 | 0.0350 | 0.0200 | 0.0030 | 0.0130 | 0.0140 | 0.0010 | 0.0060 | 0.0080 | 0.0004 |
| **d11** | 0.0020 | 0.0020 | 0.0270 | 0.0070 | 0.0020 | 0.0210 | 0.0190 | 0.0360 | 0.0550 | 0.2420 | 1.1960 | 0.0290 | 0.0080 | 0.0280 | 0.0080 | 0.0340 | 0.0290 | 0.0030 | 0.0200 | 0.0200 | 0.0009 |
| **d12** | 0.0010 | 0.0010 | 0.0100 | 0.0030 | 0.0009 | 0.0140 | 0.0080 | 0.0230 | 0.0190 | 0.0740 | 0.0450 | 1.0850 | 0.0060 | 0.0130 | 0.0040 | 0.0140 | 0.0110 | 0.0008 | 0.0080 | 0.0100 | 0.0006 |
| **d13** | 0.0040 | 0.0010 | 0.0070 | 0.0030 | 0.0020 | 0.0410 | 0.0070 | 0.0160 | 0.0100 | 0.0300 | 0.0040 | 0.0006 | 1.0560 | 0.0120 | 0.0030 | 0.0100 | 0.0160 | 0.0008 | 0.0050 | 0.0070 | 0.0003 |
| **d14** | 0.0070 | 0.0006 | 0.0760 | 0.0830 | 0.0009 | 0.0030 | 0.0410 | 0.0020 | 0.0070 | 0.0060 | 0.0040 | 0.0010 | 0.0007 | 1.0060 | 0.0100 | 0.0020 | 0.0030 | 0.0004 | 0.0030 | 0.0050 | 0.0004 |
| **d15** | 0.0240 | 0.0060 | 0.4220 | 0.0500 | 0.0060 | 0.1260 | 0.0680 | 0.0630 | 0.6670 | 0.2530 | 0.0670 | 0.0390 | 0.0210 | 0.0340 | 1.0240 | 0.0760 | 0.0700 | 0.0070 | 0.0340 | 0.0620 | 0.0030 |
| **d16** | 0.0650 | 0.0730 | 0.0570 | 0.0300 | 0.0780 | 0.1180 | 0.0760 | 0.0720 | 0.0490 | 0.0690 | 0.0310 | 0.0340 | 0.0140 | 0.1130 | 0.0430 | 1.0710 | 0.2840 | 0.0210 | 0.0860 | 0.1330 | 0.0070 |
| **d17** | 0.0020 | 0.0020 | 0.0100 | 0.0280 | 0.0020 | 0.0110 | 0.0630 | 0.0060 | 0.0090 | 0.0140 | 0.0100 | 0.0090 | 0.0040 | 0.0110 | 0.0100 | 0.0170 | 1.1480 | 0.0330 | 0.0090 | 0.0180 | 0.0020 |
| **d18** | 0.0010 | 0.0007 | 0.0060 | 0.0060 | 0.0010 | 0.0140 | 0.0130 | 0.0030 | 0.0080 | 0.0070 | 0.0070 | 0.0070 | 0.0010 | 0.0060 | 0.0110 | 0.0040 | 0.0070 | 1.0030 | 0.0190 | 0.0530 | 0.0008 |
| **d19** | 0.0008 | 0.0005 | 0.0050 | 0.0250 | 0.0005 | 0.0040 | 0.0560 | 0.0030 | 0.0030 | 0.0040 | 0.0050 | 0.0040 | 0.0010 | 0.0020 | 0.0020 | 0.0050 | 0.0040 | 0.0006 | 1.0100 | 0.0070 | 0.0010 |
| **d20** | 0.0260 | 0.0140 | 0.1280 | 0.0410 | 0.0210 | 0.3320 | 0.0920 | 0.0620 | 0.1750 | 0.0980 | 0.0570 | 0.0540 | 0.0220 | 0.1330 | 0.2520 | 0.0720 | 0.1400 | 0.0140 | 0.3590 | 1.3230 | 0.0190 |
| **d21** | 0.0390 | 0.0190 | 0.0710 | 0.0190 | 0.0220 | 0.0660 | 0.0500 | 0.0550 | 0.0410 | 0.0430 | 0.0230 | 0.0410 | 0.0760 | 0.0910 | 0.0530 | 0.0350 | 0.1160 | 0.0070 | 0.1030 | 0.0910 | 1.0300 |

**Table A1.3. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.05 (1/2)**

| $\bar{b}_{ij}$ | **d1** | **d2** | **d3** | **d4** | **d5** | **d6** | **d7** | **d8** | **d9** | **d10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0006 | 0.0001 | 0.0001 | 0.0000 ** | -0.0001 | 0.0000 | 0.0006 ** | 0.0004 | -0.0003 * | 0.0001 * |
| **d2** | 0.0004 | -0.0002 * | -0.0004 *** | -0.0001 *** | 0.0000 | 0.0000 | 0.0000 *** | 0.0000 | 0.0000 ** | 0.0000 * |
| **d3** | 0.0000 | 0.0000 | -0.0004 | -0.0004 | 0.0000 * | 0.0000 | -0.0003 | 0.0004 | -0.0004 | 0.0000 * |
| **d4** | 0.0000 | 0.0000 | 0.0000 ** | 0.0002 | 0.0000 | -0.0003 * | 0.0000 *** | 0.0000 * | 0.0000 ** | 0.0003 ** |
| **d5** | -0.0016 | -0.0002 | -0.0001 | 0.0000 ** | 0.0003 | -0.0003 | 0.0002 ** | 0.0003 | -0.0001 ** | 0.0005 |
| **d6** | 0.0004 | -0.0004 | -0.0003 | 0.0001 *** | 0.0002 | 0.0008 | 0.0004 *** | 0.0005 * | 0.0005 ** | 0.0007 ** |
| **d7** | -0.0002 | 0.0000 | -0.0002 | 0.0016 | 0.0000 | 0.0004 | 0.0006 * | 0.0001 | 0.0004 | 0.0003 ** |
| **d8** | 0.0003 | 0.0004 | 0.0004 | -0.0002 *** | -0.0005 | -0.0003 | 0.0005 *** | 0.0007 ** | 0.0001 ** | 0.0003 ** |
| **d9** | -0.0001 | 0.0000 | 0.0014 | -0.0004 ** | 0.0000 | -0.0001 ** | -0.0001 *** | 0.0002 | 0.0005 | 0.0001 |
| **d10** | -0.0002 | 0.0000 | 0.0001 | 0.0001 ** | 0.0000 | -0.0004 | 0.0002 ** | 0.0000 | 0.0003 | 0.0010 * |
| **d11** | -0.0003 | -0.0003 | -0.0002 * | -0.0004 *** | -0.0002 | 0.0000 | 0.0001 *** | 0.0001 | 0.0002 | 0.0006 |
| **d12** | -0.0004 | 0.0000 | 0.0005 ** | 0.0001 ** | 0.0000 | -0.0003 * | 0.0004 *** | 0.0002 | 0.0004 | 0.0009 *** |
| **d13** | 0.0005 | -0.0003 | 0.0003 ** | 0.0003 ** | 0.0005 | 0.0003 | -0.0001 *** | -0.0004 | -0.0003 ** | 0.0001 * |
| **d14** | 0.0002 | 0.0000 | 0.0006 | -0.0002 | 0.0000 | 0.0000 | -0.0003 | 0.0002 | -0.0002 * | 0.0002 *** |
| **d15** | -0.0001 | 0.0000 | 0.0030 ** | 0.0007 ** | -0.0004 | 0.0001 * | 0.0008 *** | -0.0001 | 0.0038 * | 0.0021 ** |
| **d16** | 0.0002 | 0.0002 | 0.0003 * | 0.0002 ** | 0.0004 | 0.0007 | 0.0004 ** | 0.0001 | 0.0003 * | 0.0001 ** |
| **d17** | -0.0003 | 0.0003 | 0.0000 *** | 0.0007 ** | -0.0001 | -0.0003 | 0.0006 ** | -0.0001 | -0.0001 * | -0.0002 * |
| **d18** | -0.0003 | 0.0000 | -0.0004 ** | 0.0002 ** | 0.0000 | 0.0003 * | -0.0002 ** | -0.0004 | 0.0000 * | 0.0004 ** |
| **d19** | 0.0000 * | 0.0000 | 0.0001 *** | 0.0004 ** | 0.0000 | 0.0001 | 0.0001 ** | 0.0003 | 0.0002 ** | -0.0005 ** |
| **d20** | -0.0003 * | -0.0001 | 0.0011 ** | 0.0006 ** | -0.0004 | 0.0015 * | 0.0002 ** | -0.0002 | 0.0019 ** | 0.0009 ** |
| **d21** | 0.0014 | 0.0024 | -0.0014 | -0.0006 | 0.0005 | -0.0005 | 0.0000 | -0.0010 | -0.0018 | -0.0003 |

**Table A1.3. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.05 (2/2)**

| $\bar{b}_{ij}$ | **d11** | **d12** | **d13** | **d14** | **d15** | **d16** | **d17** | **d18** | **d19** | **d20** | **d21** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0002 | -0.0002 | 0.0001 | -0.0002 | 0.0004 | -0.0003 | -0.0003 | -0.0005 | -0.0001 | 0.0004 | 0.0005 |
| **d2** | -0.0001 * | 0.0000 | 0.0000 | -0.0002 | 0.0000 | -0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 * | 0.0000 |
| **d3** | 0.0000 | 0.0000 | 0.0000 | -0.0005 | 0.0000 * | 0.0000 * | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 * |
| **d4** | -0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0004 * | 0.0000 | -0.0004 | 0.0003 | 0.0000 | -0.0005 | 0.0000 |
| **d5** | 0.0005 | -0.0002 | -0.0003 | -0.0003 | 0.0000 | 0.0002 | 0.0006 | 0.0001 | -0.0002 | 0.0005 | -0.0005 |
| **d6** | -0.0005 | -0.0003 * | 0.0000 | 0.0005 | -0.0002 | 0.0004 | -0.0003 | -0.0002 | -0.0001 | -0.0004 * | 0.0000 |
| **d7** | 0.0001 * | -0.0003 | 0.0000 | 0.0000 | 0.0002 | 0.0002 | 0.0001 | -0.0001 * | -0.0001 | -0.0002 | 0.0000 |
| **d8** | 0.0001 | 0.0004 | 0.0000 | -0.0003 | 0.0003 | 0.0003 | -0.0002 | 0.0005 | 0.0000 | 0.0003 * | -0.0002 * |
| **d9** | -0.0003 ** | 0.0000 | 0.0000 | -0.0003 | 0.0001 | -0.0004 | -0.0002 | 0.0003 | -0.0004 | 0.0000 * | 0.0000 * |
| **d10** | -0.0001 | 0.0002 | -0.0003 | -0.0005 | -0.0004 | 0.0003 | 0.0000 | -0.0003 | 0.0004 | 0.0000 * | 0.0000 * |
| **d11** | -0.0003 | 0.0000 | -0.0003 | -0.0004 | 0.0002 | -0.0005 | -0.0001 | -0.0003 | 0.0004 | 0.0001 | 0.0000 |
| **d12** | 0.0003 | 0.0005 | -0.0005 ** | -0.0002 | 0.0000 | -0.0004 | 0.0001 | 0.0000 | -0.0004 | -0.0005 *** | 0.0000 * |
| **d13** | -0.0003 | 0.0000 | -0.0006 * | 0.0000 | 0.0002 | 0.0002 | 0.0005 | 0.0000 | -0.0004 | -0.0004 | 0.0000 |
| **d14** | 0.0003 ** | 0.0000 | 0.0000 | -0.0005 | -0.0003 | -0.0001 | -0.0003 | 0.0000 | 0.0005 | 0.0000 | 0.0000 |
| **d15** | 0.0002 * | 0.0000 | 0.0000 | 0.0004 | -0.0002 | 0.0003 | 0.0004 | -0.0005 | -0.0002 | -0.0007 *** | -0.0003 |
| **d16** | -0.0001 * | 0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0001 | -0.0002 | -0.0002 | 0.0003 | -0.0002 | 0.0004 |
| **d17** | 0.0003 | 0.0001 | 0.0002 | 0.0001 | -0.0003 | 0.0004 | 0.0002 | -0.0005 | -0.0001 | 0.0000 | 0.0001 |
| **d18** | 0.0000 | -0.0001 | -0.0001 | 0.0000 | -0.0004 | 0.0000 | 0.0002 | -0.0002 | -0.0003 | 0.0004 * | 0.0000 |
| **d19** | 0.0003 | 0.0003 | -0.0005 | -0.0005 | -0.0001 | -0.0001 * | 0.0003 | 0.0000 | 0.0000 | -0.0002 * | -0.0001 |
| **d20** | 0.0004 | 0.0000 | -0.0003 | -0.0005 | 0.0009 | 0.0002 | 0.0009 | -0.0002 | 0.0005 | 0.0000 * | 0.0001 |
| **d21** | 0.0001 | 0.0003 | -0.0002 | -0.0002 | 0.0003 | 0.0002 | 0.0005 | 0.0001 | 0.0000 | -0.0003 | -0.0004 |

**Table A1.4. Average Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.10**

| $\bar{l}_{ij}$ | **d1** | **d2** | **d3** | **d4** | **d5** | **d6** | **d7** | **d8** | **d9** | **d10** | **d11** | **d12** | **d13** | **d14** | **d15** | **d16** | **d17** | **d18** | **d19** | **d20** | **d21** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 1.1940 | 0.0520 | 0.0440 | 0.0260 | 0.0820 | 0.0110 | 0.0610 | 0.0280 | 0.0100 | 0.0610 | 0.0100 | 0.0020 | 0.0030 | 0.0370 | 0.0120 | 0.0200 | 0.0180 | 0.0010 | 0.0080 | 0.0200 | 0.0040 |
| **d2** | 0.0030 | 1.0120 | 0.0010 | 0.0060 | 0.0040 | 0.0009 | 0.0140 | 0.0008 | 0.0004 | 0.0020 | 0.0010 | 0.0003 | 0.0002 | 0.0010 | 0.0004 | 0.0010 | 0.0010 | 0.0001 | 0.0004 | 0.0009 | 0.0001 |
| **d3** | 0.0001 | 0.0004 | 1.0380 | 0.0010 | 0.0001 | 0.0009 | 0.0030 | 0.0020 | 0.0010 | 0.0009 | 0.0010 | 0.0002 | 0.0001 | 0.0010 | 0.0004 | 0.0006 | 0.0020 | 0.0002 | 0.0005 | 0.0008 | 0.0000 |
| **d4** | 0.0003 | 0.0002 | 0.0020 | 1.0210 | 0.0002 | 0.0010 | 0.0010 | 0.0008 | 0.0030 | 0.0030 | 0.0020 | 0.0006 | 0.0004 | 0.0006 | 0.0040 | 0.0008 | 0.0110 | 0.0020 | 0.0010 | 0.0020 | 0.0001 |
| **d5** | 0.8210 | 0.6700 | 0.0790 | 0.0350 | 1.2160 | 0.0450 | 0.0850 | 0.0660 | 0.0320 | 0.0970 | 0.0230 | 0.0050 | 0.0060 | 0.0730 | 0.0180 | 0.0400 | 0.0590 | 0.0090 | 0.0190 | 0.0390 | 0.0030 |
| **d6** | 0.0150 | 0.0040 | 0.0280 | 0.0070 | 0.0070 | 1.2240 | 0.0190 | 0.0400 | 0.0300 | 0.0360 | 0.0110 | 0.0010 | 0.0070 | 0.0370 | 0.0080 | 0.0260 | 0.0330 | 0.0020 | 0.0130 | 0.0220 | 0.0009 |
| **d7** | 0.0060 | 0.0006 | 0.0680 | 0.4680 | 0.0009 | 0.0030 | 1.0530 | 0.0060 | 0.0090 | 0.0080 | 0.0060 | 0.0010 | 0.0009 | 0.0120 | 0.0110 | 0.0030 | 0.0090 | 0.0010 | 0.0030 | 0.0060 | 0.0004 |
| **d8** | 0.0150 | 0.0100 | 0.0820 | 0.0120 | 0.0180 | 0.0280 | 0.0290 | 1.1880 | 0.0260 | 0.0340 | 0.0130 | 0.0100 | 0.0150 | 0.0400 | 0.0090 | 0.0190 | 0.0290 | 0.0040 | 0.0190 | 0.0250 | 0.0020 |
| **d9** | 0.0010 | 0.0007 | 0.2870 | 0.0090 | 0.0008 | 0.0080 | 0.0210 | 0.0100 | 1.0850 | 0.0140 | 0.0030 | 0.0008 | 0.0040 | 0.0130 | 0.0020 | 0.0060 | 0.0140 | 0.0020 | 0.0020 | 0.0040 | 0.0002 |
| **d10** | 0.0010 | 0.0010 | 0.0300 | 0.0050 | 0.0010 | 0.0100 | 0.0130 | 0.0210 | 0.0180 | 1.1970 | 0.0210 | 0.0050 | 0.0350 | 0.0210 | 0.0030 | 0.0130 | 0.0140 | 0.0010 | 0.0060 | 0.0080 | 0.0004 |
| **d11** | 0.0020 | 0.0020 | 0.0280 | 0.0080 | 0.0020 | 0.0210 | 0.0190 | 0.0360 | 0.0560 | 0.2450 | 1.1970 | 0.0290 | 0.0080 | 0.0290 | 0.0080 | 0.0350 | 0.0290 | 0.0030 | 0.0200 | 0.0190 | 0.0009 |
| **d12** | 0.0010 | 0.0010 | 0.0100 | 0.0030 | 0.0009 | 0.0140 | 0.0080 | 0.0230 | 0.0190 | 0.0750 | 0.0450 | 1.0850 | 0.0070 | 0.0130 | 0.0040 | 0.0140 | 0.0110 | 0.0008 | 0.0080 | 0.0100 | 0.0006 |
| **d13** | 0.0040 | 0.0010 | 0.0070 | 0.0030 | 0.0020 | 0.0410 | 0.0070 | 0.0160 | 0.0110 | 0.0300 | 0.0040 | 0.0006 | 1.0560 | 0.0120 | 0.0030 | 0.0100 | 0.0160 | 0.0008 | 0.0050 | 0.0070 | 0.0003 |
| **d14** | 0.0070 | 0.0006 | 0.0760 | 0.0840 | 0.0009 | 0.0030 | 0.0420 | 0.0020 | 0.0070 | 0.0060 | 0.0040 | 0.0010 | 0.0007 | 1.0070 | 0.0100 | 0.0020 | 0.0030 | 0.0004 | 0.0030 | 0.0050 | 0.0004 |
| **d15** | 0.0240 | 0.0060 | 0.4270 | 0.0510 | 0.0060 | 0.1270 | 0.0690 | 0.0640 | 0.6750 | 0.2560 | 0.0680 | 0.0390 | 0.0210 | 0.0340 | 1.0240 | 0.0760 | 0.0700 | 0.0080 | 0.0340 | 0.0620 | 0.0030 |
| **d16** | 0.0650 | 0.0740 | 0.0580 | 0.0310 | 0.0780 | 0.1180 | 0.0770 | 0.0720 | 0.0500 | 0.0700 | 0.0310 | 0.0340 | 0.0140 | 0.1130 | 0.0430 | 1.0710 | 0.2870 | 0.0210 | 0.0860 | 0.1320 | 0.0070 |
| **d17** | 0.0020 | 0.0020 | 0.0100 | 0.0280 | 0.0020 | 0.0110 | 0.0640 | 0.0060 | 0.0090 | 0.0140 | 0.0100 | 0.0090 | 0.0040 | 0.0110 | 0.0100 | 0.0170 | 1.1490 | 0.0340 | 0.0090 | 0.0180 | 0.0020 |
| **d18** | 0.0010 | 0.0007 | 0.0070 | 0.0060 | 0.0010 | 0.0140 | 0.0130 | 0.0030 | 0.0080 | 0.0070 | 0.0070 | 0.0070 | 0.0010 | 0.0060 | 0.0120 | 0.0040 | 0.0070 | 1.0030 | 0.0190 | 0.0530 | 0.0008 |
| **d19** | 0.0008 | 0.0005 | 0.0050 | 0.0250 | 0.0005 | 0.0040 | 0.0570 | 0.0030 | 0.0030 | 0.0050 | 0.0050 | 0.0040 | 0.0010 | 0.0030 | 0.0020 | 0.0050 | 0.0040 | 0.0006 | 1.0100 | 0.0070 | 0.0010 |
| **d20** | 0.0270 | 0.0140 | 0.1300 | 0.0410 | 0.0210 | 0.3340 | 0.0940 | 0.0630 | 0.1780 | 0.0990 | 0.0570 | 0.0540 | 0.0220 | 0.1340 | 0.2540 | 0.0720 | 0.1410 | 0.0140 | 0.3620 | 1.3210 | 0.0190 |
| **d21** | 0.0400 | 0.0210 | 0.0710 | 0.0190 | 0.0210 | 0.0650 | 0.0500 | 0.0550 | 0.0400 | 0.0420 | 0.0230 | 0.0410 | 0.0770 | 0.0910 | 0.0530 | 0.0350 | 0.1170 | 0.0070 | 0.1040 | 0.0910 | 1.0300 |

**Table A1.5. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.10 (1/2)**

| $\bar{b}_{ij}$ | **d1** | **d2** | **d3** | **d4** | **d5** | **d6** | **d7** | **d8** | **d9** | **d10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0016 | 0.0011 * | 0.0001 ** | 0.0010 *** | 0.0009 * | 0.0000 | 0.0016 *** | 0.0004 ** | -0.0003 *** | 0.0011 *** |
| **d2** | 0.0004 | -0.0002 *** | -0.0004 *** | -0.0001 *** | 0.0000 | 0.0000 | 0.0000 *** | 0.0000 | 0.0000 *** | 0.0000 *** |
| **d3** | 0.0000 | 0.0000 | 0.0006 | -0.0004 ** | 0.0000 | 0.0000 | -0.0003 * | 0.0004 * | -0.0004 * | 0.0000 *** |
| **d4** | 0.0000 ** | 0.0000 | 0.0000 *** | 0.0002 | 0.0000 | -0.0003 * | 0.0000 *** | 0.0000 *** | 0.0000 *** | 0.0003 *** |
| **d5** | 0.0024 | 0.0038 | 0.0009 *** | 0.0010 *** | 0.0013 | -0.0003 | 0.0022 *** | 0.0003 ** | 0.0009 *** | 0.0015 *** |
| **d6** | 0.0004 * | -0.0004 * | 0.0007 *** | 0.0001 *** | 0.0002 * | 0.0018 ** | 0.0004 *** | 0.0005 ** | 0.0005 *** | 0.0007 *** |
| **d7** | -0.0002 ** | 0.0000 | 0.0008 ** | 0.0046 | 0.0000 | -0.0006 | 0.0006 ** | 0.0001 | 0.0004 ** | 0.0003 *** |
| **d8** | 0.0003 | 0.0004 | 0.0004 * | 0.0008 *** | 0.0005 | 0.0007 * | 0.0005 *** | 0.0027 *** | 0.0001 *** | 0.0003 *** |
| **d9** | -0.0001 | 0.0000 | 0.0034 * | 0.0006 *** | 0.0000 | -0.0001 ** | 0.0009 *** | 0.0002 ** | 0.0015 ** | 0.0001 *** |
| **d10** | -0.0002 | 0.0000 | 0.0001 ** | 0.0001 *** | 0.0000 | -0.0004 | 0.0002 *** | 0.0000 | 0.0003 ** | 0.0030 *** |
| **d11** | -0.0003 | -0.0003 | 0.0008 *** | 0.0006 *** | -0.0002 | 0.0000 | 0.0001 *** | 0.0001 | 0.0012 * | 0.0036 ** |
| **d12** | -0.0004 | 0.0000 | 0.0005 *** | 0.0001 *** | 0.0000 | -0.0003 ** | 0.0004 *** | 0.0002 | 0.0004 ** | 0.0019 *** |
| **d13** | 0.0005 | -0.0003 | 0.0003 *** | 0.0003 *** | 0.0005 | 0.0003 * | -0.0001 *** | -0.0004 | 0.0007 *** | 0.0001 *** |
| **d14** | 0.0002 | 0.0000 | 0.0006 * | 0.0008 * | 0.0000 | 0.0000 | 0.0007 ** | 0.0002 *** | -0.0002 *** | 0.0002 *** |
| **d15** | -0.0001 * | 0.0000 | 0.0080 *** | 0.0017 *** | -0.0004 | 0.0011 ** | 0.0018 *** | 0.0009 * | 0.0118 ** | 0.0051 *** |
| **d16** | 0.0002 | 0.0012 ** | 0.0013 *** | 0.0012 *** | 0.0004 | 0.0007 ** | 0.0014 *** | 0.0001 | 0.0013 *** | 0.0011 *** |
| **d17** | -0.0003 * | 0.0003 | 0.0000 *** | 0.0007 *** | -0.0001 | -0.0003 | 0.0016 *** | -0.0001 ** | -0.0001 *** | -0.0002 *** |
| **d18** | -0.0003 *** | 0.0000 | 0.0006 *** | 0.0002 *** | 0.0000 | 0.0003 ** | -0.0002 *** | -0.0004 ** | 0.0000 *** | 0.0004 *** |
| **d19** | 0.0000 | 0.0000 * | 0.0001 *** | 0.0004 *** | 0.0000 * | 0.0001 | 0.0011 *** | 0.0003 * | 0.0002 *** | 0.0005 *** |
| **d20** | 0.0007 *** | -0.0001 | 0.0031 *** | 0.0006 *** | -0.0004 | 0.0035 ** | 0.0022 *** | 0.0008 ** | 0.0049 *** | 0.0019 *** |
| **d21** | 0.0024 | 0.0044 | -0.0014 | -0.0006 | -0.0005 | -0.0015 | 0.0000 | -0.0010 | -0.0028 | -0.0013 |

**Table A1.5. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.10 (2/2)**

| $\bar{b}_{ij}$ | **d11** | | **d12** | | **d13** | | **d14** | | **d15** | | **d16** | | **d17** | | **d18** | | **d19** | | **d20** | | **d21** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0002 | * | -0.0002 | * | 0.0001 | ** | -0.0002 | | 0.0004 | | -0.0003 | | 0.0007 | ** | -0.0005 | * | -0.0001 | | -0.0006 | *** | 0.0005 | |
| **d2** | -0.0001 | *** | 0.0000 | | 0.0000 | | -0.0002 | | 0.0000 | | -0.0002 | | 0.0000 | ** | 0.0000 | *** | 0.0000 | * | 0.0000 | *** | 0.0000 | * |
| **d3** | 0.0000 | ** | 0.0000 | | 0.0000 | | -0.0005 | | 0.0000 | ** | 0.0000 | | 0.0002 | ** | 0.0000 | | 0.0000 | | 0.0000 | *** | 0.0000 | *** |
| **d4** | -0.0003 | | 0.0000 | | 0.0000 | * | 0.0000 | | 0.0004 | ** | 0.0000 | * | 0.0006 | ** | 0.0003 | ** | 0.0000 | | -0.0005 | *** | 0.0000 | * |
| **d5** | 0.0005 | ** | -0.0002 | | -0.0003 | ** | 0.0007 | | 0.0000 | | 0.0002 | | 0.0006 | ** | 0.0001 | | -0.0002 | | -0.0005 | | -0.0005 | |
| **d6** | 0.0005 | ** | -0.0003 | * | 0.0000 | | 0.0005 | | -0.0002 | | 0.0004 | | 0.0007 | ** | -0.0002 | ** | -0.0001 | | -0.0004 | *** | 0.0000 | * |
| **d7** | 0.0001 | *** | -0.0003 | | 0.0000 | | 0.0000 | | 0.0002 | | 0.0002 | | 0.0001 | *** | -0.0001 | *** | -0.0001 | | -0.0002 | * | 0.0000 | * |
| **d8** | 0.0001 | * | 0.0004 | | 0.0000 | * | 0.0007 | | 0.0003 | | 0.0003 | | -0.0002 | | 0.0005 | | 0.0000 | | -0.0007 | *** | -0.0002 | * |
| **d9** | -0.0003 | *** | 0.0000 | | 0.0000 | | -0.0003 | | 0.0001 | | -0.0004 | | -0.0002 | ** | 0.0003 | ** | -0.0004 | | 0.0000 | *** | 0.0000 | *** |
| **d10** | -0.0001 | ** | 0.0002 | | -0.0003 | | 0.0005 | | -0.0004 | | 0.0003 | | 0.0000 | ** | -0.0003 | ** | 0.0004 | | 0.0000 | *** | 0.0000 | *** |
| **d11** | 0.0007 | | 0.0000 | | -0.0003 | ** | 0.0006 | | 0.0002 | | 0.0005 | | -0.0001 | * | -0.0003 | | 0.0004 | | -0.0009 | *** | 0.0000 | * |
| **d12** | 0.0003 | | 0.0005 | | 0.0005 | *** | -0.0002 | | 0.0000 | | -0.0004 | * | 0.0001 | ** | 0.0000 | | -0.0004 | | -0.0005 | *** | 0.0000 | * |
| **d13** | -0.0003 | ** | 0.0000 | | -0.0006 | | 0.0000 | | 0.0002 | | 0.0002 | | 0.0005 | | 0.0000 | * | -0.0004 | | -0.0004 | * | 0.0000 | |
| **d14** | 0.0003 | *** | 0.0000 | | 0.0000 | ** | 0.0005 | | -0.0003 | | -0.0001 | | -0.0003 | *** | 0.0000 | ** | 0.0005 | | 0.0000 | *** | 0.0000 | * |
| **d15** | 0.0012 | ** | 0.0000 | | 0.0000 | * | 0.0004 | ** | -0.0002 | | 0.0003 | | 0.0004 | ** | 0.0005 | ** | -0.0002 | | -0.0007 | *** | -0.0003 | * |
| **d16** | -0.0001 | *** | 0.0000 | | 0.0000 | | 0.0001 | | 0.0001 | | 0.0001 | | 0.0028 | | -0.0002 | | 0.0003 | | -0.0012 | *** | 0.0004 | |
| **d17** | 0.0003 | | 0.0001 | | 0.0002 | | 0.0001 | | -0.0003 | | 0.0004 | | 0.0012 | | 0.0005 | | -0.0001 | | 0.0000 | * | 0.0001 | * |
| **d18** | 0.0000 | | -0.0001 | | -0.0001 | | 0.0000 | | 0.0006 | | 0.0000 | | 0.0002 | *** | -0.0002 | * | -0.0003 | | 0.0004 | *** | 0.0000 | |
| **d19** | 0.0003 | | 0.0003 | | -0.0005 | | 0.0005 | | -0.0001 | | -0.0001 | | 0.0003 | | 0.0000 | * | 0.0000 | | -0.0002 | *** | -0.0001 | |
| **d20** | 0.0004 | *** | 0.0000 | | -0.0003 | | 0.0005 | | 0.0029 | * | 0.0002 | | 0.0019 | *** | -0.0002 | | 0.0035 | | -0.0020 | *** | 0.0001 | |
| **d21** | 0.0001 | | 0.0003 | | 0.0008 | | -0.0002 | | 0.0003 | | 0.0002 | | 0.0015 | | 0.0001 | | 0.0010 | | -0.0003 | | -0.0004 | |

**Table A1.6. Average Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.15**

| $\bar{l}_{ij}$ | **d1** | **d2** | **d3** | **d4** | **d5** | **d6** | **d7** | **d8** | **d9** | **d10** | **d11** | **d12** | **d13** | **d14** | **d15** | **d16** | **d17** | **d18** | **d19** | **d20** | **d21** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 1.1980 | 0.0530 | 0.0450 | 0.0270 | 0.0840 | 0.0110 | 0.0620 | 0.0280 | 0.0110 | 0.0620 | 0.0100 | 0.0020 | 0.0030 | 0.0380 | 0.0120 | 0.0200 | 0.0180 | 0.0010 | 0.0080 | 0.0190 | 0.0040 |
| **d2** | 0.0030 | 1.0130 | 0.0010 | 0.0070 | 0.0040 | 0.0009 | 0.0150 | 0.0008 | 0.0004 | 0.0020 | 0.0010 | 0.0003 | 0.0002 | 0.0010 | 0.0004 | 0.0010 | 0.0010 | 0.0001 | 0.0004 | 0.0009 | 0.0001 |
| **d3** | 0.0001 | 0.0004 | 1.0380 | 0.0010 | 0.0001 | 0.0009 | 0.0030 | 0.0020 | 0.0010 | 0.0010 | 0.0010 | 0.0002 | 0.0001 | 0.0010 | 0.0004 | 0.0006 | 0.0020 | 0.0002 | 0.0005 | 0.0008 | 0.0000 |
| **d4** | 0.0003 | 0.0002 | 0.0020 | 1.0210 | 0.0002 | 0.0010 | 0.0010 | 0.0008 | 0.0030 | 0.0030 | 0.0020 | 0.0006 | 0.0004 | 0.0006 | 0.0040 | 0.0008 | 0.0110 | 0.0020 | 0.0010 | 0.0020 | 0.0001 |
| **d5** | 0.8330 | 0.6800 | 0.0810 | 0.0370 | 1.2210 | 0.0450 | 0.0880 | 0.0670 | 0.0320 | 0.0990 | 0.0230 | 0.0050 | 0.0060 | 0.0740 | 0.0180 | 0.0400 | 0.0600 | 0.0090 | 0.0190 | 0.0360 | 0.0030 |
| **d6** | 0.0150 | 0.0050 | 0.0280 | 0.0070 | 0.0070 | 1.2260 | 0.0200 | 0.0410 | 0.0310 | 0.0370 | 0.0110 | 0.0010 | 0.0070 | 0.0370 | 0.0080 | 0.0260 | 0.0330 | 0.0020 | 0.0130 | 0.0220 | 0.0009 |
| **d7** | 0.0060 | 0.0006 | 0.0690 | 0.4750 | 0.0009 | 0.0030 | 1.0540 | 0.0060 | 0.0090 | 0.0080 | 0.0060 | 0.0010 | 0.0009 | 0.0120 | 0.0110 | 0.0030 | 0.0090 | 0.0010 | 0.0030 | 0.0050 | 0.0004 |
| **d8** | 0.0150 | 0.0100 | 0.0840 | 0.0120 | 0.0180 | 0.0280 | 0.0300 | 1.1910 | 0.0270 | 0.0350 | 0.0130 | 0.0100 | 0.0150 | 0.0400 | 0.0090 | 0.0190 | 0.0300 | 0.0040 | 0.0190 | 0.0250 | 0.0020 |
| **d9** | 0.0010 | 0.0007 | 0.2910 | 0.0090 | 0.0008 | 0.0080 | 0.0210 | 0.0100 | 1.0870 | 0.0140 | 0.0030 | 0.0008 | 0.0040 | 0.0140 | 0.0020 | 0.0060 | 0.0150 | 0.0020 | 0.0020 | 0.0040 | 0.0002 |
| **d10** | 0.0010 | 0.0010 | 0.0310 | 0.0050 | 0.0010 | 0.0100 | 0.0130 | 0.0210 | 0.0180 | 1.2000 | 0.0220 | 0.0050 | 0.0360 | 0.0210 | 0.0030 | 0.0130 | 0.0140 | 0.0010 | 0.0060 | 0.0070 | 0.0004 |
| **d11** | 0.0020 | 0.0020 | 0.0280 | 0.0080 | 0.0020 | 0.0210 | 0.0200 | 0.0370 | 0.0560 | 0.2490 | 1.1990 | 0.0290 | 0.0090 | 0.0290 | 0.0080 | 0.0350 | 0.0300 | 0.0030 | 0.0200 | 0.0190 | 0.0009 |
| **d12** | 0.0010 | 0.0010 | 0.0100 | 0.0030 | 0.0009 | 0.0140 | 0.0080 | 0.0230 | 0.0190 | 0.0760 | 0.0450 | 1.0850 | 0.0070 | 0.0130 | 0.0040 | 0.0150 | 0.0110 | 0.0008 | 0.0080 | 0.0100 | 0.0006 |
| **d13** | 0.0040 | 0.0010 | 0.0070 | 0.0030 | 0.0020 | 0.0410 | 0.0070 | 0.0170 | 0.0110 | 0.0310 | 0.0040 | 0.0006 | 1.0560 | 0.0120 | 0.0030 | 0.0100 | 0.0160 | 0.0009 | 0.0050 | 0.0070 | 0.0003 |
| **d14** | 0.0070 | 0.0006 | 0.0770 | 0.0860 | 0.0009 | 0.0030 | 0.0430 | 0.0020 | 0.0080 | 0.0060 | 0.0040 | 0.0010 | 0.0007 | 1.0070 | 0.0110 | 0.0020 | 0.0030 | 0.0004 | 0.0020 | 0.0050 | 0.0004 |
| **d15** | 0.0250 | 0.0060 | 0.4360 | 0.0520 | 0.0060 | 0.1280 | 0.0710 | 0.0640 | 0.6870 | 0.2600 | 0.0690 | 0.0390 | 0.0220 | 0.0340 | 1.0240 | 0.0760 | 0.0720 | 0.0080 | 0.0340 | 0.0600 | 0.0030 |
| **d16** | 0.0670 | 0.0760 | 0.0590 | 0.0320 | 0.0800 | 0.1190 | 0.0790 | 0.0730 | 0.0500 | 0.0710 | 0.0320 | 0.0340 | 0.0140 | 0.1140 | 0.0430 | 1.0710 | 0.2910 | 0.0220 | 0.0870 | 0.1310 | 0.0070 |
| **d17** | 0.0020 | 0.0020 | 0.0110 | 0.0290 | 0.0020 | 0.0110 | 0.0650 | 0.0060 | 0.0090 | 0.0150 | 0.0100 | 0.0090 | 0.0040 | 0.0110 | 0.0100 | 0.0170 | 1.1510 | 0.0340 | 0.0090 | 0.0170 | 0.0020 |
| **d18** | 0.0010 | 0.0007 | 0.0070 | 0.0060 | 0.0010 | 0.0140 | 0.0140 | 0.0030 | 0.0080 | 0.0070 | 0.0070 | 0.0070 | 0.0010 | 0.0060 | 0.0120 | 0.0040 | 0.0070 | 1.0030 | 0.0200 | 0.0520 | 0.0008 |
| **d19** | 0.0008 | 0.0004 | 0.0050 | 0.0260 | 0.0005 | 0.0040 | 0.0580 | 0.0030 | 0.0030 | 0.0050 | 0.0050 | 0.0040 | 0.0010 | 0.0030 | 0.0020 | 0.0050 | 0.0040 | 0.0006 | 1.0100 | 0.0070 | 0.0010 |
| **d20** | 0.0270 | 0.0140 | 0.1340 | 0.0430 | 0.0220 | 0.3370 | 0.0960 | 0.0630 | 0.1830 | 0.1010 | 0.0580 | 0.0540 | 0.0220 | 0.1350 | 0.2570 | 0.0720 | 0.1440 | 0.0140 | 0.3660 | 1.3190 | 0.0190 |
| **d21** | 0.0410 | 0.0220 | 0.0700 | 0.0190 | 0.0210 | 0.0650 | 0.0500 | 0.0540 | 0.0380 | 0.0420 | 0.0230 | 0.0410 | 0.0770 | 0.0910 | 0.0530 | 0.0350 | 0.1190 | 0.0070 | 0.1060 | 0.0910 | 1.0300 |

**Table A1.7. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.15 (1/2)**

| $\bar{b}_{ij}$ | d1 | | d2 | | d3 | | d4 | | d5 | | d6 | | d7 | | d8 | | d9 | | d10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0056 | *** | 0.0021 | *** | 0.0011 | *** | 0.0020 | *** | 0.0029 | *** | 0.0000 | | 0.0026 | *** | 0.0004 | *** | 0.0007 | *** | 0.0021 | *** |
| **d2** | 0.0004 | ** | 0.0008 | *** | -0.0004 | *** | 0.0009 | *** | 0.0000 | * | 0.0000 | | 0.0010 | *** | 0.0000 | *** | 0.0000 | *** | 0.0000 | *** |
| **d3** | 0.0000 | ** | 0.0000 | | 0.0006 | ** | -0.0004 | *** | 0.0000 | | 0.0000 | * | -0.0003 | *** | 0.0004 | ** | -0.0004 | *** | 0.0000 | *** |
| **d4** | 0.0000 | *** | 0.0000 | *** | 0.0000 | *** | 0.0002 | * | 0.0000 | * | -0.0003 | | 0.0000 | *** | 0.0000 | *** | 0.0000 | *** | 0.0003 | *** |
| **d5** | 0.0144 | * | 0.0138 | * | 0.0029 | *** | 0.0030 | *** | 0.0063 | ** | -0.0003 | | 0.0052 | *** | 0.0013 | *** | 0.0009 | *** | 0.0035 | *** |
| **d6** | 0.0004 | *** | 0.0006 | *** | 0.0007 | *** | 0.0001 | *** | 0.0002 | *** | 0.0038 | ** | 0.0014 | *** | 0.0015 | *** | 0.0015 | *** | 0.0017 | *** |
| **d7** | -0.0002 | *** | 0.0000 | | 0.0018 | *** | 0.0116 | ** | 0.0000 | | -0.0006 | * | 0.0016 | *** | 0.0001 | ** | 0.0004 | *** | 0.0003 | *** |
| **d8** | 0.0003 | | 0.0004 | | 0.0024 | *** | 0.0008 | *** | 0.0005 | | 0.0007 | ** | 0.0015 | *** | 0.0057 | *** | 0.0011 | *** | 0.0013 | *** |
| **d9** | -0.0001 | *** | 0.0000 | ** | 0.0074 | *** | 0.0006 | *** | 0.0000 | | -0.0001 | *** | 0.0009 | *** | 0.0002 | *** | 0.0035 | *** | 0.0001 | *** |
| **d10** | -0.0002 | | 0.0000 | * | 0.0011 | *** | 0.0001 | *** | 0.0000 | | -0.0004 | | 0.0002 | *** | 0.0000 | * | 0.0003 | *** | 0.0060 | *** |
| **d11** | -0.0003 | | -0.0003 | ** | 0.0008 | *** | 0.0006 | *** | -0.0002 | | 0.0000 | | 0.0011 | *** | 0.0011 | ** | 0.0012 | *** | 0.0076 | *** |
| **d12** | -0.0004 | | 0.0000 | | 0.0005 | *** | 0.0001 | *** | 0.0000 | | -0.0003 | ** | 0.0004 | *** | 0.0002 | ** | 0.0004 | *** | 0.0029 | *** |
| **d13** | 0.0005 | ** | -0.0003 | *** | 0.0003 | *** | 0.0003 | *** | 0.0005 | ** | 0.0003 | ** | -0.0001 | *** | 0.0006 | ** | 0.0007 | *** | 0.0011 | *** |
| **d14** | 0.0002 | ** | 0.0000 | *** | 0.0016 | *** | 0.0028 | *** | 0.0000 | ** | 0.0000 | | 0.0017 | *** | 0.0002 | *** | 0.0008 | *** | 0.0002 | *** |
| **d15** | 0.0009 | *** | 0.0000 | *** | 0.0170 | *** | 0.0027 | *** | -0.0004 | | 0.0021 | ** | 0.0038 | *** | 0.0009 | *** | 0.0238 | *** | 0.0091 | *** |
| **d16** | 0.0022 | *** | 0.0032 | *** | 0.0023 | *** | 0.0022 | *** | 0.0024 | ** | 0.0017 | ** | 0.0034 | *** | 0.0011 | * | 0.0013 | *** | 0.0021 | *** |
| **d17** | -0.0003 | *** | 0.0003 | | 0.0010 | *** | 0.0017 | *** | -0.0001 | | -0.0003 | | 0.0026 | *** | -0.0001 | *** | -0.0001 | *** | 0.0008 | *** |
| **d18** | -0.0003 | *** | 0.0000 | | 0.0006 | *** | 0.0002 | *** | 0.0000 | | 0.0003 | ** | 0.0008 | *** | -0.0004 | *** | 0.0000 | *** | 0.0004 | *** |
| **d19** | 0.0000 | | 0.0000 | *** | 0.0001 | *** | 0.0014 | *** | 0.0000 | *** | 0.0001 | | 0.0021 | *** | 0.0003 | *** | 0.0002 | *** | 0.0005 | *** |
| **d20** | 0.0007 | *** | -0.0001 | | 0.0071 | *** | 0.0026 | *** | 0.0006 | | 0.0065 | *** | 0.0042 | *** | 0.0008 | *** | 0.0099 | *** | 0.0039 | *** |
| **d21** | 0.0034 | | 0.0054 | | -0.0024 | | -0.0006 | | -0.0005 | | -0.0015 | | 0.0000 | | -0.0020 | | -0.0048 | | -0.0013 | |

**Table A1.7. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.15 (2/2)**

| $\bar{b}_{ij}$ | **d11** | **d12** | **d13** | **d14** | **d15** | **d16** | **d17** | **d18** | **d19** | **d20** | **d21** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0002 *** | -0.0002 *** | 0.0001 *** | 0.0008 * | 0.0004 | -0.0003 | 0.0007 *** | -0.0005 ** | -0.0001 * | -0.0016 *** | 0.0005 |
| **d2** | -0.0001 *** | 0.0000 | 0.0000 ** | -0.0002 *** | 0.0000 | -0.0002 * | 0.0000 *** | 0.0000 *** | 0.0000 *** | -0.0001 *** | 0.0000 *** |
| **d3** | 0.0000 *** | 0.0000 | 0.0000 ** | -0.0005 | 0.0000 *** | 0.0000 | 0.0002 *** | 0.0000 *** | 0.0000 * | 0.0000 *** | 0.0000 *** |
| **d4** | -0.0003 ** | 0.0000 * | 0.0000 *** | 0.0000 | 0.0004 *** | 0.0000 *** | 0.0006 *** | 0.0003 *** | 0.0000 | -0.0005 *** | 0.0000 *** |
| **d5** | 0.0005 *** | -0.0002 * | -0.0003 *** | 0.0017 ** | 0.0000 | 0.0002 * | 0.0016 *** | 0.0001 *** | -0.0002 | -0.0035 *** | -0.0005 |
| **d6** | 0.0005 *** | -0.0003 *** | 0.0000 | 0.0005 ** | -0.0002 | 0.0004 | 0.0007 *** | -0.0002 *** | -0.0001 | -0.0004 *** | 0.0000 * |
| **d7** | 0.0001 *** | -0.0003 * | 0.0000 | 0.0000 | 0.0002 | 0.0002 | 0.0001 *** | -0.0001 *** | -0.0001 * | -0.0012 *** | 0.0000 *** |
| **d8** | 0.0001 *** | 0.0004 | 0.0000 *** | 0.0007 ** | 0.0003 | 0.0003 | 0.0008 ** | 0.0005 ** | 0.0000 | -0.0007 *** | -0.0002 * |
| **d9** | -0.0003 *** | 0.0000 | 0.0000 ** | 0.0007 ** | 0.0001 | -0.0004 | 0.0008 *** | 0.0003 *** | -0.0004 | 0.0000 *** | 0.0000 *** |
| **d10** | 0.0009 *** | 0.0002 | 0.0007 * | 0.0005 | -0.0004 | 0.0003 | 0.0000 *** | -0.0003 *** | 0.0004 | -0.0010 *** | 0.0000 *** |
| **d11** | 0.0027 * | 0.0000 | 0.0007 *** | 0.0006 * | 0.0002 | 0.0005 | 0.0009 *** | -0.0003 *** | 0.0004 | -0.0009 *** | 0.0000 *** |
| **d12** | 0.0003 ** | 0.0005 | 0.0005 *** | -0.0002 | 0.0000 | 0.0006 ** | 0.0001 *** | 0.0000 *** | -0.0004 | -0.0005 *** | 0.0000 *** |
| **d13** | -0.0003 *** | 0.0000 * | -0.0006 | 0.0000 * | 0.0002 | 0.0002 | 0.0005 *** | 0.0000 *** | -0.0004 | -0.0004 *** | 0.0000 * |
| **d14** | 0.0003 *** | 0.0000 | 0.0000 *** | 0.0005 | 0.0007 ** | -0.0001 * | -0.0003 *** | 0.0000 *** | -0.0005 | 0.0000 *** | 0.0000 * |
| **d15** | 0.0022 *** | 0.0000 | 0.0010 *** | 0.0004 *** | -0.0002 | 0.0003 | 0.0024 *** | 0.0005 *** | -0.0002 | -0.0027 *** | -0.0003 * |
| **d16** | 0.0009 *** | 0.0000 | 0.0000 * | 0.0011 | 0.0001 * | 0.0001 | 0.0068 ** | 0.0008 *** | 0.0013 * | -0.0022 *** | 0.0004 |
| **d17** | 0.0003 *** | 0.0001 | 0.0002 | 0.0001 | -0.0003 | 0.0004 | 0.0032 ** | 0.0005 ** | -0.0001 | -0.0010 *** | 0.0001 * |
| **d18** | 0.0000 ** | -0.0001 | -0.0001 ** | 0.0000 | 0.0006 ** | 0.0000 * | 0.0002 *** | -0.0002 *** | 0.0007 ** | -0.0006 *** | 0.0000 |
| **d19** | 0.0003 * | 0.0003 | -0.0005 | 0.0005 ** | -0.0001 | -0.0001 | 0.0003 *** | 0.0000 * | 0.0000 * | -0.0002 *** | -0.0001 |
| **d20** | 0.0014 *** | 0.0000 | -0.0003 | 0.0015 | 0.0059 ** | 0.0002 | 0.0049 *** | -0.0002 | 0.0075 ** | -0.0040 *** | 0.0001 |
| **d21** | 0.0001 | 0.0003 | 0.0008 ** | -0.0002 | 0.0003 | 0.0002 | 0.0035 * | 0.0001 | 0.0030 | -0.0003 | -0.0004 |

**Table A1.8. Average Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.20**

| $\bar{l}_{ij}$ | d1 | d2 | d3 | d4 | d5 | d6 | d7 | d8 | d9 | d10 | d11 | d12 | d13 | d14 | d15 | d16 | d17 | d18 | d19 | d20 | d21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 1.2050 | 0.0560 | 0.0460 | 0.0280 | 0.0860 | 0.0110 | 0.0640 | 0.0290 | 0.0110 | 0.0640 | 0.0100 | 0.0020 | 0.0030 | 0.0380 | 0.0120 | 0.0210 | 0.0180 | 0.0010 | 0.0080 | 0.0180 | 0.0040 |
| **d2** | 0.0030 | 1.0130 | 0.0020 | 0.0070 | 0.0040 | 0.0009 | 0.0150 | 0.0008 | 0.0005 | 0.0020 | 0.0010 | 0.0003 | 0.0002 | 0.0010 | 0.0004 | 0.0010 | 0.0010 | 0.0001 | 0.0004 | 0.0008 | 0.0001 |
| **d3** | 0.0001 | 0.0004 | 1.0390 | 0.0020 | 0.0001 | 0.0009 | 0.0040 | 0.0020 | 0.0020 | 0.0010 | 0.0010 | 0.0002 | 0.0001 | 0.0020 | 0.0004 | 0.0006 | 0.0020 | 0.0002 | 0.0005 | 0.0007 | 0.0000 |
| **d4** | 0.0003 | 0.0002 | 0.0020 | 1.0220 | 0.0002 | 0.0010 | 0.0010 | 0.0009 | 0.0030 | 0.0030 | 0.0020 | 0.0006 | 0.0004 | 0.0006 | 0.0040 | 0.0008 | 0.0110 | 0.0020 | 0.0010 | 0.0020 | 0.0001 |
| **d5** | 0.8550 | 0.6970 | 0.0840 | 0.0400 | 1.2280 | 0.0450 | 0.0930 | 0.0690 | 0.0330 | 0.1030 | 0.0240 | 0.0050 | 0.0070 | 0.0750 | 0.0180 | 0.0410 | 0.0620 | 0.0090 | 0.0180 | 0.0320 | 0.0030 |
| **d6** | 0.0160 | 0.0050 | 0.0290 | 0.0080 | 0.0070 | 1.2290 | 0.0200 | 0.0410 | 0.0320 | 0.0380 | 0.0110 | 0.0010 | 0.0070 | 0.0380 | 0.0080 | 0.0260 | 0.0340 | 0.0020 | 0.0130 | 0.0210 | 0.0009 |
| **d7** | 0.0070 | 0.0007 | 0.0710 | 0.4850 | 0.0010 | 0.0030 | 1.0560 | 0.0060 | 0.0090 | 0.0080 | 0.0060 | 0.0010 | 0.0009 | 0.0120 | 0.0110 | 0.0030 | 0.0100 | 0.0010 | 0.0030 | 0.0040 | 0.0004 |
| **d8** | 0.0150 | 0.0100 | 0.0860 | 0.0130 | 0.0180 | 0.0280 | 0.0310 | 1.1950 | 0.0280 | 0.0360 | 0.0130 | 0.0090 | 0.0160 | 0.0410 | 0.0090 | 0.0190 | 0.0310 | 0.0040 | 0.0190 | 0.0240 | 0.0020 |
| **d9** | 0.0010 | 0.0008 | 0.2970 | 0.0090 | 0.0008 | 0.0080 | 0.0220 | 0.0100 | 1.0890 | 0.0150 | 0.0030 | 0.0008 | 0.0040 | 0.0140 | 0.0020 | 0.0060 | 0.0150 | 0.0020 | 0.0020 | 0.0030 | 0.0002 |
| **d10** | 0.0010 | 0.0010 | 0.0320 | 0.0060 | 0.0010 | 0.0100 | 0.0140 | 0.0220 | 0.0190 | 1.2060 | 0.0220 | 0.0050 | 0.0370 | 0.0210 | 0.0030 | 0.0130 | 0.0150 | 0.0010 | 0.0060 | 0.0070 | 0.0004 |
| **d11** | 0.0020 | 0.0020 | 0.0290 | 0.0080 | 0.0020 | 0.0210 | 0.0210 | 0.0370 | 0.0580 | 0.2550 | 1.2020 | 0.0290 | 0.0090 | 0.0290 | 0.0080 | 0.0350 | 0.0310 | 0.0030 | 0.0200 | 0.0170 | 0.0009 |
| **d12** | 0.0010 | 0.0010 | 0.0100 | 0.0030 | 0.0009 | 0.0150 | 0.0080 | 0.0230 | 0.0200 | 0.0780 | 0.0460 | 1.0860 | 0.0070 | 0.0140 | 0.0040 | 0.0150 | 0.0120 | 0.0009 | 0.0090 | 0.0090 | 0.0006 |
| **d13** | 0.0040 | 0.0010 | 0.0070 | 0.0030 | 0.0020 | 0.0420 | 0.0080 | 0.0170 | 0.0110 | 0.0320 | 0.0040 | 0.0006 | 1.0570 | 0.0120 | 0.0030 | 0.0100 | 0.0160 | 0.0009 | 0.0060 | 0.0070 | 0.0002 |
| **d14** | 0.0070 | 0.0006 | 0.0790 | 0.0880 | 0.0009 | 0.0030 | 0.0440 | 0.0020 | 0.0080 | 0.0060 | 0.0040 | 0.0010 | 0.0007 | 1.0070 | 0.0110 | 0.0020 | 0.0040 | 0.0004 | 0.0020 | 0.0040 | 0.0004 |
| **d15** | 0.0250 | 0.0060 | 0.4490 | 0.0540 | 0.0070 | 0.1300 | 0.0740 | 0.0650 | 0.7060 | 0.2670 | 0.0700 | 0.0390 | 0.0220 | 0.0350 | 1.0240 | 0.0770 | 0.0740 | 0.0080 | 0.0340 | 0.0580 | 0.0030 |
| **d16** | 0.0690 | 0.0780 | 0.0610 | 0.0340 | 0.0820 | 0.1200 | 0.0830 | 0.0740 | 0.0520 | 0.0730 | 0.0320 | 0.0340 | 0.0140 | 0.1160 | 0.0440 | 1.0720 | 0.2990 | 0.0220 | 0.0880 | 0.1290 | 0.0070 |
| **d17** | 0.0020 | 0.0020 | 0.0110 | 0.0310 | 0.0020 | 0.0110 | 0.0680 | 0.0060 | 0.0100 | 0.0150 | 0.0100 | 0.0090 | 0.0040 | 0.0110 | 0.0100 | 0.0170 | 1.1550 | 0.0350 | 0.0090 | 0.0160 | 0.0020 |
| **d18** | 0.0010 | 0.0007 | 0.0070 | 0.0060 | 0.0010 | 0.0140 | 0.0140 | 0.0040 | 0.0090 | 0.0070 | 0.0070 | 0.0070 | 0.0010 | 0.0060 | 0.0120 | 0.0040 | 0.0070 | 1.0030 | 0.0200 | 0.0520 | 0.0008 |
| **d19** | 0.0008 | 0.0004 | 0.0050 | 0.0270 | 0.0005 | 0.0040 | 0.0600 | 0.0030 | 0.0030 | 0.0050 | 0.0050 | 0.0040 | 0.0010 | 0.0030 | 0.0020 | 0.0050 | 0.0040 | 0.0006 | 1.0100 | 0.0070 | 0.0010 |
| **d20** | 0.0270 | 0.0140 | 0.1400 | 0.0450 | 0.0220 | 0.3400 | 0.0990 | 0.0640 | 0.1900 | 0.1050 | 0.0590 | 0.0540 | 0.0230 | 0.1370 | 0.2610 | 0.0730 | 0.1480 | 0.0140 | 0.3720 | 1.3160 | 0.0190 |
| **d21** | 0.0420 | 0.0240 | 0.0700 | 0.0200 | 0.0210 | 0.0640 | 0.0500 | 0.0540 | 0.0360 | 0.0410 | 0.0230 | 0.0410 | 0.0780 | 0.0910 | 0.0540 | 0.0350 | 0.1230 | 0.0060 | 0.1080 | 0.0910 | 1.0300 |

**Table A1.9. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.20 (1/2)**

| $\bar{b}_{ij}$ | **d1** | | **d2** | | **d3** | | **d4** | | **d5** | | **d6** | | **d7** | | **d8** | | **d9** | | **d10** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0126 | *** | 0.0051 | *** | 0.0021 | *** | 0.0030 | *** | 0.0049 | *** | 0.0000 | * | 0.0046 | *** | 0.0014 | *** | 0.0007 | *** | 0.0041 | *** |
| **d2** | 0.0004 | *** | 0.0008 | *** | 0.0006 | *** | 0.0009 | *** | 0.0000 | *** | 0.0000 | | 0.0010 | *** | 0.0000 | *** | 0.0000 | *** | 0.0000 | *** |
| **d3** | 0.0000 | *** | 0.0000 | ** | 0.0016 | *** | 0.0006 | *** | 0.0000 | | 0.0000 | ** | 0.0007 | *** | 0.0004 | *** | 0.0006 | *** | 0.0001 | *** |
| **d4** | 0.0000 | *** | 0.0000 | *** | 0.0000 | *** | 0.0012 | *** | 0.0000 | *** | -0.0003 | | 0.0000 | *** | 0.0000 | *** | 0.0000 | *** | 0.0003 | *** |
| **d5** | 0.0364 | *** | 0.0308 | *** | 0.0059 | *** | 0.0060 | *** | 0.0133 | *** | -0.0003 | | 0.0102 | *** | 0.0033 | *** | 0.0019 | *** | 0.0075 | *** |
| **d6** | 0.0014 | *** | 0.0006 | *** | 0.0017 | *** | 0.0011 | *** | 0.0002 | *** | 0.0068 | *** | 0.0014 | *** | 0.0015 | *** | 0.0025 | *** | 0.0027 | *** |
| **d7** | 0.0008 | *** | 0.0000 | ** | 0.0038 | *** | 0.0216 | *** | 0.0000 | | -0.0006 | *** | 0.0036 | *** | 0.0001 | *** | 0.0004 | *** | 0.0003 | *** |
| **d8** | 0.0003 | *** | 0.0004 | *** | 0.0044 | *** | 0.0018 | *** | 0.0005 | ** | 0.0007 | ** | 0.0025 | *** | 0.0097 | *** | 0.0021 | *** | 0.0023 | *** |
| **d9** | -0.0001 | *** | 0.0000 | *** | 0.0134 | *** | 0.0006 | *** | 0.0000 | | -0.0001 | *** | 0.0019 | *** | 0.0002 | *** | 0.0055 | *** | 0.0011 | *** |
| **d10** | -0.0002 | | 0.0000 | ** | 0.0021 | *** | 0.0011 | *** | 0.0000 | | -0.0004 | | 0.0012 | *** | 0.0010 | ** | 0.0013 | *** | 0.0120 | *** |
| **d11** | -0.0003 | | -0.0003 | *** | 0.0018 | *** | 0.0006 | *** | -0.0002 | | 0.0000 | | 0.0021 | *** | 0.0011 | *** | 0.0032 | *** | 0.0136 | *** |
| **d12** | -0.0004 | * | 0.0000 | ** | 0.0005 | *** | 0.0001 | *** | 0.0000 | | 0.0007 | ** | 0.0004 | *** | 0.0002 | *** | 0.0014 | *** | 0.0049 | *** |
| **d13** | 0.0005 | *** | -0.0003 | *** | 0.0003 | *** | 0.0003 | *** | 0.0005 | *** | 0.0013 | *** | 0.0009 | *** | 0.0006 | *** | 0.0007 | *** | 0.0021 | *** |
| **d14** | 0.0002 | *** | 0.0000 | *** | 0.0036 | *** | 0.0048 | *** | 0.0000 | *** | 0.0000 | | 0.0027 | *** | 0.0002 | *** | 0.0008 | *** | 0.0002 | *** |
| **d15** | 0.0009 | *** | 0.0000 | *** | 0.0300 | *** | 0.0047 | *** | 0.0006 | ** | 0.0041 | *** | 0.0068 | *** | 0.0019 | *** | 0.0428 | *** | 0.0161 | *** |
| **d16** | 0.0042 | *** | 0.0052 | *** | 0.0043 | *** | 0.0042 | *** | 0.0044 | *** | 0.0027 | *** | 0.0074 | *** | 0.0021 | ** | 0.0033 | *** | 0.0041 | *** |
| **d17** | -0.0003 | *** | 0.0003 | | 0.0010 | *** | 0.0037 | *** | -0.0001 | | -0.0003 | | 0.0056 | *** | -0.0001 | *** | 0.0009 | *** | 0.0008 | *** |
| **d18** | -0.0003 | *** | 0.0000 | | 0.0006 | *** | 0.0002 | *** | 0.0000 | * | 0.0003 | *** | 0.0008 | *** | 0.0006 | *** | 0.0010 | *** | 0.0004 | *** |
| **d19** | 0.0000 | | 0.0000 | *** | 0.0001 | *** | 0.0024 | *** | 0.0000 | *** | 0.0001 | | 0.0041 | *** | 0.0003 | *** | 0.0002 | *** | 0.0005 | *** |
| **d20** | 0.0007 | *** | -0.0001 | | 0.0131 | *** | 0.0046 | *** | 0.0006 | | 0.0095 | *** | 0.0072 | *** | 0.0018 | *** | 0.0169 | *** | 0.0079 | *** |
| **d21** | 0.0044 | | 0.0074 | | -0.0024 | | 0.0004 | | -0.0005 | | -0.0025 | | 0.0000 | | -0.0020 | | -0.0068 | | -0.0023 | |

**Table A1.9. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.20 (2/2)**

| $\bar{b}_{ij}$ | **d11** | | **d12** | | **d13** | | **d14** | | **d15** | | **d16** | | **d17** | | **d18** | | **d19** | | **d20** | | **d21** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0002 | *** | -0.0002 | *** | 0.0001 | *** | 0.0008 | *** | 0.0004 | | 0.0007 | * | 0.0007 | *** | -0.0005 | ** | -0.0001 | *** | -0.0026 | *** | 0.0005 | |
| **d2** | -0.0001 | *** | 0.0000 | | 0.0000 | *** | -0.0002 | *** | 0.0000 | | -0.0002 | *** | 0.0000 | *** | 0.0000 | *** | 0.0000 | *** | -0.0002 | *** | 0.0000 | *** |
| **d3** | 0.0000 | *** | 0.0000 | | 0.0000 | *** | 0.0005 | ** | 0.0000 | *** | 0.0000 | | 0.0002 | *** | 0.0000 | *** | 0.0000 | ** | -0.0001 | *** | 0.0000 | *** |
| **d4** | -0.0003 | *** | 0.0000 | ** | 0.0000 | *** | 0.0000 | * | 0.0004 | *** | 0.0000 | *** | 0.0006 | *** | 0.0003 | *** | 0.0000 | | -0.0005 | *** | 0.0000 | *** |
| **d5** | 0.0015 | *** | -0.0002 | *** | 0.0007 | *** | 0.0027 | *** | 0.0000 | | 0.0012 | *** | 0.0036 | *** | 0.0001 | *** | -0.0012 | * | -0.0075 | *** | -0.0005 | |
| **d6** | 0.0005 | *** | -0.0003 | *** | 0.0000 | * | 0.0015 | *** | -0.0002 | | 0.0004 | | 0.0017 | *** | -0.0002 | *** | -0.0001 | | -0.0014 | *** | 0.0000 | *** |
| **d7** | 0.0001 | *** | -0.0003 | *** | 0.0000 | ** | 0.0000 | | 0.0002 | | 0.0002 | | 0.0011 | *** | -0.0001 | *** | -0.0001 | *** | -0.0022 | *** | 0.0000 | *** |
| **d8** | 0.0001 | *** | -0.0006 | | 0.0010 | *** | 0.0017 | *** | 0.0003 | | 0.0003 | | 0.0018 | *** | 0.0005 | ** | 0.0000 | | -0.0017 | *** | -0.0002 | * |
| **d9** | -0.0003 | *** | 0.0000 | | 0.0000 | *** | 0.0007 | *** | 0.0001 | | -0.0004 | | 0.0008 | *** | 0.0003 | *** | -0.0004 | | -0.0010 | *** | 0.0000 | *** |
| **d10** | 0.0009 | *** | 0.0002 | | 0.0017 | *** | 0.0005 | ** | -0.0004 | | 0.0003 | ** | 0.0010 | *** | -0.0003 | *** | 0.0004 | | -0.0010 | *** | 0.0000 | *** |
| **d11** | 0.0057 | *** | 0.0000 | | 0.0007 | *** | 0.0006 | *** | 0.0002 | | 0.0005 | ** | 0.0019 | *** | -0.0003 | *** | 0.0004 | * | -0.0029 | *** | 0.0000 | *** |
| **d12** | 0.0013 | *** | 0.0015 | * | 0.0005 | *** | 0.0008 | *** | 0.0000 | | 0.0006 | *** | 0.0011 | *** | 0.0000 | *** | 0.0006 | * | -0.0015 | *** | 0.0000 | *** |
| **d13** | -0.0003 | *** | 0.0000 | * | 0.0004 | | 0.0000 | *** | 0.0002 | | 0.0002 | | 0.0005 | *** | 0.0001 | *** | 0.0006 | | -0.0004 | *** | 0.0000 | *** |
| **d14** | 0.0003 | *** | 0.0000 | | 0.0000 | *** | 0.0005 | ** | 0.0007 | *** | -0.0001 | *** | 0.0007 | *** | 0.0000 | *** | -0.0005 | | -0.0010 | *** | 0.0000 | *** |
| **d15** | 0.0032 | *** | 0.0000 | | 0.0010 | *** | 0.0014 | *** | -0.0002 | | 0.0013 | | 0.0044 | *** | 0.0005 | *** | -0.0002 | | -0.0047 | *** | -0.0003 | *** |
| **d16** | 0.0009 | *** | 0.0000 | | 0.0000 | *** | 0.0031 | ** | 0.0011 | ** | 0.0011 | ** | 0.0148 | *** | 0.0008 | *** | 0.0023 | ** | -0.0042 | *** | 0.0004 | |
| **d17** | 0.0003 | *** | 0.0001 | | 0.0002 | * | 0.0001 | * | -0.0003 | | 0.0004 | | 0.0072 | *** | 0.0015 | *** | -0.0001 | | -0.0020 | *** | 0.0001 | * |
| **d18** | 0.0000 | *** | -0.0001 | | -0.0001 | *** | 0.0000 | ** | 0.0006 | *** | 0.0000 | *** | 0.0002 | *** | -0.0002 | *** | 0.0007 | *** | -0.0006 | *** | 0.0000 | |
| **d19** | 0.0003 | *** | 0.0003 | | -0.0005 | | 0.0005 | *** | -0.0001 | * | -0.0001 | | 0.0003 | *** | 0.0000 | *** | 0.0000 | ** | -0.0002 | *** | -0.0001 | |
| **d20** | 0.0024 | *** | 0.0000 | | 0.0007 | ** | 0.0035 | ** | 0.0099 | *** | 0.0012 | *** | 0.0089 | *** | -0.0002 | | 0.0135 | *** | -0.0070 | *** | 0.0001 | |
| **d21** | 0.0001 | | 0.0003 | | 0.0018 | *** | -0.0002 | | 0.0013 | | 0.0002 | | 0.0075 | ** | -0.0009 | | 0.0050 | ** | -0.0003 | | -0.0004 | |

**Table A1.10. Average Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.25**

| $\bar{l}_{ij}$ | **d1** | **d2** | **d3** | **d4** | **d5** | **d6** | **d7** | **d8** | **d9** | **d10** | **d11** | **d12** | **d13** | **d14** | **d15** | **d16** | **d17** | **d18** | **d19** | **d20** | **d21** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 1.2160 | 0.0600 | 0.0500 | 0.0330 | 0.0910 | 0.0100 | 0.0910 | 0.0300 | 0.0110 | 0.0670 | 0.0100 | 0.0020 | 0.0030 | 0.0400 | 0.0120 | 0.0210 | 0.0200 | 0.0010 | 0.0070 | 0.0150 | 0.0040 |
| **d2** | 0.0030 | 1.0140 | 0.0020 | 0.0080 | 0.0040 | 0.0009 | 0.0230 | 0.0009 | 0.0005 | 0.0020 | 0.0010 | 0.0003 | 0.0003 | 0.0010 | 0.0004 | 0.0010 | 0.0010 | 0.0001 | 0.0003 | 0.0006 | 0.0001 |
| **d3** | 0.0001 | 0.0004 | 1.0400 | 0.0020 | 0.0001 | 0.0009 | 0.0050 | 0.0020 | 0.0020 | 0.0010 | 0.0010 | 0.0002 | 0.0001 | 0.0020 | 0.0004 | 0.0006 | 0.0020 | 0.0002 | 0.0005 | 0.0007 | 0.0000 |
| **d4** | 0.0003 | 0.0002 | 0.0020 | 1.0220 | 0.0003 | 0.0010 | 0.0020 | 0.0009 | 0.0030 | 0.0030 | 0.0020 | 0.0006 | 0.0004 | 0.0006 | 0.0040 | 0.0008 | 0.0120 | 0.0020 | 0.0010 | 0.0020 | 0.0001 |
| **d5** | 0.8920 | 0.7250 | 0.0900 | 0.0480 | 1.2400 | 0.0440 | 0.1350 | 0.0720 | 0.0340 | 0.1090 | 0.0240 | 0.0050 | 0.0070 | 0.0790 | 0.0170 | 0.0420 | 0.0670 | 0.0100 | 0.0170 | 0.0250 | 0.0040 |
| **d6** | 0.0160 | 0.0050 | 0.0310 | 0.0100 | 0.0080 | 1.2320 | 0.0310 | 0.0420 | 0.0330 | 0.0390 | 0.0110 | 0.0010 | 0.0070 | 0.0390 | 0.0080 | 0.0260 | 0.0360 | 0.0020 | 0.0130 | 0.0190 | 0.0008 |
| **d7** | 0.0070 | 0.0007 | 0.0740 | 0.5040 | 0.0010 | 0.0030 | 1.0850 | 0.0060 | 0.0100 | 0.0090 | 0.0060 | 0.0010 | 0.0009 | 0.0130 | 0.0110 | 0.0030 | 0.0100 | 0.0010 | 0.0020 | 0.0020 | 0.0004 |
| **d8** | 0.0160 | 0.0110 | 0.0900 | 0.0160 | 0.0190 | 0.0280 | 0.0480 | 1.2000 | 0.0290 | 0.0370 | 0.0140 | 0.0090 | 0.0160 | 0.0420 | 0.0090 | 0.0190 | 0.0330 | 0.0040 | 0.0200 | 0.0220 | 0.0020 |
| **d9** | 0.0010 | 0.0008 | 0.3060 | 0.0110 | 0.0008 | 0.0080 | 0.0300 | 0.0100 | 1.0930 | 0.0150 | 0.0040 | 0.0008 | 0.0040 | 0.0140 | 0.0020 | 0.0070 | 0.0160 | 0.0020 | 0.0020 | 0.0030 | 0.0002 |
| **d10** | 0.0010 | 0.0010 | 0.0330 | 0.0070 | 0.0010 | 0.0100 | 0.0220 | 0.0220 | 0.0190 | 1.2130 | 0.0230 | 0.0050 | 0.0380 | 0.0220 | 0.0030 | 0.0130 | 0.0160 | 0.0010 | 0.0060 | 0.0060 | 0.0004 |
| **d11** | 0.0020 | 0.0020 | 0.0310 | 0.0100 | 0.0020 | 0.0210 | 0.0310 | 0.0380 | 0.0600 | 0.2640 | 1.2070 | 0.0290 | 0.0090 | 0.0300 | 0.0080 | 0.0360 | 0.0330 | 0.0040 | 0.0200 | 0.0150 | 0.0009 |
| **d12** | 0.0010 | 0.0010 | 0.0110 | 0.0040 | 0.0009 | 0.0150 | 0.0120 | 0.0240 | 0.0200 | 0.0810 | 0.0470 | 1.0860 | 0.0070 | 0.0140 | 0.0040 | 0.0150 | 0.0120 | 0.0009 | 0.0090 | 0.0090 | 0.0006 |
| **d13** | 0.0040 | 0.0010 | 0.0080 | 0.0040 | 0.0020 | 0.0420 | 0.0110 | 0.0170 | 0.0120 | 0.0330 | 0.0050 | 0.0006 | 1.0590 | 0.0130 | 0.0030 | 0.0100 | 0.0170 | 0.0009 | 0.0060 | 0.0060 | 0.0002 |
| **d14** | 0.0080 | 0.0006 | 0.0830 | 0.0950 | 0.0009 | 0.0030 | 0.0720 | 0.0020 | 0.0080 | 0.0070 | 0.0040 | 0.0010 | 0.0007 | 1.0070 | 0.0110 | 0.0020 | 0.0040 | 0.0004 | 0.0020 | 0.0030 | 0.0004 |
| **d15** | 0.0260 | 0.0070 | 0.4690 | 0.0610 | 0.0070 | 0.1310 | 0.1100 | 0.0670 | 0.7350 | 0.2780 | 0.0720 | 0.0390 | 0.0230 | 0.0370 | 1.0250 | 0.0780 | 0.0790 | 0.0080 | 0.0350 | 0.0540 | 0.0030 |
| **d16** | 0.0740 | 0.0830 | 0.0660 | 0.0410 | 0.0850 | 0.1210 | 0.1240 | 0.0760 | 0.0540 | 0.0770 | 0.0330 | 0.0350 | 0.0150 | 0.1190 | 0.0450 | 1.0730 | 0.3150 | 0.0230 | 0.0900 | 0.1240 | 0.0070 |
| **d17** | 0.0030 | 0.0020 | 0.0130 | 0.0360 | 0.0020 | 0.0110 | 0.0880 | 0.0070 | 0.0100 | 0.0160 | 0.0100 | 0.0090 | 0.0040 | 0.0120 | 0.0110 | 0.0170 | 1.1640 | 0.0370 | 0.0090 | 0.0150 | 0.0020 |
| **d18** | 0.0010 | 0.0007 | 0.0080 | 0.0080 | 0.0010 | 0.0140 | 0.0190 | 0.0040 | 0.0090 | 0.0070 | 0.0070 | 0.0070 | 0.0010 | 0.0060 | 0.0120 | 0.0040 | 0.0080 | 1.0030 | 0.0210 | 0.0520 | 0.0008 |
| **d19** | 0.0009 | 0.0004 | 0.0070 | 0.0330 | 0.0005 | 0.0040 | 0.0890 | 0.0030 | 0.0030 | 0.0050 | 0.0050 | 0.0040 | 0.0010 | 0.0030 | 0.0020 | 0.0050 | 0.0040 | 0.0006 | 1.0100 | 0.0060 | 0.0010 |
| **d20** | 0.0280 | 0.0140 | 0.1510 | 0.0530 | 0.0220 | 0.3450 | 0.1470 | 0.0660 | 0.2020 | 0.1100 | 0.0600 | 0.0550 | 0.0230 | 0.1410 | 0.2680 | 0.0740 | 0.1580 | 0.0140 | 0.3820 | 1.3110 | 0.0190 |
| **d21** | 0.0440 | 0.0260 | 0.0720 | 0.0220 | 0.0200 | 0.0640 | 0.0610 | 0.0540 | 0.0350 | 0.0410 | 0.0230 | 0.0410 | 0.0800 | 0.0930 | 0.0550 | 0.0350 | 0.1290 | 0.0050 | 0.1110 | 0.0890 | 1.0300 |

**Table A1.11. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.25 (1/2)**

| $\bar{b}_{ij}$ | d1 | | d2 | | d3 | | d4 | | d5 | | d6 | | d7 | d8 | | d9 | | d10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0236 | *** | 0.0091 | *** | 0.0061 | *** | 0.0080 | *** | 0.0099 | *** | -0.0010 | *** | 0.0316 | 0.0024 | *** | 0.0007 | *** | 0.0071 | *** |
| **d2** | 0.0004 | *** | 0.0018 | *** | 0.0006 | * | 0.0019 | *** | 0.0000 | *** | 0.0000 | | 0.0090 | 0.0001 | *** | 0.0001 | *** | 0.0000 | *** |
| **d3** | 0.0000 | *** | 0.0000 | *** | 0.0026 | *** | 0.0006 | ** | 0.0000 | * | 0.0000 | * | 0.0017 | 0.0004 | *** | 0.0006 | *** | 0.0001 | *** |
| **d4** | 0.0000 | *** | 0.0000 | *** | 0.0000 | *** | 0.0012 | *** | 0.0000 | *** | -0.0003 | | 0.0010 | 0.0001 | *** | 0.0000 | *** | 0.0003 | *** |
| **d5** | 0.0734 | *** | 0.0588 | *** | 0.0119 | *** | 0.0140 | *** | 0.0253 | *** | -0.0013 | | 0.0522 | 0.0063 | *** | 0.0029 | *** | 0.0135 | *** |
| **d6** | 0.0014 | *** | 0.0006 | *** | 0.0037 | *** | 0.0031 | ** | 0.0012 | *** | 0.0098 | *** | 0.0124 | 0.0025 | *** | 0.0035 | *** | 0.0037 | *** |
| **d7** | 0.0008 | *** | 0.0000 | *** | 0.0068 | *** | 0.0406 | *** | 0.0000 | ** | -0.0006 | *** | 0.0326 | 0.0001 | *** | 0.0014 | *** | 0.0013 | *** |
| **d8** | 0.0013 | *** | 0.0014 | *** | 0.0084 | *** | 0.0048 | ** | 0.0015 | *** | 0.0007 | ** | 0.0195 | 0.0147 | *** | 0.0031 | *** | 0.0033 | *** |
| **d9** | -0.0001 | *** | 0.0001 | *** | 0.0224 | *** | 0.0026 | *** | 0.0000 | ** | -0.0001 | *** | 0.0099 | 0.0002 | *** | 0.0095 | *** | 0.0011 | *** |
| **d10** | -0.0002 | ** | 0.0001 | *** | 0.0031 | *** | 0.0021 | ** | 0.0000 | | -0.0004 | | 0.0092 | 0.0010 | *** | 0.0013 | *** | 0.0190 | *** |
| **d11** | -0.0003 | ** | -0.0003 | *** | 0.0038 | *** | 0.0026 | ** | -0.0002 | | 0.0000 | | 0.0121 | 0.0021 | *** | 0.0052 | *** | 0.0226 | *** |
| **d12** | -0.0004 | *** | 0.0000 | *** | 0.0015 | *** | 0.0011 | ** | 0.0000 | | 0.0007 | ** | 0.0044 | 0.0012 | *** | 0.0014 | *** | 0.0079 | *** |
| **d13** | 0.0005 | *** | -0.0003 | *** | 0.0013 | *** | 0.0013 | *** | 0.0005 | *** | 0.0013 | *** | 0.0039 | 0.0006 | *** | 0.0017 | *** | 0.0031 | *** |
| **d14** | 0.0012 | *** | 0.0001 | *** | 0.0076 | *** | 0.0118 | *** | 0.0001 | *** | 0.0000 | * | 0.0307 | 0.0002 | ** | 0.0008 | *** | 0.0012 | *** |
| **d15** | 0.0019 | *** | 0.0010 | *** | 0.0500 | *** | 0.0117 | *** | 0.0006 | *** | 0.0051 | *** | 0.0428 | 0.0039 | *** | 0.0718 | *** | 0.0271 | *** |
| **d16** | 0.0092 | *** | 0.0102 | *** | 0.0093 | *** | 0.0112 | ** | 0.0074 | *** | 0.0037 | *** | 0.0484 | 0.0041 | *** | 0.0053 | *** | 0.0081 | *** |
| **d17** | 0.0007 | *** | 0.0003 | * | 0.0030 | ** | 0.0087 | *** | -0.0001 | ** | -0.0003 | * | 0.0256 | 0.0009 | *** | 0.0009 | *** | 0.0018 | *** |
| **d18** | -0.0003 | *** | 0.0000 | | 0.0016 | *** | 0.0022 | *** | 0.0000 | ** | 0.0003 | *** | 0.0058 | 0.0006 | *** | 0.0010 | *** | 0.0004 | *** |
| **d19** | 0.0001 | | -0.0001 | *** | 0.0021 | | 0.0084 | ** | -0.0001 | *** | 0.0001 | | 0.0331 | 0.0003 | * | 0.0002 | *** | 0.0005 | *** |
| **d20** | 0.0017 | *** | -0.0001 | | 0.0241 | *** | 0.0126 | ** | 0.0006 | | 0.0145 | *** | 0.0552 | 0.0038 | *** | 0.0289 | *** | 0.0129 | *** |
| **d21** | 0.0064 | | 0.0094 | | -0.0004 | | 0.0024 | | -0.0015 | | -0.0025 | | 0.0110 | -0.0020 | | -0.0078 | | -0.0023 | |

**Table A1.11. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.20 (2/2)**

| $\bar{b}_{ij}$ | **d11** | **d12** | **d13** | **d14** | **d15** | **d16** | **d17** | **d18** | **d19** | **d20** | **d21** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0002 *** | -0.0002 *** | 0.0001 *** | 0.0028 *** | 0.0004 | 0.0007 *** | 0.0027 *** | -0.0005 ** | -0.0011 *** | -0.0056 *** | 0.0005 |
| **d2** | -0.0001 *** | 0.0000 * | 0.0000 *** | -0.0002 ** | 0.0000 | -0.0002 *** | 0.0000 *** | 0.0000 *** | -0.0001 *** | -0.0004 *** | 0.0000 |
| **d3** | 0.0000 *** | 0.0000 | 0.0000 *** | 0.0005 *** | 0.0000 *** | 0.0000 | 0.0002 *** | 0.0000 *** | 0.0000 | -0.0001 *** | 0.0000 * |
| **d4** | -0.0003 *** | 0.0000 ** | 0.0000 *** | 0.0000 ** | 0.0004 *** | 0.0000 *** | 0.0016 *** | 0.0003 *** | 0.0000 | -0.0005 *** | 0.0000 *** |
| **d5** | 0.0015 *** | -0.0002 *** | 0.0007 *** | 0.0067 *** | -0.0010 * | 0.0022 *** | 0.0086 *** | 0.0011 *** | -0.0022 *** | -0.0145 *** | 0.0005 |
| **d6** | 0.0005 *** | -0.0003 *** | 0.0000 ** | 0.0025 *** | -0.0002 | 0.0004 * | 0.0037 *** | -0.0002 *** | -0.0001 | -0.0034 *** | 0.0000 *** |
| **d7** | 0.0001 *** | -0.0003 *** | 0.0000 *** | 0.0010 | 0.0002 | 0.0002 | 0.0011 *** | -0.0001 *** | -0.0011 *** | -0.0042 *** | 0.0000 *** |
| **d8** | 0.0011 *** | -0.0006 | 0.0010 *** | 0.0027 *** | 0.0003 | 0.0003 ** | 0.0038 *** | 0.0005 *** | 0.0010 * | -0.0037 *** | -0.0002 * |
| **d9** | 0.0007 *** | 0.0000 | 0.0000 *** | 0.0007 *** | 0.0001 | 0.0006 ** | 0.0018 *** | 0.0003 *** | -0.0004 * | -0.0010 *** | 0.0000 *** |
| **d10** | 0.0019 *** | 0.0002 | 0.0027 *** | 0.0015 *** | -0.0004 | 0.0003 *** | 0.0020 *** | -0.0003 *** | 0.0004 | -0.0020 *** | 0.0000 *** |
| **d11** | 0.0107 *** | 0.0000 | 0.0007 *** | 0.0016 *** | 0.0002 | 0.0015 *** | 0.0039 *** | 0.0007 *** | 0.0004 * | -0.0049 *** | 0.0000 *** |
| **d12** | 0.0023 *** | 0.0015 ** | 0.0005 *** | 0.0008 *** | 0.0000 | 0.0006 *** | 0.0011 *** | 0.0001 *** | 0.0006 * | -0.0015 *** | 0.0000 *** |
| **d13** | 0.0007 *** | 0.0000 *** | 0.0024 * | 0.0010 *** | 0.0002 | 0.0002 | 0.0015 *** | 0.0001 *** | 0.0006 * | -0.0014 *** | 0.0000 *** |
| **d14** | 0.0003 *** | 0.0000 | 0.0000 *** | 0.0005 * | 0.0007 *** | -0.0001 ** | 0.0007 *** | 0.0000 *** | -0.0005 | -0.0020 *** | 0.0000 * |
| **d15** | 0.0052 *** | 0.0000 | 0.0020 *** | 0.0034 *** | 0.0008 | 0.0023 ** | 0.0094 *** | 0.0005 *** | 0.0008 | -0.0087 *** | -0.0003 *** |
| **d16** | 0.0019 *** | 0.0010 * | 0.0010 *** | 0.0061 *** | 0.0021 ** | 0.0021 *** | 0.0308 *** | 0.0018 *** | 0.0043 *** | -0.0092 *** | 0.0004 |
| **d17** | 0.0003 *** | 0.0001 | 0.0002 *** | 0.0011 ** | 0.0007 | 0.0004 ** | 0.0162 *** | 0.0035 *** | -0.0001 | -0.0030 *** | 0.0001 * |
| **d18** | 0.0000 *** | -0.0001 * | -0.0001 *** | 0.0000 *** | 0.0006 *** | 0.0000 *** | 0.0012 *** | -0.0002 *** | 0.0017 *** | -0.0006 *** | 0.0000 |
| **d19** | 0.0003 *** | 0.0003 | -0.0005 | 0.0005 | -0.0001 * | -0.0001 | 0.0003 *** | 0.0000 * | 0.0000 ** | -0.0012 * | -0.0001 |
| **d20** | 0.0034 *** | 0.0010 * | 0.0007 ** | 0.0075 *** | 0.0169 *** | 0.0022 *** | 0.0189 *** | -0.0002 | 0.0235 *** | -0.0120 *** | 0.0001 |
| **d21** | 0.0001 | 0.0003 | 0.0038 *** | 0.0018 | 0.0023 | 0.0002 | 0.0135 *** | -0.0019 * | 0.0080 ** | -0.0023 | -0.0004 |

**Table A1.12. Average Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.30**

| $\bar{l}_{ij}$ | d1 | d2 | d3 | d4 | d5 | d6 | d7 | d8 | d9 | d10 | d11 | d12 | d13 | d14 | d15 | d16 | d17 | d18 | d19 | d20 | d21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 1.2390 | 0.0690 | 0.0510 | 0.0340 | 0.0990 | 0.0100 | 0.0690 | 0.0310 | 0.0120 | 0.0730 | 0.0110 | 0.0020 | 0.0030 | 0.0410 | 0.0110 | 0.0210 | 0.0210 | 0.0010 | 0.0070 | 0.0120 | 0.0040 |
| **d2** | 0.0030 | 1.0150 | 0.0020 | 0.0080 | 0.0050 | 0.0009 | 0.0160 | 0.0009 | 0.0005 | 0.0020 | 0.0010 | 0.0003 | 0.0003 | 0.0010 | 0.0004 | 0.0010 | 0.0010 | 0.0001 | 0.0003 | 0.0005 | 0.0001 |
| **d3** | 0.0001 | 0.0004 | 1.0410 | 0.0020 | 0.0001 | 0.0009 | 0.0040 | 0.0020 | 0.0020 | 0.0010 | 0.0010 | 0.0002 | 0.0001 | 0.0020 | 0.0004 | 0.0006 | 0.0020 | 0.0003 | 0.0006 | 0.0006 | 0.0000 |
| **d4** | 0.0003 | 0.0002 | 0.0030 | 1.0240 | 0.0003 | 0.0010 | 0.0010 | 0.0009 | 0.0040 | 0.0030 | 0.0030 | 0.0006 | 0.0005 | 0.0006 | 0.0040 | 0.0008 | 0.0130 | 0.0020 | 0.0009 | 0.0020 | 0.0001 |
| **d5** | 0.9680 | 0.7780 | 0.0940 | 0.0500 | 1.2640 | 0.0430 | 0.1060 | 0.0750 | 0.0350 | 0.1200 | 0.0250 | 0.0050 | 0.0070 | 0.0830 | 0.0160 | 0.0430 | 0.0720 | 0.0100 | 0.0150 | 0.0140 | 0.0040 |
| **d6** | 0.0180 | 0.0060 | 0.0320 | 0.0090 | 0.0080 | 1.2370 | 0.0210 | 0.0440 | 0.0360 | 0.0420 | 0.0120 | 0.0010 | 0.0080 | 0.0410 | 0.0080 | 0.0270 | 0.0390 | 0.0030 | 0.0140 | 0.0180 | 0.0008 |
| **d7** | 0.0070 | 0.0007 | 0.0750 | 0.5300 | 0.0010 | 0.0020 | 1.0580 | 0.0060 | 0.0100 | 0.0090 | 0.0070 | 0.0010 | 0.0010 | 0.0130 | 0.0110 | 0.0030 | 0.0110 | 0.0010 | 0.0020 | 0.0001 | 0.0003 |
| **d8** | 0.0180 | 0.0120 | 0.0930 | 0.0150 | 0.0200 | 0.0280 | 0.0330 | 1.2060 | 0.0310 | 0.0400 | 0.0140 | 0.0090 | 0.0180 | 0.0440 | 0.0090 | 0.0190 | 0.0350 | 0.0040 | 0.0200 | 0.0210 | 0.0020 |
| **d9** | 0.0010 | 0.0009 | 0.3180 | 0.0110 | 0.0009 | 0.0080 | 0.0230 | 0.0110 | 1.1010 | 0.0160 | 0.0040 | 0.0008 | 0.0050 | 0.0150 | 0.0020 | 0.0070 | 0.0180 | 0.0020 | 0.0020 | 0.0020 | 0.0002 |
| **d10** | 0.0010 | 0.0010 | 0.0350 | 0.0070 | 0.0010 | 0.0100 | 0.0150 | 0.0230 | 0.0210 | 1.2260 | 0.0230 | 0.0050 | 0.0410 | 0.0230 | 0.0030 | 0.0130 | 0.0170 | 0.0020 | 0.0060 | 0.0040 | 0.0003 |
| **d11** | 0.0030 | 0.0030 | 0.0330 | 0.0100 | 0.0020 | 0.0210 | 0.0220 | 0.0390 | 0.0640 | 0.2790 | 1.2130 | 0.0290 | 0.0100 | 0.0320 | 0.0080 | 0.0360 | 0.0360 | 0.0040 | 0.0210 | 0.0130 | 0.0009 |
| **d12** | 0.0020 | 0.0010 | 0.0120 | 0.0040 | 0.0009 | 0.0150 | 0.0090 | 0.0250 | 0.0220 | 0.0860 | 0.0490 | 1.0870 | 0.0080 | 0.0150 | 0.0040 | 0.0150 | 0.0130 | 0.0009 | 0.0090 | 0.0080 | 0.0006 |
| **d13** | 0.0040 | 0.0020 | 0.0080 | 0.0040 | 0.0020 | 0.0430 | 0.0080 | 0.0180 | 0.0120 | 0.0350 | 0.0050 | 0.0005 | 1.0680 | 0.0130 | 0.0030 | 0.0100 | 0.0190 | 0.0010 | 0.0060 | 0.0050 | 0.0002 |
| **d14** | 0.0080 | 0.0007 | 0.0840 | 0.0990 | 0.0010 | 0.0030 | 0.0440 | 0.0020 | 0.0090 | 0.0070 | 0.0040 | 0.0010 | 0.0007 | 1.0070 | 0.0110 | 0.0020 | 0.0040 | 0.0004 | 0.0020 | 0.0030 | 0.0004 |
| **d15** | 0.0280 | 0.0070 | 0.4970 | 0.0630 | 0.0070 | 0.1330 | 0.0790 | 0.0690 | 0.7840 | 0.2960 | 0.0740 | 0.0390 | 0.0250 | 0.0390 | 1.0250 | 0.0790 | 0.0880 | 0.0090 | 0.0350 | 0.0490 | 0.0030 |
| **d16** | 0.0830 | 0.0930 | 0.0680 | 0.0400 | 0.0920 | 0.1220 | 0.0880 | 0.0780 | 0.0580 | 0.0820 | 0.0340 | 0.0350 | 0.0160 | 0.1240 | 0.0450 | 1.0740 | 0.3380 | 0.0250 | 0.0930 | 0.1180 | 0.0070 |
| **d17** | 0.0030 | 0.0020 | 0.0120 | 0.0370 | 0.0020 | 0.0110 | 0.0730 | 0.0070 | 0.0110 | 0.0170 | 0.0110 | 0.0090 | 0.0050 | 0.0120 | 0.0100 | 0.0170 | 1.1810 | 0.0390 | 0.0090 | 0.0120 | 0.0020 |
| **d18** | 0.0010 | 0.0008 | 0.0080 | 0.0080 | 0.0010 | 0.0140 | 0.0150 | 0.0040 | 0.0100 | 0.0080 | 0.0080 | 0.0070 | 0.0010 | 0.0070 | 0.0130 | 0.0040 | 0.0080 | 1.0040 | 0.0220 | 0.0510 | 0.0008 |
| **d19** | 0.0007 | 0.0003 | 0.0060 | 0.0320 | 0.0004 | 0.0040 | 0.0600 | 0.0030 | 0.0030 | 0.0050 | 0.0050 | 0.0040 | 0.0010 | 0.0030 | 0.0020 | 0.0050 | 0.0040 | 0.0005 | 1.0110 | 0.0060 | 0.0010 |
| **d20** | 0.0290 | 0.0140 | 0.1620 | 0.0520 | 0.0220 | 0.3510 | 0.1040 | 0.0680 | 0.2210 | 0.1190 | 0.0620 | 0.0550 | 0.0220 | 0.1460 | 0.2790 | 0.0760 | 0.1680 | 0.0150 | 0.4030 | 1.3040 | 0.0190 |
| **d21** | 0.0480 | 0.0320 | 0.0750 | 0.0270 | 0.0180 | 0.0640 | 0.0490 | 0.0540 | 0.0420 | 0.0420 | 0.0230 | 0.0410 | 0.0950 | 0.0950 | 0.0560 | 0.0360 | 0.1360 | 0.0040 | 0.1160 | 0.0870 | 1.0310 |

**Table A1.13. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.30 (1/2)**

| $\bar{b}_{ij}$ | **d1** | | **d2** | | **d3** | | **d4** | | **d5** | | **d6** | | **d7** | | **d8** | | **d9** | | **d10** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0466 | *** | 0.0181 | *** | 0.0071 | *** | 0.0090 | *** | 0.0179 | *** | -0.0010 | *** | 0.0096 | *** | 0.0034 | *** | 0.0017 | *** | 0.0131 | *** |
| **d2** | 0.0004 | *** | 0.0028 | *** | 0.0006 | *** | 0.0019 | *** | 0.0010 | *** | 0.0000 | * | 0.0020 | | 0.0001 | *** | 0.0001 | *** | 0.0000 | *** |
| **d3** | 0.0000 | *** | 0.0000 | *** | 0.0036 | *** | 0.0006 | *** | 0.0000 | ** | 0.0000 | ** | 0.0007 | | 0.0004 | *** | 0.0006 | *** | 0.0001 | *** |
| **d4** | 0.0001 | *** | 0.0000 | *** | 0.0010 | *** | 0.0032 | *** | 0.0000 | *** | -0.0003 | | 0.0000 | *** | 0.0001 | *** | 0.0010 | *** | 0.0003 | *** |
| **d5** | 0.1494 | *** | 0.1118 | *** | 0.0159 | *** | 0.0160 | *** | 0.0493 | *** | -0.0023 | * | 0.0232 | *** | 0.0093 | *** | 0.0039 | *** | 0.0245 | *** |
| **d6** | 0.0034 | *** | 0.0016 | *** | 0.0047 | *** | 0.0021 | *** | 0.0012 | *** | 0.0148 | *** | 0.0024 | ** | 0.0045 | *** | 0.0065 | *** | 0.0067 | *** |
| **d7** | 0.0008 | *** | 0.0001 | *** | 0.0078 | *** | 0.0666 | *** | 0.0001 | ** | -0.0016 | *** | 0.0056 | | 0.0001 | *** | 0.0014 | *** | 0.0013 | *** |
| **d8** | 0.0033 | *** | 0.0024 | *** | 0.0114 | *** | 0.0038 | *** | 0.0025 | *** | 0.0007 | *** | 0.0045 | ** | 0.0207 | *** | 0.0051 | *** | 0.0063 | *** |
| **d9** | -0.0001 | *** | 0.0002 | *** | 0.0344 | *** | 0.0026 | *** | 0.0001 | *** | -0.0001 | *** | 0.0029 | *** | 0.0012 | *** | 0.0175 | *** | 0.0021 | *** |
| **d10** | -0.0002 | * | 0.0001 | *** | 0.0051 | *** | 0.0021 | *** | 0.0000 | | -0.0004 | | 0.0022 | ** | 0.0020 | *** | 0.0033 | *** | 0.0320 | *** |
| **d11** | 0.0007 | ** | 0.0007 | *** | 0.0058 | *** | 0.0026 | *** | -0.0002 | | 0.0000 | | 0.0031 | ** | 0.0031 | *** | 0.0092 | *** | 0.0376 | *** |
| **d12** | 0.0006 | *** | 0.0000 | *** | 0.0025 | *** | 0.0011 | *** | 0.0000 | | 0.0007 | ** | 0.0014 | * | 0.0022 | *** | 0.0034 | *** | 0.0129 | *** |
| **d13** | 0.0005 | *** | 0.0007 | *** | 0.0013 | *** | 0.0013 | *** | 0.0005 | *** | 0.0023 | *** | 0.0009 | ** | 0.0016 | *** | 0.0017 | *** | 0.0051 | *** |
| **d14** | 0.0012 | *** | 0.0001 | *** | 0.0086 | *** | 0.0158 | *** | 0.0001 | *** | 0.0000 | * | 0.0027 | | 0.0002 | *** | 0.0018 | *** | 0.0012 | *** |
| **d15** | 0.0039 | *** | 0.0010 | *** | 0.0780 | *** | 0.0137 | *** | 0.0006 | *** | 0.0071 | *** | 0.0118 | ** | 0.0059 | *** | 0.1208 | *** | 0.0451 | *** |
| **d16** | 0.0182 | *** | 0.0202 | *** | 0.0113 | *** | 0.0102 | *** | 0.0144 | *** | 0.0047 | *** | 0.0124 | ** | 0.0061 | *** | 0.0093 | *** | 0.0131 | *** |
| **d17** | 0.0007 | *** | 0.0003 | ** | 0.0020 | *** | 0.0097 | *** | -0.0001 | ** | -0.0003 | * | 0.0106 | *** | 0.0009 | *** | 0.0019 | *** | 0.0028 | *** |
| **d18** | -0.0003 | *** | 0.0000 | | 0.0016 | *** | 0.0022 | *** | 0.0000 | *** | 0.0003 | *** | 0.0018 | *** | 0.0006 | *** | 0.0020 | *** | 0.0014 | *** |
| **d19** | -0.0001 | * | -0.0001 | *** | 0.0011 | *** | 0.0074 | *** | -0.0001 | *** | 0.0001 | | 0.0041 | | 0.0003 | *** | 0.0002 | *** | 0.0005 | *** |
| **d20** | 0.0027 | *** | -0.0001 | | 0.0351 | *** | 0.0116 | *** | 0.0006 | * | 0.0205 | *** | 0.0122 | ** | 0.0058 | *** | 0.0479 | *** | 0.0219 | *** |
| **d21** | 0.0104 | | 0.0154 | | 0.0026 | | 0.0074 | | -0.0035 | | -0.0025 | | -0.0010 | | -0.0020 | | -0.0008 | | -0.0013 | |

**Table A1.13. Average biases in the Leontief inverse obtained by randomizing the aggregated version of the officially published Supply and Use Tables, Denmark 2003, $\rho$=0.30 (2/2)**

| $\bar{b}_{ij}$ | **d11** | **d12** | **d13** | **d14** | **d15** | **d16** | **d17** | **d18** | **d19** | **d20** | **d21** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **d1** | 0.0012 *** | -0.0002 *** | 0.0001 | 0.0038 *** | -0.0006 * | 0.0007 *** | 0.0037 *** | -0.0005 ** | -0.0011 *** | -0.0086 *** | 0.0005 |
| **d2** | -0.0001 *** | 0.0000 ** | 0.0001 ** | -0.0002 *** | 0.0000 | -0.0002 *** | 0.0000 *** | 0.0000 *** | -0.0001 *** | -0.0005 *** | 0.0000 *** |
| **d3** | 0.0000 *** | 0.0000 | 0.0000 ** | 0.0005 *** | 0.0000 *** | 0.0000 ** | 0.0002 *** | 0.0000 *** | 0.0000 ** | -0.0002 *** | 0.0000 *** |
| **d4** | 0.0007 *** | 0.0000 ** | 0.0001 ** | 0.0000 ** | 0.0004 *** | 0.0001 *** | 0.0026 *** | 0.0003 *** | -0.0001 | -0.0005 *** | 0.0000 *** |
| **d5** | 0.0025 *** | -0.0002 *** | 0.0007 *** | 0.0107 *** | -0.0020 *** | 0.0032 *** | 0.0136 *** | 0.0011 *** | -0.0042 *** | -0.0255 *** | 0.0005 * |
| **d6** | 0.0015 *** | -0.0003 *** | 0.0010 *** | 0.0045 *** | -0.0002 | 0.0014 ** | 0.0067 *** | 0.0008 *** | 0.0009 | -0.0044 *** | 0.0000 *** |
| **d7** | 0.0011 *** | -0.0003 *** | 0.0001 * | 0.0010 | 0.0002 | 0.0002 | 0.0021 *** | -0.0001 *** | -0.0011 *** | -0.0062 *** | -0.0001 *** |
| **d8** | 0.0011 *** | -0.0006 | 0.0030 ** | 0.0047 *** | 0.0003 | 0.0003 *** | 0.0058 *** | 0.0005 *** | 0.0010 ** | -0.0047 *** | -0.0002 *** |
| **d9** | 0.0007 *** | 0.0000 * | 0.0010 ** | 0.0017 *** | 0.0001 | 0.0006 *** | 0.0038 *** | 0.0003 *** | -0.0004 | -0.0020 *** | 0.0000 *** |
| **d10** | 0.0019 *** | 0.0002 | 0.0057 *** | 0.0025 *** | -0.0004 *** | 0.0003 *** | 0.0030 *** | 0.0007 *** | 0.0004 | -0.0040 *** | 0.0000 *** |
| **d11** | 0.0167 *** | 0.0000 | 0.0017 *** | 0.0036 *** | 0.0002 | 0.0015 *** | 0.0069 *** | 0.0007 *** | 0.0014 ** | -0.0069 *** | -0.0001 *** |
| **d12** | 0.0043 *** | 0.0025 ** | 0.0015 ** | 0.0018 *** | 0.0000 | 0.0006 *** | 0.0021 *** | 0.0001 *** | 0.0006 ** | -0.0025 *** | 0.0000 *** |
| **d13** | 0.0007 *** | -0.0001 * | 0.0114 | 0.0010 *** | 0.0002 | 0.0002 ** | 0.0035 *** | 0.0002 *** | 0.0006 | -0.0024 *** | 0.0000 *** |
| **d14** | 0.0003 *** | 0.0000 | 0.0001 *** | 0.0005 *** | 0.0007 *** | -0.0001 *** | 0.0007 *** | 0.0001 *** | -0.0005 | -0.0020 *** | 0.0000 *** |
| **d15** | 0.0072 *** | 0.0000 | 0.0040 *** | 0.0054 *** | 0.0008 | 0.0033 *** | 0.0184 *** | 0.0015 *** | 0.0008 | -0.0137 *** | -0.0003 *** |
| **d16** | 0.0029 *** | 0.0010 ** | 0.0020 *** | 0.0111 *** | 0.0021 ** | 0.0031 *** | 0.0538 *** | 0.0038 *** | 0.0073 *** | -0.0152 *** | 0.0004 |
| **d17** | 0.0013 *** | 0.0001 | 0.0012 | 0.0011 *** | -0.0003 | 0.0004 *** | 0.0332 *** | 0.0055 *** | -0.0001 | -0.0060 *** | 0.0001 * |
| **d18** | 0.0010 *** | -0.0001 ** | -0.0001 | 0.0010 *** | 0.0016 *** | 0.0000 *** | 0.0012 *** | 0.0008 *** | 0.0027 *** | -0.0016 *** | 0.0000 * |
| **d19** | 0.0003 *** | 0.0003 | -0.0005 | 0.0005 * | -0.0001 | -0.0001 | 0.0003 *** | -0.0001 *** | 0.0010 *** | -0.0012 *** | -0.0001 |
| **d20** | 0.0054 *** | 0.0010 ** | -0.0003 | 0.0125 *** | 0.0279 *** | 0.0042 *** | 0.0289 *** | 0.0008 | 0.0445 *** | -0.0190 *** | 0.0001 |
| **d21** | 0.0001 | 0.0003 | 0.0188 | 0.0038 | 0.0033 | 0.0012 | 0.0205 ** | -0.0029 *** | 0.0130 *** | -0.0043 * | 0.0006 |

## Appendix 2: Output multipliers results[8]

| $\rho=$ 0.05 | | Leontief inverse approach | | *SUBE* (OLS) | |
|---|---|---|---|---|---|
| | True | Multiplier | Bias | Multiplier | Bias |
| **d1** | 2.2231 | 2.2232 | 0.0001 *** | 2.2210 | -0.0021 |
| **d2** | 1.8637 | 1.8656 | 0.0019 *** | 1.8400 | -0.0237 |
| **d3** | 2.4677 | 2.4720 | 0.0043 *** | 2.4490 | -0.0187 |
| **d4** | 1.8751 | 1.8780 | 0.0029 *** | 1.8880 | 0.0129 ** |
| **d5** | 1.4634 | 1.4634 | -0.0001 *** | 1.4630 | -0.0004 |
| **d6** | 2.0789 | 2.0808 | 0.0019 *** | 2.0790 | 0.0001 |
| **d7** | 1.7840 | 1.7880 | 0.0040 *** | 1.7900 | 0.0060 |
| **d8** | 1.7004 | 1.7016 | 0.0012 *** | 1.6980 | -0.0024 |
| **d9** | 2.2502 | 2.2554 | 0.0053 *** | 2.2430 | -0.0072 |
| **d10** | 2.2813 | 2.2889 | 0.0076 *** | 2.2840 | 0.0027 |
| **d11** | 1.5381 | 1.5390 | 0.0009 *** | 1.5380 | -0.0001 |
| **d12** | 1.3290 | 1.3295 | 0.0004 *** | 1.3300 | 0.0010 |
| **d13** | 1.2840 | 1.2813 | -0.0027 *** | 1.2830 | -0.0010 |
| **d14** | 1.6852 | 1.6816 | -0.0037 *** | 1.6810 | -0.0042 * |
| **d15** | 1.4868 | 1.4878 | 0.0010 *** | 1.4870 | 0.0002 |
| **d16** | 1.4685 | 1.4693 | 0.0009 *** | 1.4670 | -0.0015 * |
| **d17** | 2.0130 | 2.0150 | 0.0019 *** | 2.0120 | -0.0010 |
| **d18** | 1.1146 | 1.1129 | -0.0017 *** | 1.1150 | 0.0004 |
| **d19** | 1.7282 | 1.7279 | -0.0003 *** | 1.7280 | -0.0002 |
| **d20** | 1.8613 | 1.8598 | -0.0015 *** | 1.8610 | -0.0003 * |
| **d21** | 1.0764 | 1.0760 | -0.0004 *** | 1.0760 | -0.0004 |

| $\rho=$ 0.10 | | Leontief inverse approach | | *SUBE* (OLS) | |
|---|---|---|---|---|---|
| | True | Multiplier | Bias | Multiplier | Bias |
| **d1** | 2.2231 | 2.2302 | 0.0071 *** | 2.2150 | -0.0081 |
| **d2** | 1.8637 | 1.8736 | 0.0099 *** | 1.7740 | -0.0897 |
| **d3** | 2.4677 | 2.4880 | 0.0203 *** | 2.4730 | 0.0053 |
| **d4** | 1.8751 | 1.8890 | 0.0139 *** | 1.8950 | 0.0199 |
| **d5** | 1.4634 | 1.4654 | 0.0019 *** | 1.4620 | -0.0014 |
| **d6** | 2.0789 | 2.0838 | 0.0049 *** | 2.0810 | 0.0021 |
| **d7** | 1.7840 | 1.7990 | 0.0150 *** | 1.7940 | 0.0100 |
| **d8** | 1.7004 | 1.7056 | 0.0052 *** | 1.7070 | 0.0066 |
| **d9** | 2.2502 | 2.2704 | 0.0203 *** | 2.2570 | 0.0068 |
| **d10** | 2.2813 | 2.3019 | 0.0206 *** | 2.2900 | 0.0087 |
| **d11** | 1.5381 | 1.5420 | 0.0039 *** | 1.5380 | -0.0001 |
| **d12** | 1.3290 | 1.3295 | 0.0004 *** | 1.3290 | 0.0000 |
| **d13** | 1.2840 | 1.2833 | -0.0007 *** | 1.2690 | -0.0150 |
| **d14** | 1.6852 | 1.6886 | 0.0033 *** | 1.6760 | -0.0092 |
| **d15** | 1.4868 | 1.4908 | 0.0040 *** | 1.4880 | 0.0012 |
| **d16** | 1.4685 | 1.4703 | 0.0019 *** | 1.4660 | -0.0025 |
| **d17** | 2.0130 | 2.0240 | 0.0109 *** | 2.0100 | -0.0030 |
| **d18** | 1.1146 | 1.1150 | 0.0003 *** | 1.1150 | 0.0004 |
| **d19** | 1.7282 | 1.7319 | 0.0037 *** | 1.7280 | -0.0002 |
| **d20** | 1.8613 | 1.8527 | -0.0085 *** | 1.8610 | -0.0003 * |
| **d21** | 1.0764 | 1.0760 | -0.0004 *** | 1.0760 | -0.0004 |

[8] Asterisks show the significance level of biases: 10% (*), 5% (**) and 1% (***).

| $\rho=$ | 0.15 | Leontief inverse approach | | | *SUBE* (OLS) | | |
|---|---|---|---|---|---|---|---|
| | **True** | **Multiplier** | **Bias** | | **Multiplier** | **Bias** | |
| **d1** | 2.2231 | 2.2502 | 0.0271 | *** | 2.2190 | -0.0041 | |
| **d2** | 1.8637 | 1.8896 | 0.0260 | *** | 1.8340 | -0.0297 | |
| **d3** | 2.4677 | 2.5140 | 0.0463 | *** | 2.5950 | 0.1273 | |
| **d4** | 1.8751 | 1.9080 | 0.0329 | *** | 1.9020 | 0.0269 | |
| **d5** | 1.4634 | 1.4754 | 0.0120 | *** | 1.4620 | -0.0014 | |
| **d6** | 2.0789 | 2.0908 | 0.0119 | *** | 2.0810 | 0.0021 | |
| **d7** | 1.7840 | 1.8180 | 0.0340 | *** | 1.7960 | 0.0120 | |
| **d8** | 1.7004 | 1.7127 | 0.0123 | *** | 1.7060 | 0.0056 | |
| **d9** | 2.2502 | 2.2914 | 0.0413 | *** | 2.2700 | 0.0198 | |
| **d10** | 2.2813 | 2.3240 | 0.0426 | *** | 2.2790 | -0.0023 | |
| **d11** | 1.5381 | 1.5480 | 0.0099 | *** | 1.5380 | -0.0001 | |
| **d12** | 1.3290 | 1.3295 | 0.0004 | | 1.3240 | -0.0050 | |
| **d13** | 1.2840 | 1.2863 | 0.0023 | ** | 1.2810 | -0.0030 | |
| **d14** | 1.6852 | 1.6936 | 0.0083 | ** | 1.6680 | -0.0172 | |
| **d15** | 1.4868 | 1.4948 | 0.0080 | ** | 1.4840 | -0.0028 | |
| **d16** | 1.4685 | 1.4713 | 0.0029 | ** | 1.4690 | 0.0005 | |
| **d17** | 2.0130 | 2.0410 | 0.0280 | *** | 1.9980 | -0.0150 | |
| **d18** | 1.1146 | 1.1160 | 0.0013 | *** | 1.1140 | -0.0006 | |
| **d19** | 1.7282 | 1.7389 | 0.0107 | ** | 1.7300 | 0.0018 | |
| **d20** | 1.8613 | 1.8397 | -0.0216 | *** | 1.8610 | -0.0003 | |
| **d21** | 1.0764 | 1.0759 | -0.0004 | *** | 1.0760 | -0.0004 | |

| $\rho=$ | 0.20 | Leontief inverse approach | | | *SUBE* (OLS) | | |
|---|---|---|---|---|---|---|---|
| | **True** | **Multiplier** | **Bias** | | **Multiplier** | **Bias** | |
| **d1** | 2.2231 | 2.2842 | 0.0611 | *** | 2.2130 | -0.0101 | |
| **d2** | 1.8637 | 1.9137 | 0.0500 | *** | 1.8890 | 0.0253 | |
| **d3** | 2.4677 | 2.5560 | 0.0883 | *** | 2.7640 | 0.2963 | |
| **d4** | 1.8751 | 1.9390 | 0.0639 | *** | 1.9030 | 0.0279 | |
| **d5** | 1.4634 | 1.4874 | 0.0240 | *** | 1.4640 | 0.0006 | |
| **d6** | 2.0789 | 2.1008 | 0.0219 | *** | 2.0910 | 0.0121 | * |
| **d7** | 1.7840 | 1.8490 | 0.0650 | *** | 1.7770 | -0.0070 | |
| **d8** | 1.7004 | 1.7247 | 0.0243 | *** | 1.7040 | 0.0036 | |
| **d9** | 2.2502 | 2.3295 | 0.0793 | *** | 2.2500 | -0.0002 | |
| **d10** | 2.2813 | 2.3600 | 0.0786 | *** | 2.2820 | 0.0007 | |
| **d11** | 1.5381 | 1.5550 | 0.0169 | *** | 1.5360 | -0.0021 | |
| **d12** | 1.3290 | 1.3294 | 0.0004 | *** | 1.3270 | -0.0020 | |
| **d13** | 1.2840 | 1.2923 | 0.0084 | *** | 1.2710 | -0.0130 | |
| **d14** | 1.6852 | 1.7036 | 0.0183 | *** | 1.6550 | -0.0302 | |
| **d15** | 1.4868 | 1.5008 | 0.0140 | *** | 1.4890 | 0.0022 | |
| **d16** | 1.4685 | 1.4764 | 0.0079 | *** | 1.4650 | -0.0035 | |
| **d17** | 2.0130 | 2.0720 | 0.0590 | *** | 2.0110 | -0.0020 | |
| **d18** | 1.1146 | 1.1160 | 0.0014 | *** | 1.1140 | -0.0006 | |
| **d19** | 1.7282 | 1.7489 | 0.0206 | *** | 1.7370 | 0.0088 | |
| **d20** | 1.8613 | 1.8186 | -0.0427 | *** | 1.8600 | -0.0013 | |
| **d21** | 1.0764 | 1.0759 | -0.0005 | *** | 1.0760 | -0.0004 | |

| $\rho=$ | 0.25 | Leontief inverse approach | | | SUBE (OLS) | | |
|---|---|---|---|---|---|---|---|
| | **True** | **Multiplier** | **Bias** | | **Multiplier** | **Bias** | |
| **d1** | 2.2231 | 2.3443 | 0.1212 | *** | 2.2020 | -0.0211 | |
| **d2** | 1.8637 | 1.9558 | 0.0922 | *** | 1.9550 | 0.0913 | |
| **d3** | 2.4677 | 2.6370 | 0.1693 | *** | 2.7160 | 0.2483 | |
| **d4** | 1.8751 | 2.0280 | 0.1529 | *** | 1.8770 | 0.0019 | |
| **d5** | 1.4634 | 1.5085 | 0.0451 | *** | 1.4670 | 0.0036 | |
| **d6** | 2.0789 | 2.1088 | 0.0299 | ** | 2.0840 | 0.0051 | |
| **d7** | 1.7840 | 2.2360 | 0.4520 | *** | 1.7760 | -0.0080 | |
| **d8** | 1.7004 | 1.7438 | 0.0434 | ** | 1.7070 | 0.0066 | |
| **d9** | 2.2502 | 2.3825 | 0.1323 | ** | 2.2710 | 0.0208 | |
| **d10** | 2.2813 | 2.4140 | 0.1327 | *** | 2.2870 | 0.0057 | |
| **d11** | 1.5381 | 1.5690 | 0.0309 | ** | 1.5330 | -0.0051 | |
| **d12** | 1.3290 | 1.3314 | 0.0024 | | 1.3290 | 0.0000 | |
| **d13** | 1.2840 | 1.2994 | 0.0154 | *** | 1.2750 | -0.0090 | |
| **d14** | 1.6852 | 1.7276 | 0.0424 | *** | 1.6600 | -0.0252 | |
| **d15** | 1.4868 | 1.5108 | 0.0240 | | 1.4880 | 0.0012 | |
| **d16** | 1.4685 | 1.4824 | 0.0139 | ** | 1.4710 | 0.0025 | |
| **d17** | 2.0130 | 2.1360 | 0.1230 | *** | 2.0080 | -0.0050 | |
| **d18** | 1.1146 | 1.1201 | 0.0055 | *** | 1.1160 | 0.0014 | |
| **d19** | 1.7282 | 1.7638 | 0.0356 | | 1.7050 | -0.0232 | |
| **d20** | 1.8613 | 1.7793 | -0.0820 | *** | 1.8600 | -0.0013 | |
| **d21** | 1.0764 | 1.0768 | 0.0005 | | 1.0760 | -0.0004 | |

| $\rho=$ | 0.30 | Leontief inverse approach | | | SUBE (OLS) | | |
|---|---|---|---|---|---|---|---|
| | **True** | **Multiplier** | **Bias** | | **Multiplier** | **Bias** | |
| **d1** | 2.2231 | 2.4660 | 0.2429 | *** | 2.2230 | -0.0001 | |
| **d2** | 1.8637 | 2.0370 | 0.1733 | *** | 1.9070 | 0.0433 | |
| **d3** | 2.4677 | 2.7110 | 0.2433 | *** | 2.6540 | 0.1863 | |
| **d4** | 1.8751 | 2.0660 | 0.1909 | *** | 1.7940 | -0.0811 | * |
| **d5** | 1.4634 | 1.5480 | 0.0846 | *** | 1.4660 | 0.0026 | |
| **d6** | 2.0789 | 2.1220 | 0.0431 | *** | 2.0670 | -0.0119 | |
| **d7** | 1.7840 | 1.8970 | 0.1130 | ** | 1.7780 | -0.0060 | |
| **d8** | 1.7004 | 1.7640 | 0.0636 | *** | 1.7020 | 0.0016 | |
| **d9** | 2.2502 | 2.4870 | 0.2368 | *** | 2.3760 | 0.1258 | ** |
| **d10** | 2.2813 | 2.5080 | 0.2267 | *** | 2.2880 | 0.0067 | |
| **d11** | 1.5381 | 1.5870 | 0.0489 | *** | 1.5320 | -0.0061 | |
| **d12** | 1.3290 | 1.3330 | 0.0040 | * | 1.3360 | 0.0070 | |
| **d13** | 1.2840 | 1.3360 | 0.0520 | ** | 1.2950 | 0.0110 | |
| **d14** | 1.6852 | 1.7540 | 0.0688 | *** | 1.6940 | 0.0088 | |
| **d15** | 1.4868 | 1.5210 | 0.0342 | *** | 1.4780 | -0.0088 | * |
| **d16** | 1.4685 | 1.4910 | 0.0225 | *** | 1.4770 | 0.0085 | ** |
| **d17** | 2.0130 | 2.2260 | 0.2130 | *** | 2.0060 | -0.0070 | |
| **d18** | 1.1146 | 1.1280 | 0.0134 | *** | 1.1190 | 0.0044 | |
| **d19** | 1.7282 | 1.7940 | 0.0658 | *** | 1.7140 | -0.0142 | |
| **d20** | 1.8613 | 1.7320 | -0.1293 | *** | 1.8610 | -0.0003 | |
| **d21** | 1.0764 | 1.0760 | -0.0004 | | 1.0760 | -0.0004 | |

## Appendix 3: $CO_2$ emission multipliers results[9]

| $\rho=$ 0.05 | True | Leontief inverse approach Multiplier | Bias | | SUBE (OLS) Multiplier | Bias | |
|---|---|---|---|---|---|---|---|
| **d1** | 1.0370 | 1.0460 | 0.0090 | * | 1.0360 | -0.0010 | |
| **d2** | 1.1840 | 1.1890 | 0.0050 | * | 1.1350 | -0.0490 | ** |
| **d3** | 1.1200 | 1.1230 | 0.0030 | | 1.1160 | -0.0040 | |
| **d4** | 1.0660 | 1.0660 | 0.0000 | | 1.0670 | 0.0010 | |
| **d5** | 1.0640 | 1.0650 | 0.0010 | | 1.0630 | -0.0010 | * |
| **d6** | 1.0490 | 1.0520 | 0.0030 | | 1.0500 | 0.0010 | |
| **d7** | 1.0760 | 1.0780 | 0.0020 | | 1.0760 | 0.0000 | |
| **d8** | 1.1870 | 1.1870 | 0.0000 | | 1.1860 | -0.0010 | |
| **d9** | 1.0150 | 1.0210 | 0.0060 | | 1.0020 | -0.0130 | *** |
| **d10** | 1.1090 | 1.1110 | 0.0020 | | 1.1100 | 0.0010 | |
| **d11** | 1.2180 | 1.2190 | 0.0010 | | 1.2190 | 0.0010 | |
| **d12** | 1.1260 | 1.1270 | 0.0010 | | 1.1300 | 0.0040 | * |
| **d13** | 1.2160 | 1.2170 | 0.0010 | | 1.2160 | 0.0000 | |
| **d14** | 1.1660 | 1.1680 | 0.0020 | | 1.1640 | -0.0020 | |
| **d15** | 1.0280 | 1.0290 | 0.0010 | | 1.0280 | 0.0000 | |
| **d16** | 1.1390 | 1.1390 | 0.0000 | | 1.1390 | 0.0000 | |
| **d17** | 1.0350 | 1.0430 | 0.0080 | ** | 1.0350 | 0.0000 | |
| **d18** | 1.0550 | 1.0550 | 0.0000 | ** | 1.0550 | 0.0000 | |
| **d19** | 1.1460 | 1.1430 | -0.0030 | * | 1.1450 | -0.0010 | |
| **d20** | 1.0950 | 1.0930 | -0.0020 | ** | 1.0950 | 0.0000 | |
| **d21** | 1.0780 | 1.0780 | 0.0000 | | 1.0780 | 0.0000 | |

| $\rho=$ 0.10 | True | Leontief inverse approach Multiplier | Bias | | SUBE (OLS) Multiplier | Bias | |
|---|---|---|---|---|---|---|---|
| **d1** | 1.0370 | 1.0620 | 0.0250 | ** | 1.0320 | -0.0050 | |
| **d2** | 1.1840 | 1.1990 | 0.0150 | ** | 1.1130 | -0.0710 | * |
| **d3** | 1.1200 | 1.1330 | 0.0130 | | 1.1190 | -0.0010 | |
| **d4** | 1.0660 | 1.0690 | 0.0030 | | 1.0710 | 0.0050 | |
| **d5** | 1.0640 | 1.0690 | 0.0050 | | 1.0630 | -0.0010 | * |
| **d6** | 1.0490 | 1.0570 | 0.0080 | * | 1.0500 | 0.0010 | |
| **d7** | 1.0760 | 1.0820 | 0.0060 | | 1.0790 | 0.0030 | |
| **d8** | 1.1870 | 1.1880 | 0.0010 | | 1.1860 | -0.0010 | |
| **d9** | 1.0150 | 1.0350 | 0.0200 | * | 0.9980 | -0.0170 | ** |
| **d10** | 1.1090 | 1.1180 | 0.0090 | | 1.1140 | 0.0050 | |
| **d11** | 1.2180 | 1.2210 | 0.0030 | | 1.2190 | 0.0010 | |
| **d12** | 1.1260 | 1.1270 | 0.0010 | | 1.1310 | 0.0050 | ** |
| **d13** | 1.2160 | 1.2190 | 0.0030 | * | 1.2170 | 0.0010 | |
| **d14** | 1.1660 | 1.1710 | 0.0050 | | 1.1640 | -0.0020 | |
| **d15** | 1.0280 | 1.0310 | 0.0030 | | 1.0280 | 0.0000 | |
| **d16** | 1.1390 | 1.1390 | 0.0000 | | 1.1390 | 0.0000 | |
| **d17** | 1.0350 | 1.0570 | 0.0220 | ** | 1.0350 | 0.0000 | |
| **d18** | 1.0550 | 1.0560 | 0.0010 | *** | 1.0550 | 0.0000 | |
| **d19** | 1.1460 | 1.1400 | -0.0060 | | 1.1450 | -0.0010 | |
| **d20** | 1.0950 | 1.0870 | -0.0080 | *** | 1.0950 | 0.0000 | |
| **d21** | 1.0780 | 1.0780 | 0.0000 | | 1.0780 | 0.0000 | |

[9] Asterisks show the significance level of biases: 10% (*), 5% (**) and 1% (***).

| $\rho=$ | 0.15 | Leontief inverse approach | | | *SUBE* (OLS) | | |
|---|---|---|---|---|---|---|---|
| | **True** | **Multiplier** | **Bias** | | **Multiplier** | **Bias** | |
| **d1** | 1.0370 | 1.0910 | 0.0540 | *** | 1.0400 | 0.0030 | |
| **d2** | 1.1840 | 1.2140 | 0.0300 | *** | 1.0680 | -0.1160 | |
| **d3** | 1.1200 | 1.1520 | 0.0320 | ** | 1.1220 | 0.0020 | |
| **d4** | 1.0660 | 1.0760 | 0.0100 | | 1.0850 | 0.0190 | |
| **d5** | 1.0640 | 1.0760 | 0.0120 | ** | 1.0630 | -0.0010 | |
| **d6** | 1.0490 | 1.0630 | 0.0140 | * | 1.0500 | 0.0010 | |
| **d7** | 1.0760 | 1.0910 | 0.0150 | ** | 1.0840 | 0.0080 | |
| **d8** | 1.1870 | 1.1920 | 0.0050 | | 1.1860 | -0.0010 | |
| **d9** | 1.0150 | 1.0600 | 0.0450 | ** | 0.9920 | -0.0230 | * |
| **d10** | 1.1090 | 1.1300 | 0.0210 | * | 1.1010 | -0.0080 | |
| **d11** | 1.2180 | 1.2250 | 0.0070 | | 1.2190 | 0.0010 | |
| **d12** | 1.1260 | 1.1290 | 0.0030 | | 1.1330 | 0.0070 | ** |
| **d13** | 1.2160 | 1.2210 | 0.0050 | ** | 1.2180 | 0.0020 | |
| **d14** | 1.1660 | 1.1770 | 0.0110 | * | 1.1620 | -0.0040 | |
| **d15** | 1.0280 | 1.0340 | 0.0060 | | 1.0280 | 0.0000 | |
| **d16** | 1.1390 | 1.1410 | 0.0020 | | 1.1380 | -0.0010 | |
| **d17** | 1.0350 | 1.0790 | 0.0440 | *** | 1.0350 | 0.0000 | |
| **d18** | 1.0550 | 1.0570 | 0.0020 | *** | 1.0550 | 0.0000 | |
| **d19** | 1.1460 | 1.1390 | -0.0070 | | 1.1460 | 0.0000 | |
| **d20** | 1.0950 | 1.0760 | -0.0190 | *** | 1.0950 | 0.0000 | |
| **d21** | 1.0780 | 1.0780 | 0.0000 | | 1.0780 | 0.0000 | |

| $\rho=$ | 0.20 | Leontief inverse approach | | | *SUBE* (OLS) | | |
|---|---|---|---|---|---|---|---|
| | **True** | **Multiplier** | **Bias** | | **Multiplier** | **Bias** | |
| **d1** | 1.0370 | 1.1400 | 0.1030 | *** | 1.0360 | -0.0010 | |
| **d2** | 1.1840 | 1.2350 | 0.0510 | *** | 1.0270 | -0.1570 | |
| **d3** | 1.1200 | 1.1860 | 0.0660 | *** | 1.1270 | 0.0070 | |
| **d4** | 1.0660 | 1.0870 | 0.0210 | ** | 1.0770 | 0.0110 | |
| **d5** | 1.0640 | 1.0880 | 0.0240 | *** | 1.0630 | -0.0010 | * |
| **d6** | 1.0490 | 1.0700 | 0.0210 | ** | 1.0510 | 0.0020 | |
| **d7** | 1.0760 | 1.1050 | 0.0290 | *** | 1.0670 | -0.0090 | |
| **d8** | 1.1870 | 1.1990 | 0.0120 | | 1.1860 | -0.0010 | |
| **d9** | 1.0150 | 1.1060 | 0.0910 | *** | 1.0320 | 0.0170 | |
| **d10** | 1.1090 | 1.1480 | 0.0390 | ** | 1.1060 | -0.0030 | |
| **d11** | 1.2180 | 1.2310 | 0.0130 | ** | 1.2180 | 0.0000 | |
| **d12** | 1.1260 | 1.1300 | 0.0040 | | 1.1350 | 0.0090 | ** |
| **d13** | 1.2160 | 1.2250 | 0.0090 | *** | 1.2200 | 0.0040 | |
| **d14** | 1.1660 | 1.1860 | 0.0200 | ** | 1.1630 | -0.0030 | |
| **d15** | 1.0280 | 1.0390 | 0.0110 | | 1.0280 | 0.0000 | |
| **d16** | 1.1390 | 1.1440 | 0.0050 | | 1.1380 | -0.0010 | |
| **d17** | 1.0350 | 1.1140 | 0.0790 | *** | 1.0320 | -0.0030 | |
| **d18** | 1.0550 | 1.0580 | 0.0030 | *** | 1.0550 | 0.0000 | |
| **d19** | 1.1460 | 1.1380 | -0.0080 | | 1.1480 | 0.0020 | |
| **d20** | 1.0950 | 1.0590 | -0.0360 | *** | 1.0950 | 0.0000 | |
| **d21** | 1.0780 | 1.0780 | 0.0000 | | 1.0780 | 0.0000 | |

| $\rho=$ | 0.25 | Leontief inverse approach | | | *SUBE* (OLS) | | |
|---|---|---|---|---|---|---|---|
| | True | Multiplier | Bias | | Multiplier | Bias | |
| **d1** | 1.0370 | 1.2600 | 0.2230 | *** | 1.0390 | 0.0020 | |
| **d2** | 1.1840 | 1.2680 | 0.0840 | *** | 0.9060 | -0.2780 | |
| **d3** | 1.1200 | 1.2620 | 0.1420 | *** | 1.1300 | 0.0100 | |
| **d4** | 1.0660 | 1.1070 | 0.0410 | *** | 1.0580 | -0.0080 | |
| **d5** | 1.0640 | 1.1090 | 0.0450 | *** | 1.0630 | -0.0010 | |
| **d6** | 1.0490 | 1.0780 | 0.0290 | ** | 1.0490 | 0.0000 | |
| **d7** | 1.0760 | 1.1290 | 0.0530 | *** | 1.0640 | -0.0120 | |
| **d8** | 1.1870 | 1.2110 | 0.0240 | ** | 1.1920 | 0.0050 | |
| **d9** | 1.0150 | 1.2380 | 0.2230 | ** | 1.1080 | 0.0930 | * |
| **d10** | 1.1090 | 1.1780 | 0.0690 | *** | 1.0900 | -0.0190 | |
| **d11** | 1.2180 | 1.2390 | 0.0210 | ** | 1.2190 | 0.0010 | |
| **d12** | 1.1260 | 1.1320 | 0.0060 | * | 1.1340 | 0.0080 | |
| **d13** | 1.2160 | 1.2310 | 0.0150 | *** | 1.2250 | 0.0090 | |
| **d14** | 1.1660 | 1.2000 | 0.0340 | *** | 1.1460 | -0.0200 | |
| **d15** | 1.0280 | 1.0470 | 0.0190 | | 1.0250 | -0.0030 | |
| **d16** | 1.1390 | 1.1500 | 0.0110 | ** | 1.1390 | 0.0000 | |
| **d17** | 1.0350 | 1.1790 | 0.1440 | *** | 1.0380 | 0.0030 | |
| **d18** | 1.0550 | 1.0610 | 0.0060 | *** | 1.0560 | 0.0010 | |
| **d19** | 1.1460 | 1.1360 | -0.0100 | | 1.1620 | 0.0160 | |
| **d20** | 1.0950 | 1.0250 | -0.0700 | *** | 1.0950 | 0.0000 | |
| **d21** | 1.0780 | 1.0780 | 0.0000 | | 1.0780 | 0.0000 | |

| $\rho=$ | 0.30 | Leontief inverse approach | | | *SUBE* (OLS) | | |
|---|---|---|---|---|---|---|---|
| | True | Multiplier | Bias | | Multiplier | Bias | |
| **d1** | 1.0370 | 1.2230 | 0.1860 | | 1.0410 | 0.0040 | |
| **d2** | 1.1840 | 1.3200 | 0.1360 | *** | 0.8740 | -0.3100 | |
| **d3** | 1.1200 | 0.8330 | -0.2870 | | 1.1300 | 0.0100 | |
| **d4** | 1.0660 | 1.1260 | 0.0600 | ** | 1.0590 | -0.0070 | |
| **d5** | 1.0640 | 1.1470 | 0.0830 | *** | 1.0640 | 0.0000 | |
| **d6** | 1.0490 | 1.1040 | 0.0550 | *** | 1.0420 | -0.0070 | |
| **d7** | 1.0760 | 1.1340 | 0.0580 | * | 1.0690 | -0.0070 | |
| **d8** | 1.1870 | 1.2220 | 0.0350 | ** | 1.1870 | 0.0000 | |
| **d9** | 1.0150 | 0.2380 | -0.7770 | | 1.1210 | 0.1060 | ** |
| **d10** | 1.1090 | 1.2080 | 0.0990 | *** | 1.0870 | -0.0220 | |
| **d11** | 1.2180 | 1.2490 | 0.0310 | *** | 1.2190 | 0.0010 | |
| **d12** | 1.1260 | 1.1360 | 0.0100 | ** | 1.1310 | 0.0050 | |
| **d13** | 1.2160 | 1.2380 | 0.0220 | *** | 1.2400 | 0.0240 | |
| **d14** | 1.1660 | 1.2070 | 0.0410 | | 1.1530 | -0.0130 | |
| **d15** | 1.0280 | 1.1450 | 0.1170 | | 1.0250 | -0.0030 | |
| **d16** | 1.1390 | 1.1560 | 0.0170 | * | 1.1380 | -0.0010 | |
| **d17** | 1.0350 | 0.8990 | -0.1360 | | 1.0310 | -0.0040 | |
| **d18** | 1.0550 | 1.0530 | -0.0020 | | 1.0560 | 0.0010 | |
| **d19** | 1.1460 | 1.1490 | 0.0030 | | 1.2010 | 0.0550 | * |
| **d20** | 1.0950 | 1.0410 | -0.0540 | | 1.0950 | 0.0000 | |
| **d21** | 1.0780 | 1.0770 | -0.0010 | | 1.0780 | 0.0000 | |

## Appendix 4: Industry/Product classification

| Code | Industry/Commodity |
|---|---|
| 01 | Products of agriculture, hunting and related services |
| 02 | Fish and other fishing products; services incidental of fishing |
| 03 | Coal, uranium and other mining and quarrying products |
| 04 | Crude petroleum & natural gas; and incidental related services |
| 05 | Food products and beverages; Tobacco |
| 06 | Textiles, leather, wood, cork, pulp, paper and paper products |
| 07 | Coke, refined petroleum products and nuclear fuels |
| 08 | Chemicals, rubber and plastics |
| 09 | Other non-metallic mineral products |
| 10 | Metallurgy and fabricated metal products |
| 11 | Machinery and equipment; electrical machinery & apparatus |
| 12 | Office mach. & computers; radio, TV and communication equip. medical and precision instruments; transport equipment |
| 13 | Furniture; other manufactured goods; secondary raw materials |
| 14 | Electricity, gas, steam and hot water |
| 15 | Construction work |
| 16 | Trade; hotel and restaurant services |
| 17 | Land transport |
| 18 | Water transport |
| 19 | Air transport |
| 20 | Other services |
| 21 | Public Admin. Education and Health & social work services |