Accounting foundations for interregional commodity-by-industry input-output models*

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Abstract. Several procedures for generating interregional commodity flow matrices have been developed in the US recent years (see, e.g., Canning and Wang 2005, Jackson et al. 2006, Lindall, Olsen and Alward 2006). Despite the fact that these methods derive from the commodity-by-industry framework, very little attention has been given recently to the fundamental conceptual issues that must be confronted to generate a consistently defined interregional model or to conduct an interregional impacts assessment using an appropriate interregional framework. This paper revives the focus on interregional modeling issues initiated by Oosterhaven (1984), identifies and elaborates on these and additional issues, and traces the development of the accounting foundations from single-region inter-industry through interregional commodity-by-industry accounts. Its contribution lies in the provision of a high-level perspective on these frameworks that in the process both clarifies and simplifies key conceptual issues and operational decisions.

1. Introduction

Regional and interregional input-output (IO) models have long occupied central positions in regional science research. From its inception, IO modeling at the regional level has been dominated by a focus on industry-based analysis. This has been the case especially in the United States, despite the 1972 shift from industry-based to commodity-by-industry-based data reporting practices at the national level. The understandable reluctance to shift emphasis on the part of regional analysts is due in large part to the preponderance of industry-based data on national and regional employment, income, hours worked, etc., and the paucity of similar commodity-based data. Nevertheless, analysts constructing regional IO tables rarely if ever rely on primary data, resorting instead to regionalizing national accounts via one of a number of methods. Hence, unless one purchases IO data from a commercial source, or works with BEA's multiplier-only RIMS data, working with the national industry and commodity data has become a practical necessity for regional IO modeling.

One option in dealing with the national commodity-by-industry accounts is to first assume either commodity- or industry-based technology and construct a national industry by industry table from the Make and Use tables, then regionalize using industry-based regional data and a location quotient, supply-demand pool, regional purchase coefficient, GRIT, or similar method. There is ample treatment of such methods in the literature (Kuehn, Procter and Braschler 1985, West 1990, Stevens et al. 1983, Stevens et al. 1988). An alternative is to use region-specific data to generate regionalized versions of the national Make and Use tables, then construct the desired commodity-by-industry, industry-by-industry, or other single region account format based on either the commodity or industry technology assumption. Jackson (1998) and Lahr (2001) have addressed this latter approach to regional accounts construction. In a US interregional context, Canning and Wang (2005) presented a method for generating interregional IO data, Jackson et al (2006) described an approach to estimating interregional commodity flows, Lindall et al. (2006) discussed multi-region models in the IMPLAN framework, and Schwarm et al. (2006) and Robinson and Liu (2006) provided comparisons of the results of selected techniques to published flow data and to one another. Yet no works to date focus directly on the conceptual implications of modeling decisions and assumptions in the context of the interregional IO and the commodity-by-industry format of the U.S. national benchmark accounts (U.S. Department of Commerce, 2007).

Perhaps somewhat curiously, with the notable exception of Oosterhaven (1984), an enumeration and elaboration of an approach to constructing *interregional* IO accounts from the commodity-by-industry foundation framework and basic interregional commodity flow data is lacking from the literature. Oosterhaven's development of a family of square and rectangular models identified and addressed a large number of issues that also surface in this paper, and provided some very specific and highly detailed approaches to interregional model implementation. It is perhaps because of the low-level (e.g., detailed) approach taken in his paper that it has received less than its due attention. Indeed, the recently published Second Edition of Miller and Blair describes interregional models only in industry-by-industry settings, and states in a chapter footnote that "In a commodity-by-industry accounting setting, one would deal with Use matrices" (2009, pg 375), implying that this is a simple, straightforward and transparent switch. To our knowledge, however, there is little in the literature that provides a concise, comprehensive high-level presentation of relevant accounts and their interrelationships for analysts constructing such models.

The purpose of this letter, therefore, is to provide an explicit treatment of national commodity-by-industry data in the construction and use of interregional IO models. Rather than focus on methods for estimating the interregional interaction, *per se,* this paper focuses attempts a clear and concise description of organization of basic data for interregional commodity-by-industry settings.[1]

2. National models, regional models, and extensions of single-region assumptions for interregional models

To lay necessary groundwork, we first very briefly revisit the historically conventional national inter-industry IO framework, focusing only on aspects critical to the subsequent discussion and assuming reader familiarity with conventional IO modeling notation.

[1] Space constraints prohibit a point-by-point comparison with Oosterhaven (1984). There is considerable but not complete overlap in content, but the two papers have much less in presentation-level comparability and therefore broader purpose.

The first of these concerns the accounting of imports in terms of data organization. In the familiar $X - AX = Y$, the RHS term is final demand, modified by negative values corresponding to imports. Matrix A thus comprises inter-industry technical coefficients. However, when moving to a subnational regional context, a matrix of trade coefficients, R, is estimated such that $a_{ij} = r_{ij} + m_{ij}$, where the coefficients m_{ij} are import coefficients. Since matrix R now accounts explicitly for imports, final demands need not be purged of imports to retain the output balance equations. Table 1 represents the shift in accounting framework diagrammatically.

National Framework

	Industries	Final Demand with Imports
Industries		
Value Added		

Regional Framework

	Industries	Final Demand no Imports
Industries		
Value Added		
Imports		

Table 1. National and regional accounting frameworks

The shift to a commodity-by-industry accounting framework follows an analogous pattern. The conventional framework shown in Table 2 is the one used by most statistical reporting agencies worldwide, with the domestic (F), export (x), and import (m) components of commodity final demand shown explicitly. Matrices U, V, W, and E are Use, Make, Value Added and Final Demand, respectively, and q and g are commodity and industry total output, and the prime symbol denotes transpose. The Use matrix depicts column industry use (purchases) of each row commodity; the Make matrix depicts the column commodity output of each row industry; value added includes all payments sectors; Final Demand depicts row commodity final demand by column final demand activity, such as consumption, investment, government expenditures, and exports (which are zero in a closed system). For simplicity, we will assume in the discussion that follows that a) final demand columns have been aggregated to a single column, likewise that b) the rows of W have been aggregated to a single row, and c) the number of commodities is equal to the number of industries.

	Commodity	Industry	Final Demand	Total
Commodity		U	$E = F \mid x \mid (\text{-}m)$	q
Industry	V			g
Value Added		W		
Total	q'	g'		

Table 2. Conventional commodity-by-industry framework (Source: Jackson 1998)

This framework, and indeed the model solution equations that derive from this organizational data structure (e.g., Miller and Blair 1985, 2009), corresponds conceptually to the national framework in Table 1, in which the final demand partition includes negative elements corresponding to commodity imports. The foundation for a regionalized commodity-by-industry framework developed by Jackson (1998) is presented in Table 3, with imports shown as a commodity input source.

	Commodity	Industry	Final Demand	Total
Commodity		U	$F \mid x$	$s = q + m$
Industry	V			g
Value Added		W		
Output	q'	g'		
Imports	m'			
Total	s'			

Table 3. Commodity-by-industry framework with imports as a commodities source (Source: Jackson 1998)

In the framework of Table 3, the Use matrix corresponds to technical relationships, and the final demands are those that stem from local sources and from export demand. The commodity row sums now equal total commodities used (domestically and for exports), and are equal to total regional commodity supply, s. Total regional commodity supply also equals the commodity column sums. One can then generate an inter-industry counterpart to R, for example, by using $\tilde{D}B = V\hat{s}^{-1}U\hat{g}^{-1}$ rather than the standard $DB = V\hat{q}^{-1}U\hat{g}^{-1}$. The effect of D in the latter is to reallocate commodities used by industries to the industries that produced them irrespective of geographic origin. The effect of $\tilde{D}$ in the former is to reallocate commodities used by industries to the industries that produced them by respective domestic or rest-of-world (import) sources.[2] However, Table 3 does not correspond directly to the regional framework of Table 1. To approximate that framework requires additional reorganization of the data as shown in Table 4. Here, both *U* and final demand in the Commodity row are purged of imports, which now appear in the 'Imports (a)' row (or row partition if commodity detail is retained) of industry use (the *m* suffix on *U* or *V* is used to distinguish between Use-comparable and Make-compatible imports. Computing $DB = V\hat{q}^{-1}U\hat{g}^{-1}$ from the accounts in Table 4 would now generate a counterpart to the regional trade coefficients matrix, *R*.

	Commodity	Industry	Final Demand	Total
Commodity		U	$F \mid x$	$q = s - m$
Industry	V			g
Value Added		W		
Imports (a)		Um		
Total Output	q'	g'		
Imports (b)	m'			
Total Supply	s'			

Table 4. Regional commodity-by-industry framework

2.1 Many-Region IO

Two approaches to handling many-region models are well entrenched in the literature. The first is the interregional model (Isard 1951), in which there is a complete enumeration of all flows among all sectors.[3] In the IRIO, the coefficients are regional trade coefficients, not regional technical coefficients. The second approach to many-region models is the multiregional IO model, or MRIO. Often called the Chenery-Moses model, this formulation is attributed to Chenery (1953) and Moses (1955), who developed essentially the same structure independently.

[2] Although there has been a great deal of debate in the literature concerning the appropriateness of one versus the other technology assumption (see inter alia, de Mesnard 2004), we will not engage in such debate here, though what follows may eventually contribute to the basis for that discussion. In this paper, we arbitrarily adopt the industry-based technology assumption, and further, use only the industry-by-industry form of the possible solutions. A parallel presentation of the issues below using the commodity-based assumption is a straightforward exercise, and will be presented elsewhere.

[3] There are numerous sources describing most of the established frameworks referenced in this paper.

The MRIO approach begins with a set of regional *technical* coefficients tables as the basic building blocks, as opposed to the regional input coefficients tables of the IRIO. To take advantage of the kinds of data likely to be available, a set of trade tables is developed by first estimating trade flows by region, then ascribing the general flow relationships to individual industries. Of most relevance to the current discussion is that the final demand vectors in the IRIO and MRIO specifications also are not identical. For the IRIO approach, region-specific final demand is explicitly identified, while in the MRIO approach, final demand for each region's output is determined by using the trade tables to allocate final demands to regions of origin.[4]

2.2 Commodity-by-industry interregional issues

The construction of interregional IO models most commonly involves the combination of interregional commodity flow (trade) data with production accounts in the form of commodity-by-industry frameworks. Methods are devised to merge the information in the two datasets. The most fundamental issue to be addressed in making the accounting framework transition to a many-region model is that neither the Make nor the Use tables, in their conventional single-region formats, nor indeed any aspect of the production accounts beyond mere imports and exports, are geographically specific. Consider a rudimentary two-region, closed-system accounting framework that simply reflects the addition of a second region, as shown in Table 5.

	Commodity	Commodity	Industry	Industry	Final Demand	Total Output
Commodity			$U1$		$E1$	$q1$
Commodity				$U2$	$E2$	$q2$
Industry	$V1$					$g1$
Industry		$V2$				$g2$
Value Added			$W1$	$W2$		
Total Output	$q1'$	$q2'$	$g1'$	$g2'$		

Table 5. Rudimentary closed 2-region commodity-by-industry framework

While Table 5 is correct, in the sense that its row and column sums retain consistency, it provides very little information about the *interregional* flows of commodities, which is of course critically important, if not the *raison d'être,* for interregional modeling. If there were trade between these two regions, it would be embedded in the final demand entries, both exports and

[4] In conventional notation, CY in MRIO approximates Y in IRIO.

imports. Given the trade flow, not only by commodity but by industry of destination, we could begin to account for interregional flows by reallocating the interregional inter-industry portions of export final demands to the Use partitions of the receiving regions *and* removing them from the initial receiving regions Use tables, such that for example, $U_{12} + U_{22} = U_2$. The partitioned Use table effectively represents interregional commodity-by-industry flow. To retain consistency in the commodity output balance, final demand will include domestic final demand, exports to the other region, and (-) imports from the other region. In this example, exports from one region are final demand imports to the other. Momentarily looking past this configuration, the result will be the accounts presented in Table 6.

	Commodity	Commodity	Region 1 Industry	Region 2 Industry	Final Demand	Total
Region 1 Commodity			$U11$	$U12$	$F1 \mid x1$	$s1 = q1 + x2$
Region 2 Commodity			$U21$	$U22$	$F2 \mid x2$	$s2 = q2 + x1$
Region 1 Industry	$V1$					$g1$
Region 2 Industry		$V2$				$g2$
Value Added			$W1$	$W2$		
Total Output	$q'1$	$q'2$	$g'1$	$g'2$		
Imports	$x2'$	$x1'$				
Total Supply	$s1'$	$s2'$				

Table 6. Closed 2-region commodity-by-industry framework with Use origins and destinations

Since this formulation parallels that of Table 3, the Make and Use block matrices could be standardized by their column sums, and an interregional *R* computed as their *DB* product. This coefficients matrix would be appropriate in conjunction with final demands for each region's commodity output, as in the historical IRIO modeling framework.

In transforming the accounts in Table 5 to those in Table 6, we modified the Use tables from representations of technical relationships to representations of trade relationships. However, it would have been equally feasible to modify the Make data to correspond to industry of origin and destination of commodity. Adding destination-specific information to the Make table accounts would result in the framework shown in Table 7. In this framework, V_{11} and V_{12} now denote commodities produced by Region 1 industries that are available in Regions 1 and 2. Note that by adding origin and destination detail to the Make tables, the column partition-sum identities no longer hold, i.e., $s_1+s_2=q_1+q_2$, but s_1 and s_2 need not equal q_1 and q_2, respectively. The Use partitions are technical relationships, and final demands include imports. A column-standardized partitioned Make pre-multiplying the industry output-standardized partitioned-Use table will generate an interregional inter-industry trade coefficients table analogous to single region *R*.

	Region 1 Commodity	Region 2 Commodity	Region 1 Industry	Region 2 Industry	Final Demand	Total
Region 1 Commodity			$U1$		$F1 \mid x1$	$s1 = q1 + m1$
Region 2 Commodity				$U2$	$F2 \mid x2$	$s2 = q2 + m2$
Region 1 Industry	$V11$	$V12$				$g1$
Region 2 Industry	$V21$	$V22$				$g2$
Value Added			$W1$	$W2$		
Total Output	$s1$	$s2$	$g1'$	$g2'$		

Table 7. Closed 2-region commodity-by-industry framework with Make origins and destinations

Opening either system to the rest of the world is conceptually straightforward, although it will become clear that simpler changes are required to the geographically specific Make-based framework (Make-regionalized). The Use-regionalized framework of Table 6 opened to the rest of the world is shown in Table 8. The imports row in column partitions 1 and 2 represents commodities produced by the rest of the world, including the other region) available for use in each region. The imports row in column partitions 3 and 4 are the values of all imports available for use in production. Excluding imports from final demand and adding them to the row total output transforms regional output *q* into regional supply, *s*. The commodity row sums equal commodity output, and the relationships among supply and output are shown in the Total column.

	Region 1 Commodity	Region 2 Commodity	Region 1 Industry	Region 2 Industry	Final Demand	Total Output
Region 1 Commodity			$U11$	$U12$	$F1 \mid x1$	$q1 = s1$ - $Vm1$ - $x2$
Region 2 Commodity			$U21$	$U22$	$F2 \mid x2$	$q2 = s2$ - $Vm2$ - $x1$
Region 1 Industry	$V1$					$g1$
Region 2 Industry		$V2$				$g2$
Value Added			$W1$	$W2$		
Total Output	$q1$	$q2$				
Imports	$Vm1' + x2'$	$Vm2' + x1'$	$Um1$	$Um2$		
Total	$s1'$	$s2'$	$g1'$	$g2'$		

Table 8. Open 2-region commodity-by-industry framework with Use origins and destination

The first two (commodity) block rows of the Make-regionalized system in Table 9 report commodities used by intermediate and final demand in each region during the accounting period. Some of the commodities will come from outside each region, either from the other region or from the rest of the world. Final demands for commodities for the respective regions include a negative entry for imports, such that row sums equal total commodity output, *q*. The Make matrix will be standardized by total regional supply, resulting in a D matrix that reallocates commodity demand to geographically specific sources.

	Region 1 Commodity	Region 2 Commodity	Region 1 Industry	Region 2 Industry	Final Demand	Total
Region 1 Commodity			$U1$		$F1 \mid x1$	$s1 = q1 - Vm1$
Region 2 Commodity				$U2$	$F2 \mid x2$	$s2 = q2 - Vm2$
Region 1 Industry	$V11$	$V12$				$g1$
Region 2 Industry	$V21$	$V22$				$g2$
Value Added			$W1$	$W2$		
Total Output	$q1$	$q2$				
Imports	$Vm1$	$Vm2$				
Total Supply	$s1'$	$s2'$	$g1'$	$g2'$		

Table 9. Open 2-region commodity-by-industry framework with Make origins and destinations

3. Modeling Decisions.

Given these accounting systems, how does the development of modeling solutions parallel the single-region framework solutions? Again for the sake of simplicity in exposition, we focus here only on the industry-based technology assumption. To clarify the choice of approach, consider the following formulations.

Let

$$Vr = \begin{bmatrix} Vr_{11} & Vr_{12} \\ Vr_{21} & Vr_{22} \end{bmatrix}, \; S = \begin{bmatrix} s_1 & s_2 \end{bmatrix}, \; \tilde{D} = Vr\hat{S}^{-1}, \qquad (1)$$

$$V = \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix}, \; Q = \begin{bmatrix} q_1 & q_2 \end{bmatrix}, \; D = V\hat{Q}^{-1}, \qquad (2)$$

and let

$$U = \begin{bmatrix} U_1 & 0 \\ 0 & U_2 \end{bmatrix}, \; G = \begin{bmatrix} g_1 & g_2 \end{bmatrix}, \; B = U\hat{G}^{-1}, \qquad (3)$$

$$Ur = \begin{bmatrix} Ur_{11} & Ur_{12} \\ Ur_{21} & Ur_{22} \end{bmatrix}, \; G = \begin{bmatrix} g_1 & g_2 \end{bmatrix}, \; \tilde{B} = U\hat{G}^{-1}, \qquad (4)$$

The use of transformation matrices D and $\tilde{D}$ as pre-multipliers for commodity column vectors or for matrices with commodity rows transforms commodity space into industry space.

The use of the latter (D) will transform within blocks of rows corresponding to the partitions, while the former transformation matrix ($\tilde{D}$) can operate within and across blocks.

Specifically, the pre-multiplication by $\tilde{D}$ has the effect of allocating each commodity used to its industry and geographical source (including ROW sources for open systems). Hence, matrix $\tilde{D}B$ is an interregional inter-industry trade table whose transactions are derived by transforming aspatial production function specifications (B) using system-wide market shares ($\tilde{D}$). The partitioned matrix product is shown in Equation 5, below.

$$\begin{bmatrix} \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{D}_{21} & \tilde{D}_{22} \end{bmatrix} \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} = \begin{bmatrix} \tilde{D}_{11}B_1 & \tilde{D}_{12}B_2 \\ \tilde{D}_{21}B_1 & \tilde{D}_{22}B_2 \end{bmatrix} \qquad (5)$$

Matrix $\tilde{D}B$ is derived by transforming a spatially explicit production function using market shares specific to the region from which the commodity is sourced. The pre-multiplication by D has the effect of allocating each commodity supplied to a region to its source industry, based on the supplying region' own market share structure. Matrix $D\tilde{B}$ is thus an interregional inter-industry trade table whose transactions are derived by allocating commodities appearing in spatial production function specifications ($\tilde{B}$) using region-specific market shares (D). The partitioned matrix product is shown in Equation 6, below.

$$\begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} \tilde{B}_{11} & \tilde{B}_{12} \\ \tilde{B}_{21} & \tilde{B}_{22} \end{bmatrix} = \begin{bmatrix} D_1\tilde{B}_{11} & D_1\tilde{B}_{12} \\ D_2\tilde{B}_{21} & D_2\tilde{B}_{22} \end{bmatrix} \qquad (6)$$

Equation 7 below illustrates and accentuates the nonsensical nature of attempting to use both regionalized Make and Use matrices; each partition of the product matrix clearly includes irrational terms, such as $\tilde{B}_{21}$ in product matrix partition 1-1, or $\tilde{B}_{22}$ in product matrix partition 1-2.

$$\begin{bmatrix} \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{D}_{21} & \tilde{D}_{22} \end{bmatrix} \begin{bmatrix} \tilde{B}_{11} & \tilde{B}_{12} \\ \tilde{B}_{21} & \tilde{B}_{22} \end{bmatrix} = \begin{bmatrix} \tilde{D}_{11}\tilde{B}_{11} + \tilde{D}_{12}\tilde{B}_{21} & \tilde{D}_{11}\tilde{B}_{12} + \tilde{D}_{12}\tilde{B}_{22} \\ \tilde{D}_{21}\tilde{B}_{11} + \tilde{D}_{22}\tilde{B}_{21} & \tilde{D}_{21}\tilde{B}_{12} + \tilde{D}_{22}\tilde{B}_{22} \end{bmatrix} \quad (7)$$

Formulations using DB and $\tilde{D}\tilde{B}$ generate either undesired (effectively aspatial) or nonsensical results, narrowing the choice to one between the Use- and Make-regionalized formulations. The decision as to which of the formulations is appropriate should be made on conceptual and theoretical grounds, but also in recognition of potential data constraints. Again, we focus attention only on the industry technology assumption and industry-by-industry target dimensions. Additional issues will undoubtedly arise from the assumption of commodity technology or in the maths of alternative target dimensions, and these are left to others to develop. The protocol for doing so, however, is identical to the one we have laid out in this paper.

Given that IO models ultimately assume constant structure, the Use-regionalized formulation represents a system in which region-specific industrial production functions are the driving force behind the interregional frameworks generated. In a demand driven framework, it seems likely that establishments that have identified extra-regional sources of imports would indeed increase the size of their existing input orders according to increased production demands. The Use-regionalized system can thus be argued to correspond more closely to a demand rather than supply driven system. The Make-regionalized formulation, in contrast, implies a system in which increases in an industry's total output will result in proportional increases in each purchasing industry and region. Hence, it can be argued to more closely approximate a supply-driven system.

However, we also note that the use of the consolidated Make matrix applies the aggregate region-specific industrial commodity output distribution (irrespective of destination) to regional industry production used in all regions. For the two-region closed system example, this is of little consequence, but could potentially take on greater importance – and hence introduce more error – as the number of regions and corresponding intervening interregional distances – and therefore spatial variation in production – increase. It might also be the case empirically, for example, that a large portion of an industry's primary commodity output is exported great distances, while its secondary commodities are produced and sold to a more localized market.

Nevertheless, from the standpoint of rational economic behavior, the relationships in the Use-regionalized framework rest on the foundation of production and demand relationships and support it over the alternative.

Partly countermanding the conceptual advantages of the Use-regionalized framework are practical considerations. The Use-regionalized framework requires more extensive data, including an imports matrix,[5], for which additional assumptions and modeling mechanisms may have to be introduced for allocating these across sub-national regions.

4. Summary

This letter has provided a high-level perspective on frameworks underlying many-region IO models founded on commodity-by-industry data. The workable options are identified by beginning with conventional single-region inter-industry frameworks and extending to many region commodity-by-industry frameworks. The two choices are mutually exclusive since regionalized Make and Use data cannot be used in the same interregional modeling formulation.

This discussion supports the Use-regionalized approach, provided that the necessary supporting data are available, and that suitable mechanisms can be identified for allocating national industry imports to subnational regions. The preference is based on the foundation of production behavior consistent with the demand-driven IO model rather than market share behavior, which appears to be more consistent with a supply-driven IO model. The paper identifies a broad set of relevant issues and implications of alternative approaches to the construction of interregional models, some of which are underscore those introduced in Oosterhaven (1984),[6] and provides an initial set of mechanisms and protocol for moving forward.

[5] Dietzenbacher, Albino and Kuhtz 2005 critique the use of US-type Make-Use systems with embedded imports. Note that their criticisms raised can be at least partly addressed by reformulating the US accounts as shown in Table 3, above.

[6] More formal theoretical development at greater levels of detail should begin with a review of Oosterhaven (1984). However, the optional use of total commodity supply rather than total commodity output may require some extensions and modifications of findings presented there.

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