Entropy-based benchmarking methods

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SEVENTH FRAMEWORK PROGRAMME

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- Benchmarking problem: combine the relative strengths of the inconsistent low- and high-frequency series

FIGURE: Benchmarking problem example (Source: Bloem et al. (2001), QNA Manual, IMF, Washington DC, p. 91)

Date	Indicator: The Indicator	Indicator: Period-to Period rate of Change	Annual Data 2000A	Annual BI Ratio 2000A
q1 1998	98.2			
q2 1998	100.3	2.6%		
q3 1998	102.2	1.4%		
q4 1998	100.8	−1.4%		
Sum	**402.0**		**4,000.0**	**9.950**
q1 1999	99.0	−1.8%		
q2 1999	101.6	2.6%		
q3 1999	102.7	1.1%		
q4 1999	101.5	−1.2%		
Sum	**404.8**	**0.7%**	**4,161.4**	**10.280**
q1 2000	100.5	−1.0%		
q2 2000	103.0	2.5%		
q3 2000	103.5	0.5%		
q4 2000	101.5	−1.9%		
Sum	**408.5**	**0.9%**	**4,100.0**	**10.037**

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- Mathematical methods: deterministic, binding annual constraints
  - Denton (1971), Causey and Trager (1981)
- Statistical methods: stochastic, binding/nonbinding constraints
  - Regression methods: Chow and Lin (1971), ARIMA and generalized regression-based methods (Dagum and Cholette, 2006)

Assessment and applications

- Chen (2007): 60 series from the US national economic accounts
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- Denton procedures: BEA, Statistics Netherlands
- Causey-Trager procedure: US Census Bureau (Brown 2010, Titova et al. 2010)

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  - Motivation: abundant series are volatile and/or include both positive and negative values

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- Extended pro-rata distribution (semiGRAS method)
- Entropy-based benchmarking methods

Notations

$t = 1, \ldots, T$ years

$i = 1, \ldots, I$ sub-periods for each year

$\mathbf{z}_t = I \times 1$ vector of observed indicator series for year t

$\mathbf{x}_t = I \times 1$ vector of estimated benchmarked series for year t

$\mathbf{z} = (\mathbf{z}'_1, \mathbf{z}'_2, \ldots, \mathbf{z}'_T)'$

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Consider annual constraints $\boldsymbol{\imath}'\mathbf{x}_t = y_t$ for all t, or

$$\begin{bmatrix} \boldsymbol{\imath}' & \mathbf{0}' & \vdots & \mathbf{0}' \\ \mathbf{0}' & \boldsymbol{\imath}' & \vdots & \mathbf{0}' \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}' & \mathbf{0}' & \cdots & \boldsymbol{\imath}' \end{bmatrix} \mathbf{x} = \mathbf{B}\mathbf{x} = \mathbf{y}$$

Denton (1971)

- The *additive first difference* (AFD) and *proportional first difference* (PFD) variants are:

$$f_{AFD} = (x_1 - z_1)^2 + \sum_{j=2}^{N} \left[(x_j - z_j) - (x_{j-1} - z_{j-1})\right]^2, \qquad (1)$$

$$f_{PFD} = \left(\frac{x_1}{z_1} - 1\right)^2 + \sum_{j=2}^{N} \left(\frac{x_j}{z_j} - \frac{x_{j-1}}{z_{j-1}}\right)^2, \qquad (2)$$

subject to $\mathbf{Bx} = \mathbf{y}$.

Causey and Trager (1981)

- Growth rate preservation principle:

$$f_{CT} = \sum_{j=2}^{N} \left(\frac{x_j}{x_{j-1}} - \frac{z_j}{z_{j-1}} \right)^2 . \quad (3)$$

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- For all methods, changing sign is possible!

Apply the Generalized RAS idea to benchmarking (Günlük-Şenesen and Bates 1988,Junius and Oosterhaven 2003)

Result

The solution of the benchmarking problem is $\mathbf{X} = \mathbf{P}\hat{\mathbf{s}} - \mathbf{N}\hat{\mathbf{s}}^{-1}$, *where the annual adjustment factors* $\mathbf{s}$ *are derived from*

$$\mathbf{s} = 0.5 \times \widehat{\mathbf{P}'\imath}^{-1} \left(\mathbf{y} + \sqrt{\mathbf{y} \circ \mathbf{y} + 4 \times (\mathbf{P}'\imath) \circ (\mathbf{N}'\imath)} \right),$$

where $\mathbf{Z} = (\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_T)$, $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_T)$ *and* $\mathbf{Z} = \mathbf{P} - \mathbf{N}$.

Level preservation principle ⇒ step problem due to discontinuities between years

Proportional first difference preservation

- $\frac{x_j}{z_j} = \frac{x_{j-1}}{z_{j-1}} + \varepsilon_{j0}$, denote $g_j = \frac{z_j}{z_{j-1}}$ and $\varepsilon_j = z_{j-1}\varepsilon_{j0}$, then

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- $x_j = g_j x_{j-1} + \varepsilon_j$ for all $j = 2, \ldots N(= IT)$
- $$\begin{bmatrix} x_N - g_N x_{N-1} = \varepsilon_N \\ x_{N-1} - g_{N-1} x_{N-2} = \varepsilon_{N-1} \\ \vdots \\ x_2 - g_2 x_1 = \varepsilon_2 \end{bmatrix} \Leftrightarrow$$

$$\begin{bmatrix} 1 & -g_N & 0 & \cdots & 0 \\ 0 & 1 & -g_{N-1} & & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & -g_2 \end{bmatrix} \begin{bmatrix} x_N \\ x_{N-1} \\ \vdots \\ x_1 \end{bmatrix} = \begin{bmatrix} \varepsilon_N \\ \varepsilon_{N-1} \\ \vdots \\ \varepsilon_1 \end{bmatrix}$$

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- PFD constraints: $\mathbf{C}_P \tilde{\mathbf{x}} = \varepsilon$

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- OLS?
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- Treat $\tilde{x}_j$ as a discrete random variable with a compact support and M possible outcomes $\mathbf{r}_j = (r_{j1}, \ldots, r_{jM})'$ with $2 \leq M < \infty$, i.e.,

$$\tilde{x}_j = \sum_{m=1}^{M} r_{jm} p_{jm} = \mathbf{r}_j' \mathbf{p}_j.$$

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- The GCE estimator is

$$\min_{\mathbf{p},\mathbf{w}} \; I(\mathbf{p},\mathbf{q},\mathbf{w},\mathbf{u}) = \mathbf{p}' \log(\mathbf{p}/\mathbf{q}) + \mathbf{w}' \log(\mathbf{w}/\mathbf{u}) \tag{4}$$

subject to

$$\tilde{\mathbf{y}} = \boldsymbol{\Gamma}\mathbf{Z}\tilde{\mathbf{x}} + \mathbf{V}\mathbf{w}, \tag{5}$$

$$\boldsymbol{\imath} = (\mathbf{I} \otimes \boldsymbol{\imath}')\mathbf{p}, \tag{6}$$

$$\boldsymbol{\imath} = (\mathbf{I} \otimes \boldsymbol{\imath}')\mathbf{w}, \tag{7}$$

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- Work in progress

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$$AARPD = \frac{100}{N-1} \sum_{j=2}^{N} \left| \left(\frac{x_j}{x_{j-1}} - \frac{z_j}{z_{j-1}} \right) / \frac{z_j}{z_{j-1}} \right|.$$

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$$\mathbf{r}_j' = (0.1, 0.55, 1, 1.45, 1.9) \times z_j \text{ with } \mathbf{q}_j' = (0.05\ 0.05\ 0.98\ 0.05\ 0.05)$$

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$\mathbf{v}'_k = (-1.7, -0.85, 0, 0.85, 1.7) \times \sigma_{\mathbf{z}}$ with
$\mathbf{u}'_k = (0.05\ 0.05\ 0.98\ 0.05\ 0.05)$ except for the annual constraints with
$\mathbf{u}'_k = (0\ 0\ 1\ 0\ 0)$

Results

Table: Some aggregate indicators

	f_{CT}	*AALD*	*AACD*	*AAPD* (%)	*AARPD* (%)
SemiGRAS	0.069	15.000	9.868	2.719	5.439
AFD Denton	1.198	18.324	5.972	17.510	13.260
AFD Entropy	0.323	16.774	6.270	10.360	9.440
PFD Denton	0.144	17.563	10.565	6.971	6.088
PFD Entropy	0.078	15.557	10.277	4.508	5.962
CT	0.044	16.553	10.348	3.761	5.761

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    - Controlling for sign change
2. Plausible competitors to current benchmarking methods

Thanks for your attention!