Analyzing Industrial Structure Change in the Input-Output Economic System Based on Q-analysis: an Application to Chinese's Economy from 1992 to 2005

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Abstract: The topological principles of the well-known Atkin Q-analysis are applied to the analysis of interconnectedness of sectors in input-output systems**.** This paper presents an attempt to integrate important coefficients(IC), Q-analysis and graph theory in the perspective of breadth and depth of connections between sectors, to find the changes of key sectors and analyze structural chains of highest dimension as the most significant input-output industrial clusters. Important coefficients analysis presents a transition from quantitative matrices to binary incidence matrices based on the maximum information content. While depending on above binary incidence matrices, structural Q-analysis provides a set of structural sectors which reflect the incidence relation of sectors and structural chains of highest dimension, and also provides a new way for visualizing economic complexity through the process of structural economic complication. Using a set of annual 19-sectors constant- price input-output tables, the application to Chinese's economic structure analysis in the period of 1992 to 2005 revealed an increasing pulling function of other service industry to other sectors. Secondly, there is strong asymmetry between the forward and backward linkages structures by the Q-analysis, and the key structural chains mainly composed of chemical industry、non-mental mineral product industry and metal smelting、steel wire products industry both in the forward and backward linkages,; besides that, a much more stable and complex economic structure in the perspective of forward linkages than that in the perspective of backward linkages.

1. Introduction

The purpose of this paper is to integrate important coefficients(IC) and Q-analysis to find the changes of key sectors and analyze structural chains of highest dimension as the most significant input-output industrial clusters in the perspective of depth and span. Also, a visualizing economic complexity will be revealed in the application of graph theory. The central concern is the deepening of economic complexity through structural changes generated by the improvement of technology and the change of final demand. These structural changes are often complex and difficult to extract, so, new tools for illustration, interpretation and visualization provide the potential for greater insights into the nature of these changes.

Recently, structural analysis has come to be one of the more important applications of input–output analysis. Traditionally, qualitative input–output analysis (QIOA) developed by Schnabl and Holub (1979) and Holub and Schnabl (1985) split the intermediate transaction flow (say, T) into several layers based on the Euler power series of technical coefficient matrix A. With a critical filter value, the sliced transaction matrices can be transferred to Boolean matrices with 0 or 1 entries, showing the pattern of economic links among sectors in each layer up to a certain layer. Since the filter is chosen subjectively according to some criteria, some information may be lost depending on the nature of the filter critical value when the decomposition is conducted. Then, Dewhurst(1993) made some efforts on analyzing the production pattern ,revealing the sectoral interdependence in an economy ,by decomposing intermediate transactions in an input-output system.

In contrast to QIOA, structural decomposition analysis (SDA) has received much more emphasis. Rose and Casler (1996) provided a detailed review of SDA. While new applications on SDA can be found in Albala-Bertrand (1999), Alcala et al. (1999), Mukhopadhyay and Chakraborty (1999), Wier and Hasler (1999), Dietzenbacher et al. (2000), Hitomi et al. (2000), Jacobsen (2000), Casler (2001), Dietzenbacher (2001), and Milana (2001) among others, these new applications still follow the traditional SDA, which decomposes observed changes into determinant parts, like technology, final demand or synergistic interactions between these two components.

Two alternative approaches, the superposition flow decomposition(Sonis,1980; Sonis and Hewings ,1998,2001;Guo Dong, Sonis and Hewings,2003,2007) and the structural Q-analysis, proposed by Atkin (1974,1981) and developed further by Sonis and Hewings (1998,2000, 2001,2003,2005,2007) had been applied in decomposing the strrcture hierarchically. On the one hand, superposition flow decomposition examines the degree to which the structure of flows might be decomposed into a set of weighed subflows, in which the subflows can be expressed in the form of extreme tendencies. Q-analysis, on the other hand, analyses the structure of a relationship or interdependency between sets of groups, such as some groups in a social context. The main central philosophy of Q-analysis is considering all the elements of a set as some different polyhedrons, all of these polyhedrons connect by the facts which shared by each of them. Atkin(1974) illustrates the application of the Q-analysis by giving an example of how a group of people connect with each other based on some common hobbies and characteristics.

Note that in Q-analysis, the structure of the social interdependencies is represented by binary matrices. Following this idea, taking economic sectors as the sets in Q-analysis, provides an option to explore the intersectoral relationships in an economy. Usually, the intersectoral links in an economy can be shown in two ways: forward linkage and backward linkage (Chenery and Watanabe, 1958), so does the Q-analysis.

This paper is organized as follows: section two describe the methodology of structural Q-analysis and important coefficients (IC) .Following this exposition, the application to Chinese's production structure from 1992 to 2005 will be made. Some summary remarks complete the paper.

2. The Description of the Methodology

2.1 A brief introduction of Structural Q-analysis

The following methodological description of the procedure of Q-analysis is taken from the Atkin studies(Atkin,1974; see also, Sonis, Hewings and Dong Guo, 2007,2005,2003;Sonis and Hewings,2000,1998;Sonis,1998, and Sonis ,Hewings and Bronstein,1994).

The topological principle of Q-analysis, supposed by Atkin (1974) describes the structure of relationships. Suppose there are two finite sets X and Y, each of which has elements x_i (i=1,2,⋯, m) and y_i (i=1,2,⋯n) , and every element of X and Y considered to be a convex polyhedron. Suppose from set Y to set X there is a relation $\lambda \subset Y \times X$, which is a binary zero-one matrix $\Lambda = \lambda_{ij}$, defined as an incidence matrix. So such a relation defines a simplicial complex K, which can be denoted by $K_Y(X;\lambda)$; the pattern is shown in figure 1.

Note that the inverse relation $\lambda^{-1} \subset X \times Y$ defines a simplicial complex $K_X(Y;\lambda^{-1})$, whose pattern is shown in figure 2.

λ	X
Y	(λ_{ij})

Figure 1 Pattern of $K_Y(X;\lambda)$

λ^{-1}	Y
X	(λ_{ij}^{-1})

Figure 2 Pattern of $K_X(Y;\lambda^{-1})$

In simplicial complex $K_X(Y;\lambda^{-1})$, a p-simplex, σ_p, is constructed with distinctive (p+1) vertices which are elements of X vertex set. It is the same way in the simplicial complex $K_X(Y;\lambda^{-1})$.

In the context of input-output tables, we consider the consume sectors as a set X and the consumed sectors as a set Y, each of which has elements x_i (i=1,2,$\cdots$, m) and y_i (i=1,2,$\cdots$n).

The forward linkage reveals the structural relation of any element of set Y, means taking y_i as a q_i -dimensional polyhedron with $q_i + 1$ vertices in the set X and researching the number of common faces and the connection mode between y_i and y_j ($i \neq j$). All the consumed sectors y_k, ($k = 1, 2, \cdots \mathrm{n}$) is called the forward linkages simplicial family, hence, the backward linkage is analogical.

2.2 q-connectivity

Two simplexes, σ_i and σ_j, are q-near only if they share at least q+1 vertices. Thus, two sectors y_i and y_j ($i \neq j$) are q-connected only if they share at least q+1 economic sectors accepting the inputs from the sectors y_i and y_j,which defined as q-connected simplexes. A series of such q-connected simplexes connected with each other is a q-connectivity forming a q-chain. It is not difficult to see that if σ_i and σ_j are q-connected then they are also (q-1)-, (q-2)-,$\cdots$,1-,0-connected in $K_Y(X;\lambda)$. Looking for the q-connected components, for a fixed value of q, means that we are looking for all simplices σ_p with $p \geq q$ to see if they share any common faces of dimension greater than or equal to q.

2.3 The procedure of the linkage Q-analysis

Following Atkin (1974) the operational basis of forward linkage for Q-analysis is given by a shared face matrix SF of the form:

$$SF = \Lambda\Lambda^T - U$$

where Λ is the incidence matrix corresponding to the chosen slicing procedure, Λ^T is its transpose and U is the matrix with unit entries. The components of the shared face matrix SF are the dimensions of the maximal mutual faces for each pair of sectors y_i and y_j.

The operational algorithm of Q-analysis includes following iterative steps for each dimension q, q=0,1,⋯N, where N is the maximal dimension of simplices from the simplicial complex:

(1) Identify the sectors and their corresponding simplices whose dimension are equal or larger than q; the maximal dimension of simplices are on the main diagonal of the shared face matrix SF.

(2) Constructed all distinct q-chains as previous step, the number of distinct q-chains is denoted as Q_q .The vector $Q = \{Q_N, Q_{N-1}, \cdots, Q_0\}$ is called the structural vector of the simplicial K(Y) and the maximal q-value N is a dimension of this complex.

So far, the interaction matrix of the forward linkage Q-analysis has been presented. From the backward linkage perspective, the same slicing procedure can be adopted .Thus,

$$S^T F = \Lambda^T \Lambda - U$$

The components of the shared face matrix $S^T F$ are the dimensions of the maximal mutual faces for each pair of sectors x_i and x_j. Moreover, the backward linkages input-output Q-analysis can be performed analogously with the help of the conjugate shared face matrix.

2.4 Slicing procedure

In Q-analysis, construction a binary matrix to describe the relationship between two defined sets is called the slicing procedure. However, input-output tables describe the inter-sectoral relationship in an economy with definite numerical data, rather than in binary form. So chosing a proper slicing way to translate these non-binary matrices into binary matrices is one of the key parts of Q-analysis. In the context of input-output tables, there are many slicing methods have been tried before to transfer numerical data into the form of binary matrixes to show a certain sectoral structure. Sonis and Hewings (2000) removed 50% of the smallest components of the Leontief inverse matrix which called Hierarchical rank-size analysis when Q-analysing Israel’s economic structure. In the variable filter approach used in minimal flow analysis in QIOA noted earlier (Holub et al.,1985; Schnabl,1994), usually the filter value is subjectively set to be 0.2. However, it comes with the cost of losing some important information in a complicated economic system. Further, there is the problem of the appropriate definition of the filter. Even though
standard SDA can decompose the total output changes between different time periods into three parts, it cannot provide the sectoral interaction relationship behind the changes.

On the basis of above deficiencies of these slicing approaches, hence, this paper intends to adopt information method based on the idea of important coefficients to choose some important coefficients from direct consumption matrix (A) which include enough information of Leontief inverse matrix (B) and construct a binary matrix avoiding missing important messages of the complicated economic system.

Generally, important coefficients refers to the small part of direct consumption coefficients which play an important role in the economic system (take the Leontief inverse matrix or total output for example). The main coefficient means it is only limited to the direct consumption coefficient and it is major or not depending on the extent of impact on the economic system. The field of influence and tolerable limits , based on Sherman-Morrison formula proposed by Xu&Madden（1991）, are the most common methods to analyse major coefficient. But both of them have some limitations. For the field of influence approach, it only measures the impact of each coefficient roles in the Leontief inverse based on the absolute change of each coefficient with considering the coefficient scale. However, the tolerable limits approach only inspects the relationship between one sector and total output and completely ignores the impact on other sectors. Thus, we adopt a new method, information method, to select the main coefficients which is a ideal approach to analysis the which direct consumption coefficient can take a greatest impact on the formation of current economic system.

In the information method, there are two matrixes-influence matrix (A) and the influenced matrix (to be B or total output matrix), the main idea is to choose a main coefficients set from A which contains as little coefficients in a given information contained proportion (p). The information contained proportion means that we get a new matrix A^p of which some coefficients are keeping original as A, and others are 0. So, how much information of A contained in A^p is called the information contained proportion. Take Leontief inverse matrix (B) as influenced object for example, the information contained proportion P could be as follows:

$$P = 1 - \frac{\sum |b_{ij} - b_{ij}^p|}{\sum b_{ij}} \quad P = 1 - \frac{\sum |b_{ij} - b_{ij}^p|}{\sum b_{ij}}$$

b_{ij}^p is the element of matrix B^p calculated by matrix A^p

Obviously, matrix A^p contains all the information of A when P equals 1 only if A^p equals to A. The value of P depends on the needs of analysis and is usually subjectively set to be 75%, 80% or 85%, the more the non-zero coefficients of A^p, the larger the P. Generally speaking, we set p equals to 60%-75% as the main coefficients couldn’t contain enough information of A in a smaller p and there are too many coefficients in a lager p. The relationship between information contained proportion and the rate of main coefficients accounting for total coefficients in different years is shown in table 1.Hence, we set 75% as a filter value of p. All the main coefficients of each year were presented from table 8 to 15. So far the binary matrix is obtained by the slicing procedure when letting the non-zero coefficients of A^p to be 1 and others are still 0.

Table 1 The information contained proportion and main coefficients scale.

P	2005	2002	1997	1992
75%	33.5%	29.9%	28.3%	28.3%
80%	39.6%	36.6%	35.2%	34.9%
85%	46.8%	44.0%	42.9%	42.7%
90%	54.9%	52.4%	51.8%	51.8%

3. Applications to the Chinese Economy:1992-2005

3.1 Classical Backward and Forward linkages

Avoiding the influence coursed by price fluctuation which goes with time, a set of annual 19-sector constant-price input-output tables, aggregated from the 62-sector constant-price tables, from 1992 to 2005 will be used to explore the structural changes in the Chinese economy. The sectors' definitions are shown in table 2.

Table 2 Sector definitions in the Chinese input-output tables

Number	Sector
1	**Agriculture**
2	**Coal Mining and Washing Industry**
3	**Petroleum and Natural Gas Extraction**
4	**other Mining Industry**
5	**Food, Textiles, Paper-making and Furniture Industry**
6	**Petroleum Refining and Coking**
7	**Chemical Industry**
8	**Non-metallic Mineral Product Industry**
9	**Metal Smelting and Metal Products**
10	**Engineering Industry**
11	**Transportation Facilities Manufacturing**
12	**Electronic and Electrical Equipment Manufacturing**
13	**Instrument and Meter, Office Machinery Manufacturing and other Industry**
14	**Electricity, Gas and Water Production and Supply**
15	**Construction Industry**
16	**Transportation, Storage Post**
17	**Wholesale and Retail Accommodations Industry**
18	**Real Estate Financing and Insurance industry**
19	**Other Service Industry**

In the classical input-output analysis, forward and backward linkage coefficients are often used to reveal the impetus of each sector on the social promotion which calculated with the help of Rasmussen indices presenting:

1. Forward linkage :

$$FL_i = \frac{1}{n}\sum_j b_{ij} / \frac{1}{n^2}\sum_{ij} b_{ij}$$

2. Backward linkage:

$$BL_j = \frac{1}{n}\sum_i b_{ij} / \frac{1}{n^2}\sum_{ij} b_{ij}$$

where b_{ij} is element of Leontief inverse matrix and all the dynamics of linkages from 1992 to 2005 are presented in table 3 and table 4.

Table 3. Dynamics of Forward linkage , 1992-2005.

Rank size hierarchy	1992	Sector No.	1997	Sector No.	2002	Sector No.	2005	Sector No.
1	1.7848	17	1.8163	9	1.8634	9	1.8021	9
2	1.7254	9	1.8116	7	1.7488	7	1.6779	7
3	1.6950	5	1.4687	5	1.3444	5	1.3690	5
4	1.3464	7	1.2188	17	1.2768	12	1.3298	12
5	1.3334	3	1.0978	1	1.1652	17	1.1498	19
6	1.3086	18	1.0748	10	1.0666	16	1.1348	16
7	1.1335	1	1.0746	12	1.0250	19	1.1194	14
8	1.0192	16	0.9599	3	0.9955	10	1.0944	17
9	0.9038	6	0.9548	18	0.9619	3	0.9746	10
10	0.8850	10	0.9491	16	0.9570	1	0.9575	3
11	0.8359	14	0.9275	14	0.9454	14	0.9228	1
12	0.7366	12	0.8301	19	0.8871	6	0.8573	6
13	0.7277	8	0.8073	6	0.8733	18	0.8008	11
14	0.6846	2	0.7951	11	0.8486	11	0.7110	18
15	0.6439	4	0.7484	2	0.6750	2	0.6833	2
16	0.6166	11	0.7171	4	0.6531	4	0.6804	4
17	0.6083	13	0.6733	8	0.6249	8	0.6779	8
18	0.5948	19	0.6312	13	0.6121	13	0.6096	13
19	0.4165	15	0.4437	15	0.4761	15	0.4476	15

Table 4. Dynamics of Backward linkage, 1992-2005.

Rank size hierarchy	1992	Sector No.	1997	Sector No.	2002	Sector No.	2005	Sector No.
1	1.4948	12	1.3582	12	1.3370	12	1.2411	12
2	1.4119	10	1.2261	9	1.2236	11	1.2135	9
3	1.3766	9	1.2090	11	1.2006	10	1.2035	7
4	1.3389	11	1.1995	7	1.1878	15	1.1695	2
5	1.2751	8	1.1490	15	1.1837	9	1.1516	10
6	1.2691	13	1.1380	10	1.1368	7	1.1075	11
7	1.1084	7	1.1370	8	1.0814	5	1.1042	15
8	0.9944	4	1.1209	5	1.0397	13	1.0351	6
9	0.9792	5	1.0440	4	1.0336	8	1.0216	4
10	0.9241	6	1.0209	6	1.0029	6	1.0173	13
11	0.9085	15	0.9894	13	0.9352	4	1.0150	14
12	0.8582	16	0.9616	14	0.9315	19	1.0047	5
13	0.8333	2	0.8985	2	0.9310	16	1.0000	8
14	0.8077	17	0.8792	17	0.9016	14	0.9552	19
15	0.7944	14	0.8085	19	0.8782	17	0.8609	16
16	0.7487	19	0.7691	1	0.8703	2	0.8122	17
17	0.7081	18	0.7686	16	0.7923	1	0.7476	1
18	0.6333	1	0.6846	18	0.6768	18	0.7232	3
19	0.5353	3	0.6380	3	0.6558	3	0.6161	18

As usually defined in conventional key sectors theory, the sector i is considered as a key sector if both of its FL_i and BL_j are larger than one. A forward linkage oriented sector means $FL_i > 1$ and $BL_j < 1$; and a backward linkage oriented sector means $FL_i < 1$ and $BL_j > 1$; a weakly linked sector mean both FL_i and BL_j are less than one. As depicted in table 3 and table 4, from 1992 to 2005, Metal Smelting and Metal Products (9), Chemical Industry (7), Food, Textiles, Paper-making and Furniture Industry (5), Electronic and Electrical Equipment Manufacturing (12) were emerging to be the key sectors of Chinese economical system. Almost only defined as a forward linkage oriented sector, Agriculture (1) had a small pulling impetus on other sectors, which BL_j was about 0.7 and FL_i was nearly 1 from 1992 to 2005. Besides, Construction Industry (15) transformed into a backward linkage oriented sector from a weakly linked sector, which was rising in the hierarchy of backward linkages during the study period.

Table 3 and table 4 shows that the hierarchy of forward linkages is more stable then the hierarchy of backward linkages. In the hierarchy of forward linkage Metal Smelting and Metal Products (9), Chemical Industry (7), Food, Textiles, Paper-making and Furniture Industry (5) are always on the top 3 of hierarchy, Petroleum and Natural Gas Extraction (3) and Real Estate Financing and Insurance industry (18) are always on the bottom; in the hierarchy of backward linkages Electronic and Electrical Equipment Manufacturing (12) is always on the top, while Metal Smelting and Metal Products (9), Engineering Industry (10), Transportation Facilities Manufacturing(11) are always up and down and all other sectors changed their place in the hierarchy. This change in both hierarchies signified the qualitative change in the economic relationship between the sectors during time.

3.2 Structural Q-analysis of Chinese input-output system, 1992-2005

Taking 2005 for example, based on the direct consumption matrix (A) and Leontief inverse matrix (B) of input-output table, 121 coefficients, accounting for one third of all coefficients, in A contain three fourths information of B. Then the incidence matrix Λ could be formed on the ground of such important coefficients, which is depicted in the table 5. Following previous section, the consumption sectors and the consumed sectors are respectively defined as set X and Y, the subscript quantity of each sector consistent with the number in the table 2.

For the forward linkage, as a practical algorithm for producing the connectivities of the simplices in $K_Y(X;\lambda)$ shown in Table 6 we can proceed as above mentioned in 2.3.

For the backward linkage ,by a similar reasoning we obtain table 7 for the connectivities of the simplices in $K_X(Y;\lambda^{-1})$ by forming the matrix product $\Lambda^T\Lambda$ and subtracting 1 from each element.

In the tables we represent q=-1(disconnection) by the symbol -, for ease of reference. Since Λ is a 19 $\times$ 19 matrix it follows that both of $\Lambda\Lambda^T$ and $\Lambda^T\Lambda$ are 19 $\times$ 19.

Table 5 The incidence matrix of 2005

λ	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19
Y1	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
Y2	0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0
Y3	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
Y4	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
Y5	1	0	0	0	1	0	1	1	0	0	0	1	1	0	0	0	1	0	1
Y6	0	0	1	1	0	1	1	0	1	0	0	0	0	1	0	1	0	0	0
Y7	1	1	0	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	1
Y8	0	0	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0
Y9	0	1	1	1	0	0	1	1	1	1	1	1	1	0	1	0	0	0	0
Y10	0	1	1	1	0	0	1	1	1	1	1	1	0	1	0	1	0	0	0
Y11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0
Y12	0	1	1	0	0	0	0	0	0	1	0	1	1	1	0	0	1	0	1
Y13	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0
Y14	0	1	1	1	0	0	1	1	1	0	0	0	0	1	0	0	0	0	1
Y15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y16	0	1	0	1	1	1	1	1	1	1	0	0	0	1	0	1	0	0	1
Y17	0	1	0	0	1	0	1	0	1	1	0	1	0	1	0	0	1	0	1
Y18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y19	0	1	1	1	1	0	1	0	1	1	0	1	0	1	0	1	1	1	1

Table 6 q-connectivities in $K_Y(X;\lambda)$, from $\Lambda\Lambda^T$

Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	Y13	Y14	Y15	Y16	Y17	Y18	Y19	
3	—	0	0	3	0	2	—	0	0	—	0	—	0	—	1	2	—	2	**Y1**
	3	0	0	—	2	1	0	1	2	—	1	1	2	—	3	2	—	2	**Y2**
		1	0	0	1	0	—	0	0	—	—	—	0	—	1	0	—	0	**Y3**
			3	1	2	3	1	3	3	—	—	0	3	—	3	1	—	2	**Y4**
				7	0	6	1	3	2	—	3	0	2	—	3	4	—	4	**Y5**
					6	2	0	3	5	0	1	1	4	—	5	2	—	5	**Y6**
						11	2	8	7	0	4	1	5	—	7	6	—	7	**Y7**
							3	3	2	—	0	0	1	—	1	1	—	1	**Y8**
								10	8	0	4	1	5	—	5	4	—	6	**Y9**
									10	1	4	1	6	—	7	5	—	8	**Y10**
										1	—	—	—	—	0	—	—	0	**Y11**
											7	1	3	—	3	5	—	6	**Y12**
												2	1	—	1	1	—	1	**Y13**
													7	—	6	4	—	6	**Y14**
														—	—	—	—	—	**Y15**
															10	6	—	8	**Y16**
																8	—	8	**Y17**
																	—	—	**Y18**
																		12	**Y19**

Table 7 q-connectivities in $K_X(Y;\lambda^{-1})$, from $\Lambda^T\Lambda$

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	
2	0	—	0	2	—	2	1	0	0	0	1	1	—	—	—	1	—	1	**X1**
	8	4	5	3	1	6	4	7	6	2	5	2	6	0	2	2	0	5	**X2**
		5	4	0	0	4	2	4	3	1	3	1	4	0	2	1	0	2	**X3**
			7	2	1	7	5	7	4	2	3	1	4	0	3	0	0	3	**X4**
				5	0	5	2	3	3	0	3	1	2	—	1	3	0	4	**X5**
					3	2	0	2	0	—	—	—	2	—	1	—	—	0	**X6**
						11	6	8	5	2	5	2	5	0	3	3	0	5	**X7**
							7	6	3	2	4	2	2	1	1	0	—	3	**X8**
								11	5	2	5	2	7	1	3	1	0	4	**X9**
									6	2	5	2	4	0	2	2	0	4	**X10**
										3	2	1	0	0	1	—	—	0	**X11**
											7	3	3	1	1	3	0	4	**X12**
												4	1	0	—	1	—	2	**X13**
													8	—	3	2	0	4	**X14**
														1	—	—	—	—	**X15**
															4	0	0	1	**X16**
																4	0	3	**X17**
																	0	0	**X18**
																		6	**X19**

Referring to Table 6 and Table 7 we can obtain the Q-analysis for both $K_Y(X;\lambda)$ and $K_X(Y;\lambda^{-1})$ as the following set of equivalence classes:

$K_Y(X;\lambda)$

q **-value**	Q_q **-value**	**Components**
12	1	{19}
11	2	{19},{7}
10	5	{19},{7},{9},{10},{16}
9	5	{19},{7},{9},{10},{16}
8	1	{19,17,16,10,9,7}
7	4	{19,17,16,10,9,7},{5},{12},{14}
6	2	{19,17,16,14,12,10,9,7,5},{6}
5	1	{19,17,16,14,12,10,9,7,5,6}
4	1	{19,17,16,14,12,10,9,7,5,6}
3	1	{19,17,16,14,12,10,9,7,5,6,4,2,1,8}
2	1	{19,17,16,14,12,10,9,7,5,6,4,2,1,8}
1	1	{19,17,16,14,12,10,9,7,5,6,4,2,1,8,13,3,11}
0	1	{19,17,16,14,12,10,9,7,5,6,4,2,1,8,13,3,11}

The structure vector is

$$Q = \{\overset{12}{1}\quad 2\quad 5\quad 5\quad 1\quad 4\quad 2\quad 1\quad 1\quad 1\quad 1\quad 1\quad \overset{0}{1}\}$$

$K_X(Y;\lambda^{-1})$

q **-value**	Q_q **-value**	**Components**
11	2	{7},{9}
10	2	{7},{9}
9	2	{7},{9}
8	3	{7,9},{14},{2}
7	3	{7,9,2,14,4},{12},{8}
6	3	{7,9,2,14,4,8,10},{12},{19}
5	2	{7,9,2,14,4,8,10,5,12,19},{3}
4	4	{7,9,2,14,4,8,10,5,12,19,3},{13},{16},{17}
3	2	{7,9,2,14,4,8,10,5,12,19,3,16,17,13},{6}
2	1	{7,9,2,14,4,8,10,5,12,19,3,16,17,13,1,6,11}
1	1	{7,9,2,14,4,8,10,5,12,19,3,16,17,13,1,6,11,15}
0	1	{all}

The structure vector is

$$Q = \{\overset{11}{2}\quad 2\quad 2\quad 3\quad 3\quad 3\quad 2\quad 4\quad 2\quad 1\quad 1\quad \overset{0}{1}\}$$

The structure vector Q measures the stability of the economic system in the connect width of each sector. If we find a situation in which $Q_0 > 1$, and all communication of sectors has ceased. In other words, all economic sectors are naturally divided into several irrelevant sets and all communication among the sets would be impossible when $Q_0 > 1$. From the structure vector of $K_Y(X;\lambda)$, all sectors, excepting Construction Industry and Real Estate Financing and Insurance industry, are connected until we reach the level of $q = 1$, which means Construction Industry and Real Estate Financing and Insurance industry have relatively weaker forward influence than other sectors so as to the forward influence couldn’t be reflected in the structure vector rather than other sectors don’t needs inputs from these two sectors. And Other Service Industry (19), Wholesale and Retail Accommodations Industry (17), Transportation, Storage Post (16), Chemical Industry (7) form a key department chain at $q = 8$, the value of $Q_8 = 1$ which reveals that these sectors are highly associate and to be the major inputs source of other sectors. However, Agriculture don’t have such extensive contact as the above mentioned department, which is a major source of Agriculture, Food, Textiles, Paper-making and Furniture Industry, Chemical Industry, Wholesale and Retail Accommodations Industry.

From the structure vector of $K_X(Y;\lambda^{-1})$, all sectors are connected until we reach the level of $q = 0$, which means that a chain of 0-connection exists which includes any pair. Chemical Industry (7), Metal Smelting and Metal Products (9), Electricity, Gas and Water Production and Supply (14), Coal Mining and Washing Industry (2), other Mining Industry (4), Non-metallic Mineral Product Industry (8), Engineering Industry (10) form a key department chain at $q = 6$ to require inputs from other sectors. It is also depicted that until $q = 2$, the value of $Q_2 = 1$ in the structure vector of $K_X(Y;\lambda^{-1})$ while at $q = 8$ and

$q = 5, 4, 3, 2, 1, 0$, the value of $Q_8 = Q_5 = Q_4 = Q_3 = Q_2 = Q_1 = Q_0 = 1$ in the structure vector of $K_Y(X;\lambda)$ which reflect that the backward linkage structure is less stable than forward linkage structure. The comparison between the results of forward and backward linkage Q-analysis suggests that there is strong asymmetry between the forward and backward linkages structures.

On the other hand, Q-analysis exposit the complexity of economic system and the connection of each section in the perspective of associated breadth, but, further study should be conducted in the perspective of associated depth. Based on classical forward and backward linkage theory, the strength of forward and backward linkage among sectors is measured by the corresponding correlation coefficients. Thus, we also adopt correlation coefficients to measure the associated depth of economic sectors.

The whole backward and forward linkage between sectors are depicted in the figure 3 and figure 4 in Appendix. And the sectors pointed by arrowheads are respectively consumed and consumption sectors the figures. Decided by the completely consumption coefficients, the extent of sectoral inter-correlation is revealed by the thickness of connections lines between sectors, the more coarse the associated deeper, and vice versa. It is also shown that Other Service Industry (19), Wholesale and Retail Accommodations Industry (17), Transportation, Storage Post (16), Chemical Industry (7) are major sectors of forward linkage, but Agriculture is a key sector in the forward linkage as the extent of forward linkage is still deep with the connection sectors.

It is shown in table 9 that although only Agriculture 、Food, Textiles, Paper-making and Furniture Industry 、Chemical Industry、Wholesale and Retail Accommodations Industry consume most of agricultural products and agriculture is the main raw materials for above forward connected sectors whose forward linkage coefficients with agriculture are 1.2367、0.3157、0.1276 and 0.1176 . So, Agriculture is still a key sector in the forward linkage structure.

As to Construction Industry and Real Estate Financing and Insurance industry, the forward linkage is not significant revealed in the Q-analysis for the forward associated coefficients of Construction Industry and Real Estate Financing and Insurance industry are all too small to be selected. On the one hand , these two sectors play an more important role in the Residents sector than in the others which means that a residents-endogenous model can much more comprehensively reflect the forward linkage of these two sectors. On the other hand , the construction industry is defined as construction activities of some special buildings ,excepting the preparation work of constructing by China's national economy industry classification standards so as to the connection to the other sectors is unapparent.

The computational processes of Q-analysis in 1992, 1997 and 2002 are presented in the Appendix.

4. Conclusion

The analysis of sectoral structure in Chinese's economy during 1992 to 2005 suggests that the economy is becoming complicated in the sense of the complex of sectoral structure. Seen from the structure vector of forward linkage structure, a 6-connectivity was formed in 1992,1997 and 2002 , but in 2005 a 8-connectivity was formed which means that over time the value Q_q =1 when the value of q is relatively large and the closer links among sectors. It is also depicted that Food, Textiles, Paper-making and Furniture Industry, Chemical Industry, Metal Smelting and Metal Products, Engineering Industry, Other Service Industry are the key input sectors to form a department chain of forward linkage and Other service is playing an increasing prominent role. Besides that, Agriculture is still a key sector in the forward linkage structure to provide materials to other sectors although the association breadth was lower than some manufacturing departments.

In the perspective of backward linkage structure, Chemical Industry, Metal Smelting and Metal Products, Non-metallic Mineral Product Industry, Engineering Industry, other Mining Industry, Electricity, Gas and Water Production and Supply are the key sectors, consuming product from other sectors, to form a department chain of backward linkage. At the same time, The completely consumption coefficients of Construction Industry to the connected sectors are also large , so it still plays an important role in the economic system.

The objective of this paper is three fold. Information method was used to selected the important coefficients of the direct consumption matrixes to avoid missing some important information. Although this approach has some shortcomings, As for QIOA and Hierarchical rank-size analysis, this approach has less subjective facts.

Secondly, the ideas of combinatorial topology in the form of Atkin's Q-analysis were used in the analysis of economic interactions between the sectors which revealed in the complex system of forward and backward linkages. Meanwhile, the methodology has been illustrated with 19 sectors tables, which is much more valuable in application than that with very aggregated tables. Thus, this method is especially important for the analysis of the structural complication in input-output systems.

Then, the relationship of sectors has depicted in the graphs of backward and forward linkage with 19 sectors, which providing a forceful support of sector complexity.

The methodology can provide some important ,complementary insights into the structure of economies only in the perspective of breadth of the link between sectors. Considering the value of forward and backward linkage coefficients of each sector is a complementary of Q-analysis, which in the perspective of depth of the link between sectors. So, combined these two perspectives in the analysis of economic interactions using connection graphs is the biggest innovation of this paper.

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Appendix

Table 8 Selected Matrices of direct input in 2005

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.1568				0.1652		0.0383										0.0467		
2	0.0838				0.2880		0.0412	0.0443				0.0194	0.0795				0.1210		0.0928
3	0.0656	0.0421		0.0724	0.0644		0.3804	0.0553	0.0186	0.0494	0.0547	0.0712	0.0774						0.0654
4		0.0341				0.0308			0.0134					0.1655					
5						0.6114	0.0248												
6				0.0815			0.0210	0.0551	0.0987										
7			0.0279	0.0612		0.0360	0.0360		0.0256					0.0511		0.1299			
8								0.0917	0.0211			0.0277			0.2008				
9		0.1034	0.0324	0.0344			0.0180	0.0487	0.3550	0.2157	0.1134	0.1094	0.0895		0.1377				
10		0.0661	0.0302	0.0480			0.0129	0.0328	0.0223	0.1772	0.0785	0.0230		0.0504		0.0227			
11											0.2735					0.0690			
12		0.0454	0.0243							0.0624		0.3819	0.1158	0.0542			0.0249		0.0809
13									0.0359				0.0541	0.0251					
14		0.1438	0.0617	0.0926			0.0627	0.0731	0.0565					0.0884					0.0236
15																			
16		0.0714		0.0704	0.0284	0.0452	0.0380	0.0519	0.0429	0.0348				0.0401		0.1256			0.0335
17		0.0512			0.0438		0.0399		0.0360	0.0410		0.0378		0.0445			0.0482		0.0671
18																			
19		0.0789	0.0442	0.0538	0.0312		0.0354		0.0270	0.0346		0.0362		0.0372		0.0282	0.0821	0.0736	0.0842

Table 9 Selected Matrices of Leontief Inverse in 2005

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	1.2367				0.3157		0.1276										0.1176		
2		1.0926				0.0664			0.0639					0.2134					
3						0.6852	0.1154												
4				1.1154			0.0577	0.0925	0.1883										
5	0.1842				1.5024		0.1800	0.1462				0.1360	0.2035				0.2398		0.2190
6			0.0569	0.1238		1.0942	0.1113		0.1034					0.1035		0.1798			
7	0.1768	0.1951		0.2175	0.2323		1.7180	0.1909	0.1600	0.2059	0.2159	0.2818	0.2431						0.2115
8								1.1208	0.0571			0.0720			0.2447				
9		0.3069	0.1187	0.1664			0.1374	0.1797	1.6688	0.5084	0.3617	0.3671	0.2516		0.3264				
10		0.1490	0.0672	0.1153			0.0762	0.0940	0.1018	1.2734	0.1757	0.0951		0.1235		0.0709			
11											1.4073					0.1270			
12		0.1755	0.0878							0.1930		1.6890	0.2521	0.1772			0.0887		0.1954
13									0.0818				1.0811	0.0527					
14		0.2449	0.1055	0.1764			0.1743	0.1581	0.1747					1.1805					0.0819
15																			
16		0.1723		0.1557	0.0970	0.1166	0.1379	0.1346	0.1542	0.1302				0.1235		1.1924			0.1004
17		0.1458			0.1167		0.1356		0.1327	0.1315		0.1387		0.1216			1.1018		0.1378
18																			
19		0.1779	0.0896	0.1347	0.1006		0.1339		0.1251	0.1254		0.1381		0.1237		0.0864	0.1392	0.1119	1.1597

Table 10 Selected Matrices of direct input in 2002

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.1622				0.1608		0.0289								0.0783		0.0518		
2														0.1517					
3						0.5753													
4				0.0578				0.0545	0.0732										
5	0.0634				0.2843		0.0299	0.0441				0.0191	0.0809				0.1157		0.0935
6				0.0518		0.0425	0.0348		0.0266					0.0488		0.1377			
7	0.0725			0.0716	0.0723		0.3730	0.0656	0.0199	0.0522	0.0611	0.0797	0.0776						0.0520
8								0.0810	0.0157			0.0223			0.1124				
9		0.0689	0.0248				0.0160	0.0516	0.3533	0.2162	0.1226	0.1156	0.0886		0.1718				
10		0.0413	0.0233	0.0422				0.0341	0.0229	0.1815	0.0844	0.0245		0.0390	0.0454	0.0260			
11											0.2845					0.0711	0.0231		
12										0.0621	0.0274	0.3882	0.0988	0.0337			0.0256		0.0608
13									0.0314				0.0520						
14		0.0699	0.0414	0.0724			0.0436	0.0561	0.0452					0.0510					
15																			
16				0.0514	0.0267	0.0406	0.0312	0.0543	0.0380	0.0310				0.0409		0.1115			
17					0.0540		0.0409	0.0530	0.0397	0.0442	0.0387	0.0452		0.0442			0.0532		0.0594
18																0.0542	0.0638	0.1001	
19					0.0265		0.0263		0.0231			0.0307					0.0616	0.0613	0.0621

Table 11 Selected Matrices of Leontief Inverse in 2002

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	1.2333				0.3032		0.0927								0.1413		0.1183		
2														0.1774					
3						0.6333													
4				1.0767				0.0813	0.1329										
5	0.1412				1.4775		0.1246	0.1294				0.1218	0.1862				0.2207		0.1948
6				0.0962		1.0850	0.0934		0.0885					0.0839		0.1844			
7	0.1807			0.1845	0.2417		1.6686	0.1932	0.1344	0.1980	0.2262	0.2928	0.2294						0.1687
8								1.1003	0.0377			0.0549			0.1395				
9		0.1805	0.0811				0.1018	0.1670	1.6347	0.4956	0.3811	0.3731	0.2322		0.3712				
10		0.0825	0.0458	0.0888				0.0845	0.0840	1.2704	0.1836	0.0911		0.0842	0.1057	0.0720			
11											1.4262					0.1283	0.0522		
12										0.1742	0.1213	1.6886	0.2085	0.1029			0.0784		0.1418
13									0.0653				1.0727						
14		0.1070	0.0611	0.1163			0.1068	0.1100	0.1175					1.0930					
15																			
16				0.1044	0.0832	0.0856	0.0979	0.1184	0.1165	0.1055				0.0899		1.1657			
17					0.1294		0.1181	0.1214	0.1221	0.1309	0.1270	0.1490		0.0969			1.1064		0.1194
18																0.0937	0.1013	1.1290	
19					0.0791		0.0859		0.0865			0.1087					0.1052	0.0919	1.1109

Table 12 Selected Matrices of direct input in 1997

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.1658	0.0486			0.2087		0.0408										0.0464		
2								0.0506						0.1776					
3						0.5800	0.0246												
4				0.0993			0.0171	0.0693	0.0778										
5	0.0585				0.3015		0.0653	0.0665				0.0242	0.0865				0.1104		0.0562
6						0.0437	0.0206		0.0273					0.0474		0.0839			
7	0.0728	0.0353	0.0268	0.0773	0.0629		0.3642	0.0692	0.0215	0.0394	0.0575	0.1182	0.0825						0.0567
8								0.0927	0.0167			0.0246			0.1673				
9		0.0697		0.0274			0.0132	0.0616	0.3251	0.1943	0.1515	0.1646	0.0824		0.1463				
10		0.0484	0.0335	0.0685			0.0140	0.0341	0.0301	0.1753	0.0854	0.0291		0.0418					
11											0.2525					0.0522	0.0324		
12		0.0292							0.0097	0.0590	0.0253	0.2946	0.0571	0.0374	0.0510		0.0318		
13									0.0355				0.0481						
14		0.0573		0.0692			0.0455	0.0526	0.0526					0.0625					
15																			
16				0.0744			0.0254	0.0528	0.0362							0.0606			
17				0.0465	0.0594		0.0487	0.0680	0.0377	0.0351		0.0518		0.0477	0.0703		0.0758		
18									0.0345								0.0663	0.1409	
19									0.0244										

Table 13 Selected Matrices of Leontief Inverse in 1997

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	1.2474	0.0959			0.4045		0.1484										0.1259		
2								0.0925						0.2128					
3						0.6346	0.0819												
4				1.1301			0.0430	0.1081	0.1442										
5	0.1399				1.5265		0.2144	0.1858				0.1523	0.2030				0.2204		0.1356
6						1.0774	0.0624		0.0788					0.0785		0.1070			
7	0.1753	0.1297	0.0761	0.2116	0.2312		1.6628	0.2169	0.1471	0.1713	0.2135	0.3604	0.2267						0.1510
8								1.1169	0.0407			0.0577			0.2016				
9		0.1778		0.1371			0.0951	0.1856	1.5755	0.4325	0.4066	0.4313	0.1986		0.3227				
10		0.0947	0.0577	0.1356			0.0667	0.0987	0.1062	1.2644	0.1852	0.1063		0.0970					
11											1.3621					0.0846	0.0598		
12		0.0759							0.0677	0.1385	0.0940	1.4651	0.1134	0.0966	0.1165		0.0725		
13									0.0721				1.0678						
14		0.0944		0.1196			0.1087	0.1122	0.1267					1.1062					
15																			
16				0.1277			0.0802	0.1124	0.1062							1.0870			
17				0.1165	0.1418		0.1395	0.1525	0.1261	0.1149		0.1629		0.1075	0.1582		1.1309		
18									0.1031								0.1079	1.1801	
19									0.0757										

Table 14 Selected Matrices of direct input in 1992

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	0.1120				0.2405		0.0563										0.0362		
2									0.0246					0.1397					
3			0.0832			0.7585	0.0409							0.0682					
4				0.0900				0.0464	0.0712										
5	0.0416				0.2824		0.0901	0.1054	0.0238				0.2699				0.1190	0.0250	
6				0.0414			0.0214	0.0640	0.0266					0.0491		0.2073			
7	0.0782			0.0704	0.0407		0.2673	0.0657	0.0181	0.0365	0.0749	0.0949	0.0812				0.0158		0.0564
8								0.1031	0.0171						0.1067			0.0371	
9		0.0348					0.0161	0.0549	0.3952	0.2208	0.1064	0.1737	0.0725		0.1204		0.0119		
10		0.0428	0.0197	0.0470				0.0319	0.0217	0.1806	0.1094	0.0425							
11											0.1881					0.0492			
12										0.0692	0.0285	0.2297							
13													0.0936						
14				0.0831			0.0374	0.0801	0.0497										
15																			
16							0.0255		0.0241								0.1512		
17		0.0666		0.0857	0.0557	0.0583	0.0848	0.1299	0.1072	0.1288	0.1502	0.1678	0.1253	0.0467		0.0607	0.0350	0.0664	0.0902
18					0.0295		0.0736	0.0654	0.0911	0.0877	0.0746	0.1084					0.0235	0.1218	
19																	0.0282	0.0342	

Table 15 Selected Matrices of Leontief Inverse in 1992

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	1.1667				0.4191		0.1752										0.1146		
2									0.0793					0.1645					
3			1.1080			0.8744	0.1329							0.1565					
4				1.1183				0.0815	0.1461										
5	0.1044				1.4816		0.2618	0.2981	0.1851				0.5474				0.2271	0.0969	
6				0.0944			0.0714	0.1385	0.1061					0.0846		0.2398			
7	0.1402			0.1626	0.1466		1.4330	0.1865	0.1206	0.1609	0.2151	0.2638	0.2254				0.0741		0.1181
8								1.1479	0.0652						0.1397			0.0577	
9		0.1253					0.0940	0.1959	1.7472	0.5570	0.3614	0.4854	0.2176		0.2673		0.0587		
10		0.0765	0.0329	0.0923				0.0865	0.0840	1.2746	0.2049	0.1168							
11											1.2529					0.0694			
12										0.1363	0.0818	1.3316							
13													1.1256						
14				0.1299			0.0830	0.1441	0.1343										
15																			
16							0.0935		0.1259								0.1875		
17		0.1557		0.2058	0.1422	0.1206	0.2136	0.2996	0.3160	0.3586	0.3681	0.4205	0.2902	0.1218		0.1471	1.1093	0.1314	0.1601
18					0.0926		0.1725	0.1808	0.2498	0.2603	0.2307	0.2981					0.0726	1.1729	
19																	0.0422	0.0499	

Table 16 The incidence matrix of 2002

λ	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19
Y1	1	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	1	0	0
Y2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Y3	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Y4	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
Y5	1	0	0	0	1	0	1	1	0	0	0	1	1	0	0	0	1	0	1
Y6	0	0	0	1	0	1	1	0	1	0	0	0	0	1	0	1	0	0	0
Y7	1	0	0	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	1
Y8	0	0	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0
Y9	0	1	1	0	0	0	1	1	1	1	1	1	1	0	1	0	0	0	0
Y10	0	1	1	1	0	0	0	1	1	1	1	1	0	1	1	1	0	0	0
Y11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0
Y12	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	1	0	1
Y13	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
Y14	0	1	1	1	0	0	1	1	1	0	0	0	0	1	0	0	0	0	0
Y15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y16	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	1	0	0	0
Y17	0	0	0	0	1	0	1	1	1	1	1	1	0	1	0	0	1	0	1
Y18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
Y19	0	0	0	0	1	0	1	0	1	0	0	1	0	0	0	0	1	1	1

Table 17 q-connectivities in $K_Y(X;\lambda)$, from $\Lambda\Lambda^T$ 2002

Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	Y13	Y14	Y15	Y16	Y17	Y18	Y19	
4	—	—	—	3	0	2	0	1	0	0	0	—	0	—	1	2	0	2	**Y1**
	0	—	—	—	0	—	—	—	0	—	0	—	0	—	0	0	—	—	**Y2**
		0	—	—	0	—	—	—	—	—	—	—	—	—	0	—	—	—	**Y3**
			2	0	1	2	1	1	2	—	—	0	2	—	2	1	—	0	**Y4**
				7	0	6	1	3	1	0	3	0	1	—	2	5	0	4	**Y5**
					5	2	0	1	3	0	0	0	3	—	5	2	0	1	**Y6**
						10	2	6	5	0	4	1	3	—	5	7	—	4	**Y7**
							3	3	3	—	0	0	1	—	1	2	—	1	**Y8**
								9	7	—	3	1	4	—	3	5	—	2	**Y9**
									10	1	3	0	5	—	5	5	0	1	**Y10**
										2	1	—	—	—	0	1	1	0	**Y11**
											6	0	0	—	1	5	0	2	**Y12**
												1	0	—	0	0	—	0	**Y13**
													6	—	4	3	—	1	**Y14**
														—	—	—	—	—	**Y15**
															8	5	0	2	**Y16**
																9	0	5	**Y17**
																	2	1	**Y18**
																		6	**Y19**

Table 18 q-connectivities in $K_X(Y;\lambda^{-1})$, from $\Lambda^T\Lambda$ 2002

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	
2	—	—	0	2	—	2	1	0	0	0	1	1	—	0	—	1	—	1	**X1**
	2	2	1	—	—	1	2	2	1	1	1	0	1	1	0	—	—	—	**X2**
		2	1	—	—	1	2	2	1	1	1	0	1	1	0	—	—	—	**X3**
			5	1	1	3	4	5	2	1	1	0	3	0	2	—	—	0	**X4**
				5	0	5	3	3	2	1	3	1	1	0	0	3	0	3	**X5**
					2	1	0	1	0	—	—	—	1	—	1	—	—	—	**X6**
						8	5	6	3	2	4	2	3	1	1	3	0	3	**X7**
							8	7	4	3	5	2	3	2	1	1	—	2	**X8**
								10	4	3	5	2	4	2	2	1	0	2	**X9**
									5	4	4	2	3	1	1	1	—	2	**X10**
										5	4	2	2	1	1	2	—	2	**X11**
											7	3	2	2	0	3	0	4	**X12**
												4	0	0	—	1	—	2	**X13**
													6	0	2	1	—	1	**X14**
														3	0	0	—	—	**X15**
															4	1	0	—	**X16**
																6	1	3	**X17**
																	1	0	**X18**
																		4	**X19**

Referring to Table 17 and Table 18 we can obtain the Q-analysis for both $K_Y(X;\lambda)$ and $K_X(Y;\lambda^{-1})$ as the following set of equivalence classes:

$K_Y(X;\lambda)$

q **-value**	Q_q **-value**	**Components**
10	2	{7},{10}
9	4	{7},{10},{9},{17}
8	5	{7},{10},{9},{17},{16}
7	4	{7,17},{10,9},{16},{5}
6	5	{7,17,5,9,10},{16},{12},{14},{19}
5	1	{7,17,5,9,10,16,12,14,6,19}
4	2	{7,17,5,9,10,16,12,14,6,19},{1}
3	1	{7,17,5,9,10,16,12,14,6,19,1,8}
2	3	{7,17,5,9,10,16,12,14,6,19,1,8,4},{11},{18}
1	1	{7,17,5,9,10,16,12,14,6,19,1,8,4,11,18,13}
0	1	{7,17,5,9,10,16,12,14,6,19,1,8,4,11,18,13,2,3}

The structure vector is

$$Q = \{\overset{10}{2} \quad 4 \quad 5 \quad 4 \quad 5 \quad 1 \quad 2 \quad 1 \quad 3 \quad 1 \quad \overset{0}{1}\}$$

$K_X(Y;\lambda^{-1})$

q -value	Q_q -value	Components
10	1	{9}
9	1	{9}
8	3	{9},{7},{8}
7	3	{9,8},{7},{12}
6	4	{9,8,7},{12},{14},{17}
5	5	{9,8,7,12,4,5},{14},{17},{10},{11}
4	4	{9,8,7,12,4,5,10,14,11,19},{17},{13},{16}
3	3	{9,8,7,12,4,5,10,14,11,19,17,13},{16},{15}
2	2	{9,8,7,12,4,5,10,14,11,19,17,13,2,3,15,16,1},{6}
1	1	{all}
0	1	{all}

The structure vector is

$$Q = \{\overset{10}{1} \quad 1 \quad 3 \quad 3 \quad 4 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad \overset{0}{1}\}$$

Table 19 The incidence matrix of 1997

λ	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19
Y1	1	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
Y2	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
Y3	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
Y4	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
Y5	1	0	0	0	1	0	1	1	0	0	0	1	1	0	0	0	1	0	1
Y6	0	0	0	0	0	1	1	0	1	0	0	0	0	1	0	1	0	0	0
Y7	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	1
Y8	0	0	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0
Y9	0	1	0	1	0	0	1	1	1	1	1	1	1	0	1	0	0	0	0
Y10	0	1	1	1	0	0	1	1	1	1	1	1	0	1	0	0	0	0	0
Y11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0
Y12	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	0	1	0	0
Y13	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
Y14	0	1	0	1	0	0	1	1	1	0	0	0	0	1	0	0	0	0	0
Y15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y16	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	0	0
Y17	0	0	0	1	1	0	1	1	1	1	0	1	0	1	1	0	1	0	0
Y18	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0
Y19	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0

Table 20 q-connectivities in $K_Y(X;\lambda)$, from $\Lambda\Lambda^T$ 1997

Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	Y13	Y14	Y15	Y16	Y17	Y18	Y19	
4	—	0	0	3	0	3	—	1	1	0	1	—	1	—	0	2	0	—	**Y1**
	1	—	0	0	0	0	0	0	1	—	0	—	1	—	0	1	—	—	**Y2**
		1	0	0	1	0	—	0	0	—	—	—	0	—	0	0	—	—	**Y3**
			3	1	1	3	1	3	3	—	0	0	3	—	3	3	0	0	**Y4**
				7	0	6	1	3	2	0	2	0	1	—	1	4	0	—	**Y5**
					4	1	0	1	2	0	1	0	2	—	2	2	0	0	**Y6**
						12	2	8	8	0	5	1	4	—	3	6	0	0	**Y7**
							3	3	2	—	2	0	1	—	1	3	0	0	**Y8**
								9	7	0	6	1	4	—	3	6	0	0	**Y9**
									9	0	5	0	5	—	3	6	0	0	**Y10**
										2	1	—	—	—	0	0	0	—	**Y11**
											8	1	2	—	0	5	1	0	**Y12**
												1	0	—	0	0	0	0	**Y13**
													5	—	3	4	0	0	**Y14**
														—	—	—	—	—	**Y15**
															4	3	0	0	**Y16**
																9	1	0	**Y17**
																	2	0	**Y18**
																		0	**Y19**

Table 21 q-connectivities in $K_X(Y;\lambda^{-1})$, from $\Lambda^T\Lambda$ 1997

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	
2	1	0	0	2	—	2	1	0	0	0	1	1	—	—	—	1	—	1	**X1**
	5	1	3	1	—	4	3	4	3	3	3	2	2	1	—	1	—	0	**X2**
		1	1	0	—	1	1	1	1	1	1	0	0	—	—	—	—	0	**X3**
			6	1	—	6	6	6	3	2	3	1	2	1	0	0	—	0	**X4**
				3	—	3	2	1	1	0	2	1	0	0	—	2	—	1	**X5**
					1	1	—	0	—	—	—	—	0	—	0	—	—		**X6**
						10	7	7	3	2	4	2	3	1	1	2	—	1	**X7**
							9	7	3	2	5	2	3	2	0	1	—	1	**X8**
								12	4	3	5	3	4	3	1	2	0	0	**X9**
									4	3	4	2	2	2		1	—	0	**X10**
										4	3	2	1	1	0	1	—	0	**X11**
											6	3	2	3	—	2	—	1	**X12**
												4	0	1	—	1	—	1	**X13**
													5	1	0	1	—	—	**X14**
														3	—	1	—	—	**X15**
															2	0	—	—	**X16**
																5	0	0	**X17**
																	0	—	**X18**
																		1	**X19**

Referring to Table 20 and Table 21 we can obtain the Q-analysis for both $K_Y(X;\lambda)$ and $K_X(Y;\lambda^{-1})$ as the following set of equivalence classes:

$K_Y(X;\lambda)$

q **-value**	Q_q **-value**	**Components**
12	1	{7}
11	1	{7}
10	1	{7}
9	4	{7},{9},{10},{17}
8	3	{7,9,10},{17},{12}
7	4	{7,9,10},{17},{12},{5}
6	1	{7,9,10,5,17,12}
5	1	{7,9,10,5,17,12,14}
4	4	{7,9,10,5,17,12,14},{16},{6},{1}
3	2	{7,9,10,5,17,12,14,1,4,16,8},{6}
2	1	{7,9,10,5,17,12,14,1,4,16,8,6},{11},{18}
1	1	{7,9,10,5,17,12,14,1,4,16,8,6,2,3,11,13,18}
0	1	{7,9,10,5,17,12,14,1,4,16,8,6,2,3,11,13,18,19}

The structure vector is

$$Q = \{\overset{12}{1} \quad 1 \quad 1 \quad 4 \quad 3 \quad 4 \quad 1 \quad 1 \quad 4 \quad 2 \quad 1 \quad 1 \quad \overset{0}{1}\}$$

$K_X(Y;\lambda^{-1})$

q **-value**	Q_q **-value**	**Components**
12	1	{9}
11	1	{9}
10	2	{9},{7}
9	3	{9},{7},{8}
8	3	{9},{7},{8}
7	1	{9,7,8}
6	2	{9,7,8,4},{12}
5	4	{9,7,8,4,12},{2},{14},{17}
4	4	{9,7,8,4,12,2,10,14},{17},{11},{13}
3	2	{9,7,8,4,12,2,10,14,11,13,15,5},{17}
2	2	{9,7,8,4,12,2,10,14,11,13,15,5,17,1,7},{16}
1	1	{9,7,8,4,12,2,10,14,11,13,15,5,17,1,7,16,19}
0	1	{all }

The structure vector is

$$Q=\{\overset{12}{1}\quad 1\quad 2\quad 3\quad 3\quad 1\quad 2\quad 4\quad 4\quad 2\quad 2\quad 1\quad \overset{0}{1}\}$$

Table 22 The incidence matrix of 1992

λ	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19
Y1	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
Y2	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
Y3	0	0	1	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0
Y4	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
Y5	1	0	0	0	1	0	1	1	1	0	0	0	1	0	0	0	1	1	0
Y6	0	0	0	1	0	0	1	1	1	0	0	0	0	1	0	1	0	0	0
Y7	1	0	0	1	1	0	1	1	1	1	1	1	1	0	0	0	1	0	1
Y8	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0
Y9	0	1	0	0	0	0	1	1	1	1	1	1	1	0	1	0	1	0	0
Y10	0	1	1	1	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
Y11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0
Y12	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
Y13	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Y14	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
Y15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Y16	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0
Y17	0	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
Y18	0	0	0	0	1	0	1	1	1	1	1	1	0	0	0	0	1	1	0
Y19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0

Table 23 q-connectivities in $K_Y(X;\lambda)$, from $\Lambda\Lambda^T$ 1992

Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	Y13	Y14	Y15	Y16	Y17	Y18	Y19	
3	—	0	—	3	0	3	—	1	—	—	—	—	0	—	1	2	2	0	**Y1**
	1	0	0	0	1	0	0	0	0	—	—	—	0	—	0	1	0	—	**Y2**
		3	—	0	1	0	—	0	0	—	—	—	0	—	0	2	0	—	**Y3**
			2	1	2	2	1	1	2	—	—	—	2	—	0	2	1	—	**Y4**
				7	2	6	2	4	1	—	—	0	2	—	2	6	5	1	**Y5**
					5	3	1	2	2	0	—	—	3	—	1	5	2	—	**Y6**
						11	1	7	5	0	2	0	3	—	2	10	7	0	**Y7**
							3	2	1	—	—	—	1	—	0	2	2	0	**Y8**
								9	5	0	2	0	2	—	2	8	6	0	**Y9**
									7	0	2	—	2	—	0	6	4	—	**Y10**
										1	0	—	—	—	—	1	0	—	**Y11**
											2	—	—	—	—	2	2	—	**Y12**
												0	—	—	—	0	—	—	**Y13**
													3	—	1	3	2	—	**Y14**
														—	—	—	—	—	**Y15**
															2	2	2	0	**Y16**
																15	8	1	**Y17**
																	8	1	**Y18**
																		1	**Y19**

Table 24 q-connectivities in $K_X(Y;\lambda^{-1})$, from $\Lambda^T\Lambda$ 1992

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	
2	—	—	0	2	—	2	1	1	0	0	0	1	—	—	—	2	0	0	**X1**
	2	0	1	0	0	1	2	2	2	2	2	1	0	0	0	1	0	0	**X2**
		1	0	—	0	0	0	0	0	0	0	—	0	—	—	—	—	—	**X3**
			5	1	0	3	5	5	2	2	2	1	1	—	1	1	0	1	**X4**
				4	0	4	3	3	2	2	2	2	0	—	0	4	2	1	**X5**
					1	1	0	0	0	0	0	0	1	—	0	0	0	0	**X6**
						9	6	7	3	3	3	3	2	0	1	6	2	1	**X7**
							9	9	4	4	4	3	1	1	1	4	3	1	**X8**
								11	4	4	4	3	2	1	1	5	3	1	**X9**
									5	5	5	2	0	0	0	3	1	1	**X10**
										6	5	2	0	0	1	3	1	1	**X11**
											5	2	0	0	0	3	1	1	**X12**
												4	0	0	0	3	1	1	**X13**
													3	—	1	0	0	0	**X14**
														1	—	0	0	—	**X15**
															2	0	0	0	**X16**
																7	3	1	**X17**
																	4	0	**X18**
																		1	**X19**

Referring to Table 23 and Table 24 we can obtain the Q-analysis for both $K_Y(X;\lambda)$ and $K_X(Y;\lambda^{-1})$ as the following set of equivalence classes:

$K_Y(X;\lambda)$

q **-value**	Q_q **-value**	**Components**
15	1	{17}
11	2	{17},{7}
10	1	{17,7}
9	2	{17,7},{9}
8	1	{17,7,9,18}
7	3	{17,7,9,18},{10},{5}
6	1	{17,7,9,18,10,5}
5	1	{17,7,9,18,10,5,6}
4	1	{17,7,9,18,10,5,6}
3	3	{17,7,9,18,10,5,6,14,1},{3},{8}
2	1	{17,7,9,18,10,5,6,14,1,3,8,4,12,16}
1	1	{17,7,9,18,10,5,6,14,1,3,8,4,12,16,2,11,19}
0	1	{17,7,9,18,10,5,6,14,1,3,8,4,12,16,2,11,19,13}

The structure vector is

$$Q = \{\overset{15}{1}\quad \overset{11}{2}\quad 1\quad 2\quad 1\quad 3\quad 1\quad 1\quad 1\quad 3\quad 1\quad 1\quad \overset{0}{1}\}$$

$K_X(Y;\lambda^{-1})$

q **-value**	Q_q **-value**	**Components**
11	1	{9}
10	1	{9}
9	2	{9,8};{7}
8	2	{9,8};{7}
7	2	{9,8,7};{17}
6	2	{9,8,7,17};{11}
5	2	{9,8,7,17,4};{11,10,12}
4	3	{9,8,7,17,4,11,10,12,5};{13};{18}
3	2	{9,8,7,17,4,11,10,12,5,13,18};{14}
2	2	{9,8,7,17,4,11,10,12,5,13,18,2,14,1};{16}
1	2	{9,8,7,17,4,11,10,12,5,13,18,2,14,1,15,16,19,6};{3}
0	1	{all}

The structure vector is

$$Q = \{\overset{11}{1}\quad 1\quad 2\quad 2\quad 2\quad 2\quad 2\quad 3\quad 2\quad 2\quad 2\quad \overset{0}{1}\}$$

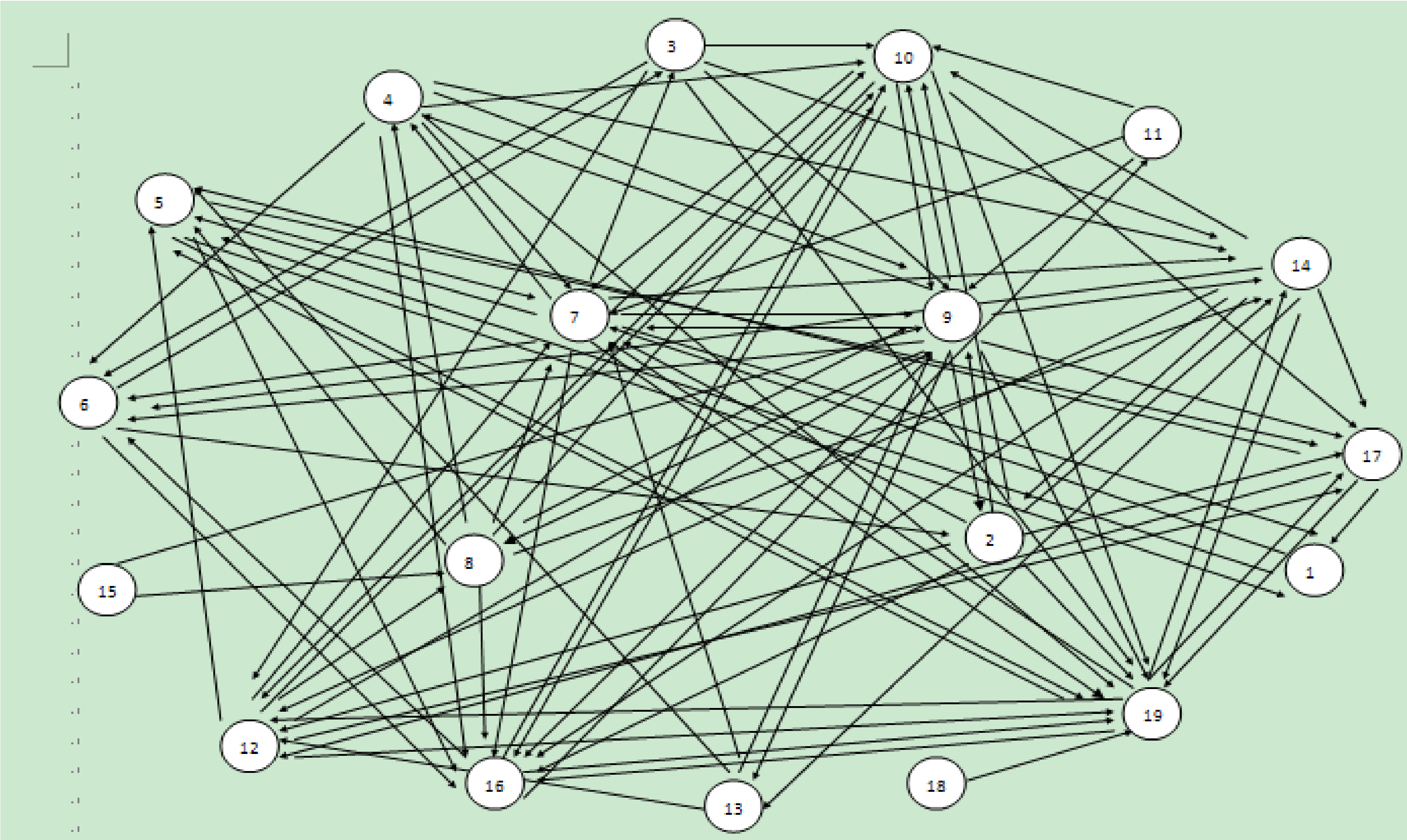

Figure 3 The backward linkage between sectors in 2005

Figure 4 The forward linkage between sectors in 2005