

## **Which model for measuring aggregate change should be used in structural decomposition analysis?**

Topic: Environmental IO models 6

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### Introduction

Structural decomposition analysis (SDA), like index number theory (INT), aims at disentangling an aggregate change in a variable into its  $r$  factors. To give an example: in their seminal 1998 article, Dietzenbacher and Los (D&L, 1998) decompose sectoral labor cost into  $r=4$  factors: labor cost per unit; technical changes; final demand mix; and final demand level.

Change can be measured as a ratio or as a difference, which, in INT, leads to indices and indicators, respectively (see Balk, 2008), and which, in SDA, leads to multiplicative and additive decompositions, respectively.

An important question is: which model should be used?

### Part I Theoretical viewpoint

D&L (1998) give as models the two polar decompositions, which in INT are the models of Laspeyres and of Paasche, respectively; the average of all elementary decompositions, which in INT is the (generalized) Fisher model, and the average of the two polar decompositions.

We will give the axioms and the tests that are used in INT. The models of Laspeyres and Paasche do not pass the tests of time reversal and the factor reversal, whereas the average of the two elementary decompositions does not pass the factor reversal test. From a theoretical viewpoint these models are not satisfactory. The average of all elementary decompositions, the generalized Fisher model, passes these tests and, consequently, is a candidate for adoption. De Boer (2008) proposed to use the Montgomery decomposition for an additive decomposition and he proposed the Sato-Vartia decomposition for a multiplicative decomposition (de Boer, 2009). These decompositions are candidates as well. But the Montgomery approach can also be used for a multiplicative decomposition (called Montgomery-Vartia in INT), and the Sato-Vartia approach to additive decomposition. Last, but not least, we mention the Törnqvist index which is a likely candidate as well.

### Part II Application viewpoint

In the quest of the model to be used, besides the theoretical viewpoint, we also pay attention to the application viewpoint: such as simplicity of the model, zero value robustness (can it handle zeros?) and change in sign robustness (a weaker requirement than the negative value robustness, introduced by Chung and Lee, 2001).

### Part III Empirical applications

The first application is the example of Dietzenbacher and Los (1998) that we mentioned in the introduction. The second one is the decomposition of the change of carbon dioxide emissions in the Netherlands (2005 as compared to 2004) into four factors: carbon dioxide emitted per unit of

production in industries; technical changes; share of industries in final demand; and gross domestic product. We give the results for a multiplicative decomposition using the models of Montgomery-Vartia; Sato-Vartia and generalized Fisher, as well as the results for an additive decomposition using the models of Montgomery; additive Sato-Vartia and additive generalized Fisher (which, alternatively, can be named "generalized Bennet").

It turns out that the differences between the various models are minor.

## References

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