PPP's for SDR's? Towards a coherent measure of global inflation

Abstract

If it is a lesson learnt from the current financial-economic crisis that global markets and global money require global regulation then it is also true that global regulation requires global economic data. Purchasing power parities (PPP's) are well suited to meet this new need, because they are being established at regular time intervals, and their scope is world-wide, in principle. Their use has so far been restricted to the real economy, providing international volume comparisons for products, at the elementary level of aggregation, for national industries, at a higher level, and finally for domestic product and national income, at the level of countries as a whole. At this highest level of aggregation the bridge from the real to the financial economy is reached, because the general price level, which serves as the measure of national inflation is equal (grosso modo) to the inverse of the purchasing power of the national currency. The paper develops this track.

The recently aired political proposal to replace the US-dollar in its role as a universal means of payment by some more diversified system such as the Special Drawing Rights (SDR's) employed by the IMF warrants an equally universe measure of monetary dynamics. Such a measure – so the claim of the paper – may be based on the existing sytem of PPP's, adding to it an appropriate rule of normalisation which places the measure of world inflation within a world acounting framework in a coherent way. The paper explains this rule in theory, and by means of a small black-board example for purpose of illustration.

World Financial System

The recent financial crisis and its spillover into economic recession have confronted the world with a long-standing, but still unanswered question: what kind of international reserve currency do we need to secure global financial stability and facilitate world economic growth?

There were in history various institutional arrangements, including the Silver Standard, the Gold Standard, the Gold Exchange Standard and the Bretton Woods system. The underlying problem, however, as the ongoing financial crisis demonstrates, is far from being solved, and proves an inherent weakness of the current international monetary system. The crisis calls for creative reform of the existing international monetary system towards an international reserve currency with a stable value, rule-based issuance and manageable supply, so as to achieve the objective of safeguarding global economic and financial stability. What we need is, in the words of the governor the People's Bank of China, "an international reserve currency that is disconnected from individual nations and is able to remain stable in the long run". (Zhou Xiaochuan 2009)

Countries issuing currencies which are used as reserve by others are constantly confronted with the dilemma between achieving their domestic monetary policy goals and meeting other countries' demand for reserve currencies. On the one hand, the monetary authorities cannot simply focus on domestic goals without carrying out their international responsibilities; on the other hand, they cannot pursue different domestic and international objectives at the same time. They may either fail to adequately meet the demand of a growing global economy for liquidity as they try to ease inflation pressures at home, or create excess liquidity in the global markets by overly stimulating domestic demand.

The dollar reserve system may not be the only source of global financial instability, but it contributes to it. The question is, will the global economy lurch from the current system to another – such as the two currency reserve system towards which the world now seems to be moving – equally beset with problems? There is a remarkably simple solution, one which was recognized long ago by Keynes: the international community can provide a new form of fiat money to act as reserves. The countries of the world would agree to exchange the fiat money for their own currency, in times of crisis. (Stiglitz 2007, p.260) The desirable goal of reforming the international monetary system is to create an international reserve currency that is disconnected from individual nations and is able to remain stable in the long run, thus removing the inherent deficiencies caused by using credit-based national currencies.

Special consideration may be given to the SDR in playing a greater role. The SDR has the features and potential to act as a super-sovereign reserve currency. The scope of using the SDR should be broadened, so as to enable it to fully satisfy the member countries' demand for a reserve currency. The SDR, which is now only used between governments and international institutions, could become a widely accepted means of payment in international trade and financial transactions. (Zhou Xiaochuan 2009)

Any currency, whether national or international needs a control for inflation. At present, these are submitted implicitly by the currencies employed in the Special

Drawing Rights (US$, €, Pound Sterling, Yen). But using the national rates of inflation of these countries as control implies that inflation of all other countries goes unnoticed, as long as exchange rates respond and shield it from outside. Purchasing power parities remain unaffected. A world money requires a measure of world inflation. It can be based on the already existing system of international purchasing power parities, which must be completed into incorporating a measure of change over time. The paper provide a theoretical structure for such a concept.

Purchasing power parities

The compilation of purchasing power parities began as a research project at the University of Pennsylvania and has since grown to become a regular activity of official statistical bodies, such as OECD, EUROSTAT and UN. Its purpose is to provide a sound empirical basis for international comparison of different national gross domestic products. (OECD 2005, p.1f) Each GDP being compiled and expressed in its own national currency, such comparison requires a rule of transforming different currencies into each other. The straight-forward way is to resort to official exchange rates, as they are determined either on the markets of foreign exchange or by the respective governments. This has been general practice before the advent of purchasing power parities. Exchange rates, however, are highly volatile, making a comparison of yearly GDPs dependent on a daily changing index. In addition they only allow comparison of nominal GDP, and not of its complement real GDP, which is the universal measure of domestic production. Between the two there is the national price level, which differs between countries. For eliminating such differences in national price levels purchasing power parities are required.

In their simple form purchasing power parities (PPP's) are price relatives that show the ratio of the prices in national currencies of the same product in different countries. For example, if the price of a hamburger in France is 2.84 euros and in the United States it is 2.20 dollars, the PPP for hamburgers between France and the United States is 2.84 euros per 2.20 dollars or 1.29 euros to the dollar. In other words, for every dollar spent on hamburgers in the United States, 1.29 euros would have to be spent in France in order to obtain the same quantity or quality – or volume – in hamburgers. (OECD 2005 p.2). Determination of such elementary PPPs forms the first stage of the compilation process. The second stage is situated at the level of product groups where the price relatives calculated for products in the group are combined in an unweighted average for the group as a whole. The third stage is attained at the level of aggregate GDP where the expenditures of each GDP component are compiled. Comparing GDP components (e.g. capital formation) at purchasing power parity means comparing the same volume of aggregate product.[1]

This paper addresses the third stage of aggregation. It is at this stage where the real economy and its monetary complement meet in the statistical system so that an extension of PPP's beyond GDP to measuring a world price level, and its movement, the rate of world inflation, must start from here. The rationale for the extension is simple: The axiom that nominal GDP equals real GDP multiplied by the general price level is accepted world wide. It is obvious in countries which take the implied GDP deflator as the measure of their general price level. In other countries, using rather the

[1] OECD 2005 cannot abstain from asserting that both volumes "will, in principle, provide equivalent satisfaction or utility" (p.2). The advantage of applying such microeconomic variables, which are unobservable by definition, to macroeconomic aggregates such as exports or capital formation has never really been proven. We prefer, therefore, to stay with the concept of product "volume" as defined in statistical systems. By the way, avoiding the term "equal" and using the term "equivalent" instead, the authors themselves admit to a certain uneasiness in this direct micro-macro identification.

Consumer Price Index, the relationship applies with a small transformation. In this way the present concept of a world GDP established by means of purchasing power parities may also generate its own complement of a world price level, and - more interesting, - its change over time.

The institutional backup for implementing such an extension is hardly visible, yet, at present. But to visionaries, such as quoted above, it appears at the horizon of the future, looking at Special Drawing Rights from the IMF as a possible point of departure. Whatever the national currencies by which such monetary asset will be backed may be, it is hardly acceptable that only their price levels enter into the concern of global monetary policy. Rather, all national price levels must be included in such a measure in a fair share, in the same way as their national GDP's contribute to global GDP today.

Index number theory

Searching for a measure of the general price level, or of its movement, the rate of inflation, one invariably runs into an index number problem. The problem was invented by Irving Fisher in the early 20th century when the science of physics stood at its height of glamour and served as the model of every other science. Economics, in particular, adopted from it the zeal of pursuing a mathematical track which, for good or for bad, it has never left since. Fisher imitated physics, - not only by building a physical model of circulating fluids of an economy, still to be admired at Harvard University,- but by introducing the axiomatic method of building a theory. He postulated five axioms an economic index number would have to obey. In a second step he tried the kown index number formulas against these axioms, and finally decided for one index, which has been called “ideal” ever since. He did so in spite of the fact that the index violates one of the previously erected axioms, a procedure which is not really alike to physics. Nevertheless, it established the index number problem, and with it a new discipline of index number theory, which has since grown and flourished as an independent field of research, although one has learned not to “search for the holy grail of index number theory” (Balk 2008) anymore.

In this paper we avoid the index number problem, and want to remain rather at a relatively low level of sophistication. The idea of a world price level, and a corresponding rate of inflation, is still at a stage of infancy where clarification of concepts, and design of statistical procedures are discussed rather than the choice of a certain index number formula. Were we to take our path through index number theory we would have to consider and choose between seven methods for making multilateral comparisons: the star method, the democratic and plutocratic weight methods, the GEKS method, the own share method, the average basket method and the GK method (Diewert 2008, p. 208). Fortunately, only two of these are of relevance in practice, and we will discuss them only. A similar situation prevails in respect to the dimension of time. The list of possible index number formulae suggested for comparison over time has grown almost to infinity, for all practical purposes, (as well as the number of tests supporting them) where again only a few have found their way into statistical organisations. It is unlikely that other formulae than the one’s already in use will be applied when a rate of global inflation is to be determined.

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System of aggregation

The fundamental axiom underlying a compilation of aggregate purchasing power parities consists of the hypothesis that a unit of product represents the same economic value, independent of where it is produced or to whom it is sold, all over the world ("a potato is a potato"). The difficulties of making this assumption come to true are almost overwhelming to one who dares dive into the "technical notes" explaining the compilation process. Nevertheless, once the hypothesis is accepted it makes sense to speak of a "world price" as an analytical means for comparing national prices, eliminating the effect of varying exchange rates on product comparison, and to search for a way to measure it. The procedure requires each country to provide a set of national annual prices for a selection of representative and comparable products chosen from a common basket of goods and services that covers the whole range of final expenditure on GDP and a detailed breakdown of final expenditure on GDP according to a common classification. (EUROSTAT, OECD 2005, p. 223) Let $v_i^j(t)$ be the value of a product flow in classification grouping *i* of country *j* at time *t*. It is denominated in the currency of country *j*, which is the currency in which its national accounts are compiled. Let $p_i^j(t)$ be a corresponding price index number furnished by the department of price statistics. It is then possible to derive, as a complement, a "volume"

(1) $$q_i^j(t) = \frac{v_i^j(t)}{p_i^j(t)}.$$

Both, the price index number and the volume are relative variables. They acquire their full meaning when attached to some base year *t=0* which is selected arbitrarily. It is worth noting that in respect to their formal characteristics price and volume are not symmetric. Price is an "intensive" variable: when two national markets of the same price are added the price of the combined market remains the same. The volume, however, doubles. It is what in physics is called an "extensive" variable, a distinction which pure mathematical index number theory does not yet recognize. Once the data demanded by equation 1 haved been statistically established their aggregation is performed by means of the following system of equations, where Latin letters stand for the data, and Greek letters for the unknowns:

(2)
$$\begin{aligned} \pi_i \sum_j q_i^j - \sum_j \varepsilon^j v_i^j &= 0 \,,\ i = 1,... \\ \sum_i \pi_i q_i^j - \varepsilon^j \sum_i v_i^j &= 0 \,,\ j = 1,... \end{aligned}$$

In equations 2, $\pi_i\ (t)$ defines a world price of product i [SDR's/unit of product] at time t, and $\varepsilon^j (t)$ the exchange prevailing at purchasing power parity of the currency of country j [SDR's/unit of national currency]. We call it the "parity exchange rate" in order to distinguish it from the market or actual exchange rate. Equations 2 form a system of linear homogeneous equations. Their number is given by the sum of the number of commodity classes figuring in the comparison, and the number of countries

$$\begin{aligned} \pi_i \sum_j q_i^j - \sum_j \varepsilon^j v_i^j &= 0 \,,\ i = 1,... \\ \sum_i \pi_i q_i^j - \varepsilon^j \sum_i v_i^j &= 0 \,,\ j = 1,... \end{aligned}$$

participating. It is a Geary-Khamis (GK) system such as is being used by the United Nations[2].

We have opted against the Eltetö-Köves-Szulc (EKS) system, the obvious alternative, for the following reason. It is known that the conflict between the two systems is one of transitivity versus characteristicity in multilateral and bilateral comparisons. (Daban, Doménech, Molinas 1997, p. 33) The GK-system assures transitivity of comparison which means that a direct comparison of two countries gives the same figures as constructing the comparison through a third country. The EKS system, on the other

[2] Symbols and definition of variables are slightly at variance wirh offical PPP notation. For establishing the connection, see the appendix.

hand, strives for the optimal character of the basket of products taken as representative of the patterns of expenditure in two countries. It requires that the weights used in the comparison be solely based on spending patterns of these two countries. Hence, extending thesystem to include an additional country does not affect the overall pattern, while in the GK-system its representative basket changes with every country joining the group.

The UN is an organisation comprehending all nations of the world. Even if not all nations participate in the PPP project, yet, this is the perspective. The number of countries is naturally fixed. The situation is different for exclusive "clubs" such as EU and OECD where the selection of countries is open to variation, and rivalry between two similar countries is more at the order of the day. A change in membership is unlikely for the UN, representing the world a whole, while the opposite is true for OECD and EU. It is thus natural they have adopted different systems, each choosing the one which best suits its specific needs. One may say the EKS method represents the multilateral model of interrelated, but independent economies, while the GK-method reflects the global view of a single, unified world economy. We conclude in pursuing the goal of a measure of global inflation, the representative basket for which is global GDP, the GK-system in use at the UN appears as the appropriate tool to work with in this paper. Besides this institutional argument, the formal quality of consistency in aggregation (additivity) inherent in the GK-method, and not in the EKS-method, adds another reason to prefer the first to the second in the vision of a future world accounting system.

Normalising the system

Equations 2 form a linear homogeneous system. It can be solved if its rank is lower than the number of equations, which is assured by its construction. But its solution is determined up to a scaling factor only. The system delivers a set of proportions, not of absolute values. The absolute value of the unknowns is being determined by adding an equation of normalisation. Such normalisation is not an arbitrary decision; for it implies more than simply finding a numeraire. It defines the unit of measurement. A simple rule such as setting the parity exchange rate of one country equal to one does not suffice here. For assume country *k* is that country. Taking its currency as numeraire means

$$\varepsilon^k = 1 \ . \tag{3}$$

It follows from the second set of equations 2 and equation 1 that for this particular country the system reduces to

$$\sum_i \pi_i q_i^k = \sum_i v_i^k = \sum_i p_i^k q_i^k \quad . \tag{4}$$

GDP of country k valued at world prices equals GDP valued at its national prices. World prices would be tied to the price level of country k, which is non-sense. We must look for a more sophisticated normalisation rule.

In actual PPP work one transforms different national values into values of one nation (usually the US) by means of the nominal exchange rates established on corresponding markets. If e^j is the market exchange rate (units of SDR/units of national currency) nominal world GDP in currency k is then given by

(5) $$Y^k = \sum_j \frac{e^j}{e^k} \sum_i v_i^j \quad .$$

In the GK-system, on the other hand, world GDP is given by

(6) $$Y = \sum_i \pi_i \sum_j q_i^j = \sum_j \varepsilon^j \sum_i v_i^j \quad .$$

Normalising equation 6 to equation 5 means

(7) $$\sum_j \varepsilon^j \sum_i v_i^j = \sum_j \frac{e^j}{e^k} \sum_i v_i^j \ .$$

It follows that, at given prices and volumes at the national level, world GDP may differ depending on the market exchange rates existing between the numeraire currency and all others. This is unimportant as long as one remains within one point of time, which is the present application of the GK system. But in a coherent method of monitoring purchasing power in geographic space as well as over time, as is envisaged in a world accounting system, normalisation to the currency of one country creates an unwarranted statistical bias.

The way out towards a sensible normalisation can be found by following up a theorem exposed by E. Diewert. After showing that the level approach to index number theory is impossible he concludes that "instead of trying to decompose the value of the aggregate into price and quantity components for a single period, we instead attempt to decompose a value ratio pertaining to two periods…into a price change component P and a quantity change component Q.” (Diewert 2008, p.191) In this sense, all variables figuring in the GK-system are variables depending on time. Their time series are regularly observed and recorded in statistical offices. While actual measurement makes them discrete it is admissable, for theoretical purposes, to treat them as continuous, which simplifies the argument. Let a dot above the variable denote the first derivative with respect to time. Applying this operation to equation 6 yields

$$\dot{Y} = \sum_j \varepsilon^j \sum_i \dot{v}_i^j + \sum_i v_i^j \sum_j \dot{\varepsilon}^j \quad . \tag{8}$$

The new normalisation proposed here with the purpose of unifying inflation accounting across space and over time, simultaneaously, makes use of the decomposition that is naturally produced by differentiating a mathematic product (Reich 2001, p. 101). The first term describes the movement of national nominal values at constant parity exchange rates, and the second term describes the movement of the rates weighted at constant GDP. Clearly, a pure movement of “exchange” rates, ceteris (p_i^j , q_i^j) paribus, - as the name says - must not alter overall nominal world GDP. Hence we may stipulate

$$\sum_j \dot{\varepsilon}^j \sum_i v_i^j = 0 \quad . \tag{9}$$

This is a symmetric rule of normalisation assigning no special role to any country. The rationale behind is the following. A change in nominal GDP is measured in Special Drawing Rights, - all exchange rates being defined as SDR’s per unit of national currency. A movement in nominal world GDP may occur for two reasons. Either one of the volumes q_i^j moves, or one of the prices p_i^j . A mere variation in exchange rates between currencies cannot cause an increase in nominal GDP, as exchange rates compare currencies and not production.

If equation 9 holds compilation of a time series begins with choosing a certain reference year, valuing all national GDPs in SDR's of that year on the by means of the then existing market exchange rates. From the reference year on, equation 9 takes over. The yearly changes in parity exchange rates, determined by equations 2, are gauged in such a way that their weighted sum is zero. The procedure can be illustrated by resorting to the discrete case, which is what one finds in statistical practice. A discrete approximation to equation 9 may be

(10) $$\sum_{i,j} v_i^j(t) d\varepsilon^j \cong \sum_{i,j} v_i^j(t) \Delta\varepsilon^j(t) = 0 \ .$$

where the finite difference $\Delta\varepsilon^j(t)$ is defined as

(11) $$\Delta\varepsilon^j(t) = \varepsilon^j(t) - \varepsilon^j(t-1) \ .$$

Inserting equation 11 into equation 10 yields

(12) $$\sum_{i,j} \varepsilon^j(t) v_i^j(t) = \sum_{i,j} \varepsilon^j(t-1) v_i^j(t) \ .$$

This is a Laspeyres type index normalising the movement of parity exchange rates; its Paasche complement would be just as legitimate to use, of course (or their average as a "superlative" index, for that matter, approximating the underlying continuous fnction to the second degree). This year's nominal world GDP comes out as the sum of this year's national GDPs valued at last year's parity exchange rates, which value sets the scale for this year's parity exchange rates. Repeating the rationale, a shift in exchange rates can indicate neither a movement in world product nor in world inflation, by itself, because an exchange rate expresses a relationship between currencies only.

With normalisation 12, GK-system 2 is determined up to the choice of a reference year which provides the unit of measurement (SDT's of year 0) so that we may define world growth and world inflation in the following simple way,

(13) $$\frac{dQ}{Q} = \sum_{i,j} \frac{\pi_i dq_i^j}{\sum_{k,l} \pi_k q_k^l}$$

(14) $$\frac{dP}{P} = \sum_{i,j} \frac{q_i^j d\pi_i}{\sum_{k,l} \pi_k q_k^l}$$

The definition combines valuation at purchasing power parities with an accounting for inflation in a systematic and coherent way. It means that the measure of growth determined within a world system differs from its measure at the national level (loss of characticity), becausd world prices are used instead of national prices. So does the measure of inflation; obtained within the model of world system it looks different than when measured at the national level. The reason is not difficult to find. Measurement variables are always defined within a theoretical system based on certain assumptions. The crucial assumption here concerns the value of a product. In a world system this value is deemed to be the same in all countries ("a potatot is a potato"), which definitely is not true when you consider growth and inflation on the basis of purely national systems. Which model to apply, the national or the global market model, is a matter of judgement. When a world currency is being thought about as a realistic possibility the assumption that a product incorporates the same economic value, independent of where on earth and at what time it is being observed, is a necessary theoretical implication.

Example of compilation

Table 1 provides a simple application of the model to an "economy" of two countries and two products. Volumes are fixed at $q_i^j(t) = 100$ for all products and countries in order to demonstrate the pure effect of price changes. The value of world product $Y(t)$ is assumed at 1000 SDR's in base year $t = 0$. SDR's of year 0 are thus defined as the measuring rod of value throughout the example. Price indexes begin at $p_i^j(0) = 1.00$ which is the usual convention for a base year. The nominal values $v_i^j(0) = p_i^j(0) \times q_i^j(0)$ are therefore equal to 100, as well. They are given in national currencies ($). Equations 2 are then specified in the following way:

(15)
$$\begin{aligned}
&\pi_1(100+100) - (\varepsilon^A \times 100 + \varepsilon^B \times 100) = 0 \\
&\pi_2(100+100) - (\varepsilon^A \times 100 + \varepsilon^B \times 100) = 0 \\
&(\pi_1 \times 100 + \pi_2 \times 100) - \varepsilon^A(100+100) = 0 \\
&(\pi_1 \times 100 + \pi_2 \times 100) - \varepsilon^B(100+100) = 0
\end{aligned}$$

The system is normalised on the basis of equation 6, namely by

(16)
$$Y(0) = 1000[SDR's].$$

In other words, we assume that 2.500 SDR's are paid for a national dollar of each country A and B, in year 0, which solves the system of equations 15 and 16. The world price $\pi_i(0)$ of each product is 2.500 SDR's per unit of product for both products.

Next year (t = 1) the price index of product 1 in country A is assumed to rise by 10 percent. The nominal value v_1^A follows and rises from100 to 110 [national $], as the volume q_1^A has not changed. The corresponding PPP system for t = 1 looks as follows:

$$(17)\qquad \begin{aligned} &\pi_1(100+100)-(\varepsilon^A\times 110+\varepsilon^B\times 100)=0\\ &\pi_2(100+100)-(\varepsilon^A\times 100+\varepsilon^B\times 100)=0\\ &(\pi_1\times 100+\pi_2\times 100)-\varepsilon^A(110+100)=0\\ &(\pi_1\times 100+\pi_2\times 100)-\varepsilon^B(100+100)=0 \end{aligned}$$

This is the point to introduce a new normalisation. In the present PPP system world GDP at world prices is normalised to world GDP in US$, national GDP's being converted at actual exchange rates. A change, ceteris paribus, in one of these rates would change nominal world GDP without necessaily indicating a process of inflation. Instead, it may be adequate to apply a normalisation according to equation 12, which is explicitly designed to exclude such unwarranted variation:

$$(18)\qquad \begin{aligned} Y(1) &= \sum_j \varepsilon^j(1)\sum_i v_i^j(1)=\sum_j \varepsilon^j(0)\sum_i v_i^j(1)\\ &= 2.5\times(110+100)+2.5\times(100+100)=1025\,[SDR's] \end{aligned}$$

As a result world price of product 1 increases, following the national price increase, $\pi_1(1) = 2.624$ [SDR's/unit of product], while world price of product 2 remains almost constant, $\pi_2(1) = 2.501$ [SDR's/unit of product]. Parity exchange rates, however, both move. The currency of country A is devalued, the currency of country B is revalued. World nominal GDP increases to 1025 [SDR's]; as real GDP has not changed, by definition, this is a pure change of the world price level. World inflation amounts to 2.5 percent per year. The SDR's used to measure nominal world GDP in equations 18 are those of year 0, based on actual exchange rates of year 0. If world GDP measured at actual SDR exchange rates of year 1 had increased more, say to 1040 [SDR's of year 1], the balance of 15 to the compiled PPP-GDP of 1025 [SDR's of year 0] would signify a devalution of SDR's against all national currencies.

In year t = 2 product 2 is assumed to raise its price in country A, and in years t = 3 and t = 4 country B follows (table 1). The final outcome after four years of price movement is a price level of 1.102 which is slightly more than if all movements had been performed together, probably a result of the well known effect of using a Laspeyres index for approximating the actual continuous movement of the observables.

By the way, the example has a baring for the present compilation method, as well. If you choose to denominate all data in US-dollars, instead of SDR's, equation 18 will give a figure for world GDP, which will differ from world GDP compiled at actual exchange rates in that the effect of currency revalutions is excluded. World GDP valued at actual exchange rates to the dollar may turn out larger than last years, without any change in volume or price, simply because the dollar has devalued against the other currencies. The failure of national GDP times series to meet with their global compalition series, demanding recurrent bench-marking to each other, may be due in part to a neglect of this exchange rate effect.

Table 1: Price level of an economy of two countries and two goods at constant volumes [$q_i^j(t) = 100$]

	National price indexes (p_i^j)		World price index (π_i)	World GDP at PPP [$\pi_i (q_i^A + q_i^B)$]
	Country A	Country B		
t = 0				
Product 1	1.00	1.00	2.500	500
Product 2	1.00	1.00	2.500	500
Parity exchange rate (ε^j)	2.500	2.500		1000
t = 1				
Product 1	1.10	1.00	2.624	525
Product 2	1.00	1.00	2.501	500
Parity exchange rate (ε^j)	2.440	2.563		1025
t = 2				
Product 1	1.1	1.0	2.624	525
Product 2	1.1	1.0	2.624	525
Parity exchange rate (ε^j)	2.385	2.624		1050
t = 3				
Product 1	1.1	1.1	2.753	551
Product 2	1.1	1.0	2.625	525
Parity exchange rate (ε^j)	2.445	2.561		1076
t = 4				
Product 1	1.1	1.1	2.753	556
Product 2	1.1	1.1	2.753	556
Parity exchange rate (ε^j)	2.503	2.503		1102

Conclusion

The system of equations introduced in this paper extends the ordinary measurement of purchasing power parities between nations into a model of the global economy based on the law of product equivalence, independently of where the product is produced or consumed, which is the characteristic feature of a global market (law of one price). The proposed extension is simple in mathematical terms, because it concerns only the normalisation of the Geary-Khmis system. It is suggested to set the aggregate change in parity exchange rates equal to zero, for the reason that a mere movement of exchange rates must not affect measurement neither of product growth nor of money inflation.

Appendix

In OECD terms the GK method is defined through the the system of interrelated equations below (OECD 2005 p. 130):

$$P_i = \frac{\sum_{j=1}^{M} (p_{ij} q_{ij}) / PPP_j}{\sum_{j=1}^{M} q_{ij}} \tag{20}$$

and

$$PPP_j = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i q_{ij}} \; . \tag{21}$$

p_{ij} and q_{ij} are price and quantity of product i in countr j, corresponding to p_i^j and q_i^j in this paper. The unknowns P_i denote the international price of product i and corresponds to π_i in this paper while PPP_j is the purchasing power parity of an aggregate such as

GDP in country *j*. These equations may be rearanged to yield equations 22 and 23, namely

(22) $$P_i \sum_j q_{ij} - \sum_j \frac{v_{ij}}{PPP_j} = 0$$

and

(23) $$\sum_i P_i q_{ij} - \frac{1}{PPP_j} \sum_i v_{ij} = 0$$

where $v_{ij} = p_{ij} q_{ij}$ in accordance with equation 1. If we replace P_i by π_i and PPP_j by $1/\varepsilon_j$ and change the second subindex *j* into a superindex we attain equations 2. The reason for using ε rather than 1/PPP is that it produces a linear system of equations and has a simple interpretation as the parity exchange rate of a national currency *j* in international currency [SDR's/national currency unit].

The parity exchange rate ε^j is related to what is usually called the (dimensionless) real exchange rate r^j in the following way:

(24) $$\varepsilon^j = r^j e^j \text{ [SDR's/ unit of national currency]},$$

where e^j is the market exchange rate. If $r^j > 1$ the national currency of country *j* is undervalued on the foreign exchange market, its market exchange rate e^j is below parity, and the reverse is true in the opposite case.

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