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Structural Decomposition Analysis
Sense and Sensitivity

by

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Summary

In recent years Structural Decomposition Analysis (SDA) has become an important tool for Statistics Denmark to break down the observed changes in physical variables like energy consumption or CO_2-emissions into the changes in their physical and economic determinants.

Recently we have found that there are some challenges to the continued use of this method that we need to get a better knowledge about and some practical solutions to. The extent of some of the uncertainties attached to the calculations need to evaluated and some of the decisions that are necessary to take in advance must be considered carefully.

Therefore this project is concerned with an investigation of the sensibility of the structural decomposition analysis to a number of the most important challenges to the method.

The report starts with a careful look at the methodology behind the SDA and how the formulas are applied in statistics Denmark.

First of all there is n important decision to be made about the decompositions form that is applied. Should one arbitrarily picked form be used or the average all possible forms or perhaps just some of the forms. The report shows the statistics on some of the solutions and one of them is recommended.

A number of issues that are important to the analysis are discussed and measured in tables and graphics and recommendations are put forward. These have to do with the choice of base year for the calculation, the importance of the aggregation level and a few others.

The perhaps most important part of the report is about the indispensable basis for these calculations, namely a full set of input-output tables in fixed prices. An SDA needs the economic variables to be in quantities i.e. fixed prices because otherwise price effects will dominate the results to some unknown extent. The traditional set of tables in the prices of a fixed base year is on their way out and must be placed by another set.

The report works with three sets of fixed price variables, prices from fixed base year, previous year's prices and chained value matrices. The latter are inherently non-additive but a method is advised to calculate correction matrices that when they are applied to the chained value matrices make them additive again and usable in input-output calculations.

1. Introduction

In recent years Structural Decomposition Analysis (SDA) has become an important tool to break down the observed changes in physical variables like energy consumption or CO_2-emissions into the changes in their physical and economic determinants. This methodology is also applied by Statistics Denmark. Thus, each year the annual publication of the Danish Environmental Accounts contains some pages about the update of our SDA of CO_2-emissions by Danish industries from 1966 to the latest possible year. This calculation draws on our NEMEA tables and input-output tables in general. Moreover SDA is used in various other papers and presentations by statistics Denmark and considered an important contribution to understanding the development.

What is SDA?

Decomposition analysis is a so-called comparative-static technique and can be utilised on various types of statistics including macro variables of the full economy. Adding the term "Structural" to get "Structural Decomposition Analysis" indicates the inclusion of the detailed structure of production as well as final demand by input-output tables and models. In analyses of the complex interaction between the economy and the environment it is very important to get all the details in the production and consumption structure provided by the detailed input-output tables included in the analysis.

The objective is to determine the influence of structural changes on the changes in environmental variables. The analysis is based on a number of determinants and the effects of the single determinants can be distinguished. Results show that results in decreasing emissions in one area can be offset or nullified by increases in other areas so that overall it appears that nothing is gained. For compiling policy advice it is therefore very important to be able to distinguish the effects of the various determinants which is actually what the SDA offers.

Other studies of sense and sensitivity

In Dietzenbacher and Los (1998), which is the most cited article in this area, the authors addressed the questions of sense and sensitivity as well. They concentrated on different solutions to the non-uniqueness problem and on the choice of aggregation level. The analysis dealt entirely with economic variables. This report differs from the Dietzenbacher and Los (1998) study by looking at the sensitivity of more factors and by basing itself on a mixture of physical and economic variables.

What is the background for suggesting this project?

The initial work on the current SDA analysis by Statistics Denmark started in 2003 as a project financed by Eurostat (DG Eurostat/B1 Grant agreement no. 200141200007). Working with this type of analysis since then we have gathered some experience about restrictions and drawbacks related to the method that may question its overall usability. One of the things we have found is that a number of the methodological choices to be made prior to running the particular analysis can affect the outcome and even the conclusions of the analysis. The purpose of this project is to identify and measure the sensitivity of the analysis to a number of those choices.

The background for our interest for this topic is as mentioned the experience that we have gathered in terms of the following aspects of the analysis

- In order to do a full decomposition it is crucial for the analysis to work with some kind of average of the decomposition forms and not just choose one of the forms arbitrarily. Doing so can lead to false conclusions.

- We have found that the level of aggregation of the industries can have considerable effect on the results and one of the aims of this project to map and quantify this effect and provide some advice about what to do about it. A set of SDA's will be compiled for various aggregation levels and the effects on the results will be recorded.

- Another effect is that adding or leaving out certain variables from the analysis has quite unequal effects on the remaining variables. This is another source for uncertainty about the final results that we will try to map by calculating a large number of different analyses.

- Working with the input-output tables of domestic production one positive effect e.g. on the emissions of CO_2 is the change towards more imported input and final demand (that does not give rise to any domestic emissions) – possibly outsourcing. This effect is rarely accounted for in SDA's so in this project we will look at the possibility to include this effect among the effects from the domestic variables.

- In the Danish case we have annual input-output tables al the way back to 1966. But working with these long time series in constant 2000 prices we have found that the base year of the calculation are important for the results. Thus the importance of the various variables in a given period is different when using e.g. 1966 as the stating point as compared to 1990 as the starting point. This is probably the base year effect that has initiated the change towards the compilation and use of chain weighted variables. It is expected to see this effect disappear as the traditional fixed price matrices are exchanged by matrices in previous year's prices.

- An SDA requires the economic variables to be measured in quantities. It usually means that variables are measured in prices of a fixed base year. But in recent years variables measured in previous years prices have been introduced as well. Finally, quantity variables can be expressed as fixed prices, chained values. This report carries out SDA's based on all three types of quantity variables and the sensitivity of the calculation to the choice between the methods is quantified.

  The use of economic variables in previous year's prices requires some changes to the calculation method, and the use of variables in fixed prices, chained values requires major work on the data. It includes the calculation of matrices and vectors in fixed prices, chained values and a subsequent adjustment of all cells in order to reintroduce the applicability of standard summation rules in the matrices and vectors. Both task are described in detail in the report.

2. Structural Decomposition Analyses in Statistics Denmark. Methods and Applications.

Introduction

The chapter is an introduction to the current use of Structural Decomposition Analysis (SDA) in Statistics Denmark. It will present the method being used and some of the results that have been published. For the last 5 years or so the annual publication of the Danish Environmental Accounts has contained a quite comprehensive structural decomposition analysis of the development in the emissions of CO_2, SO_2 and NO_x in Denmark. The SDA methodology has also been used in other ad-hoc reports and papers.

2.1 The method

Started with a project for Eurostat

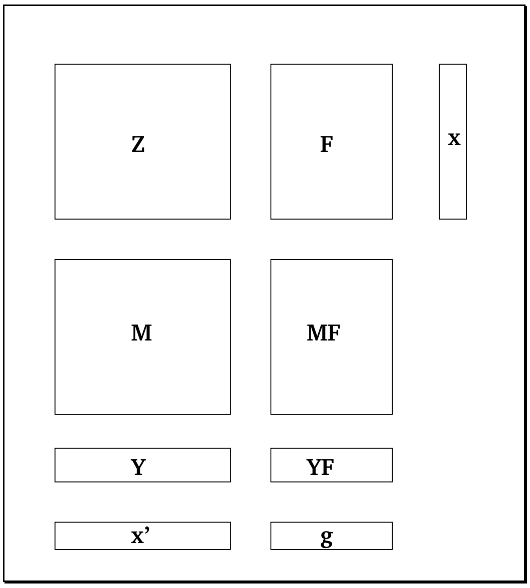
The methodology behind the Danish calculations was first developed in 2003 as a part of a project financed by Eurostat (DG Eurostat/B1 Grant agreement no. 200141200007). Apart from the report submitted to Eurostat the outcome of the project was presented in a conference (Rørmose, P. & Olsen, T., 2005).

The method was developed in the project on the basis of especially three contributions from the literature Dietzenbacher and Los (1998), Wier (1998), de Haan (2001) and Seibel (2003).

The input-output framework

As mentioned earlier the basic part of an SDA is the input-output framework. A very brief schematic of the set of Danish input-out put tables would look like

Figure 1. Overview of the relevant part of the Danish set of Input-output tables



where

Z intermediate deliveries from industry to industry
F deliveries from domestic industries to various categories of final demand
x vector of total output
M imports delivered to Danish industries
Y taxes and gross value added
g total final demand by category

The vector of total final demand delivered by each industry is obtained by summing over the columns of **F,** where **i** is a summation vector.

$$\mathbf{f} = \mathbf{Fi} \tag{1}$$

The vector of total final demand by category is found by summing down over the rows of F, MF and YF matrices.

$$\mathbf{g} = \mathbf{iF} + \mathbf{iMF} + \mathbf{iYF} \tag{2}$$

In the usual way the basic input-output model can be derived

$$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1} \tag{3}$$

where

A technical coefficients or input-output coefficients

The matrix **A** is obtained by dividing each row of the matrix of deliveries **Z** by the vector of total output **x**. It means that the equation can be written

$$\mathbf{x} = \mathbf{Ax} + \mathbf{f} \tag{4}$$

By rearranging the terms the basic input-output model is derived

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} \tag{5}$$

where **I** is the identity matrix with ones in the diagonal.

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{Lf} = [l_{ij}] \tag{6}$$

The term $\mathbf{L} = (\mathbf{I}\text{-}\mathbf{A})^{-1}$ is called the *Leontief inverse* or the *total requirements matrix.*

The basic methodological idea

The principle behind structural decomposition can be illustrated in the case of a two-determinant multiplicative function

$$\mathbf{x} = \mathbf{Lf} \tag{7}$$

If we express the left hand side in absolute change terms $\Delta\mathbf{x}$, we can make an additive decomposition with the right hand side also expressed in absolute terms. There are other ways to decompose (7), and the choice of decomposition form depends on the objective of the analysis. But for SDA the additive version with both sides expressed in absolute terms is by far the most common, and it is the one used in the decompositions in Statistics Denmark.

The decompositions dealt with in this project starts with a multiplicative function like (7). It is often possible to add more elements to the analysis by splitting the determinants into more factors as long as the total product is still the same. That will be investigated further below.

After differentiation of (7) by the product rule the additive decomposition form is obtained

$$\Delta\mathbf{x} = \mathbf{L}\Delta\mathbf{f} + \mathbf{f}\Delta\mathbf{L} \tag{8}$$

Thus, the change in the right hand side of (7) can be decomposed into two parts that depends both on the changes in **L** and **f** and the relative size of the weights **L** and **f**.

The non-completeness problem

However, the choice of weights is a very fundamental question, because once we need to rewrite (8) into continuous time we will need to apply some index to the determinants and the equation will not necessarily hold any more.

$$\Delta \mathbf{x} \neq \mathbf{L}_t \Delta \mathbf{f} + \mathbf{f}_t \Delta \mathbf{L} \quad \text{Paasche index} \tag{9}$$

$$\Delta \mathbf{x} \neq \mathbf{L}_{t-1} \Delta \mathbf{f} + \mathbf{f}_{t-1} \Delta \mathbf{L} \quad \text{Laspeyres index} \tag{10}$$

where Δ now means the change in the variable from period *t-1* to *t*.

Both of these cases are examples of decompositions that are not full. It means that either the change in y is overestimated (the case of using the Paasche index) or underestimated (the case of using the Laspeyres index). The reason that the non-equal sign is used is that there is a residual term that is either accounted for twice or not accounted for. In some SDA's this term is taken in as a part of the explanation and in others it is there, but no attention is paid to it. But it can be significant in some studies and also quite difficult to give an exact interpretation of, so most SDA studies is now designed to get rid of it.

.. and its solution

The solution is to mix the Paasche and the Laspeyres index.

$$\Delta \mathbf{x} = \mathbf{L}_t \Delta \mathbf{f} + \mathbf{f}_{t-1} \Delta \mathbf{L} \quad \text{Paasche-Laspeyres index} \tag{11}$$

$$\Delta \mathbf{x} = \mathbf{L}_{t-1} \Delta \mathbf{f} + \mathbf{f}_t \Delta \mathbf{L} \quad \text{Laspeyres-Paasche index} \tag{12}$$

Now there are no residuals and the equal sign applies. So the solution to this problem is to use a mix of the two indexes.

The non-uniqueness problem

Equations (11) and (12) are both complete which means that they explain all the changes in **x** in terms of the changes in **L** and **f**. However, the solution is not unique. It means that the two equations will have different things to say about how much the change in the two variables have contributed to the change in **y**. Thus, if the relative increase in **L** ($\mathbf{L}_t$ / $\mathbf{L}_{t-1}$) is different from the relative increase in **f** ($\mathbf{f}_t$ / $\mathbf{f}_{t-1}$) the two equations (11) and (12) will have different results for the contributions of **L** and **f** to the change in **x**. Here two determinants and two different decomposition forms. It is shown by Dietzenbacher and Los (1998) that in the general *n*-determinants case there is *n!* different forms.

... its consequences

It means that if one of the n! forms are picked arbitrarily the results for the n determinants can be very different from the results calculated on the basis of another arbitrarily picked decomposition form.

The equations shown above only have two determinants, but when this method is used in practice a larger number of determinants will normally be included. Thus, the annual Danish SDA includes 8 determinants, increases the number of possible decomposition forms to 8! = 40,320. In (Rørmose and Olsen, (2005)) tables with standard deviations of the results show quite large variations between the upper and lower bounds. Chapter 4 treats these problems in an empirical way based on the Danish SDA. In some instances variations are so large that there is a risk of drawing the wrong conclusions unless attention is paid to choose the right method.

...and its solution

Except from not paying attention to this problem there are (at least) two other solutions to this. One is to take the average of all possible decomposition forms. This is being done in the Danish case, where the 8 determinants generate 40,320 different forms. This is a lot of calculation, but fortunately it has been shown in the literature (Dietzenbacher and Los, (1998)) that even though there is n! different forms they are actually not all different. In fact, in the case of n determinants there is only 2^{n-1} different forms. In the Danish case that equals 128 different decomposition equations. But the 40,320 forms are not represented evenly over the 128 different forms. In order to calculate how many of the 128 different forms there exist in the material the following equation from Rørmose and Olsen (2005) or Seibel (2003) can be utilised

$$q = \frac{(n-1)!}{[(n-1-k)!k!]} \quad , \quad k \in \{0; n-1\} \tag{13}$$

where

q the number of different decomposition equations given *k* and *n*.
n the number of determinants
k the number of *t-1* subscripts, running from 0 to n-1

The parameter *k* indicates the number of *t-1* subscripts in the mixed Paasche – Laspeyres index. Thus the run of *k* from 0 to n-1 indicates the gradual change from a pure Paasche based decomposition form to a pure Laspeyeres based decomposition form. In a decomposition form with n determinants there is only one Δ-term and the remaining n-1 terms are weights with an index of either *t* or *t-1* subscripts.

But more important it is to calculate the number of each of the different decomposition forms. In a SDA that only calculates the 2^{n-1} different forms instead of the n! forms this number can be regarded as a weight that must be applied to each of the 2^{n-1} different forms in order to calculate a weighted average. The number can be deduced from the following formula, which is actually just the denominator from (13)

$$w = (n-1-k)!k! \quad , \quad k \in \{0; n-1\} \tag{14}$$

Applying (13) and (14) to the Danish case with 8 determinants gives rise to the following table

Table 1. **The number of different decomposition forms where n=8**

k (number of t-1)	Number of possible forms given k *(n-1)! / [(n-1-k)!·k!]*	w=(n-1-k)!·k!
0	1	5040
1	7	720
2	21	240
3	35	144
4	35	144
5	21	240
6	7	720
7	1	5040
	128	40,320

What the table says is that it is necessary to calculate the only the 128 truly different forms and in the effort to calculate a weighted average of these forms it is necessary to know the numbers in the third column that indicates how many of each of the 128 decomposition forms there are. The column in the middle indicates how many possibilities there are to construct a decomposition form with *k* instances of *t-1*subscripts (which at the same time means that there is *n-1-k* instances of *t* subscripts).

Note that for all *k,* multiplication of columns 2 and 3 gives 5040 indicating that t and t-1 are represented an equal amount of times in the full calculation.

2.2 Data for the Danish SDA

Input-output data

The starting point of the Danish decomposition analyses is the set of Danish input-output tables from 1966 to currently 2007 supplemented with emission data from Danish NAMEA system covering the same time span plus 1-2 years.

Intermediate consumption and final demand matrices

The matrix of domestically produced intermediate consumption **Z** has the dimension 130 by 130 industries so it is quite detailed. It is compiled on the basis of approximately 2350 different product balances. The balances are the basis of the entire Danish national accounts and therefore produced carefully in details every year and are therefore an excellent basis for the annual compilation of input-output tables.

The matrix of domestically produced final demand **F** has the dimension 130 industries by 106 categories of final demand which are

Final demand matrix **F** (130 by 106)

- 73 categories of private consumption
- 21 categories of government consumption
- 10 categories of fixed capital formation
- 1 column of changes in stocks
- 1 column of exports

Three different price levels

All of the Danish input output data is available in current prices, fixed prices (base year 2000) and previous years prices. The fixed prices are constructed from the bottom by deflating every one of the 2350 different product balances before constructing a new set of input-output tables in fixed prices.

Energy and emission data

One of the main attractions about SDA's is that it is possible to combine physical variables like e.g. employment, energy consumption and emissions of various substances with purely economic variables.

Detailed information in the energy accounts

In order to be able to estimate the greenhouse gas emissions related to the use of fossil fuels in the industries and the households, it is important to have information on the consumption of energy broken down by energy types, industries and households. This is done in the Danish energy accounts.

The Danish Energy Account is compiled in the autumn and is published approximately 11 month after the end of the year in question.

130 industries and 40 types of energy

At the most detailed level, the Danish energy accounts include information on the supply and use of energy by the 130 industry classification used for the Danish national accounts. Further, the detailed accounts include a breakdown by 40 different types of energy.

Physical and monetary information at various measuring units

The Danish energy accounts include information on energy flows at physical units, tera joules (TJ) and in addition, it includes energy flows at various mass and volume units (tonnes, cubic metres, etc.), and monetary information at various price levels (basic prices, trade margins, taxes and subsidies, VAT, and purchasers prices).

The emission data

The Danish air emissions are compiled every spring as a part of the Danish Environmental Accounts approximately 16 month after the end of the statistical year. The accounts comprise not only greenhouse gases, but eight different substances in total. CO_2, SO_2, NO_x, CH_4, N_2O, NMVOC, NH_3, and CO, are distinguished in the accounts.

Energy related air emissions

For energy related emissions, the Danish air emission accounts include a breakdown of the emissions by the same 40 types of energy, which are included in the Danish energy accounts, cf. Annex A.2. Furthermore, all information on emissions is broken down by 130 industries and households.

The primary sources used to compile the Danish air emissions accounts are the Danish energy accounts and emission factors and emission inventories obtained from the Danish National Environmental Research Institute (NERI).

Generally, the air emissions are estimated at a detailed level (i.e. for each type of energy and each industry and households) by multiplying the energy use by a technical emissions factor. The general procedure is described in the box.

2.3 A Danish SDA

In order to get a picture as detailed as possible of the factors behind the development in the energy-related emissions of CO_2 by Danish industries the Danish SDA consists of as much as 8 determinants. However, only six of them contribute to the general picture. The ones that do not are emission coefficients, and the second is just a summation vector which is necessary to have in the construction.

$$\mathbf{p} = \mathbf{C} \otimes \mathbf{M} \cdot \mathbf{i} \otimes \mathbf{k} \otimes \mathbf{L} \cdot \mathbf{G} \cdot \mathbf{o}' \otimes z \tag{15}$$

where

$\mathbf{p}$	A vector of emissions by industry calculated as $\mathbf{Si}$, where $\mathbf{i}$ means summation
$\mathbf{S}$	a matrix (130 by 40) of emissions by industry and type of energy
$\mathbf{C}$	is a matrix of emission coefficients compiled as $\mathbf{S}(\mathbf{E}^{-1})$
$\mathbf{E}$	a (130 by 40) matrix of consumption of energy by industry and type of energy
$\mathbf{k}$	a vector of energy intensity by industry $\mathbf{Ei}(\mathbf{x}^{-1})$
$\mathbf{x}$	total output by industry
$\mathbf{L}$	the Leontief inverse. $(\mathbf{I}-\mathbf{A})^{-1}$
$\mathbf{G}$	a matrix of final demand coefficients $\mathbf{F}(\hat{\mathbf{g}}^{-1})$
$\mathbf{F}$	final demand by industry and category
$\mathbf{g}$	total final demand by category
$\mathbf{o}$	a column vector of final demand categories of total final demand $tr(\mathbf{g}(\mathbf{gi}^{-1}))$
z	a scalar $\mathbf{gi}$
•	standard matrix multiplication
$\otimes$	element by element multiplication

Emission coefficients

The first determinant is $\mathbf{SE}^{-1}$ which is emission coefficients. It is constant by definition when it comes to CO_2 and therefore it does not contribute to changes in emissions.

Relative consumption of energy

As the second determinant every row of the matrix of energy consumption is divided by its own row sum to get a matrix that represents the share of each type of energy that each industry uses. Thus, when an industry changes its energy consumption from one type of energy to another it will be recorded as changes in this matrix.

Summation vector

Third determinant is just a (40 by 1) summation vector which is necessary to bind some of the matrices together in a chain.

Energy intensity

The fourth determinant is a vector of energy intensity by industry. It is calculated as the row sums of the energy matrix divided by total output by industry.

Leontief inverse

The fifth determinant is the inverted matrix $\mathbf{(I\text{-}A)}^{-1}$ which represents the possible changes in the production structure of each of the 130 industries. The structure may change towards a larger share of inputs that either generates more emissions or less emissions.

Final demand structure

The sixth determinant represents the structure of the final demand by all of the 106 different categories of final demand. This is a coefficient matrix and changes occur when the composition of the final demand categories changes. The matrix is generated by dividing the flow matrix of final demand with the column totals.

Composition of final demand

The seventh determinant is a vector with shares of each of the column totals of final demand in the total of final demand. Thus, when the relative share of exports in the total final demand increases it will be represented by this determinant.

Total final demand

The eighth and final determinant is a scalar of the total final demand.

In the case of just two determinants like (7) the two decomposition equations (11) and (12) is easily found. But in the case of 8 determinants it is not easy to manage the 40,320 equations. Fortunately the problems can be reduced to 128 different equations as described earlier, but it still requires some work to get it right.

Setting up the model

Starting from the basic equation (15) what we want to do is to look at the change that has happened in the left hand side variable from one period to another period i.e.

$$\Delta \mathbf{p} = \mathbf{p_t} - \mathbf{p_{t-1}} \tag{16}$$

so that means

$$\begin{aligned}\Delta \mathbf{p} = {} & \mathbf{C}_t \otimes \mathbf{M}_t \cdot \mathbf{i} \otimes \mathbf{k}_t \otimes \mathbf{L}_t \cdot \mathbf{G}_t \cdot \mathbf{o}'_t \otimes z_t - \\ & \mathbf{C}_{t-1} \otimes \mathbf{M}_{t-1} \cdot \mathbf{i} \otimes \mathbf{k}_{t-1} \otimes \mathbf{L}_{t-1} \cdot \mathbf{G}_{t-1} \cdot \mathbf{o}'_{t-1} \otimes z_{t-1}\end{aligned} \tag{17}$$

Additive identity splitting

By using the additive identity splitting technique it is possible to rewrite this equation into an additive equation where each of the 8 determinants appear once with a weight factor attached to it

$$\begin{aligned}\Delta \mathbf{p} = {} & \Delta \mathbf{C}_t \otimes \mathbf{M}_t \cdot \mathbf{i} \otimes \mathbf{k}_t \otimes \mathbf{L}_t \cdot \mathbf{G}_t \cdot \mathbf{o}'_t \otimes z_t + \\ & \mathbf{C}_{t-1} \otimes \Delta \mathbf{M}_t \cdot \mathbf{i} \otimes \mathbf{k}_t \otimes \mathbf{L}_t \cdot \mathbf{G}_t \cdot \mathbf{o}'_t \otimes z_t + \\ & \ldots\ldots \\ & \mathbf{C}_{t-1} \otimes \mathbf{M}_{t-1} \cdot \mathbf{i} \otimes \mathbf{k}_{t-1} \otimes \mathbf{L}_{t-1} \cdot \mathbf{G}_{t-1} \cdot \mathbf{o}'_{t-1} \otimes \Delta z_t\end{aligned} \tag{18}$$

Notice that now we have an equation with 8 terms each one with only one of the determinants as a difference and the other 7 with either a *t* or a *t-1*attached to it. Notice also that the difference term wanders from left to right and that all the period indications to the left of the difference term are *t-1* and those to the right of it are marked *t*. This equation, however, is just one of the 128 possible equations and the 127 other ones are found by shifting the period indications around in an appropriate way.

128 "permutations"

In the programming step where the software is supposed to loop over the 128 equations the programmer could think of the difference between $2^{(n-1)}$ permutations.

> In mathematics, the notion of **permutation** is used with several slightly different meanings, all related to the act of **permuting** (rearranging in an ordered fashion) objects or values. Informally, a permutation of a set of objects is an arrangement of those objects into a particular order. For example, there are six permutations of the set {1,2,3}, namely [1,2,3], [1,3,2], [2,1,3], [2,3,1], [3,1,2], and [3,2,1]. One might define an anagram of a word as a permutation of its letters. The study of permutations in this sense generally belongs to the field of combinatorics. *Source: http://en.wikipedia.org/wiki/Permutation*

Dimension of the results

The model (15) is set up to give output in the dimension 130 industries by 8 determinants by year. But mostly when the results are published they are summed across the 130 industries to give a national total. So results by industry or groups of industries are readily available in the output.

Furthermore, through a slight change in the model the full dimension of the 106 different categories of final demand can be listed as well.

Results later As the results of the Danish SDA will be a part of a number of the next chapters no results are shown in this chapter

3. Sensitivity to the choice of base year.

As discussed previously before running a decomposition analysis on the basis of (15) a number of choices must be made. One of them is which year to use as the base or starting year for the calculation. That again depends on the purpose of the study of course.

An SDA is basically about comparing a set of variables between two years. SDA analysis carried out by Statistics Denmark has primarily aimed at construction of long time series of SDA's comparing a specific base year with all the consecutive years one by one. Mostly, the base year of the series of SDA's has been 1980 or 1990. However, comparing some of the results that we have obtained indicates that switching the base year between 1980 and 1990 it is not without effect on the results. It means the relative importance of the determinants does not seem to be entirely the same in years covered by both analyses.

Testing the importance of the base year In order to quantify these effects two decomposition analyses like equation (15) has been carried out once with 1980 as the base year and once with 1990 as the base year. These analyses have incorporated economic variables in constant prices with prices fixed as they were in 2000

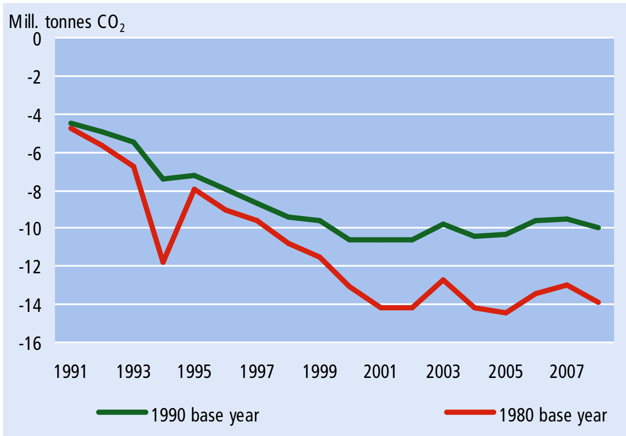
Table 2. **Contribution to the change in CO2 emissions by relative changes in the type of energy used with 1980 and 1990 as base years respectively**

	Base year 1990	Base year 1980	Level adjustet to 1980 calculation Base year 1990
	(1)	(2)	$(3)=(1)+(2)_{1991}$
	Million tonnes CO_2		
1991	0,0	-4,8	-4,8
1992	-0,4	-5,7	-5,2
1993	-0,9	-6,8	-5,7
1994	-2,8	-11,8	-7,6
1995	-2,7	-8,0	-7,5
1996	-3,4	-9,0	-8,2
1997	-4,2	-9,6	-9,0
1998	-4,9	-10,7	-9,7
1999	-5,1	-11,6	-9,9
2000	-6,1	-13,0	-10,9
2001	-6,1	-14,2	-10,9
2002	-6,1	-14,2	-10,9
2003	-5,3	-12,7	-10,1
2004	-5,9	-14,1	-10,7
2005	-5,8	-14,5	-10,6
2006	-5,1	-13,5	-9,9
2007	-5,0	-13,0	-9,8
2008	-5,5	-13,9	-10,3

The table illustrates how many tonnes of CO_2 the relative shift in type of energy used by Danish industries in isolation have contributed to the overall change in Danish emission of CO_2.

In order to be able to compare the two different decomposition analyses the level of all of the results in the 1990 analysis has been increased by the level of the result of the 1980 analysis in 1991. It is obvious by looking at the table, that it does matter which base year is used. There is a general tendency that the relative shift in type of energy used weighs more heavily when the analysis is based on 1980 as a base year. It becomes even clearer when looking at a graphical display.

Figure 2 Contribution to the change in CO2 emissions by relative changes in the type of energy used with 1980 and 1990 as base years respectively



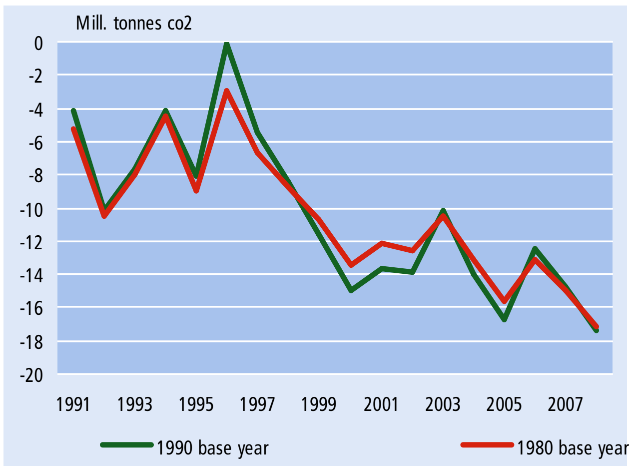
Some explanations

One thing that looks a bit erroneous is the very low value in 1994 in the 1980-based series. It is, however, not a mistake. There is a general drop in emissions in almost all 130 industries as a consequence of the relative change in fuel mix. The largest changes took place in industries like freight transport by road, supply of electricity gas and district heating. Compared to 1980 the share of fuel oil in production of electricity has dropped and the share of coal has increased. That tends to increase emissions as coal generates more CO_2 than fuel oil. But specifically in 1994 the share of fuel oil was not so much lower as it was in 1993 and 1995 and the share of coal was not so much higher in 1994 as it was in 1993 and 1995.

The other determinants

It seems like the difference between using 1980 or 1990 as the base year have not so much influence on the other determinants.

Figure 3 Contribution to the change in CO2 emissions by changes in the energy intensity with 1980 and 1990 as base years respectively



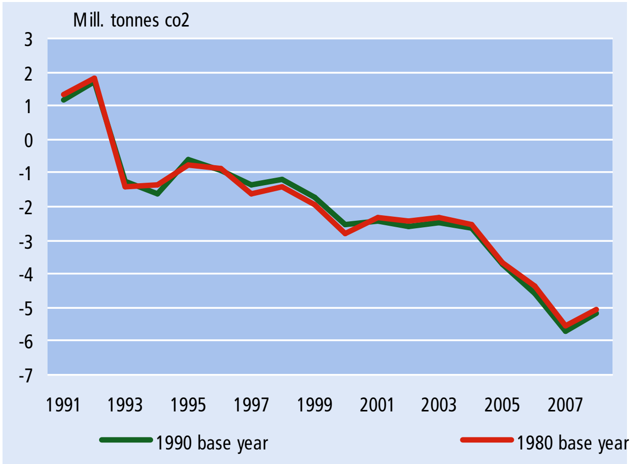
The energy intensity is one of the major contributors to the downward pressure on the total CO_2 emissions Thus, from 1980 to 2008 it has secured an 18 millions tonnes lower emission than would have prevailed if lots of work had not been done to improve the energy efficiency.

The base year effect

In this case there are some effects from changing the base year. The trend is more or less the same, but where the biggest gaps are there is a difference at about 2 million tonnes.

Figure 4

Contribution to the change in CO2 emissions by changes in the production structure represented by the Leontief inverse used with 1980 and 1990 as base years respectively



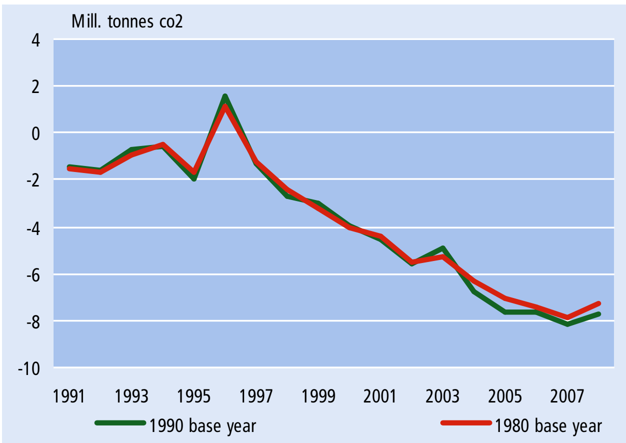
As it appears from figure 4 the general change in input structure has also contributed to draw the total emission level in a downward direction with about 5 million tonnes in 2008 as compared to 1980. The base year effect does not seem to be of any significance in this case.

Shift to more imported input may an explanation

It is a fact that as the economy gradually turns more global many industries increase the share of imported input in their production. This may be one of the explanations for the observation in figure 4. Because the input coefficients must sum to one for each industry and as the import coefficients tend to increase the coefficients for the domestically produced input must decrease. Thus as imports are left out it works like each industry can do with less and less input over the years and that will gradually drag down emissions. However, this is a bit of a false effect.

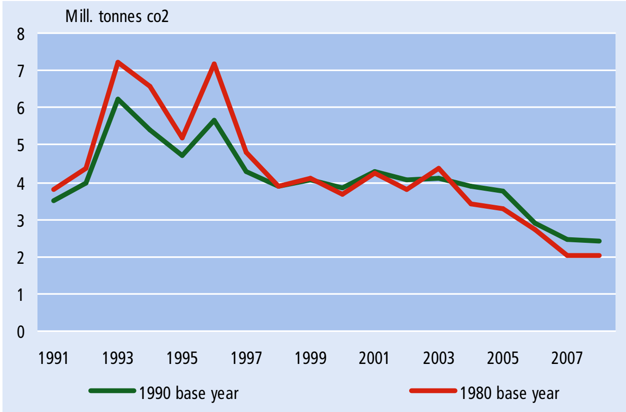
Figure 5

Contribution to the change in CO2 emissions by changes in the structure of the final demand used with 1980 and 1990 as base years respectively



Also it seems like changes in the way that the final demand is put together has a negative effect on emissions. It may actually be the case that a number of the final demand components is put together in a less CO2 intensive way, but it is more likely the case that imports have gradually taken over a larger part of the delivery of final demand components as it is also the case with input in production.

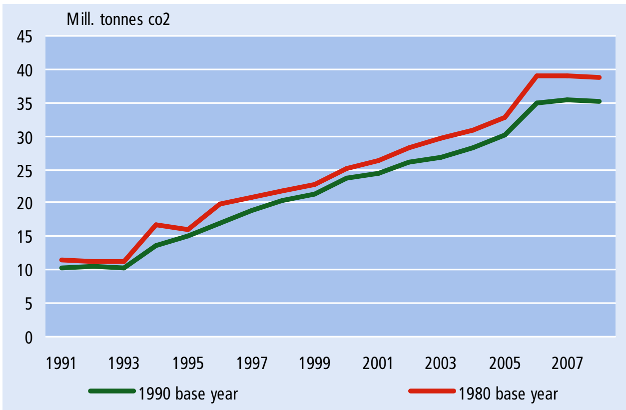
Figure 6 Contribution to the change in CO_2 emissions by changes in the relative composition of the final demand used with 1980 and 1990 as base years respectively



Exports are more polluting

Here we are looking at one of the few determinants that have had an increasing effect on emissions. The changes in the composition of the final demand have increased emissions in 2008 by 2 million tonnes but during the period in 1993 and 1996 this determinant really was pulling emissions up by about 7 million tonnes. In 1993 and especially in 1996 Denmark exported a lot of electricity. Moreover, Danish exports generally tend to be more polluting than production for domestic use.

Figure 7 Contribution to the change in CO_2 emissions by changes in the total final demand used with 1980 and 1990 as base years respectively



Finally we have the big "motor" in the equation. The change in the level of final demand is what really pulls the emissions upwards. This variable is more or less synonymous with the economic growth of the Danish economy. So it comes as no surprise that this is what really drives the emissions. Although it does not seem like a big difference between the 1980 based and the 1990 based calculation there is a difference of more than 4 million tonnes at the end of the period, which may be something to consider.

Conclusion

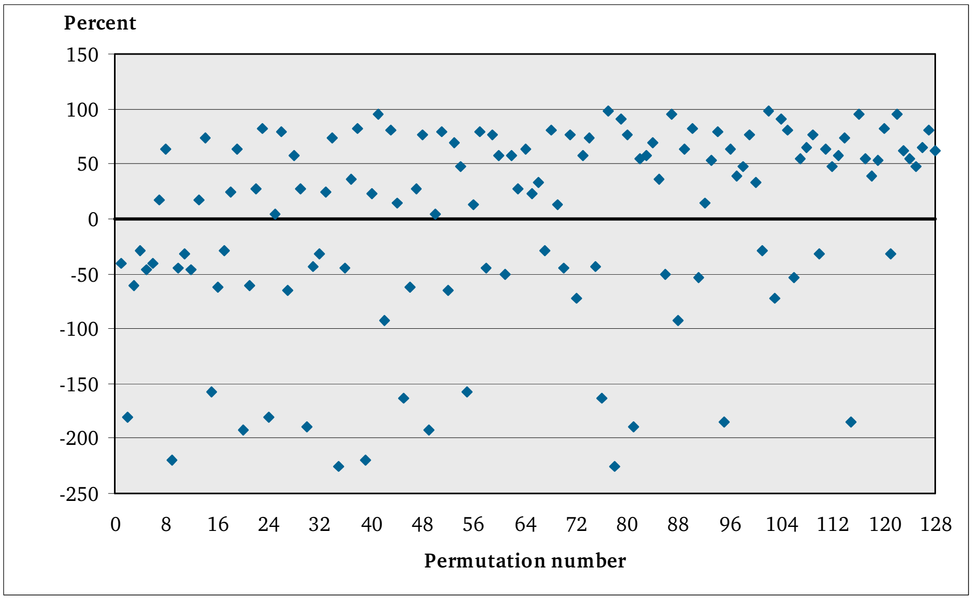
This chapter is about the effects of using different base years for the analysis, but it has to a large extent been used to introduce the effects of the single determinants. When decomposition analyses are based on traditional fixed prices like this one, base year effects will be present. It is related to the discussion of using prices from a fixed base year to compile series of variables in fixed prices. The further away it gets from the base year that less likely it is that the relative weights between the industries are still the same as in the base year. This problem has been solved with the introduction of series in previous years prices that can be applied for calculating series in fixed prices chained values. So when the calculations based on 1980 and 1990 respectively yields different results it is because that the weight put on the various industries and components were different.

4. Sensitivity to the choice of decomposition method

In the literature on Structural Decomposition Analysis it has been a fact for a number of years that picking just one of the n! different decomposition forms can yield results that are quite far away from what is expected on the average and thus, wrong decisions can be made on the basis of such analysis. Therefore we shall turn to look at some results now.

Figure 8

Contribution from changes in the composition of types of energy used. Percentage deviation from the mean of 128 permutations. Effects observed for 2007 with base year 1980.

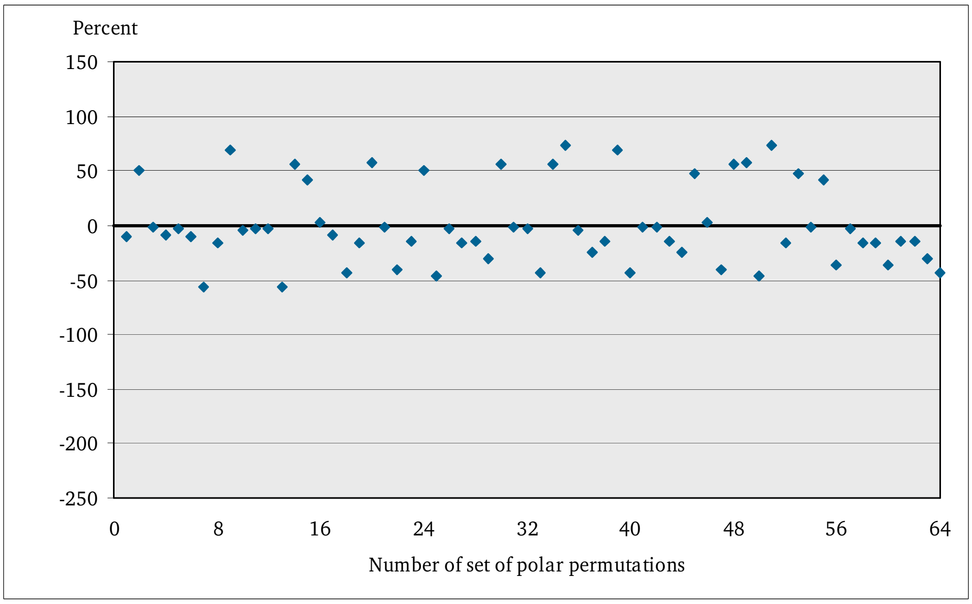


Percentage deviation from the mean

It is apparent that the calculated contribution from the changes in the composition of energy used to the changes in the CO_2 emissions 28 years after the base year in 1980 is compiled with quite some uncertainty. If one of the permutations is picked arbitrarily one could be left with a result some -225 pct. or +100 pct. away from the mean of all the 128 permutations. Obviously, this is not a picture that creates a lot of confidence in the outcome and conclusions of SDA's.

Figure 9

Contribution from changes in the composition of types of energy used. Percentage deviation from the mean of 128 permutations. Effects observed for 2007 with base year 1980.



Mirrored images

One of the suggestions in the literature is to look at one of the 64 mirrored permutations. Mirrored images are the average of two permutations that are opposite to each other. One example would be

$$\mathbf{\Delta p} = \mathbf{h_t} \otimes \mathbf{\Delta L} \cdot \mathbf{G_{t-1}} \cdot \mathbf{o_{t-1}} \otimes z_{t-1} \quad (19)$$

and its mirror image

$$\mathbf{\Delta p} = \mathbf{h_{t-1}} \otimes \mathbf{\Delta L} \cdot \mathbf{G_t} \cdot \mathbf{o_t} \otimes z_t \quad (20)$$

where

h is the emission intensity by industry $\mathbf{Si(x^{-1})}$

Notice that all of the time indexes have changed. Thus *t*'s have changed to *t-1*'s and this way becomes a mirror image.

Taking the average of these two decomposition forms brings down the deviation from the mean down quite a lot as illustrated in figure 9 above. The deviation from the mean now only lies in the band between ±50 pct. which is considerably lower. Moreover this particular determinant probably is the one where the mirrored images have the largest deviations from the mean. Results for the other determinants can be seen in the tables below or as graphs in the appendix x.

Table 3. **Statistics on 2007 values of determinants in an SDA of changes in CO_2 emissions 1980 to 2007.**

Determinant		Average	Minimum	Maximum	Standard deviation
Composition of energy consumed	A	-10.18	-33.08	-0.22	8.95
	B	-9.77	-33.08	-0.22	7.87
	C	-10.18	-17.61	-4.43	3.70
Energy intensity	A	-18.47	-33.26	-6.77	7.58
	B	-19.51	-33.26	-6.77	7.97
	C	-18.47	-23.11	-13.69	2.95
Production structure	A	-5.59	-11.83	-2.03	2.28
	B	-5.54	-11.83	-2.03	2.47
	C	-5.59	-7.31	-4.37	0.58
Final demand structure	A	-7.71	-15.13	-3.39	3.15
	B	-8.03	-15.13	-3.39	3.50
	C	-7.71	-9.28	-6.13	0.82
Composition of final demand	A	1.79	-2.38	7.47	1.78
	B	1.51	-2.38	7.47	1.67
	C	1.79	0.01	3.67	0.92
Total final demand	A	37.29	21.94	55.09	8.27
	B	37.75	21.94	55.09	10.06
	C	37.29	32.39	40.89	2.60

A: Average of the 128 unique permutations
B: Weighted average of all the 40.320 decomposition forms
C: Average of the 64 pairs of mirrored images

There are a couple of important findings from this table

- It is hard to justify the need to calculate the weighted results instead of just taking the average of the set of unique permutations.
- It will be a better idea to pick one pair of mirror images arbitrarily than one random decomposition form because the standard deviation is much lower and the outliers in shape of extreme values are eliminated to a certain extent.
- However, the recommendation to pick one pair of mirror images arbitrarily is still just second best compared to taking the average of the set of 2^{n-1} unique permutations.

5. Sensitivity to the choice of decomposition model

One aspect of SDA that can potentially weaken its credibility is if the value of the determinants changes significantly when one variable is included or left out. It has been tested by running two alternative SDA's where some of the variables from our standard 8 determinant SDA has been aggregated into one and and additional variables have been introduced. Doing that it has been the expectation that the value of the remaining determinants would be stable and not move away from their original value.

5.1 Experiment 1

In order to test the stability of the system and at the same time to test the possibility to let a new set of determinants explain the degree to which the increasing share of imported final demand can explain why the structure of final demand has contributed to lower the Danish CO_2 emissions. Just for the sake of the test the last to variables which are the composition of the final demand and the scalar of total final demand has been aggregated into one variable.

So now the model is

$$\mathbf{p} = \mathbf{C} \otimes \mathbf{M} \cdot \mathbf{i} \otimes \mathbf{k} \otimes \mathbf{L} \cdot \mathbf{J} \otimes \mathbf{V} \cdot \mathbf{g} \tag{21}$$

where

J A matrix of final demand coefficients including import $(\mathbf{F_{DK}} + \mathbf{F_M})(\hat{\mathbf{g}}^{-1})$

V **t**he share of domestically produced final demand $\mathbf{GJ}^{-1}$

Results of the SDA is given in table x, where it is compared with the results from the original SDA (15)

5.2 Experiment 2

As a second experiment equation 19 presented above is considered.

$$\mathbf{p} = \mathbf{h} \otimes \mathbf{L} \cdot \mathbf{G} \cdot \mathbf{o} \otimes z \tag{19}$$

where

h is the emission intensity by industry $\mathbf{Si}(\mathbf{x}^{-1})$

Results of the SDA is given in table x, where it is compared with the results from the original SDA (15)

Table 4 **Values of the determinants in three different SDA's of the change in CO_2 emissions between 1980 and 2007.**

	M	h	k	L	G	J	V	o	z	g
					Million tonnes CO_2					
SDA (15)	-12.97		-14.97	-5.57		-7.89		2.02	39.13	
SDA (21)	-13.20		-15.22	-5.87		-6.36	-2.22			42.68
SDA (19)		-27.51		-5.86		-7.96		1.81	39.34	

Conclusion

The result of the comparison is that the method seems to be very robust to ggregation and disaggregation of variables in the model. Splitting one variable in two other components like in the import experiment has almost no effect on the values of the other variables. The same thing is true for the aggregation of the three environmental variables into one.

6. Sensitivity related to the aggregation level of the data.

The Danish input-output tables are compiled at the level of 130 industries which is the most detailed level of the national accounts. At the same time many years of work with the Danish NAMEA tables has secured that physical data is available at the exact same level.

So most of the SDA's being carried out are on this level of aggregation. However, in some cases it might be handier to run an SDA at a higher level of aggregation. It could be that some new data that needs to be included only comes at a higher level of aggregation or there could be other reasons.

In the following we will be looking at an experiment, where the exact same SDA has been run at three different levels of aggregation, namely the 130, 52 and 26 sector levels that are standard Danish classifications. Comments to all of the figures in total appears under the last of the 4 graphs.

Figure 10 The effect of energy mix at various aggregation levels

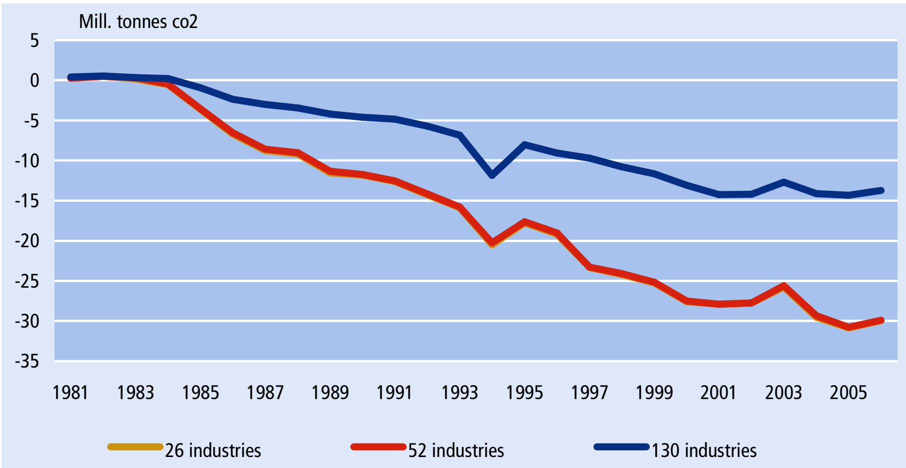


Figure 11 The effect of energy intensity at various aggregation levels

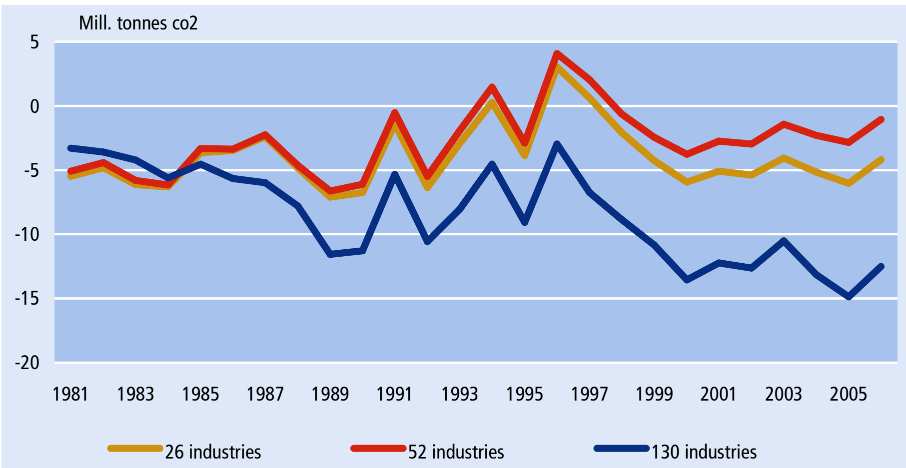


Figure 12 The effect of changes in the structure of input at various aggregation levels

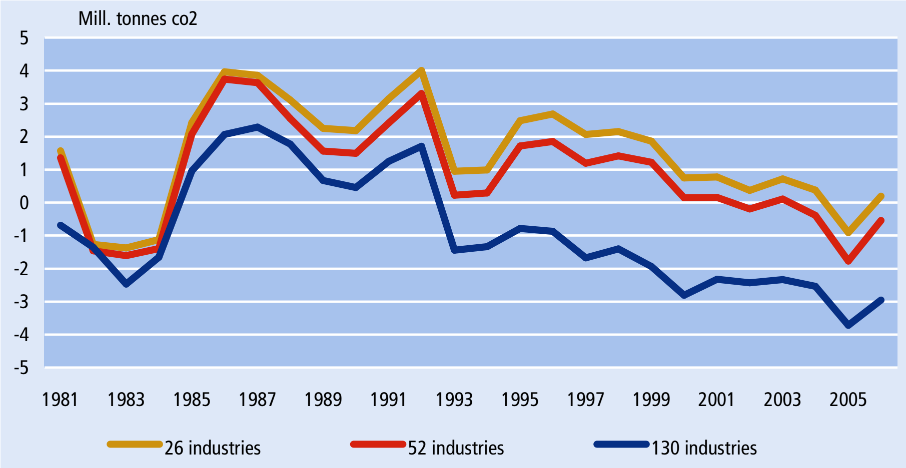
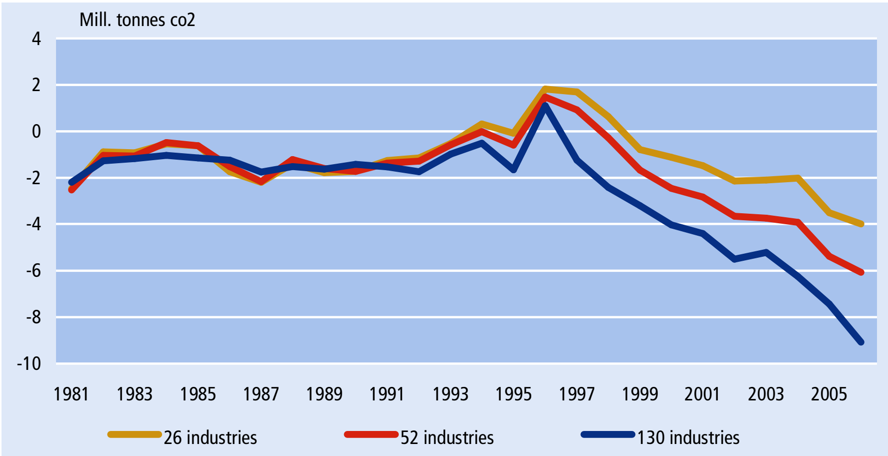


Figure 13 The effect of changes in the structure of final demand at various aggregation levels



Two graphs left out

The graphs on the composition as well as the total of the final demand have been left out since these variables have no reference to the sector level.

Huge differences but changes in emissions are fully explained

It is obvious from all four graphs that the level of aggregation has a considerable effect on the results. It should be noted that in all three cases the change in the CO_2 emissions over time is fully explained. It means that if in one analysis the explanatory power from one of the determinants is considerably lower then it will be higher for one or more of the other determinants.

Mix of energy

In the case of the contribution of the changes in the mix of energy consumed, the answer in the 26 and 52 sector based analyses would be that it has contributed with 15 million tonnes of CO_2 in 2007 as compared to 1980, whereas in the 130 sector based analysis the conclusion would be that the contribution from the determinant to the national total has been twice as big, namely 30 million tonnes.

Energy intensity

Also in the case of energy intensity there are huge differences. At the 130 sector level we see a contribution of about 12 million tonnes whereas on the 52 sector level the effect is almost completely vanished.

The rest of the determinants

Looking at the rest of the graphs it appears that the effects are more or less the same as in the first two cases. The 130 sector level sticks out more from the other two than they stick out from each other.

Figure 14 Contributions to the determinant "Changes in the composition of energy used" from two sectors

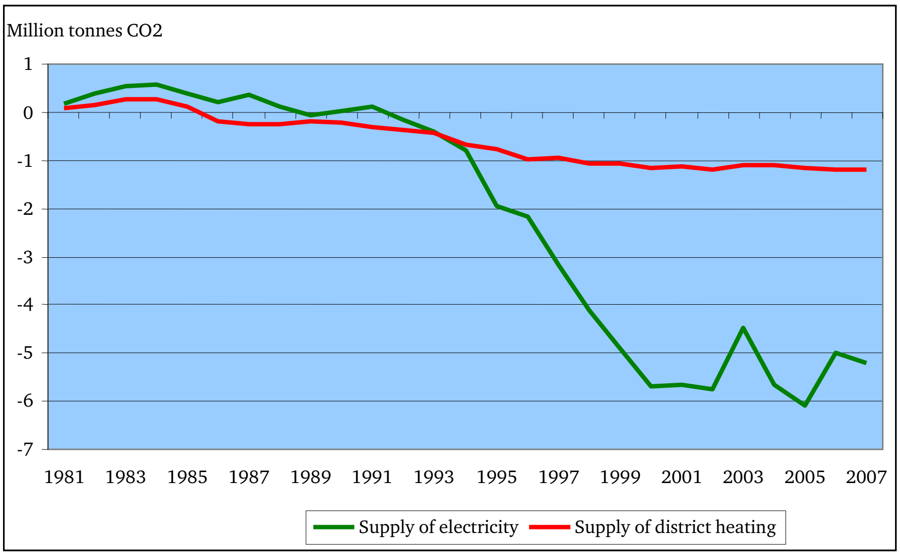
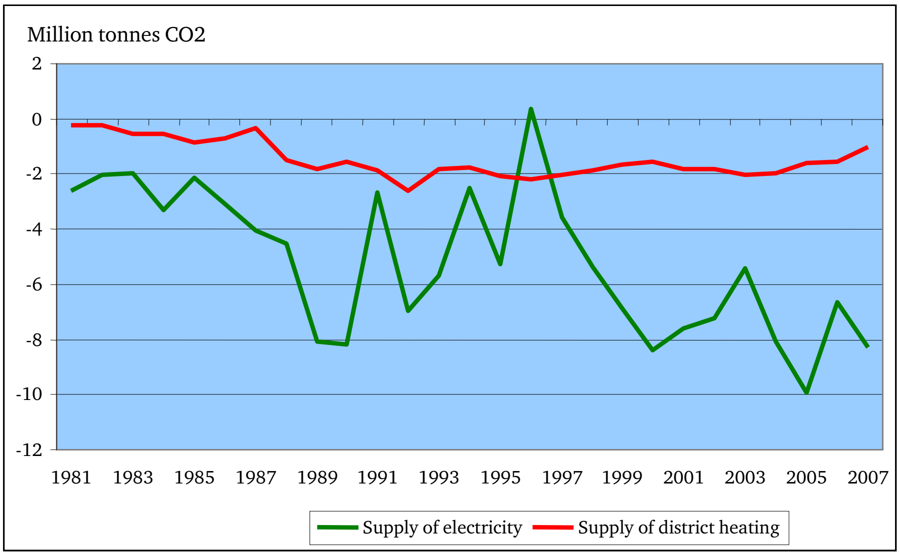


Figure 15 Contributions to the determinant "Changes in the energy intensity" from two sectors



An explanation Experience from looking at these results would say that probably an answer can be found in the sectors that supply power and district heating. Because they are responsible for a very large share of total emissions and at the 130 sector level they are split while at the 52 sector level as well as at the 26 sector level they are aggregated together. The one explanation that completely dominates in all cases is that the supply of electricity and the supply of district heating should not be put together.

The electricity supply series varies quite a lot but in an aggregation together with the district heating series this variation in the supply of electricity will be dampened. When some of the variation in the electricity supply series is dampened the variation that can actually be spotted in the CO_2 emissions series, some of the other determinants must "take over" the explanation of the variation and the general conclusions may be affected.

Conclusion It can be concluded that SDA's should always be carried out at the most disaggregated level possible. Or at least should care be taken that the most dominating sectors in terms of use of energy and emissions of CO2 should not be added together.

7. Sensitivity to the choice of deflation for the economic variables in quantities.

7.1. Economic variables in quantities - values in fixed prices.

Introduction

It is fundamental for a structural decomposition analysis that the economic variables are measured in quantities i.e. some kind of fixed prices. Otherwise, changes in prices will influence the result to an extent which makes it hard to conclude anything from the results. However, there are now more than one set of fixed price series available, based on different deflation methodologies. It may have some influence on the results of the SDA which methodology is chosen. Therefore, in this chapter the different methodologies are discussed and the extent of their influence on the results of the analysis is evaluated.

Through the deflation procedure the economic variables in the National Accounts, values are split in their price part and their quantity part (which is represented by the variable in fixed prices). Variables in fixed prices represent an evaluation of the flow of goods and services measured in the prices of some fixed base year. This implies that values in fixed prices are normally only compiled for those parts of the National Accounts that are concerned with real transactions.

Two calculations of values in fixed prices

Every year two sets of fixed price calculations are carried out at Statistics Denmark.

- The first one is that all values are calculated in the prices of the previous year. On the basis of these values growth rates that are the basis for the calculation in chained values are assessed.
- By the second calculation, values in the prices of a fixed base year (2000) are still calculated.

Which type is more relevant?

In the fixed price calculation with a fixed base year, the weights of the single prices are constant through the entire period. Therefore, the further away from the base year, the year under investigation is, the less likely is it that the weights of the base year are relevant to use for the current year. This is the major objection towards this method. On the other hand, it is possible to calculate the growth between two arbitrary years directly which is one of the major advantages of this method.

The calculation of fixed prices, chained values on the basis of previous years prices is an index composed of a series of chained Laspeyres quantity index describing the year to year growth with the prices from the starting year as weights. The growth between to periods not immediately following each other can only be found by multiplying all Laspeyres quantity index that lies between the two periods. Thus, the total index has no fixed weight structure. The changing weight structure is the main advantage of this index because it gives the most relevant estimates of the growth from period to period.

Chained quantity index not additive

A major disadvantage of the chained quantity index is that once it has been used on variables they are no longer additive as they usually are in current prices or standard fixed prices. In other words the sum of elements of a chained variable will not equal the chained sum of the same variable. This is a serious problem in relation to input-output tables because all the standard arithmetic operators cannot be used, and so in practice it is not possible to work with input-output tables in fixed prices, chained values. In addition there are even some practical problems already in the compilation of such tables. This will be discussed further later in this chapter.

… but variables in fixed prices, chained value is reality

However, many countries have gradually changed from compiling variables in fixed prices on the basis of a fixed base year to compilation of variables in previous year's prices and in fixed prices, chained values.

Almost all data now published in fixed prices, chained values

That has also happened in Statistics Denmark, where the official GDP growth has been based on GDP measured in fixed prices, chained values since 2004-2005. Almost all other economic variables published by Statistics Denmark are now measured in chained values as a supplement to current prices.

… but not input-output tables

The only noticeable exception is the input-output tables. They are published in three price levels, current prices, fixed prices, base year (2000) and previous year's prices.

In the following chapter it is investigated how SDA's can be calculated without the traditional fixed prices on the basis of a fixed base year. It is possible to carry out the analysis on the basis of input-output tables in previous year's prices right away. However, compiling and using input-output tables in chained values requires quite a bit more work. Both methods will be elaborated below.

7.1 Construction of input-output tables in fixed prices, chained values

Compilation method

Input-output vectors and matrices can be compiled in constant prices, chained values as the product of the value in the initial constant price year (base year) and the multiplicative chain of yearly Laspeyres indicies of the volume changes. The annual chain elements are compiled as the current years output measured in prices of the previous year divided by output in the previous period measured in that periods price.

$$\mathbf{x}_t^f = \mathbf{x}_s \cdot \prod_{u=s+1}^{t} \left(\frac{\mathbf{x}_u^p}{\mathbf{x}_{u-1}} \right) \tag{22}$$

where

$\mathbf{x}_t^f$ is the constant price, chained value version of total output in period t.

$\mathbf{x}_t^p$ is total output in period t measured in previous years prices

s is the base year, which in most calculations will be 2000.

The same method of course applies to all other input-output elements as well including the matrices. In principle this can be done with matrix operations, but first the entire series of matrices and vectors has to be checked for possible zero elements.

Zero elements

The calculation of constant prices, chained values does not by definition take care of possible zero elements in the chain. A full series of positive elements is normally required. Obviously, if just one of the z_{ij}'s in the chain of **Z** matrices is zero then the whole chain will brake for this element from the year where the zero first appears and all consecutive years. This is the case because of multiplicative structure of the calculation.

This is a problem about the method of calculating constant prices as a multiplicative chain of values, especially when dealing with large disaggregated input-output matrices that are inherently somewhat sparse. In the Danish case with a 130*130 matrix of intermediate deliveries there are 16900 cells, of which, around 10 pct., is zero.

Consequences of zero elements

There are various situations where zero elements influence the calculation

- If in the year under investigation the numerator i.e. the previous year value $z_{ij,t}^p$ equals zero. Then it does not matter if any of the previous elements in the chain are zero, because also the result $z_{ij,t}^f$ should equal zero.

- If $z_{ij,t}^{p} > 0$. Then the result $z_{ij,t}^{f}$ should **not** equal zero, but it may not always be the case for a couple of reasons as indicated below in the table.

Table 4 **Fictitious scenarios for calculating elements of the intermediate consumption input-output table in constant prices, chained values when certain elements are zero.**

	Result = (1)..(5) multiplied	Value, base year (1)	Chain element, period 1 (2)	Chain element, period 2 (3)	Chain element, period 3 (4)	Chain element, period t (5)
	$z_{ij,t}^{f}$	$z_{ij,0}^{f}$	$\frac{z_{ij,1}^{p}}{z_{ij,0}}$	$\frac{z_{ij,2}^{p}}{z_{ij,1}}$	$\frac{z_{ij,3}^{p}}{z_{ij,2}}$	$\frac{z_{ij,t}^{p}}{z_{ij,t-1}}$
a)	0	100	$\frac{110}{100}$	$\frac{0}{120}$	$\frac{130}{0}$	$\frac{140}{135}$
b)	0	0	$\frac{0}{0}$	$\frac{0}{0}$	$\frac{0}{0}$	$\frac{110}{0}$
c)	0	100	$\frac{0}{100}$	$\frac{0}{0}$	$\frac{0}{100}$	$\frac{110}{0}$

a) In this case the value of the cell z_{ij}^{f} in period *t* is zero, because the chain is broken in period 2, where the value is zero and in period 3 where it is not defined, because the denominator is zero. This is clearly wrong, because the series has an initial value of 100 and a value of 140 in period 4 (*t*) measured in the prices of period 3. So a remedy seems to be necessary to solve the zero-element problem.

b) In this case the only value that appears is in period t. Now it must be decided if the result should be zero because there is no value in period 0 and some of the later periods, or if it should just equal the value in period *t* measured in previous year's prices? Clearly the latter must be the right option, and therefore a solution must be found, so 110 be the result instead of 0.

c) This series (shaded grey) includes a mistake that is, however, not completely unlikely to happen, dealing with very small and insignificant numbers. Thus, the denominator in period 3 has a positive value despite the fact that the numerator in period 2 is zero. The result is zero anyway even though the initial value is 100 and the value in period *t* is 110.

A solution: replacing zeros by ones

In order to overcome the obvious problems illustrated by the scenarios above it is suggested to replace the zero-elements by ones. As long as there are no mistakes in the series (like in the grey scenario c above) replacing zeros with ones has a small influence on the result. However, if the "1" is replaced with a much smaller number like e.g. 0.0000001 the same effect will be obtained, but it will have no influence at all on the result. However for the simplicity of the example the "1" is kept.

An insertion of a one in all cells where it is required will enable a continuation of the multiplicative series. The mission of the ones is so to say to "keep the series alive" and as such, a replacement by any other number would also do the trick.

In table 5 the zero-elements in table 4 have been replaced by ones. So now it is possible to get some sensible results out of scenarios a and b. But for scenario c we get the result 1.1 even though the value in year *t* is 110. This is due to the mistake between period 2 and 3 mentioned above. So clearly the insertion of ones here helped to get a value, but it is still erroneous.

Table 5. **Fictitious scenarios for calculating elements of the intermediate consumption input-output table in constant prices, chained values when certain elements are zero As table 2, but with zero-elements replaced by ones.**

	Result = (1)..(5) multiplied	Value, base year (1)	Chain element, period 1 (2)	Chain element, period 2 (3)	Chain element, period 3 (4)	Chain element, period t (5)
	$z^f_{ij,t}$	$z^f_{ij,0}$	$\frac{z^p_{ij,1}}{z_{ij,0}}$	$\frac{z^p_{ij,2}}{z_{ij,1}}$	$\frac{z^p_{ij,3}}{z_{ij,2}}$	$\frac{z^p_{ij,t}}{z_{ij,t-1}}$
a)	123.6	100	$\frac{110}{100}$	$\frac{0}{120}$	$\frac{130}{0}$	$\frac{140}{135}$
b)	110	1	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{110}{1}$
c)	1.1	100	$\frac{1}{100}$	$\frac{1}{1}$	$\frac{1}{100}$	$\frac{110}{1}$

Even very small inconsistencies can pose a problem

So it has to be ensured that all series are "sound" in the respect that for each variable in each period from 0 to t there is a proper correspondence between the value in current prices and previous year's prices. Naturally, that is something that is being taken care of every day in the national accounts divisions, but if the inconsistent numbers are sufficiently small they will probably not be considered a problem for the national accounts. But as we have seen, it can be a problem for the calculation of chained values.

A solution

One solution that has been used in this report is to apply the "1" or actually a 1E-06 to all cells that are below some threshold. Due to rounding errors and other mathematic and computer technicalities a lot cells in the Danish input-output tables are populated with very small non-zero numbers that can disturb an exercise like this.

7.2 Lack of additivity of vectors and matrices in chained values

Now we have the vectors and matrices in fixed prices, chained values, but they are actually not very usable as we shall discuss in this section.

Consider a matrix **F** with i rows and j columns measured in current prices. The row sums of the matrix are equal to the elements of the vector **v**.

$$\mathbf{f} = \sum_{j=1}^{k} \mathbf{F}_j \tag{23}$$

Due to the additivity problem, this is not true when the vector **f** as well as the matrix **F** is expressed in constant prices chained values.

$$\mathbf{f}^f \neq \sum_{j=1}^{k} \mathbf{F}^f_j \tag{24}$$

where

$\mathbf{f}^f$ the vector **f** expressed in constant prices, chained values
$\mathbf{F}^f$ the matrix **F** expressed in constant prices, chained values

A solution

So these matrices cannot be applied in standard input-output matrix operations because results are simply not consistent. In relation to the discussion in the next paragraph it is shown how specific weights can be applied to each of the elements in the matrix in order to make it regain its additity.

7.2.1. Additivity of rows of intermediate consumption and final demand.

Consider now the two standard matrices **Z** and **F** of intermediate consumption and final demand. The row sums of the two matrices are equal to the output by industries expressed by the elements of the vector **x**.

$$\mathbf{x} = \sum_{j=1}^{k} \mathbf{Z}_j + \sum_{j=1}^{l} \mathbf{F}_j \qquad (25)$$

where

- $\mathbf{x}$ is a vector of output by industries
- $\mathbf{Z}_j$ is the matrix of deliveries of intermediate consumption from the i^{th} to the j^{th} industry.
- $\mathbf{F}_j$ is the matrix of deliveries from the i^{th} industry to the j^{th} final demand category.

The identity (25) is true for current prices, for traditional constant prices and for matrices in previous year's prices. But is it also true in the case of matrices in constant prices, chained values?

Non additivity

The constant price, chained values calculated by formula (22) above constitutes the cells of the input-output tables **Z** and **F**. When measured in current prices, traditional constant prices or in previous years prices it is possible to sum the row values to get the output by industry. However, it is not possible to do the same with matrices Z^f and F^f because they are not additive over rows. It means that the output vector in constant price, chained value is not equal to the row sums of the two matrices.

$$\mathbf{x}^f \neq \sum_{j=1}^{k} \mathbf{Z}_j^f + \sum_{j=1}^{l} \mathbf{F}_j^f \qquad (26)$$

The reason for this is that the structure of the industries, or in other words the weight with which they enter in the calculations, changes from year to year contrary to the traditional constant price calculations, where the structure of the base year is being kept constant over the series.

How to sum the rows anyway

Thus, in order to use these matrices for input-output model calculations it is necessary to apply a special summation formula. It is possible to calculate the sum of two components in constant prices, chained values and obtain the same sum as the constant price, chained value of the aggregate by using this formula. We start with (25) and suppose that it also holds in previous years prices marked "d". The value in period 1 is

$$\mathbf{dx}_1 = \sum_{j=1}^{k} \mathbf{dZ}_{j,1} + \sum_{j=1}^{l} \mathbf{dF}_{j,1} \qquad (27)$$

The chained value in year 1 is calculated as (where the superscript "f" indicates a chained value)

$$\mathbf{x}_1^f = \mathbf{x}_0^f \frac{\mathbf{dx}_1}{\mathbf{x}_0} \qquad (28)$$

If we insert (28) in (27) we get

$$\mathbf{x}_1^f = \mathbf{x}_0^f \frac{\sum_{j=1}^{k} \mathbf{dZ}_{j,1} + \sum_{j=1}^{l} \mathbf{dF}_{j,1}}{\mathbf{x}_0} \qquad (29)$$

After some more work with this formula we end up with

$$\mathbf{x}_1^f = \mathbf{x}_0^f \left[\left(\frac{\sum_{j=1}^{k} \mathbf{Z}_{j,1}^f}{\sum_{j=1}^{k} \mathbf{Z}_{j,0}} \cdot \frac{\sum_{j=1}^{k} \mathbf{Z}_{j,1}}{\mathbf{x}_0} \right) + \left(\frac{\sum_{j=1}^{k} \mathbf{F}_{j,1}^f}{\sum_{j=1}^{k} \mathbf{F}_{j,0}} \cdot \frac{\sum_{j=1}^{k} \mathbf{F}_{j,1}}{\mathbf{x}_0} \right) \right] \tag{30}$$

Use of the summation formula in input-output calculations

The summation formula will be very inconvenient to use or build into standard input-output calculation procedures that involve a lot of summation. Therefore it has been the idea in this project to apply the summation formula above to compile weights to each of the elements in the constant prices, chained values in order to make the matrices additive in exactly the same way as the traditional constant price matrices. Therefore, based on the summation formula, we introduce specific weights to each element in the matrices

$$x^f = \sum_{j=1}^{k} \boldsymbol{\alpha}_{ij} z_{ij}^f + \sum_{j=1}^{l} \boldsymbol{\beta}_{ij} f_{ij}^f \tag{31}$$

where

$\boldsymbol{\alpha}_{ij}$ is the weight to apply to the corresponding elements in the B^f matrix

$\boldsymbol{\beta}_{ij}$ is the weight to apply to the corresponding elements in the F^f matrix

The question now is how compile these weights. We apply the formula (30) to get

$$x_{i,1}^f = x_{i,0}^f \cdot \left(\left(\frac{\sum_{j=1}^{k} z_{ij,1}^f}{\sum_{j=1}^{k} z_{ij,0}^f} \cdot \frac{\sum_{j=1}^{k} z_{ij,0}}{y_{i,0}} \right) + \left(\frac{\sum_{j=1}^{k} f_{ij,1}^f}{\sum_{j=1}^{k} f_{ij,0}^f} \cdot \frac{\sum_{j=1}^{k} f_{ij,0}}{y_{i,0}} \right) \right) \tag{32}$$

The elements in the vector output by industries $x_{i,1}^f$ in constant prices, chained values equals its own previous year value $x_{i,1}^f$ times the product of row sums from both matrices weighted together in an appropriate way. Equation 32 can be organised differently with fewer summation signs

$$x_{i,1}^f = x_{i,0}^f \cdot \left(\left(\sum_{j=1}^{k} \frac{z_{ij,1}^f}{z_{ij,0}^f} \cdot \frac{z_{ij,0}}{y_{i,0}} \right) + \left(\sum_{j=1}^{k} \frac{f_{ij,1}^f}{f_{ij,0}^f} \cdot \frac{f_{ij,0}}{y_{i,0}} \right) \right) \tag{33}$$

Now the previous year value $x_{i,0}^f$ of the output vector can be included under the summation sign and the only variable in the price of the current year can be isolated and then we have

$$x_{i,1}^f = \sum_{j=1}^{k} \frac{z_{ij,0}}{z_{ij,0}^f} \cdot \frac{x_{i,0}^f}{x_{i,0}} \cdot z_{ij,1}^f + \sum_{j=1}^{k} \frac{f_{ij,0}}{f_{ij,0}^f} \cdot \frac{x_{i,0}^f}{x_{i,0}} \cdot f_{ij,1}^f \tag{34}$$

which can be reduced to

$$x_{i,1}^{f} = \cdot\sum_{j=1}^{k} \boldsymbol{\alpha}_{ij,1} z_{ij,1}^{f} + \cdot\sum_{j=1}^{k} \boldsymbol{\beta}_{ij,1} f_{ij,1}^{f} \tag{35}$$

where

$$\boldsymbol{\alpha}_{ij,1} = \frac{z_{ij,0}}{z_{ij,0}^{f}} \cdot \frac{x_{i,0}^{f}}{x_{i,0}} \quad = \frac{\dfrac{z_{ij,0}}{x_{i,0}}}{\dfrac{z_{ij,0}^{f}}{x_{i,0}^{f}}} \tag{36}$$

and

$$\boldsymbol{\beta}_{ij,1} = \frac{f_{ij,0}}{f_{ij,0}^{f}} \cdot \frac{x_{i,0}^{f}}{x_{i,0}} \quad = \frac{\dfrac{f_{ij,0}}{x_{i,0}}}{\dfrac{f_{ij,0}^{f}}{x_{i,0}^{f}}} \tag{37}$$

Thus with (35) we have arrived at the formula we were looking for, and we have found a way to calculate the weights α and β. It appears from (36) and (37) that each of the α's and β's express the relationship between the row share of the element in current prices and the row share of the element in constant prices, chained values.

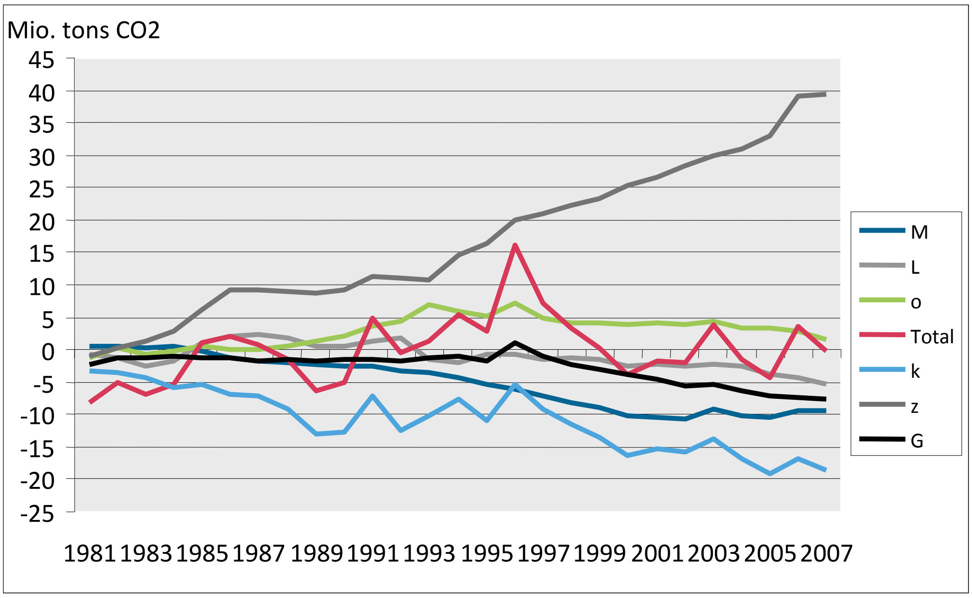
Note that the weights for period 1 input-output elements are expressed entirely in terms of values from period 0, the previous period so it is possible to use the correction for modelling work.

Thus, with correction of all the input-output matrices by correction matrices we can work with input-output algebra as we are used to.

7.3 Comparison of SDA constant prices, previous years prices and chained values

Three different SDA's have been carried out with three different sets of economic variables. The differences lie in the way that the constant prices have been compiled. The physical variables are the same in all three studies except for the energy intensity which of course is the relationship between energy consumption by industry and the total output by industry in the appropriate value.

Figure 16 SDA of the changes in CO_2 emissions in Denmark 1980 to 2007 based on economic variables measured in constant 2000 prises prices



M: Changes in composition of energy used
o: Final demand categories share of total final demand
z: Total final demand
L: Leontief inverse matrix
k: Energy intensity
G: Structure of final demand

Total: The sum of the determinants above=actual change in emission since 1980.

Already in chapter 3 there was a discussion of some of these results. In total the emissions are at exactly the same level in 2007 as they were in 1980. However, lots of factors have pulled in both upward and downward direction in the meantime.

Economic growth pulls strongly

The underlying growth of the economy represented by the total final demand would have increased the emissions in 2007 by 40 million tonnes compared to 1980, had it not been for other factors in the economy.

Smaller pull by changing shares of final demand categories

The shares of final demand categories changed in such a way that it had an increasing effect on emission. This is primarily due to growing share of exports which tend to be more emission intensive than production for domestic final demand. In the middle of the period this effect was almost 7 million tonnes, but in recent years it seems to have come down again.

Energy intensity pulls down

In the downward direction the largest contribution came from the improvement in the energy efficiency represented by a gradually declining use of energy per unit of produced output. This effect contributed with about 20 million tonnes in 2007.

Notice that the energy intensity which is the only variable that has elements of the physical side as well as the economic side are the only variable that consequently displays the same variation that can be observed in the series "Total" that is the development in the actual emissions.

Changes towards more environmentally friendly energy mix

The switch especially from fuel oil to natural gas has lowered emissions quite a bit together with the general change towards more sustainable energy in the shape of especially straw and wind power.

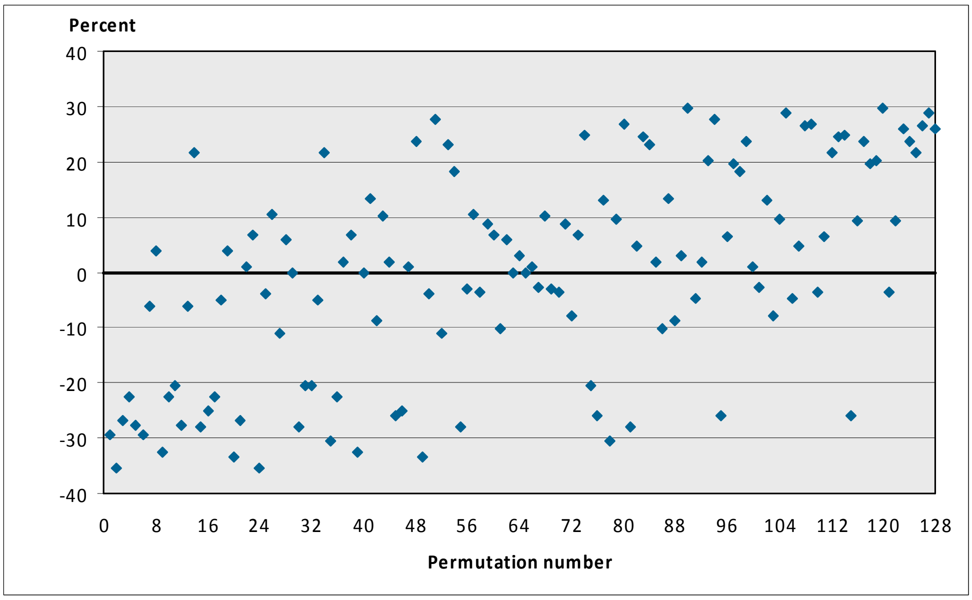
Structures also help to improve the emission account

Finally, the production structure as well as the structure of final demand has had a positive effect on the overall emission account as well. This energy mix effect has accounted for a decline in emissions of about 10 millions tons annually for the last 8-10 years.

Same results with other fixed price variables?

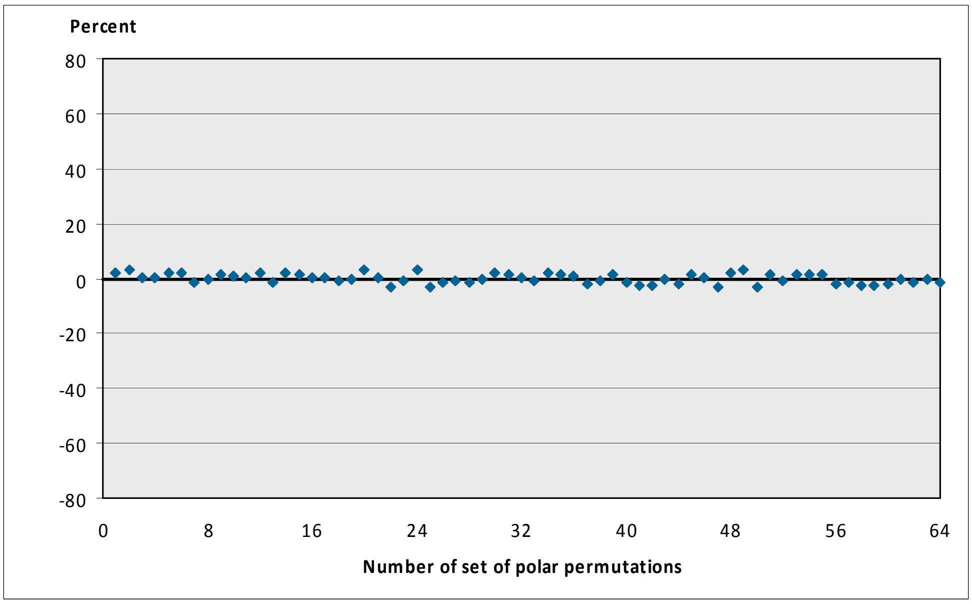
Now the fixed 2000 prices have been exchanged with variables in previous year's prices. The exiting part is whether or not these results above are robust towards this change in the economic variables. Let us first look at how we are now doing in terms of the same figure that was presented in chapter 4

Figure 17 Contribution from changes in the composition of types of energy used. Percentage deviation from the mean of 128 permutations. Effects observed for 2007 with base year 1980. Based on variables in previous year's prices.



This graph shows that the average estimate now lies within approximately ±30 pct. instead of the previous -250 pct. to + 100 pct. This is a lot better. Notice however, the "trend" in the observations that tend to grow as the permutation number increases. This is the previously mentioned drift from a Paasche index to a Laspeyres index that obviously influences the estimates.

Figure 18 Contribution from changes in the composition of types of energy used. Percentage deviation from the mean of 128 permutations. Effects observed for 2007 with base year 1980. Based on variables in previous year's prices.

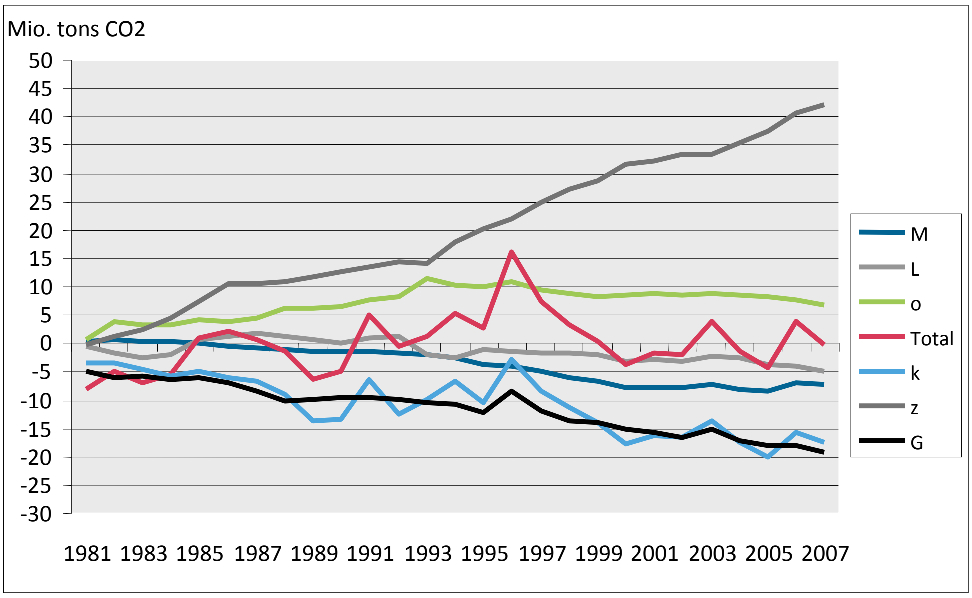


Now there really not much variation left.

Previous year's prices, slight change in the method

Variables in previous year's prices are in themselves current prices variables with an embedded price trend that is poison for an SDA. Therefore the decomposition method is marginally different from the case with fixed 2000 prices. In that case the procedure reads the base year e.g. 1980 once and then carries out the calculation by looping through the remaining years one by one and calculates an isolated result for each year. In the case of previous years prices the first loop is a comparison between 1980 and 1981 like before, but in the next loop 1982 is compared with 1981 and so on. In order to obtain a result for years after 1981 all of the years in between are necessary and have to be calculated and added together.

Figure 19 SDA of the changes in CO_2 emissions in Denmark 1980 to 2007 based on economic variables measured in previous years prices



M: Changes in composition of energy used
L: Leontief inverse matrix
o: Final demand categories share of total final demand
k: Energy intensity
z: Total final demand
G: Structure of final demand

Total: The sum of the determinants above=actual change in emission since 1980.

Results for the physical variables are the same

At first sight the results have not really changed in comparison with figure 19 above. But a closer look reveals that the results for the physical variables are pretty much the same. Only the "M" variable contributes a little less now.

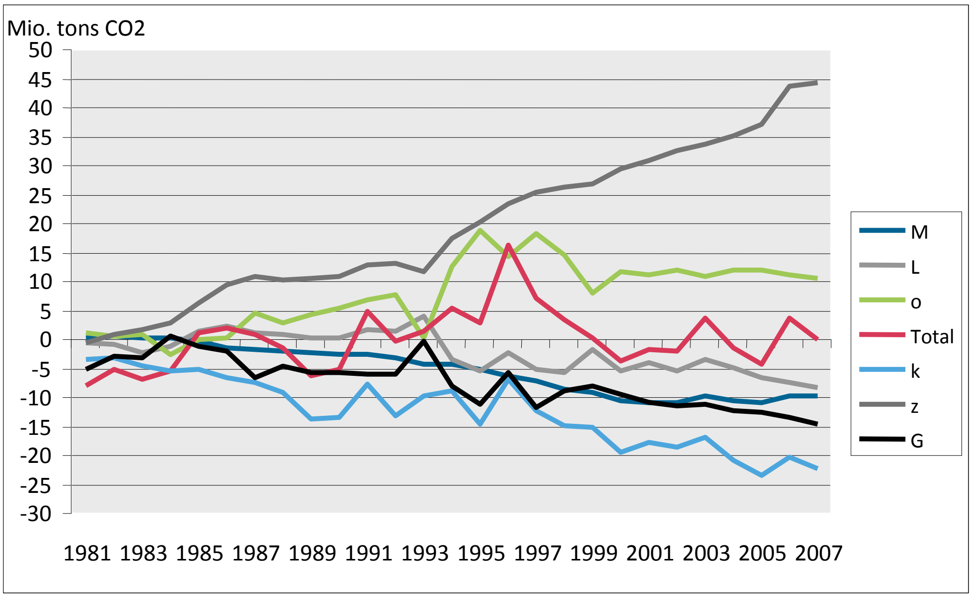
Economic variables are a more extreme

The results for the economic variables are all somewhat more extreme in this case. In 2007 the contribution from the matrix with the structure of the final demand now is the largest contributor to the downward trend in emissions. It has not been investigated in depth why this variable now turns out to be so important.

Exports dominates all determinants

It was investigated for the fixed 2000 prices case which ones of the 106 different final demand components that contributed to the 7 million tonnes decrease in emission in that SDA. A little surprisingly it turned out, that export was responsible for 5.6 out of the 7 million tonnes negative effect on emissions. Actually it also contributed to the largest decrease through the energy intensity. But at the same time it really contributed heavily to the upward trend through both the composition of final demand determinant and through the total final demand. So in the case of previous year's prices the picture is probably the same with exports pulling a lot in both directions.

Figure 20 SDA of the changes in CO_2 emissions in Denmark 1980 to 2007 based on economic variables measured in fixed prices, chained values (corrected to be additive)



M: Changes in composition of energy used
o: Final demand categories share of total final demand
z: Total final demand
L: Leontief inverse matrix
k: Energy intensity
G: Structure of final demand

Total: The sum of the determinants above=actual change in emission since 1980.

Chained value matrices

In the case of using the corrected chained value matrices one effect seems to be different from the other cases. The composition of final demand has a somewhat more dominating position, at least in the middle of the period. What really is behind this effect has not been investigated yet.

In order to get a better impression of how the three SDA's are doing under the three different types of fixed prises is investigated more in the following table.

Table 6. **Average of 128 permutations of determinants in an SDA of changes in CO_2 emissions 1980 to 2007 with three different types of fixed price calculations. 2007.**

Determinant		Average	Minimum	Maximum	Standard deviation
Composition of energy consumed	A	-10.18	-33.08	-0.22	8.95
	B	-7.07	-9.57	-4.98	1.35
	C	-10.22	-36.93	-0.12	9.37
Energy intensity	A	-18.47	-33.26	-6.77	7.58
	B	-17.90	-20.46	-15.07	1.31
	C	-22.70	-48.26	-7.51	10.67
Production structure	A	-5.59	-11.83	-2.03	2.28
	B	-4.81	-6.07	-3.51	0.82
	C	-9.79	-29.60	-2.46	7.34
Final demand structure	A	-7.71	-15.13	-3.39	3.15
	B	-19.44	-25.07	-14.16	3.50
	C	-16.80	-51.30	-4.61	11.54
Composition of final demand	A	1.79	-2.38	7.47	1.78
	B	6.84	3.03	10.57	3.26
	C	10.23	-1.58	48.12	11.49
Total final demand	A	37.29	21.94	55.09	8.27
	B	41.91	41.10	42.68	0.43
	C	43.25	23.19	82.88	12.89

A: Fixed 2000 prices
B: Previous year's prices, chained results
C: Fixed prices, chained values corrected to additivity

Results are NOT identical

It is clear that the three types of fixed prices yield results that are somewhat similar but there are also considerable differences. The biggest difference is that the standard deviation on the averages is particularly smaller than for the other experiments.

The fact that the previous years prices are based on ever changing weights and that the analysis therefore constantly works with the differences between the current and most recent years makes the averages quite certain.

Changes in the mix of energy contributes less

If the estimates from the previous year's prices analysis are to be trusted more, it seems that the average for changes in mix of energy used are overrated by the two other analyses by about 3 million tonnes in 2007.

The final demand structure looks like it is heavily underrated by the traditional fixed price method which state 8 million tonnes compared to the 20 million tonnes suggested by the previous year's price model.

Contribution by composition of final demand difficult to estimate

The determinant that deals with the how the shares of the final demand components contribute over time to the change in emissions seems to be the hardest one to estimate. There are quite different results from the three methods ranging from 2 to 10 million tonnes.

The corrected chained matrices are not good

Another conclusion is that the standard deviation on the estimates made on the basis of the chained matrices that are corrected to additivity are the absolute highest of the three experiments in the case of all 6 determinants.

Table 7. **Average of 64 mirror images of determinants in an SDA of changes in CO_2 emissions 1980 to 2007 with three different types of fixed price calculations. 2007.**

Determinant		Average	Minimum	Maximum	Standard deviation
Composition of energy consumed	A	-10,18	-17,61	-4,43	3,70
	B	-7,07	-7,30	-6,86	0,12
	C	-10,22	-19,26	-4,22	4,02
Energy intensity	A	-18,47	-23,11	-13,69	2,95
	B	-17,90	-18,08	-17,68	0,12
	C	-22,70	-30,61	-15,44	3,99
Production structure	A	-5,59	-7,31	-4,37	0,58
	B	-4,81	-5,05	-4,57	0,19
	C	-9,79	-16,30	-5,23	3,65
Final demand structure	A	-7,71	-9,28	-6,13	0,82
	B	-19,44	-19,76	-19,12	0,21
	C	-16,80	-28,41	-9,11	4,87
Composition of final demand	A	1,79	0,01	3,67	0,92
	B	6,84	6,50	7,15	0,21
	C	10,23	3,74	23,96	5,17
Total final demand	A	41,91	41,81	42,03	0,06
	B	41,91	41,81	42,03	0,06
	C	43,25	35,40	55,13	4,63

A: Fixed 2000 prices
B: Previous year's prices, chained results
C: Fixed prices, chained values corrected to additivity

The table x are the same as table x except that it deals with the mirrored images of the 128 permutations instead of the 128 themselves. The averages are the same off course, but the standard deviation is smaller. The most extreme observations are levelled out by their counterparts.

8. Conclusion

Structural Decomposition Analyses are used in various situations by Statistics Denmark. There are a number of decisions to be made before the analysis can actually be run and it can be quite challenging to explain to people what this exercise is really about and how to interpret the results. Furthermore the traditional fixed prices are on their way out and will probably not be available in the future.

Sense and sensibility

From time to time, therefore, it is on the basis of these observations only natural to ask oneself if it really makes very much sense to draw any conclusions on the basis of an SDA? Can these conclusions be trusted and can we expect to continue on the basis of alternatives to the traditional fixed price variables? In order to get an answer to these fundamental questions it seems to a very good idea to look at the sensibility of SDA's to the various aspects put forward in this report.

Overall the conclusion must be that SDA is still a very sensible way to draw information from a mixture of physical and economic variables about the development that can be observed in various variables like e.g. emission of CO_2. As an alternative to econometric methods it does supply a lot of information and the results are quite robust.

Previous years values are the way forward

Being forced more or less to change to a new set of fixed price variables seems to be no problem and in addition it seems to provide better results with a lot less deviation from the mean of the estimates. On the other hand it also seems that there is not so much reason to go through the trouble of compiling input-output matrices in chained values and correct them to a degree where they again becomes additive. The estimates are more uncertain than in the case of the traditional fixed prices and they are a lot more uncertain than with tables in previous year's values.

No more base year problems

A shift to matrices in previous years prices will also eliminate the problem with the base year, because it is always the previous years weights that are applied in the calculation and not some far away year with a completely different economic structure.

Aggregation level problem

The problem with the aggregation level is serious and remains. It is of particular importance when two or more of the most dominant sectors are aggregated. But if analyses are carried out on the most detailed level it will not make any trouble. Or at least care should be taken not to aggregate the heaviest polluting industries.

Adding or removing variables ok

Also it must be concluded that adding or deleting variables to ones SDA does not make much trouble either and it seems that the results are quite robust to such changes in the equations.

Do not compile full average of all n! forms

Finally, it must be concluded that although it is recommended in the literature to calculate the average of all n! decomposition forms there does not seem to be very much reason to do so. The average of the 2^{n-1} unique decomposition forms is normally very close to the weighted average of all forms. Even the standard deviation is more or less exactly the same.

References.

De Haan, M. (2001): A Structural Decomposition Analysis of Pollution in the Netherlands. *Economic Systems research*, Vol. 13, No. 2.

Dietzenbacher, E. & Los, B. (1998): Structural Decomposition Techniques: Sense and Sensitivity. *Economic Systems Research*, 10, pp. 307-323.

Rørmose, P. & Olsen, T. (2005) "Structural Decomposition Analysis of Air Emissions in Denmark 1980-2002. "Paper presented at the 15th International Conference on Input-Output Techniques. Beijing, China June 27 to July 1, 2005

Seibel, Steffen (2003): Decomposition Analysis of Carbon Dioxide Emission Changes in Germany - Conceptual Framework and Empirical Results. European Commission. Working Papers and Studies.

Wier, M. (1998): Sources of Change in Emissions from Energy: A Structural Decomposition Analysis. *Economic Systems Research*, 10, pp. 99-111.

Appendix 1.

Total output by industries 2006, chained 2000 prices. Comparison of directly chained results with sums obtained from chained input-output tables.

		Output, 2006. Chained 2000 prices		
		Calculated with chained io-tables	Chained directly	Differece
		DKK, millions		Percent
1	Agriculture	49.695	51.255	3,14
2	Horticulture, orchards etc.	3.827	3.840	0,33
3	Agricultural services; landscape gardeners etc.	7.820	7.926	1,35
4	Forestry	3.452	3.574	3,53
5	Fishing	3.712	3.677	-0,95
6	Extr. of crude petroleum, natural gas etc.	42.545	36.820	-13,46
7	Extr. of gravel, clay, stone and salt etc.	2.640	2.655	0,57
8	Production etc. of meat and meat products	38.332	38.324	-0,02
9	Processing etc. of fish and fish products	9.752	9.738	-0,15
10	Processing etc. of fruit and vegetables	3.755	3.760	0,14
11	Mfr. of vegetable and animal oils and fats	3.871	3.923	1,33
12	Mfr. of dairy products	25.740	25.739	0,00
13	Mfr. of starch, chocolate and sugar products	23.466	23.463	-0,01
14	Mfr. of bread, cakes and biscuits	5.105	5.109	0,07
15	Bakers' shops	3.417	3.419	0,07
16	Manufacture of sugar	-528	2.278	-531,09
17	Mfr. of beverages	8.543	8.527	-0,18
18	Manufacture of tobacco products	3.271	3.269	-0,07
19	Mfr. of textiles and textile products	7.122	7.136	0,20
20	Mfr. of wearing apparel; dressing etc. of fur	2.970	2.968	-0,06
21	Mfr. of leather and leather products	452	463	2,52
22	Mfr. of wood and wood products	15.521	15.643	0,79
23	Mfr. of pulp, paper and paper products	9.177	9.233	0,61
24	Publishing of newspapers	8.417	8.244	-2,06
25	Publishing activities, excluding newspapers	12.889	12.542	-2,69
26	Printing activities etc.	13.332	13.034	-2,23
27	Mfr. of refined petroleum products etc.	20.573	20.039	-2,60
28	Mfr. of industrial gases and inorganic basic chemicals	896	910	1,58
29	Mfr. of dyes, pigments and organic basic chemicals	5.392	5.387	-0,10
30	Manufacture of fertilizers etc.	113	118	4,02
31	Mfr. of plastics and syntethic rubber	462	461	-0,31
32	Manufacture of pesticides and other agro-chemical products	1.892	1.916	1,26
33	Mfr. of paints, printing ink and mastics	2.648	2.638	-0,37
34	Mfr. of pharmaceuticals etc.	40.801	40.491	-0,76
35	Mfr. of detergents and other chemical products	10.016	10.040	0,25
36	Mfr. of rubber products and plastic packing goods etc.	11.360	11.399	0,34
37	Mfr. of builders' ware of plastic	1.931	1.940	0,50
38	Manufacture of other plastic products n.e.c.	8.954	8.952	-0,03
39	Mfr. of glass and ceramic goods etc.	3.086	3.088	0,08

40	Mfr. of cement, bricks, tiles, flags etc.	2.296	2.306	0,44
41	Mfr. of concrete, cement, asphalt and rockwool products	13.471	13.479	0,07
42	Mfr. of basic ferrous metals	1.617	1.629	0,79
43	First processing of iron and steel	1.991	1.998	0,37
44	Mfr. of basic non-ferrous metals	2.827	2.816	-0,39
45	Casting of metal products	983	974	-0,93
46	Mfr. of construct. materials of metal etc.	24.684	24.744	0,24
47	Mfr. of hand tools, metal packaging etc.	15.542	15.571	0,19
48	Mfr. af marine engines, compressors etc.	22.118	22.608	2,22
49	Mfr. of other general purpose machinery	20.229	20.226	-0,01
50	Mfr. of agricultural and forestry machinery	5.260	5.248	-0,23
51	Mfr. of machinery for industries etc.	16.777	16.775	-0,01
52	Mfr. of domestic appliances n.e.c.	3.147	3.149	0,07
53	Mfr. of office machinery and computers	2.912	2.829	-2,86
54	Mfr. of other electrical machinery and apparatus	43.121	42.337	-1,82
55	Mfr. of radio and communicat. equipm. etc.	11.967	12.097	1,09
56	Mfr. of medical and optical instrum. etc.	20.050	20.097	0,23
57	Manufacture of motor vehicles etc.	6.999	7.033	0,48
58	Building and repairing of ships and boats	6.885	6.949	0,93
59	Mfr. of transport equipment excl. ships, motor vehicles etc.	3.237	3.226	-0,34
60	Mfr. of furniture	19.242	19.227	-0,08
61	Mfr. of toys, gold and silver articles etc.	6.405	6.402	-0,04
62	Recycling of waste and scrap	2.420	2.426	0,25
63	Production and distribution of electricity	15.437	15.703	1,73
64	Manufacture and distribution of gas	12.160	12.584	3,49
65	Steam and hot water supply	13.026	13.072	0,35
66	Collection and distribution of water	2.554	2.559	0,22
67	Construction of new buildings	65.584	65.385	-0,30
68	Repair and maintenance of buildings	54.168	54.778	1,13
69	Civil engineering	38.667	38.670	0,01
70	Construction materials	24.505	24.935	1,75
71	Sale of motor vehicles, motorcycles etc.	20.564	20.791	1,11
72	Repair and maintenance of motor vehicles	16.352	16.419	0,41
73	Service stations	1.565	1.534	-2,00
74	Ws. and commis. trade, exc. of m. vehicles	181.330	181.026	-0,17
75	Retail trade of food etc.	19.923	19.862	-0,31
76	Department stores	10.864	10.876	0,11
77	Re. sale of phar. goods, cosmetic art. etc.	4.178	4.003	-4,20
78	Re. sale of clothing, footwear etc.	11.584	11.568	-0,14
79	Other retail sale, repair work	35.038	35.000	-0,11
80	Hotels etc.	10.420	10.741	3,08
81	Restaurants etc.	30.542	30.552	0,03
82	Transport via railways	9.200	9.216	0,17
83	Other scheduled passenger land transport	8.296	8.299	0,04
84	Taxi operation and coach services	6.120	6.131	0,18
85	Freight transport by road and via pipelines	36.189	36.216	0,07
86	Water transport	136.320	136.356	0,03
87	Air transport	17.294	17.217	-0,45
88	Cargo handling, harbours etc.; travel agencies	22.706	22.714	0,03
89	Activities of other transport agencies	14.936	14.943	0,04
90	Post and telecommunications	75.391	75.294	-0,13
91	Monetary intermediation	66.632	66.331	-0,45
92	Other financial intermediation	30.640	30.862	0,72
93	Life insurance and pension funding	8.496	8.563	0,78

94	Non-life insurance	19.692	19.700	0,04
95	Activities auxiliary to finan. intermediat.	11.248	11.271	0,20
96	Real estate agents etc.	5.947	5.906	-0,68
97	Dwellings	119.133	119.133	0,00
98	Letting of non-residential buildings	40.302	40.326	0,06
99	Renting of machinery and equipment etc.	17.688	17.735	0,27
100	Computer activities exc. software consultancy and supply	15.957	15.964	0,04
101	Software consultancy and supply	39.260	39.272	0,03
102	Research and development (market)	5.003	5.009	0,13
103	Research and development (other non-market)	3.442	3.445	0,09
104	Legal activities	8.708	8.779	0,81
105	Accounting, book-keeping, auditing etc.	11.257	11.250	-0,07
106	Consulting engineers, architects etc.	43.669	43.715	0,11
107	Advertising	22.230	22.221	-0,04
108	Industrial cleaning	10.690	11.378	6,44
109	Other business activities	48.181	48.249	0,14
110	General (overall) public service activities	39.443	39.453	0,02
111	Regulation of public service activities exc. for business	16.798	16.852	0,32
112	Regulation of and contribution to more efficient operation of business	15.932	15.959	0,17
113	Provision of services to the community	42.075	42.088	0,03
114	Primary education	43.751	43.752	0,00
115	Secondary education	17.301	17.300	-0,01
116	Higher education	17.119	17.103	-0,09
117	Adult and other education (market)	2.020	2.025	0,25
118	Adult and other education (other non-market)	7.916	7.923	0,09
119	Hospital activities	57.875	57.867	-0,01
120	Medical, dental, veterinary activities etc.	24.939	24.938	0,00
121	Social institutions etc. for children	39.799	39.801	0,01
122	Social institutions etc. for adults	54.971	55.238	0,49
123	Sewage removal and disposal	5.426	5.437	0,20
124	Refuse collection and sanitation	6.571	6.575	0,06
125	Refuse dumps and refuse disposal plants	3.870	3.964	2,42
126	Activities of membership organiza. n.e.c.	16.705	16.689	-0,10
127	Recreational, cultural, sporting activities (market)	34.968	34.190	-2,22
128	Recreational, cultural, sporting activities (other non-market)	9.888	9.883	-0,05
129	Service activities n.e.c	8.543	8.596	0,62
130	Private households with employed persons	1.105	1.642	48,63