An Integrated Keynes-Leontief "Macro-Econometric and Input-Output Model"

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Abstract
The objective of this paper is to present a new model, which based on the integration of the Keynesian multiplier with the Input-Output Framework. According this model the private consumption is considered as an endogenous component of Input-Output Model. This approach gives an opportunity to consider household's consumption as Consumption Matrix (CM), not a vector as in standard Leontief's framework. Specifically, a number of conditions for construction of CM are formulated. The basic data used for this study has been the Russian input-output tables for 1997, 2000 and 2003. Using the integrated model we have presented some numerical results that can be contrasted with those derived from the standard IO method. The difference of the results supports the view that the proposed accounting procedure allows to carry out more exact calculations for an estimation of the sector multipliers.

Key words: Keynesian Macro-Econometric Model, Input-Output Model, Consumption Matrix, Russia.

1. Introduction

Input-output method is an important tool for analysis and forecasting. The input-output framework forms the basis for macroeconomic modeling, showing how changes in industry final demands influence on economics. However, the use of the standard Leontief model for evaluating of gross domestic product can be fraught with difficulty. This paper presents a new approach with an alternative, complementary way to compute effects of changes in industrial activity.

Section 2 contains an introduction to new approach, which based on Keynesian theory of multiplier and IO method. The offered model is based on the use of the special «consumption matrix». Some differences between classic IO model and new approach are described.

The formal structure of new model is described in Section 3. We show how to construct a new conceptual framework.

The main empirical results are reported in Section 4 which provides comparison between numerical results following both calculations: those derived from modified approach with those of the standard implementation of the IO method.

Finally, Section 5 presents some concluding remarks.

2. The Technology of Consumption

The main idea of offered approach based on a new household classification. There are few classifications. One of the main differences the SAM approach from standard IO method is distribution of income from primary factors to several groups of households. Pyatt and Thorbecke (1976) suppose that there are three criteria on which a household classification should be based: location; sociological considerations and the wealth. They consider three groups of households: urban, rural and estate. This approach gives an opportunity to consider household's consumption as matrix, not a vector as in standard Leontief's framework.

Define as C = (Cir) matrix of household's consumption, where i is an index of industry, which produced commodity, r is an index of household's group. This matrix describes a structure of household's consumption. Let us name this matrix as «consumption matrix».

In our approach we offer new household classification based on industry, in which household receive income (salary, profit, percent and so on). So the consumption matrix may be defined as a «consumption matrix» C = (Cij), where j is index of industry which uses production of industry i. The structure of the matrix C is analogous with the structure of the technological matrix A, so define coefficient $\underline{c_{ij}}$ = Cij/Xj as direct consumption coefficient.

It is easy to construct the matrix C, if we suppose:

1. all employed peoples receive income in one industry;
2. all members of household work in one industry;
3. households receive income from primary factors, but not receive any transfers (pensions, relief and so on)

Consider, for example, economics, which consists from 2 industries - agriculture and manufacturing. Having this assumption, we may consider, for example, coefficients C_{12} as the total sum of manufacturing's consumption of the workers (households) which employed (receive income) in agriculture.

The next stage of our analysis is to adopt real data to new approach. Let us consider a household which members have different sources of income. For example, first member works in both branches and receives 1250 dollars in agriculture and 1750 dollars in manufacturing. The second member receives 2000 dollars from manufacturing. The total income of this family is 5000 dollars. Suppose that for this household a structure of expenditures (technology of consumption) is follows: 30 % of total income is expenditures for agriculture products, 50 % - expenditures for manufacturing productions, 20 % are savings and taxes. The total income received from manufacturing is 3750 dollars and 3000 dollars from this sum used for consumption. The income received from agriculture is 1250 dollars and 1000 dollars from this sum used for consumption. So we may construct matrix MC for this household using it's technology of consumption: $C_{11} = 375$; $C_{21} = 625$; $C_{12} = 1125$; $C_{22} = 1875$. Using this procedure to another households we obtain the total matrix C by addition all separate matrixes of household's consumption C.

3. The Formal Structure of the Model

The standard Keynesian equation of general equilibrium (Keynesian macro-econometric model) can be described as follows:

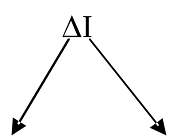
$$\underline{C} + \underline{S} + \underline{T} = Y = \underline{C} + \underline{I} + \underline{G} + \underline{NE}, \quad (1)$$

where $\underline{C}$, $\underline{S}$, $\underline{T}$, $\underline{I}$, $\underline{G}$, $\underline{NE}$ are the scalar indexes of household consumption, household saving, total sum of taxes, total sum of investments, government expenditures and net export. The left side of the equation (1) is a total supply and the right part shows a total demand.

The Keynesian multiplier k can be described as follows:

$$\Delta Y = \Delta I + c*\Delta I + c^2\Delta I + c^3\Delta I + \dots = \Delta I*k, \quad (2)$$

where ΔY is a growth of national income (total final demand), ΔI – growth of investments, $c = \Delta C/\Delta Y$ is a marginal propensity to consume, ΔC is a growth of household consumption, $k = 1/(1-c)$.



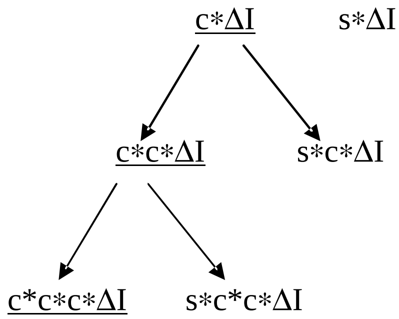


Figure 1. The Keynesian multiplier scheme

The standard IO model can be written in scalar form as follows:

$$\underline{A} + \underline{W} = \underline{X} = \underline{A} + \underline{Y}, \tag{3}$$

where $\underline{A}$ is a total sum of intermediate products, $\underline{W}$ is a total sum of added value (the total income), $\underline{Y}$ is a total final demand, $\underline{X}$ is a gross output. The left side of the equation (3) is a total supply and the right part shows a total demand.

The Leontief's inverse matrix $(E\text{-}A)^{-1}$ is the matrix multiplier of gross output, where A is an (n*n) matrix of direct input coefficients. This multiplier shows the total growth of gross output, connected with the increasing of total final demand.

It is obviously that both multipliers have some shortcomings. The matrix $(E\text{-}A)^{-1}$ not shows the growth of the total income, which is the most important macroeconomics index. So this multiplier can not be used in calculations of income's (gross domestic product) changes. The Keynesian multiplier k is a scalar multiplier, so it is not shows a growth of national income, connected with the growth of total final demand in separate industries.

Let us suppose that the total income W is equal a sum of expenditures: the household consumption, the household saving and the total sum of taxes. So we can consider the following equation of general equilibrium based on both equations (1) and (3):

$$\underline{A} + \underline{C} + \underline{S} + \underline{T} = \underline{X} = \underline{A} + \underline{C} + \underline{I} + \underline{G} \tag{4}$$

The matrix form of equation (4) is described as follows:

$$\sum_i Aij + \sum_i Cij + Sj + Tj = \sum_j Aij + \sum_j Cij + Ii + Gi, \text{ if } i = j \tag{5}$$

The left side of the equation (5) is a total supply of industry j and the right part shows a total demand.

It is not differences for producer: who is a consumer of the product - enterprise or household. For example, the sugar's plant sells sugar to the confection's factory and to member of household, which is employed in this factory. If we sum productive and non-productive consumption of some commodities, we may receive the total value of expenditures of this product $Dij=A_{ij}+C_{ij}$ (sugar and so on). So we consider both household consumption and productive expenditures as endogenous parameters.

Let us introduce new form of matrix multiplier M: (see Dondokov, 2000 a)

$$M = (E-D)^{-1}, \tag{6}$$

where $D=(\underline{d_{ij}})$, $\underline{d_{ij}}=Dij/Xj$.

This multiplier gives an opportunity to calculate whole multiplier effects. For example, if aircraft plant will produce and sell additional airplane for export, its workers will receive additional income, so they will buy more commodities and so on. The standard IO model can not gives this opportunity.

Let us consider open economics, so the equation of general equilibrium may be written as system of two equations:

$$\sum_j A_{dij} + \sum_j A_{zij} + \sum_j C_{dij} + \sum_j C_{zij} + G_i + I_i + (V_i - M_i) = X_i, \tag{7}$$

$$\sum_i A_{dij} + \sum_i A_{zij} + \sum_i C_{dij} + \sum_i C_{zij} + S_j + T_j = X_j, \tag{8}$$

where V_i is an export of industry i and M_i is an import of industry i, A_{dij} and A_{zij} are domestic and import direct inputs, C_{dij} and C_{zij} are domestic and import direct consumption's expenditures. It is obviously that the sum of domestic and import indexes is equal the total index:

$$A_{dij} + A_{zij} = A_{ij}. \tag{9}$$

$$C_{dij} + C_{zij} = C_{ij}. \tag{10}$$

Define $A_{dij}+C_{dij}$ = Dij, matrix $A_d =(\underline{a_{dij}})$ is the domestic direct input matrix, $C_d = (\underline{c_{dij}})$ is the domestic direct consumption matrix, where $\underline{a_{dij}}=A_{dij}/Xj$, $\underline{c_{dij}}=C_{dij}/Xj$. Let us name matrix D = (Dij) as matrix of total inputs (matrix of total expenditures).

The matrix multiplier Mt is described as follows:

$$M_t = (E-D)^{-1} = (E-(A_d+C_d))^{-1}. \tag{11}$$

where $D = (\underline{d_{ij}})$, $\underline{d_j}=Dij/Xj$

Let us name the multiplier Mt as multiplier of total expenditures. It shows the growth of gross domestic product connected with the growth of exogenous parameters (total sum of investments, government expenditures and net export). The private consumption is considered as an endogenous component of modified input-output model.

At the next stage we calculate coefficients Wj. They equal the shares of gross value added in gross output Xj:

$$W_j = 1-\sum_i a_{ij}, \quad (12)$$

where $a_{ij} = A_{ij}/X_j$

Let us introduce vector $W = (W_j)$. So the multiplier of income Km can be written as:

$$Km = W*Mt. \quad (13)$$

This multiplier K gives answer to the question: how to calculate growth of income (gross domestic product - GDP) connected with the growth of exogenous parameters.

This multiplier is a good tool in analysis of import leakage. The standard SAM can not provides this opportunity to evaluate this kind of leakage - total industry's leakage. For example, if sugar in our example is import product, then both confection's factory (coefficient a_{zij}) and household (coefficient c_{zij}), not only enterprise, contribute to import's growth.

4. An Empirical Comparison

The main databases for this study were provided by the Committee of Statistics of Russian Federation and include three 22-sectors IO tables for 1995, 2000 and 2002. Unfortunately, there is not adequate information about household' consumption in sectors, so we suppose that the technology of consumption is uniform for all industries.

The first columns in Table 1 show the classic Leontief's multipliers based on the domestic coefficient matrix A_d:

$$M_c = (E-A_d)^{-1} \quad (14)$$

Large output multipliers in food industry (1.96 in 1997, 2.26 in 2000 and 2.22 in 2003) may be result of small leakages or large share of intermediate consumption in gross output. A lowest value of multipliers for all years is obtained in commercial services.

The second columns of the Table 1 demonstrate the multipliers of total expenditures M_t. The highest value of multipliers are obtained in food industry too (1997, 2000) and in construction materials (2003). Small output multipliers in other productive services may be connected with large leakages.

The third columns show the multipliers of income Km. It can be seen from Table 1 that the lowest value for all years are obtained in closing and leather. This situation is a result of large import leakages in industry. By contrary of the Km, the highest value of multipliers are obtained in commercial services.

Table 1. The sector multipliers, Russia

Indusries	1997 year			2000 year			2003 year		
	Mc	Mt	Km	Mc	Mt	Km	Mc	Mt	Km
1. Electricity	1.42	1.85	1.22	1.99	2.94	1.35	2.07	3.16	1.52
2. Petroleum and natural gas	1.59	2.04	1.21	1.96	2.84	1.32	1.86	2.95	1.53
3. Coal	1.63	2.17	1.24	1.78	2.69	1.29	2.20	3.21	1.41
4. Other mining	1.46	1.94	1.23	1.53	2.41	1.33	1.55	2.60	1.44
5. Iron and Steel	1.74	2.16	1.13	2.01	2.88	1.21	2.16	3.15	1.38
6. Non-ferrous metals	1.76	2.24	1.16	2.05	2.91	1.22	2.10	3.13	1.44
7. Chemical	1.79	2.23	1.16	2.03	2.88	1.18	2.19	3.11	1.29
8. Machinery	1.71	2.18	1.14	2.06	2.94	1.21	2.14	3.09	1.32
9. Wood products	1.72	2.21	1.21	2.04	2.97	1.29	2.14	3.15	1.40
10. Construction materials	1.59	2.06	1.21	2.12	3.05	1.29	2.17	3.21	1.45
11. Closing and Leather	1.60	2.01	1.03	1.78	2.41	0.86	1.82	2.57	1.04
12. Food industry	1.96	2.46	1.18	2.26	3.14	1.20	2.22	3.19	1.34
13. Other manufactures	1.78	2.29	1.19	2.16	3.05	1.23	2.14	3.13	1.38
14. Construction	1.47	1.95	1.20	1.86	2.79	1.28	1.83	2.87	1.42
15. Agriculture	1.77	2.34	1.27	1.82	2.85	1.36	1.75	2.83	1.50
16. Transport and Communication	1.38	1.93	1.28	1.65	2.60	1.32	1.77	2.83	1.47
17. Commercial services	1.33	1.96	1.33	1.43	2.48	1.41	1.47	2.57	1.54
18. Other productive services	1.42	2.07	1.29	1.59	1.86	1.01	1.53	2.56	1.43
19. Education, medicine and culture	1.50	2.07	1.26	1.79	2.76	1.30	1.59	2.64	1.47
20. Dwelling services	1.49	2.04	1.28	1.87	2.85	1.35	1.80	2.88	1.51
21. Government and finance.	1.59	2.13	1.23	1.86	2.79	1.26	1.68	2.74	1.46
22. Science	1.65	2.15	1.16	2.06	3.01	1.32	1.95	3.01	1.48
Maximum	1.96	2.46	1.33	2.26	3.14	1.41	2.22	3.21	1.54
Minimum	1.33	1.85	1.03	1.43	1.86	0.86	1.47	2.56	1.04

5. Concluding Remarks

The new approach is an integration of Keynesian elements into Leontief's framework. The modified model should be able to compute gross domestic product in other industries.

While the SAM approach develops a disaggregated and balanced view of the circular flow of income, new method is concentrate on household's consumption. The private consumption is considered as an endogenous component of modified input-output model. The multiplier of total expenditures may be inserted to SAM model. This paper provides a (first) step in this direction.

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