Structural decomposition analyses: the differences between applying the semi-closed and the open input-output model

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Abstract. The open and the semi-closed model are two widely used types in input-output analysis. When also induced effects are important, researchers usually choose the semi-closed input-output model for their analysis. This paper examines the differences between using the semi-closed and the open input-output model in a structural decomposition analysis. The empirical part considers the decomposition of the growth in gross output and in labor compensation, both for China (1997-2007) as well as for 35 other countries and regions. Our main findings are threefold. First, we find that for gross output growth, both models yield very similar results for the factors they have in common, such as the changes in the domestic input coefficient matrix. If only the contribution of these common factors is of interest, it does not matter whether the semi-closed or the open input-output model is used. However, the semi-closed input-output model also allows for determining the effects of changes in the labor compensation coefficients on gross output growth, which is not possible in the open model. Second, for the analysis of labor compensation growth, both models include the changes in labor compensation coefficients as one of the explanatory factors and yield significantly different results. Third, unlike the open model, the semi-closed model is able to decompose sectoral consumption growth.

1 Introduction

Input-output (IO) analysis is first introduced by Leontief (1936). It is a useful tool to study industry interdependency and economic structure. In the traditional IO model, all final demand categories are treated as exogenous variables. In other words, the final demands are open to the endogenous module. Hence, the traditional IO model is also called the open IO model. The open IO model only captures the industry linkages, however other relations should also be taken into account in some contexts. If the industry-household linkage is important, researchers usually endogenize household consumption to construct the semi-closed IO model. By endogenizing the household consumption, the semi-closed IO model extends the industry relations further to industry and household relations. Thus, in some literatures the semi-closed IO model is also called the extended IO model (see Batey, Madden and Weeks, 1987; Batey and Rose, 1990). Another extension of traditional IO models is the Social Accounting Matrix (SAM) approach (see Pyatt, 1988). Broader than the semi-closed IO model, the SAM approach extends the industry relations further to institution (firms, households and public sector) relations. In IO models as well as the SAM approach, prices are assumed to be fixed. Based on the SAM database, the Computable General Equilibrium (CGE) model (see Dixon and Parmenter, 1996) further gets rid of the fixed-price assumption when modeling the behavior of different institutions.

It is clear that the semi-closed IO model is a model between the open IO model and its other two extensions (SAM approach and CGE model). This position provides the semi-closed IO model some flexibility in economic analysis. First, it is simpler and easier to implement than the SAM approach as well as the CGE model. Second, many limitations of the open IO model pointed out by SAM researchers have been overcome by working on extended IO models (See Batey and Rose, 1990). Third, the semi-closed IO model can be expressed to be a clear analytical equation. This can help to check how an outcome of a dependent variable yield, especially when a surprising outcome occurred; it also makes it easy to carry out decomposition analyses, which will be introduced later in this study. On the contrary, it is difficult to derive an analytical equation for a variable of interest from the CGE model. Therefore, the numerical simulation have to be used when carrying out decomposition analyses based on the CGE model (see Jensen-

Butler and Madsen, 2005). This is rather complicated relative to decomposing the semi-closed IO model.

Actually, the semi-closed IO model has been widely used by researchers in empirical studies, especially in policy analysis and impact analysis. In these studies, the effect of induced household consumption is usually one of the concerns, and the time horizon usually focuses on the medium-run or long-run to ensure the induced household consumption can release completely. For instance, in order to capture the effect of induced household consumption, Batey et al, (1993) and Yang et al, (2008) choose the semi-closed IO model to evaluate the socio-economic impact of large-scale projects. Dietzenbacher and Günlük-Şenesen (2003) use the semi-closed IO model to assess changes in Turkish production structure and labor income between two periods with different policy strategies. They obtain some findings that cannot be detected with the open IO model, such as the dominance of public services in sectoral gross output multipliers. The application of semi-closed IO model is even more prevalent at a regional level. One important reason is that regional economies are more open than national economies, which reduces the importance of inter-industry linkages relative to the industry-household linkage (Trigg and Madden, 1994). For instance, in the study on the region of Evros in Greece, Hewings and Romanos (1981) find that 50% of the important coefficients are related to the household sector; based on the interregional input-output analysis on the UK economy, McGregor et al (1999) find that the migration effect via the income-consumption relationship is more important than the spillover and feedback trade effect.

The structural decomposition analysis (SDA) is an important technique frequently used in input-output analysis. It can decompose a change in a variable of interest to the effects of the changes in its factors. Based on the decomposition results, researchers can analyze the contribution of each factor on the growth of the variable of interest. SDA is a useful tool to evaluate growth sources, so it has been applied to a wide range of topics. An extensive review of SDA and its early application can be found in Rose and Casler (1996). For recent applications see Wolff (2006), Kagawa et al. (2008) and Yamakawa and Peters (2011). So far, however, to the best of our knowledge all SDA applications are based on the open IO model. Schuman (1994) deems that the reason why literatures on SDA exclusively rely on the open IO model is because decomposing a structural change to isolated clear-cut sources by using the semi-closed IO model is impossible. However, in this study we show that carrying out SDAs by using the semi-closed IO model is still possible.

Empirical studies have shown the importance to take into account the industry-household linkage in corresponding contexts. Since decomposing the semi-closed IO model is possible, then a question comes up. What are the differences between applying the semi-closed model and the open model in decomposition analyses? Actually, there are some potentials for the difference. For instance, a change in domestic input coefficient matrix will cause a change in gross output via industry linkages. This is where the effect process ends in the context of an open IO model. In the semi-closed IO model, however, this change in gross output will further cause a change in labor income and thus cause a change in household consumption. The change in household consumption will further generate a multiplier effect on the gross output. Hence, the decomposition results from both models should be different, but the magnitude of the difference is not obvious. Therefore, in this study we attempt to carry out SDAs by applying both the semi-closed and open IO model to examine the differences between them. Hopefully we can obtain some findings to better guide SDA applications in practice.

The remaining content of this paper is organized as follows. Section 2 introduces the open IO model and the semi-closed IO model. In section 3, we first give the decomposition formulae for the open IO model and the semi-closed IO model. Second, we apply them on Chinese IO tables to investigate the difference between the decomposition results yield from both models in details. Third, we extend our analysis to other countries to investigate the generality of the findings obtained from the Chinese IO tables. In section 4, we show the decomposition of sectoral household consumption growth which can only be done by using the semi-closed model. Section 5 concludes.

2 Models

2.1 The open input-output model

The traditional input-output model is usually expressed as the following form (see Miller and Blair, 2009)

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{L}(\mathbf{c} + \mathbf{g}) \tag{1}$$

with

$\mathbf{x}$: gross output vector

$\mathbf{A}$: domestic input coefficient matrix

$\mathbf{f}$: a vector of final demand on domestic products, which can be further split into a vector of household consumption $\mathbf{c}$ and a vector of other final demands $\mathbf{g}$.

$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$: Leontief inverse matrix

For convenience, unless otherwise stated, when mentioning "final demand", we solely refer to the final demand on domestic products.

In equation (1), all final demand categories are treated as exogenous variables. Thus, they are open to external disturbances relative to the endogenous module $\mathbf{L}$. In this sense, the traditional input-output model is also called the open input-output model. In this study, we call it open model in short.

2.2 The semi-closed input-output model

In an economic system, households earn labor incomes from industries and spend them on the products produced by the industries in well patterned ways. Industries and households are connected together by this income-consumption relationship. The semi-closed input-output model (we call it semi-closed model in short) further takes into account this linkage between industries and households by endogenizing household consumption together with labor income. It can expressed as the following form[1]:

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{w} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & r\bar{\mathbf{c}}\mathbf{i}' \\ \hat{\mathbf{B}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{w} \end{bmatrix} + \begin{bmatrix} \mathbf{g} \\ \mathbf{0} \end{bmatrix}$$

with the solution

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{w} \end{bmatrix} = \begin{bmatrix} \mathbf{I} - \mathbf{A} & -r\bar{\mathbf{c}}\mathbf{i}' \\ -\hat{\mathbf{B}} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{g} \\ \mathbf{0} \end{bmatrix} \tag{2}$$

Compared to the open model, there are some new factors in the semi-closed model:

$\mathbf{w}$: a labor compensation vector with the same dimension as $\mathbf{x}$; the labor compensation is referred to the total bill paid to workers, including compensation of employees and income of self-employed individuals. It is used to measure the labor income of household sector.

$\hat{\mathbf{B}}$: a diagonal matrix generated from a labor compensation coefficient vector $\mathbf{b} = (b_i)_{n\times1}$. b_i denotes the labor compensation per unit gross output of sector *i*.

[1] In this version, we use sectoral labor income instead of total labor income. See Miller and Blair (2009) for the version with total labor income.

r : the ratio between total household consumption and total labor compensation

$\bar{\mathbf{c}}$: a consumption share coefficient vector measured by the consumption share of each sector in total household consumption.

$\mathbf{i}'$: a row summation vector $[1,1,\ldots 1]$

Relative to the open model, the semi-closed model provides its factors an extra effect channel to the gross output, which works via the income-consumption relationship. For instance, the open model predicts that an increase in the export will cause an increase in the gross output via industry linkages. However, the semi-closed model argues that the labor income will also increase during this process, which will cause an increase in the household consumption and further generate an multiplier effect on the gross output. This extra effect on the gross output cannot be captured by the open model.

Since the semi-closed model takes into account more linkages than the open model, we may obtain different information if carrying out SDAs by using both models. Next, we turn to examining the differences between the decomposition results yield from the semi-closed model and the open model.

3 Decompose the open model and the semi-closed model

Our decomposition objects are gross output and labor compensation. For the gross output related variables (such as imports and gas emissions), they equal the multiplication of corresponding coefficients (such as import coefficients and gas emission coefficients) and the gross output. Hence, for the decomposition of these gross output related variables, the decomposition of gross output is the core. For the labor compensation, although it is also a gross output related variable, the role of labor compensation coefficient in the semi-closed model differs from that in the open model. The labor compensation coefficient and the gross output are independent in the open model. In the semi-closed model, however, the labor compensation coefficient is a influencing factor of gross output. Considering this difference, we choose labor compensation as our second decomposition object.

3.1 Model reformulation

In the open model, the expression for gross output is given by Formula (1). It can be further expressed as:

$$\mathbf{x} = \lambda\mathbf{L}\bar{\mathbf{f}} = \lambda\mathbf{L}[\alpha\bar{\mathbf{c}} + (1-\alpha)\bar{\mathbf{g}}] \tag{3}$$

Where $\lambda = \sum_i (\mathbf{f})_i$ is the total final demand; $\bar{\mathbf{f}} = \frac{1}{\lambda}\mathbf{f}$ is a vector of final demand shares; α is the share of total household consumption in total final demand and $(1-\alpha)$ is the share of other final demand in total final demand; $\bar{\mathbf{c}}$ again is the consumption share coefficient vector; $\bar{\mathbf{g}} = \frac{\mathbf{g}}{\sum_i (\mathbf{g})_i}$ is a vector of other final demand shares. Formula (3) distinguishes the structure effect and the scale effect on the gross output. The component $\mathbf{L}[\alpha\bar{\mathbf{c}} + (1-\alpha)\bar{\mathbf{g}}]$, consisting of structural factors, gives the structure effect. We represent it as $\bar{\mathbf{x}}$, namely $\bar{\mathbf{x}} = \mathbf{L}[\alpha\bar{\mathbf{c}} + (1-\alpha)\bar{\mathbf{g}}]$. The structure effect is further blew up by the exogenous volume λ to gross output $\mathbf{x}$. As a scalar, λ gives the scale effect. It has an identical effect on the gross output of each sector, so when decomposing the gross output growth, we mainly focus on the decomposition of the structure effect $\bar{\mathbf{x}}$.

In the context of an open model, the sectoral labor compensation $\mathbf{w}$ can be expressed as

$$\mathbf{w} = \hat{\mathbf{B}}\mathbf{x} = \lambda\hat{\mathbf{B}}\mathbf{L}[\alpha\bar{\mathbf{c}} + (1-\alpha)\bar{\mathbf{g}}] \tag{4}$$

Likewise, the component $\hat{\mathbf{B}}\mathbf{L}[\alpha\bar{\mathbf{c}} + (1-\alpha)\bar{\mathbf{g}}]$ in Formula(4) gives the structure effect on the labor compensation. We represent it as $\bar{\mathbf{w}}$, namely $\bar{\mathbf{w}} = \hat{\mathbf{B}}\mathbf{L}[\alpha\bar{\mathbf{c}} + (1-\alpha)\bar{\mathbf{g}}]$. Scalar λ gives the scale effect. In the same sense, when decomposing the labor compensation growth, we mainly focus on the decomposition of the structure effect $\bar{\mathbf{w}}$.

The expressions for the gross output and the labor compensation can be derived from the semi-closed model simultaneously. Formula (2) gives the solution. In the same way, Formula (2) can be further expressed to be

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{w} \end{bmatrix} = \mu \begin{bmatrix} \mathbf{I}-\mathbf{A} & -r\bar{\mathbf{c}}\mathbf{i}' \\ -\hat{\mathbf{B}} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \bar{\mathbf{g}} \\ \mathbf{0} \end{bmatrix}, \text{ with } \mu = \sum_i (\mathbf{g})_i \tag{5}$$

Let $\begin{bmatrix} \bar{\bar{\mathbf{x}}} \\ \bar{\bar{\mathbf{w}}} \end{bmatrix} = \begin{bmatrix} \mathbf{I}-\mathbf{A} & -r\bar{\mathbf{c}}\mathbf{i}' \\ -\hat{\mathbf{B}} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \bar{\mathbf{g}} \\ \mathbf{0} \end{bmatrix}$, which is our primary concern when decomposing the growth in the gross output and the labor compensation by using the semi-closed model.

After the model reformulation, $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ become the common factors in the gross output expressions of the open model and the semi-closed model, and $\mathbf{A}$, $\bar{\mathbf{c}}$, $\bar{\mathbf{g}}$ and $\hat{\mathbf{B}}$ become the common factors in the labor compensation expressions of both models. Next, we turn to decomposing the gross output growth and the labor compensation growth by applying the open model and the semi-closed model, respectively. The multiplicative structural decomposition analysis (MSDA) will be used. Afterwards, we will see that the MSDA is very convenient for ruling out the scale effect which we are not interested in. An introduction and application of MSDA can be found in Dietzenbacher, Hoen and Los (2000).

3.2 MSDA for the open model

3.2.1 Decompose the gross output growth

We use superscript "1" and "0" to represent the terminal state and the initial state of a variable during a period of interest. Then, the growth of gross output during a period can be expressed as[2]

$$\frac{\mathbf{x}^1}{\mathbf{x}^0} = \frac{\lambda^1}{\lambda^0}\frac{\overline{\mathbf{x}}^1}{\overline{\mathbf{x}}^0}$$

In this expression, our primary concern is $\frac{\overline{\mathbf{x}}^1}{\overline{\mathbf{x}}^0}$, the gross output growth without scale effect. Since the scale effect is ruled out and only structure effect left, we call $\frac{\overline{\mathbf{x}}^1}{\overline{\mathbf{x}}^0}$ the structural growth of gross output. Easily to check, it can be decomposed into the following multiplicative form[3]:

$$\frac{\overline{\mathbf{x}}^1}{\overline{\mathbf{x}}^0} = \frac{\mathbf{L}^1[\alpha^1\overline{\mathbf{c}}^1 + (1-\alpha^1)\overline{\mathbf{g}}^1]}{\mathbf{L}^0[\alpha^0\overline{\mathbf{c}}^0 + (1-\alpha^0)\overline{\mathbf{g}}^0]}$$

$$= \frac{\mathbf{L}^1[\alpha^1\overline{\mathbf{c}}^1 + (1-\alpha^1)\overline{\mathbf{g}}^1]}{\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^1 + (1-\alpha^1)\overline{\mathbf{g}}^1]} \qquad (6.1)$$

$$\times \frac{\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^1 + (1-\alpha^1)\overline{\mathbf{g}}^1]}{\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0 + (1-\alpha^1)\overline{\mathbf{g}}^1]} \qquad (6.2)$$

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[2] For any two matrices (or vectors) $\mathbf{U} = (u_{ij})_{m\times n}$ and $\mathbf{V} = (v_{ij})_{m\times n}$, $\frac{\mathbf{U}}{\mathbf{V}}$ and $\mathbf{U}\times\mathbf{V}$ denotes elementwise division and elementwise multiplication, respectively. Namely, $\frac{\mathbf{U}}{\mathbf{V}} = \left(\frac{u_{ij}}{v_{ij}}\right)_{n\times m}$ and $\mathbf{U}\times\mathbf{V} = \left(u_{ij}\times v_{ij}\right)_{n\times m}$.

[3] The decomposition result varies across different decomposition orders of the determinants. We using the following consistent decomposition orders. For the open model: $\mathbf{A}\to\overline{\mathbf{c}}\to\overline{\mathbf{g}}\to\alpha$ (gross output); $\mathbf{A}\to\overline{\mathbf{c}}\to\overline{\mathbf{g}}\to\hat{\mathbf{B}}\to\alpha$ (labor compensation). For the semi-closed model: $\mathbf{A}\to\overline{\mathbf{c}}\to\overline{\mathbf{g}}\to\hat{\mathbf{B}}\to r$.

$$\times\frac{\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^1]}{\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^0]} \quad (6.3)$$

$$\times\frac{\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^0]}{\mathbf{L}^0[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]} \quad (6.4)$$

(6.1)-(6.4) gives the effects of $\mathbf{A}$, $\overline{\mathbf{c}}$, $\overline{\mathbf{g}}$ and α on the structural growth of gross output, respectively[4]. Corresponding to the above decomposition form, if we decompose $\frac{\overline{\mathbf{x}}^1}{\overline{\mathbf{x}}^0}$ the other way around we have the second decomposition form:

$$\frac{\overline{\mathbf{x}}^1}{\overline{\mathbf{x}}^0}=\frac{\mathbf{L}^1[\alpha^1\overline{\mathbf{c}}^1+(1-\alpha^1)\overline{\mathbf{g}}^1]}{\mathbf{L}^0[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]}$$

$$=\frac{\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]}{\mathbf{L}^0[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]} \quad (7.1)$$

$$\times\frac{\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^1+(1-\alpha^0)\overline{\mathbf{g}}^0]}{\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]} \quad (7.2)$$

$$\times\frac{\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^1+(1-\alpha^0)\overline{\mathbf{g}}^1]}{\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^1+(1-\alpha^0)\overline{\mathbf{g}}^0]} \quad (7.3)$$

$$\times\frac{\mathbf{L}^1[\alpha^1\overline{\mathbf{c}}^1+(1-\alpha^1)\overline{\mathbf{g}}^1]}{\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^1+(1-\alpha^0)\overline{\mathbf{g}}^1]} \quad (7.4)$$

Under this decomposition form, (7.1)-(7.4), likewise, gives the effects of $\mathbf{A}$, $\overline{\mathbf{c}}$, $\overline{\mathbf{g}}$ and α , respectively. These two equivalent decompositions demonstrated in (6.1-6.4) and (7.1-7.4) are called the polar decompositions. Dietzenbacher and Los (1998) shows that the average of polar decompositions is very closed to the average of all equivalent decompositions. Thus, taking the geometric average of polar decompositions (6.1-6.4) and (7.1-7.4), we obtain the final decomposition results:

The effect of $\mathbf{A}$: $E\mathbf{A}=\sqrt{(6.1)\times(7.1)}$

The effect of $\overline{\mathbf{c}}$: $E\overline{\mathbf{c}}=\sqrt{(6.2)\times(7.2)}$

The effect of $\overline{\mathbf{g}}$: $E\overline{\mathbf{g}}=\sqrt{(6.3)\times(7.3)}$

The effect of α: $E\alpha=\sqrt{(6.4)\times(7.4)}$

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[4] Since $\mathbf{L}=(\mathbf{I}-\mathbf{A})^{-1}$, the effects of $\mathbf{L}$ and $\mathbf{A}$ are equivalent.

3.2.2 Decompose the labor compensation growth

The growth of labor compensation during a period of interest can be expressed as

$$\frac{\mathbf{w}^1}{\mathbf{w}^0}=\frac{\lambda^1}{\lambda^0}\frac{\overline{\mathbf{w}}^1}{\overline{\mathbf{w}}^0}$$

In the same sense, we primarily focus on decomposing the structural growth of labor compensation $\frac{\overline{\mathbf{w}}^1}{\overline{\mathbf{w}}^0}$. We choose the following pair of polar decompositions.

Decomposition form 1:

$$\frac{\overline{\mathbf{w}}^1}{\overline{\mathbf{w}}^0}=\frac{\hat{\mathbf{B}}^1\mathbf{L}^1[\alpha^1\overline{\mathbf{c}}^1+(1-\alpha^1)\overline{\mathbf{g}}^1]}{\hat{\mathbf{B}}^0\mathbf{L}^0[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]}$$

$$=\frac{\hat{\mathbf{B}}^1\mathbf{L}^1[\alpha^1\overline{\mathbf{c}}^1+(1-\alpha^1)\overline{\mathbf{g}}^1]}{\hat{\mathbf{B}}^1\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^1+(1-\alpha^1)\overline{\mathbf{g}}^1]} \quad (8.1)$$

$$\times\frac{\hat{\mathbf{B}}^1\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^1+(1-\alpha^1)\overline{\mathbf{g}}^1]}{\hat{\mathbf{B}}^1\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^1]} \quad (8.2)$$

$$\times\frac{\hat{\mathbf{B}}^1\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^1]}{\hat{\mathbf{B}}^1\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^0]} \quad (8.3)$$

$$\times\frac{\hat{\mathbf{B}}^1\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^0]}{\hat{\mathbf{B}}^0\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^0]} \quad (8.4)$$

$$\times\frac{\hat{\mathbf{B}}^0\mathbf{L}^0[\alpha^1\overline{\mathbf{c}}^0+(1-\alpha^1)\overline{\mathbf{g}}^0]}{\hat{\mathbf{B}}^0\mathbf{L}^0[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]} \quad (8.5)$$

Decomposition form 2:

$$\frac{\overline{\mathbf{w}}^1}{\overline{\mathbf{w}}^0}=\frac{\hat{\mathbf{B}}^1\mathbf{L}^1[\alpha^1\overline{\mathbf{c}}^1+(1-\alpha^1)\overline{\mathbf{g}}^1]}{\hat{\mathbf{B}}^0\mathbf{L}^0[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]}$$

$$=\frac{\hat{\mathbf{B}}^0\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]}{\hat{\mathbf{B}}^0\mathbf{L}^0[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]} \quad (9.1)$$

$$\times\frac{\hat{\mathbf{B}}^0\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^1+(1-\alpha^0)\overline{\mathbf{g}}^0]}{\hat{\mathbf{B}}^0\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^0+(1-\alpha^0)\overline{\mathbf{g}}^0]} \quad (9.2)$$

$$\times\frac{\hat{\mathbf{B}}^0\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^1+(1-\alpha^0)\overline{\mathbf{g}}^1]}{\hat{\mathbf{B}}^0\mathbf{L}^1[\alpha^0\overline{\mathbf{c}}^1+(1-\alpha^0)\overline{\mathbf{g}}^0]} \quad (9.3)$$

$$\times \frac{\hat{\mathbf{B}}^1\mathbf{L}^1[\alpha^0\bar{\mathbf{c}}^1 + (1-\alpha^0)\bar{\mathbf{g}}^1]}{\hat{\mathbf{B}}^0\mathbf{L}^1[\alpha^0\bar{\mathbf{c}}^1 + (1-\alpha^0)\bar{\mathbf{g}}^1]} \quad (9.4)$$

$$\times \frac{\hat{\mathbf{B}}^1\mathbf{L}^1[\alpha^1\bar{\mathbf{c}}^1 + (1-\alpha^1)\bar{\mathbf{g}}^1]}{\hat{\mathbf{B}}^1\mathbf{L}^1[\alpha^0\bar{\mathbf{c}}^1 + (1-\alpha^0)\bar{\mathbf{g}}^1]} \quad (9.5)$$

Taking the geometric average of these polar decompositions, we obtain the effects of $\mathbf{A}$, $\bar{\mathbf{c}}$, $\bar{\mathbf{g}}$, $\hat{\mathbf{B}}$ and α on the structural growth of labor compensation:

The effect of $\mathbf{A}$: $E\mathbf{A} = \sqrt{(8.1)\times(9.1)}$

The effect of $\bar{\mathbf{c}}$: $E\bar{\mathbf{c}} = \sqrt{(8.2)\times(9.2)}$

The effect of $\bar{\mathbf{g}}$: $E\bar{\mathbf{g}} = \sqrt{(8.3)\times(9.3)}$

The effect of $\hat{\mathbf{B}}$: $E\hat{\mathbf{B}} = \sqrt{(8.4)\times(9.4)}$

The effect of α: $E\alpha = \sqrt{(8.5)\times(9.5)}$

When applying the SDA on a variable, it is usually assumed that the determinants of the variable are independent. In input-output analysis, however, for any sector the column sum of domestic input coefficients, import coefficient, labor compensation coefficient and other value added coefficient should equal 1. Due to this adding-up constraint, the domestic input coefficient matrix $\mathbf{A}$ and the labor compensation coefficient matrix $\hat{\mathbf{B}}$ are not strictly independent. Dietzenbacher and Los (2000) show that dependencies may cause a bias in the results of decomposition analyses. Nevertheless, the magnitude of the dependency between the column sums of $\mathbf{A}$ and $\hat{\mathbf{B}}$ is uncertain. For instance, a change in the column sums of $\mathbf{A}$ is not necessarily caused by a change in the labor compensation; it can be absorbed by any of the factors in labor compensation coefficient, import coefficient and other value added coefficient. Therefore, we need to first check the correlation between the column sums of $\mathbf{A}$ and $\hat{\mathbf{B}}$ in practice. If the correlation is not strong, then the dependency will not affect the accuracy of the decomposition formulae (8.1-8.5) and (9.1-9.5) to a large degree.

3.3 MSDA for the semi-closed model

In the context of a semi-closed model, the growth of gross output and labor compensation can be decomposed simultaneously. The growth of gross output and labor compensation during a period of interest can be expressed as

$$\frac{\begin{bmatrix}\mathbf{x}^1\\ \mathbf{w}^1\end{bmatrix}}{\begin{bmatrix}\mathbf{x}^0\\ \mathbf{w}^0\end{bmatrix}}=\frac{\mu_1}{\mu_0}\frac{\begin{bmatrix}\bar{\bar{\mathbf{x}}}^1\\ \bar{\bar{\mathbf{w}}}^1\end{bmatrix}}{\begin{bmatrix}\bar{\bar{\mathbf{x}}}^0\\ \bar{\bar{\mathbf{w}}}^0\end{bmatrix}}$$

Similarly, we call $\frac{\bar{\bar{\mathbf{x}}}^1}{\bar{\bar{\mathbf{x}}}^0}$ and $\frac{\bar{\bar{\mathbf{w}}}^1}{\bar{\bar{\mathbf{w}}}^0}$ the structural growth of gross output and the structural growth of labor compensation, respectively. We decompose them to be the following pair of multiplicative forms.

Decomposition form 1:

$$\frac{\begin{bmatrix}\bar{\bar{\mathbf{x}}}^1\\ \bar{\bar{\mathbf{w}}}^1\end{bmatrix}}{\begin{bmatrix}\bar{\bar{\mathbf{x}}}^0\\ \bar{\bar{\mathbf{w}}}^0\end{bmatrix}}=\frac{\begin{bmatrix}\mathbf{I}-\mathbf{A}^1 & -r^1\bar{\mathbf{c}}^1\mathbf{i}'\\ -\hat{\mathbf{B}}^1 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^1\\ \mathbf{0}\end{bmatrix}}{\begin{bmatrix}\mathbf{I}-\mathbf{A}^0 & -r^0\bar{\mathbf{c}}^0\mathbf{i}'\\ -\hat{\mathbf{B}}^0 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^0\\ \mathbf{0}\end{bmatrix}}$$

$$=\frac{\begin{bmatrix}\mathbf{I}-\mathbf{A}^1 & -r^1\bar{\mathbf{c}}^1\mathbf{i}'\\ -\hat{\mathbf{B}}^1 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^1\\ \mathbf{0}\end{bmatrix}}{\begin{bmatrix}\mathbf{I}-\mathbf{A}^0 & -r^1\bar{\mathbf{c}}^1\mathbf{i}'\\ -\hat{\mathbf{B}}^1 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^1\\ \mathbf{0}\end{bmatrix}} \tag{10.1}$$

$$\times\frac{\begin{bmatrix}\mathbf{I}-\mathbf{A}^0 & -r^1\bar{\mathbf{c}}^1\mathbf{i}'\\ -\hat{\mathbf{B}}^1 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^1\\ \mathbf{0}\end{bmatrix}}{\begin{bmatrix}\mathbf{I}-\mathbf{A}^0 & -r^1\bar{\mathbf{c}}^0\mathbf{i}'\\ -\hat{\mathbf{B}}^1 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^1\\ \mathbf{0}\end{bmatrix}} \tag{10.2}$$

$$\times\frac{\begin{bmatrix}\mathbf{I}-\mathbf{A}^0 & -r^1\bar{\mathbf{c}}^0\mathbf{i}'\\ -\hat{\mathbf{B}}^1 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^1\\ \mathbf{0}\end{bmatrix}}{\begin{bmatrix}\mathbf{I}-\mathbf{A}^0 & -r^1\bar{\mathbf{c}}^0\mathbf{i}'\\ -\hat{\mathbf{B}}^1 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^0\\ \mathbf{0}\end{bmatrix}} \tag{10.3}$$

$$\times\frac{\begin{bmatrix}\mathbf{I}-\mathbf{A}^0 & -r^1\bar{\mathbf{c}}^0\mathbf{i}'\\ -\hat{\mathbf{B}}^1 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^0\\ \mathbf{0}\end{bmatrix}}{\begin{bmatrix}\mathbf{I}-\mathbf{A}^0 & -r^1\bar{\mathbf{c}}^0\mathbf{i}'\\ -\hat{\mathbf{B}}^0 & \mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{g}}^0\\ \mathbf{0}\end{bmatrix}} \tag{10.4}$$

$$\times \frac{\begin{bmatrix} \mathbf{I}-\mathbf{A}^0 & -r^1\overline{\mathbf{c}}^0\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^0 \\ \mathbf{0} \end{bmatrix}}{\begin{bmatrix} \mathbf{I}-\mathbf{A}^0 & -r^1\overline{\mathbf{c}}^0\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^0 \\ \mathbf{0} \end{bmatrix}} \tag{10.5}$$

Decomposition form 2:

$$\frac{\begin{bmatrix} \overline{\overline{\mathbf{x}}}^1 \\ \overline{\overline{\mathbf{w}}}^1 \end{bmatrix}}{\begin{bmatrix} \overline{\overline{\mathbf{x}}}^0 \\ \overline{\overline{\mathbf{w}}}^0 \end{bmatrix}} = \frac{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^1\overline{\mathbf{c}}^1\mathbf{i}' \\ -\hat{\mathbf{B}}^1 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^1 \\ \mathbf{0} \end{bmatrix}}{\begin{bmatrix} \mathbf{I}-\mathbf{A}^0 & -r^0\overline{\mathbf{c}}^0\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^0 \\ \mathbf{0} \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^0\overline{\mathbf{c}}^0\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^0 \\ \mathbf{0} \end{bmatrix}}{\begin{bmatrix} \mathbf{I}-\mathbf{A}^0 & -r^0\overline{\mathbf{c}}^0\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^0 \\ \mathbf{0} \end{bmatrix}} \tag{11.1}$$

$$\times \frac{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^0\overline{\mathbf{c}}^1\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^0 \\ \mathbf{0} \end{bmatrix}}{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^0\overline{\mathbf{c}}^0\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^0 \\ \mathbf{0} \end{bmatrix}} \tag{11.2}$$

$$\times \frac{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^0\overline{\mathbf{c}}^1\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^1 \\ \mathbf{0} \end{bmatrix}}{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^0\overline{\mathbf{c}}^1\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^0 \\ \mathbf{0} \end{bmatrix}} \tag{11.3}$$

$$\times \frac{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^0\overline{\mathbf{c}}^1\mathbf{i}' \\ -\hat{\mathbf{B}}^1 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^1 \\ \mathbf{0} \end{bmatrix}}{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^0\overline{\mathbf{c}}^1\mathbf{i}' \\ -\hat{\mathbf{B}}^0 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^1 \\ \mathbf{0} \end{bmatrix}} \tag{11.4}$$

$$\times \frac{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^1\overline{\mathbf{c}}^1\mathbf{i}' \\ -\hat{\mathbf{B}}^1 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^1 \\ \mathbf{0} \end{bmatrix}}{\begin{bmatrix} \mathbf{I}-\mathbf{A}^1 & -r^0\overline{\mathbf{c}}^1\mathbf{i}' \\ -\hat{\mathbf{B}}^1 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \overline{\mathbf{g}}^1 \\ \mathbf{0} \end{bmatrix}} \tag{11.5}$$

Taking the geometric average of these polar decompositions, we can simultaneously obtain the effects of $\mathbf{A}$, $\bar{\mathbf{c}}$, $\bar{\mathbf{g}}$, $\hat{\mathbf{B}}$ and r on the structural growth of gross output and labor compensation.

The effect of $\mathbf{A}$: $E\mathbf{A} = \sqrt{(10.1)\times(11.1)}$

The effect of $\bar{\mathbf{c}}$: $E\bar{\mathbf{c}} = \sqrt{(10.2)\times(11.2)}$

The effect of $\bar{\mathbf{g}}$: $E\bar{\mathbf{g}} = \sqrt{(10.3)\times(11.3)}$

The effect of $\hat{\mathbf{B}}$: $E\hat{\mathbf{B}} = \sqrt{(10.4)\times(11.4)}$

The effect of r: $Er = \sqrt{(10.5)\times(11.5)}$

It is important to notice that the domestic input coefficient matrix $\mathbf{A}$ and the labor compensation coefficient matrix $\hat{\mathbf{B}}$ are both factors of the semi-closed model. As we discussed before, they are not strictly independent. Therefore, the correlation check between $\mathbf{A}$ and $\hat{\mathbf{B}}$ should also be done before we use decomposition formulae (10.1-10.5) and (11.1-11.5).

3.4 An application on Chinese input-output tables

3.4.1 Data description

We attempt to employ the MSDA to investigate the sources for sectoral gross output growth and sectoral labor compensation growth in China during 1997-2007. The 1997 and 2007 survey-based Chinese input-output (IO) tables published by the National Bureau of Statistics of China (NBS) are used to carry out this MSDA. These IO tables are in current prices and have 42 sectors (see the sector classification in the Appendix). Most commonly, the constant price IO tables should be used for SDAs. Unfortunately, the constant price Chinese IO tables for 1997 and 2007 are not available. In addition, our primary concern is on the differences between decomposing the semi-closed model and the open model, so it is not a crucial issue to use current price or constant price IO tables.

In Chinese IO tables, instead of labor compensation, only the compensation of employees is reported. Nevertheless, before 2004, the compensation of employees in Chinese IO tables also includes the income of self-employed individuals. Hence, the statistic scope for the compensation of employees in 1997 IO table is the same as the labor compensation we mentioned in this study. However, NBS changed the statistic scope for compensation of employees since 2004 (see Bai and Qian, 2010). For agriculture sectors, the income of self-employed individuals is still included in the compensation of employees, but the operating surplus of state-owned and collective-owned farms are accounted to compensation of employees as well. That is because obtaining detailed financial statements from these farms becomes more and more difficult. For non-agriculture sectors, however, the income of self-employed individuals is not accounted to compensation of employees any more. In this case, we need to make some adjustments on the compensation of employees in the 2007 IO table to obtain the labor compensation. To express the adjustment procedure efficiently, we first give some mathematical notations.

Let $\tilde{w}_a$ denote the compensation of employees of agriculture sector; $\tilde{\mathbf{w}}_{na}$ denote a vector of compensation of employees of non-agriculture sectors; k denote the operating surplus of state-owned and collective owned farms; x_a denote the gross output of agriculture sector.

For the agriculture sector, the operating surplus of state-owned and collective-owned farms should be subtracted from the compensation of employees. Therefore, the labor compensation of agriculture sector in 2007 (w_a^{2007})is obtained by:

$$w_a^{2007} = \tilde{w}_a^{2007} - \frac{k^{2002}}{x_a^{2002}} x_a^{2007}$$

The underlying assumption of the above formula is that the ratio between the operating surplus of state-owned as well as collective-owned farms and the gross output of agriculture sector in 2007 approximately equals to that in 2002[5]. This assumption requires the gross output of state-owned and collective-owned farms has a steady share in the total gross output of agriculture sector. Actually, according to the *China Statistical Yearbook*, the gross output share of state-owned farms in 2002 and 2007 are 2.65% and 2.97%, almost constant.

For non-agriculture sectors, the income of self-employed individuals should be added to the compensation of employees. Therefore, the labor compensation vector of non-agriculture sectors in 2007 ($\mathbf{w}_{na}^{2007}$) are obtained by:

$$\mathbf{w}_{na}^{2007} = \tilde{\mathbf{w}}_{na}^{2007} + y^{2007}\mathbf{s}$$

[5] Before 2004, 2002 is the closest year to 2007 in which the Chinese IO tables are published.

Where y^{2007} is the total non-agriculture income of self-employed individuals; $\mathbf{s}$ is a vector of self-employed income shares for non-agriculture sectors. In 2004, China conducted the first National Economic Census. Based on this census, NBS published the income of self-employed individuals by sectors the first time. The above formulae assume that the sectoral income shares of self-employed in 2007 are similar to those in 2004. This assumption seems to be strong, but this is the only information we have. After these adjustments, we have obtained the labor compensations of all sectors for the 2007 IO table.

In addition, in the IO models introduced in Section 2, the domestic and imported products are distinguished. However, the original IO tables published by NBS do not distinguish them, so we further split them by the frequently used proportional approach. Multiplying each row of the IO table with the "domestic product share in total domestic demand", we can obtain the domestic products consumed by production sector, household, government etc.. For sector i, its domestic product share in total domestic demand is defined as $\varphi_i = \frac{x_i - e_i}{x_i + m_i - e_i}$. Where x_i is the gross output of product i; m_i is the import of product i; e_i is the export of product i. The imported products can be obtained by deducting the domestic products from the original IO tables.

3.4.2 Correlation Check

As we mentioned before, the domestic input coefficient matrix $\mathbf{A}$ and the labor compensation coefficient matrix $\hat{\mathbf{B}}$ are not strictly independent. If the decomposition formulae introduced in Section 3.2 and 3.3 are used, the accuracy of our decomposition results could be adversely affected by this dependency. Hence, in this part we check the magnitude of the correlation between the column sums of the domestic input coefficient matrix and the labor compensation coefficients.

According to the adding-up constraint, a change in any of the coefficients in the identity will be absorbed by the other coefficients. We calculate the changes in these coefficients for each sector during 1997-2007 and further calculate the correlation coefficient r between the changes in these coefficients[6] (see Table 1).

[6] The correlation coefficient between X and Y is defined as:

<Insert Table 1 here>

Table 1 shows a quite strong negative correlation between the column sum of the domestic input coefficient matrix and the other value added coefficient. The correlation coefficient between them reaches -0.73. However, the correlation between the changes in other coefficients are quite weak. Especially for the column sum of the domestic input coefficient matrix and the labor compensation coefficient which cause the dependency issue of our decomposition formulae, the correlation coefficient between them is only -0.22. Hence, this weak correlation will not affect the accuracy of the decomposition formulae introduced in Section 3.2 and 3.3 to a large degree. Next, we will use these decomposition formulae to analyze the growth sources for the gross output and the labor compensation of China during 1997-2007.

3.4.3 Results and Findings

The gross output growth and the labor compensation grow during 1997-2007 are decomposed by using the open model and the semi-closed model, respectively. Since the labor compensation coefficient is defined as the labor compensation per unit gross output, the labor compensation equals labor compensation coefficient multiplying gross output. Therefore, for both the open model and the semi-closed model, when holding $\hat{\mathbf{B}}$ fixed, the effects of other factors on the gross output growth and the labor compensation growth should be the same. Our decomposition results also prove this point. For the sake of limited space, we list the decomposition results for the gross output growth and the labor compensation growth in one table. Table 2 is for the open model and Table 3 is for the semi-closed model.

<Insert Table2 here>

<Insert Table3 here>

First, we compare the decomposition results yield from both models. Formula (12) is employed to measure the absolute relative difference (ARD) between the results yield from both models sector by sector.

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

Where x_i and y_i are a series of observations of X and Y ; $\bar{x}$ and $\bar{y}$ are the sample means of X and Y .

$$\rho_i^e = \frac{|e_i^s - e_i^o|}{\sqrt{e_i^s e_i^o}} \times 100\% \quad (12)$$

Where e_i^s and e_i^o are the effects of factor e on sector i yield from decomposing the semi-closed model and the open model, respectively. It gives the absolute relative difference between e_i^s and e_i^o, and the benchmark here is the geometric average of e_i^s and e_i^o. The definition of ρ_i^e is intuitive and have some good characteristic. For instance, ρ_i^e satisfies the symmetric condition, namely exchanging the positions of e_i^s and e_i^o in the formula does not change the value of ρ_i^e; for any sector *i*, if the results yield from both models are exactly the same, then $\rho_i^e = 0$.

For the decomposition of gross output growth, Table 2 and Table 3 show that the open model and the semi-closed model yield very similar results for the factors they have in common ($\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$). The means of ARDs across all sectors for $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ are 0.33%, 2.69% and 2.04% with quite small standard deviations 0.16, 1.52 and 1.19, respectively. Decomposing the semi-closed model does not provide significantly different information for these common factors, although it provides an extra effect channel for them relative to those in the open model. In other words, the similar results indicates that the extra effects of these common factors via the industry-household linkage are very small. The reason is that the $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ are all structural factors, so changes in them will cause increases and decreases in the gross output of different sectors at the same time; the former will further cause an increase in the labor compensation and the latter will cause a decrease in the labor compensation; Therefore, aggregating them together, the effect on total labor compensation will be weakened and could further induces an insignificant change in the household consumption; finally, the insignificant change in the household consumption will cause an insignificant extra effect on the gross output. Thus, for these factors, the semi-closed model and the open model yield very similar decomposition results.

Nevertheless, if focusing on the factors that only exist in the semi-closed model, we can still find some significantly different information from decomposing the semi-closed model. For instance, in Table 3, the semi-closed model indicates that the labor compensation coefficients have negative effects on the gross output growth of all sectors. For sectors, such as *agriculture* sector (1), *manufacture of food products and tobacco processing* sector (6) and *real estate* sector

(33), the negative effects are rather significant. This information cannot be found from decomposing the open model.

For the decomposition of labor compensation growth, $\mathbf{A}$, $\bar{\mathbf{c}}$, $\bar{\mathbf{g}}$ and $\hat{\mathbf{B}}$ are the factors that the open model and the semi-closed model have in common. Since the effects of $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ on the gross output growth and the labor compensation growth are the same, similar to the situation in the decomposition of gross output growth, the effects of $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ on the labor compensation growth calculated from both models are still virtually the same. However, according to Table 2 and Table 3, the open model and the semi-closed model yield significantly different results with respect to factor $\hat{\mathbf{B}}$. The semi-closed model indicates a stronger negative effect than the open model does. The mean of the ARDs across all sectors reaches 10.31% with a standard deviation 5.94. For some sectors, the signs of the effects are even different. For instance, the open model indicates a positive effect on *manufacture of food products and tobacco processing* sector(6), whereas the semi-closed model indicates a negative effect. In the open model, the effect of $\hat{\mathbf{B}}$ on the labor compensation is basically direct. In the semi-closed model, however, as a constituent part of the industry-household linkage, $\hat{\mathbf{B}}$ is endogenized. Relative to the open model, a change in $\hat{\mathbf{B}}$ has an extra effect on the labor compensation via the income-consumption relationship. The decomposition results for the open model in Table 2 shows a relatively large negative effect of $\hat{\mathbf{B}}$ on the labor compensation of most sectors and thus a relatively large negative effect on the total labor compensation. In the semi-closed model, via the income-consumption relationship, this relatively large direct effect on the total labor compensation further generates an significantly extra effect on the labor compensation of each sector. Thus, we find a significantly different result with respect to factor $\hat{\mathbf{B}}$.

In addition, the decomposition results also show some knowledge on the growth sources of the gross output and labor compensation of China during 1997-2007. Since our decompositions are based on current price IO tables, it is important to notice that the decomposition results include the price effect as well and the growth in gross output and labor compensation is nominal growth. The decomposition results of the semi-closed model (Table 3) are used to analyze our findings about the growth sources.

For the gross output growth, *electricity and heating power production and supply* sector (23) benefits a lot from the change in domestic input coefficient matrix $\mathbf{A}$. This means due to the change in $\mathbf{A}$, industries need to expend more and more (directly and indirectly) on electricity to produce 1 unit product in monetary form. Whereas, the domestic input coefficient matrix change has an significantly negative effect on *art and craft and other manufacturing products* sector (21). One of the important reasons is probably that the production linkage of other sectors to *art and craft and other manufacturing products* sector is weakened during the process of industrialization. The maximum negative effect and positive effect of the change in consumption share $\bar{\mathbf{c}}$ is 0.64 and 1.62, on *agriculture* sector (1) and *real estate* sector (33), respectively. This indicates that the change in consumer's expenditure share slows the growth of *agriculture* sector (1) and accelerates the growth of *real estate* sector (33) to a large degree. *Telecommunication equipment, computer and other electronic equipment* sector (19) benefits a lot form the change in other final demand (export, investment etc.) share $\bar{\mathbf{g}}$. During 1997-2007, the export share of *telecommunication equipment, computer and other electronic equipment commodity* sector increased from 10.8% to 22.4%. The rapid increase in the export share drives the growth of *telecommunication equipment, computer and other electronic equipment* sector to a large degree.

Another finding is that the change in the labor compensation coefficient matrix $\hat{\mathbf{B}}$ has negative effects on all sectors' gross output growth[7]. We find that there are 34 sectors in the Chinese IO tables with 42 sectors whose labor compensation coefficients decreased during 1997-2007. Ceteris paribus, decreased labor compensation coefficients will lead to decreased total labor compensation and further lead to decreased household consumption in all sectors. The decreased household consumption will finally generate negative effects on the gross output of all sectors via the industry linkages and industry-household linkage. The sectors with larger consumption shares, such as *agriculture* sector, are usually affected more than the sectors with smaller consumption shares.

To a large degree, the widely decreased labor compensation coefficients during 1997-2007 are caused by the significantly decreased labor coefficients (the number of labors engaged to produce 1 unit gross output in current price). According to the current price Chinese IO tables, the gross output of primary, secondary and tertiary industries in 2007 are 1.92, 4.39 and 4.50 times of those in 1997. Relative to the rapid growth in gross output, the number of employees

[7] Because of rounding off, Table 3 shows that the decomposition result for the *public management and social administration* sector (42) is 1.00. Actually, it is also less than 1.

grows quite slow. According to the *China Statistical Year Book*[8], the number of employees in secondary and tertiary industries only grew 22.0% and 32.4% from 1997 to 2007. The growth rate of the primary industry is even negative, -11.8%. The significant difference in the growth rates of the gross output and the number of employees leads to significant decreases in labor coefficients. The significantly decreased labor coefficients further diminish the labor compensation coefficient to a large degree, although the average wage of employees in the national economy maintained a about 13.5% average annual growth rate during 1997-2007. For the *gas production and supply* sector (24), its gross output grew substantially over this period (increased 694% with a 179% increase in the price). However, the number of its employees only increased 7.2%. This causes its labor compensation coefficient decreased significantly from 0.20 in 1997 to 0.08 in 2007 to a large degree. *Public management and social administration* sector (42) is one of the minorities whose labor compensation coefficient increased significantly, increasing from 0.37 in 1997 to 0.48 in 2007. The primary reason is the substantial increase in the average wage of the employees in the *Public management and social administration* sector during 1997-2007. Up to 2007, the average wage of the employees in *Public management and social administration* sector is 4.04 times of that in 1997. At the same time, the gross output is 3.57 times of that in 1997.

For the labor compensation growth, as we mentioned previously, the labor compensation coefficients decreased widely during 1997-2007. Hence, Table 2 shows that the labor compensation coefficients have significantly negative effects on virtually all sectors. The maximum negative effect is on the *gas production and supply* sector (24) whose labor compensation coefficient decreased dramatically; the *Public management and social administration* sector (42) which experienced a significant increase in its labor compensation coefficient benefits most from the change in labor compensation coefficients. For factors $\mathbf{A}$, $\overline{\mathbf{c}}$ and $\overline{\mathbf{g}}$, as we mentioned before, their effects on labor compensation growth are equal to the effects on gross output growth.

3.5 Findings from other countries and regions

The MSDA application on Chinese IO tables shows that for the decomposition of gross output growth the semi-closed model and the open model yield very similar results for the factors they

[8] The data using in the following analysis are all obtained from the *China Statistical Year Book*.

have in common; however, for the decomposition of labor compensation growth both models yield significantly different results with respect to the labor compensation coefficient matrix. In this section, we further apply the MSDA on the IO tables of other countries and regions to investigate the generality of our findings.

We use the world input-output database (WIOD) as our data source[9]. WIOD provides the world input-output tables as well as the social economic account from 1995 to 2009. The world IO tables are inter-country IO tables with 35 sectors and 40 countries and regions. Since our analysis is based on national IO tables, we aggregate the 1997 and 2007 world IO tables in current price to generate the national IO tables for 40 countries and regions. The data on labor compensation is obtained from the social economic account in WIOD. For Indonesia, India, Turkey and Chinese Taipei, the data on labor compensation is not available, so we use compensation of employees as an alternative. The correlation coefficients between the column sum of the domestic input coefficient matrix and the corresponding labor compensation coefficient are calculated for each country and region, which are listed in Table 4. It can be seen that for most countries and regions the correlation is quite weak. As the decomposition results for the common factors $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ on the gross output growth and labor compensation growth are exactly the same, in this section we solely decompose the labor compensation growth by using the open model and the semi-closed model. For the results of the common factors existing in both models, we calculate the mean and standard deviation of the ARDs across all sectors for each country and region. The results are listed in Table 4 as well. In addition, for Bulgaria, Cyprus and Greece, as the ratio of other final demand on some sectors between 1997 and 2007 are negative, the multiplicative decomposition yield imaginary number. For Romania, a statistical mistake may be made on the labor compensation of agriculture sector in 2007, because it is larger than the gross output. Thus, we do not report the results for these 4 countries.

<Insert Table 4 here>

Table 4 shows that it is a very general case that the open model and the semi-closed model yield quite similar decomposition results for their common factors $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$. For almost all countries and regions the means of ARDs for $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ are less than 5% and with quite small

[9] The website for the world input-output database: www.wiod.org.

standard deviations. Furthermore, more than half of the countries and regions, the means of ARDs for $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ are less than 1%.

The special case is for Turkey. The open model and the semi-closed model yield significantly different results for factor $\bar{\mathbf{g}}$; the mean of ARDs reaches 12.20%. The reason for this special case is that the structural change in $\bar{\mathbf{g}}$ during 1997-2007 causes a substantial increase in the gross output of sectors with very high labor compensation coefficients, such as *public admin and defense as well as compulsory social secur*ity sector and *education* sector. According to the Turkish IO tables, the other final demand shares of these two sectors increased from 7.33% and 0.34% in 1997 to 13.13% and 5.77% in 2007, because of the substantial increase in government expenditures. The average labor compensation coefficients of these sectors between 1997 and 2007 are 0.76 and 0.49, obviously larger than those of other sectors[10]. Hence, this change in factor $\bar{\mathbf{g}}$ causes a quite large increase in total labor compensation. The semi-closed model further transmits the significant increase in the total labor compensation to the increase in gross output via the income-consumption relationship and thus yield a significantly different result from the open model. In addition, the same reason also causes a slightly large mean of ARDs (7.81%) for factor $\mathbf{A}$.

For the labor compensation coefficient matrix $\hat{\mathbf{B}}$, the open model and the semi-closed model also yield quite significantly different decomposition results in many other countries and regions. Especially for India, Korea, Poland and Turkey, the means of ARDs exceed 10% and the standard deviations are relatively large as well. For Austria, Germany, Spain, Italy, Latvia, Mexico, Malta and Chinese Taipei, the means of ARDs are all above 6% and with quite large standard deviations. In addition, for China, we still find that the decomposition results with respect to $\mathbf{A}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{g}}$ are very close and the results for $\hat{\mathbf{B}}$ are significantly different. This is consistent with the previous conclusions drawn from the Chinese IO tables published by the NBS.

4 Decompose sectoral household consumption growth

Another difference between applying the open model and the semi-closed model in decomposition analyses is concerning the decomposition of sectoral household consumption growth. Researchers may be interested in the contributions of the sources for the sectoral

[10] The average labor compensation coefficients of other sectors are 0.16 in 1997 and 0.14 in 2007.

household consumption growth during a period, such as the contribution of domestic input coefficient change and the contribution of consumption share change. In this section, we will show that the decomposition of sectoral household consumption growth can be done by using the semi-closed model. Different from this, the open model is unable to decompose this, since the household consumption is an exogenous variable in the open model.

Formula (2) can also be expressed as the following form by solving the inverse of the partitioned matrix.

$$\begin{bmatrix}\mathbf{x}\\ \mathbf{w}\end{bmatrix}=\begin{bmatrix}\mathbf{L}[\mathbf{I}+\mathbf{C}(\mathbf{I}-\hat{\mathbf{B}}\mathbf{L}\mathbf{C})^{-1}\hat{\mathbf{B}}\mathbf{L}] & \mathbf{L}\mathbf{C}(\mathbf{I}-\hat{\mathbf{B}}\mathbf{L}\mathbf{C})^{-1}\\ (\mathbf{I}-\hat{\mathbf{B}}\mathbf{L}\mathbf{C})^{-1}\hat{\mathbf{B}}\mathbf{L} & (\mathbf{I}-\hat{\mathbf{B}}\mathbf{L}\mathbf{C})^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{g}\\ \mathbf{0}\end{bmatrix}$$

Where $\mathbf{L}=(\mathbf{I}-\mathbf{A})^{-1}$ and $\mathbf{C}=r\bar{\mathbf{c}}\mathbf{i}'$

The expression for household consumption is:

$$\mathbf{c}=\mathbf{C}\mathbf{w}=\mathbf{C}(\mathbf{I}-\hat{\mathbf{B}}\mathbf{L}\mathbf{C})^{-1}\hat{\mathbf{B}}\mathbf{L}\mathbf{g}=r\bar{\mathbf{c}}\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}\mathbf{L}r\bar{\mathbf{c}}\mathbf{i}')^{-1}\hat{\mathbf{B}}\mathbf{L}\mathbf{g} \tag{13}$$

Similarly, the household consumption $\mathbf{c}$ can be written as:

$$\mathbf{c}=\mu\bar{\bar{\mathbf{c}}}$$

Where $\bar{\bar{\mathbf{c}}}=r\bar{\mathbf{c}}\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}\mathbf{L}r\bar{\mathbf{c}}\mathbf{i}')^{-1}\hat{\mathbf{B}}\mathbf{L}\bar{\mathbf{g}}$. It indicates the sectoral household consumption generated by 1 unit other final demands with the same shares in $\bar{\mathbf{g}}$.

Then the household consumption growth can be expressed as:

$$\frac{\mathbf{c}^1}{\mathbf{c}^0}=\frac{\mu_1}{\mu_0}\frac{\bar{\bar{\mathbf{c}}}^1}{\bar{\bar{\mathbf{c}}}^0}$$

Likewise, our primary concern is the decomposition of $\frac{\bar{\bar{\mathbf{c}}}^1}{\bar{\bar{\mathbf{c}}}^0}$. we choose the following pair of polar decompositions.

Decomposition form 1:

$$\frac{\bar{\bar{\mathbf{c}}}^1}{\bar{\bar{\mathbf{c}}}^0}=\frac{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^1}{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^0}$$

$$=\frac{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}\mathbf{g}^1}{r^0\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1r^0\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^1} \tag{14.1}$$

$$=\frac{r^0\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1r^0\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^1}{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^1} \tag{14.2}$$

$$= \frac{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1 r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^1}{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^1 r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^1\bar{\mathbf{g}}^1} \quad (14.3)$$

$$= \frac{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^1 r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^1\bar{\mathbf{g}}^1}{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^1} \quad (14.4)$$

$$= \frac{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^1}{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^0} \quad (14.5)$$

Decomposition form 2:

$$\frac{\bar{\bar{\mathbf{c}}}^1}{\bar{\bar{\mathbf{c}}}^0} = \frac{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1 r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^1}{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^0}$$

$$= \frac{r^1\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^1\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^0}{r^0\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^0\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^0} \quad (15.1)$$

$$= \frac{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^0}{r^1\bar{\mathbf{c}}^0\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^1\bar{\mathbf{c}}^0\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^0} \quad (15.2)$$

$$= \frac{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^0 r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^0\bar{\mathbf{g}}^0}{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^0\mathbf{L}^0 r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^0\mathbf{L}^0\bar{\mathbf{g}}^0} \quad (15.3)$$

$$= \frac{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1 r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^0}{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^0 r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^0\bar{\mathbf{g}}^0} \quad (15.4)$$

$$= \frac{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1 r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^1}{r^1\bar{\mathbf{c}}^1\mathbf{i}'(\mathbf{I}-\hat{\mathbf{B}}^1\mathbf{L}^1 r^1\bar{\mathbf{c}}^1\mathbf{i}')^{-1}\hat{\mathbf{B}}^1\mathbf{L}^1\bar{\mathbf{g}}^0} \quad (15.5)$$

Take the geometric average of these two decompositions to obtain the effects of r, $\bar{\mathbf{c}}$, $\hat{\mathbf{B}}$, $\mathbf{A}$ and $\bar{\mathbf{g}}$:

$$Er = \sqrt{(14.1)\times(15.1)}$$

$$E\bar{\mathbf{c}} = \sqrt{(14.2)\times(15.2)}$$

$$E\hat{\mathbf{B}} = \sqrt{(14.3)\times(15.3)}$$

$$E\mathbf{A} = \sqrt{(14.4)\times(15.4)}$$

$$E\bar{\mathbf{g}} = \sqrt{(14.5)\times(15.5)}$$

<Insert Table 5 here>

In the semi-closed model, the factors $\mathbf{A}$, r, $\hat{\mathbf{B}}$ and $\overline{\mathbf{g}}$ affect the sectoral household consumption by affecting the total labor compensation of the household sector. As only one household sector is incorporated in our semi-closed model, they have an identical effect on all sectors' consumption growth. Among these factors, Table 5 shows that $\hat{\mathbf{B}}$ has a significantly negative effect. As mentioned before, this is caused by the widely deceased labor compensation coefficients during 1997-2007. r and $\overline{\mathbf{g}}$ have slightly negative effects, whereas $\mathbf{A}$ has a slightly positive effect. If the household sector is further disaggregated into several different household sectors according to their different characteristics, the effects of these factors on different sectors will be different. The consumption share vector $\overline{\mathbf{c}}$ has different effects on each sector. For instance, it has a quite large positive effect on *petroleum processing, coking and nuclear fuel processing* sector (11) and *information communication, computer service and software* sector (29). Although, the prices of these sectors are rising over time, the increasing popularity of household vehicles, cell phones and computers is probably an important reason for the considerable growth in the consumption of these sectors as well. The development of information communication meanwhile leads to a dramatic decrease in the demand on post, so we find a significantly negative effect on *post* sector(28). With the improvement of people's living standard, the share of *textile* goods (7) and *agriculture* goods (1) in total household consumption decreases and thus leads to a considerable negative effect on the consumption growth of these sectors.

5 Conclusion

The semi-closed model takes into account the industry-household linkage which works via the income-consumption relationship. Compared to the open model, this provides an extra effect channel for the factors in the semi-closed model. From this perspective, decomposing the semi-closed model could provide us different information than decomposing the open model. Therefore, the comparison between decomposing the semi-closed model and the open model is our primary concern in this study. By means of decomposing the gross output growth and labor compensation growth of China as well as 35 other countries and regions, we obtain the following findings.

First, for the decomposition of gross output growth of China, the open model and the semi-closed model yield very similar results for the domestic input coefficient matrix, the consumption share vector and the other final demand share vector, which are the factors they have in common. The decomposition analyses for 35 other countries and regions also shows the generality of this finding. Hence, when decomposing the gross output growth, if only these common factors are of interest, it does not matter whether the semi-closed model or the open model is used. Nevertheless, the semi-closed model can also help to figure out the contribution of the changes in labor compensation coefficients on gross output growth, which is shown to be quite significant in the Chinese empirical study. However, this cannot be done by using the open model.

Second, for the decomposition of labor compensation growth of China, both models yield significantly different results for the labor compensation coefficient matrix, which is one of the factors they have in common. The decomposition analyses for many other countries and regions also show this significant difference. Hence, for the decomposition analysis of labor compensation growth, if researchers believe that it is important to take into account the industry-household linkage, then the semi-closed model should be used.

Third, unlike the open model, the sectoral household consumption growth can be decomposed by using the semi-closed model. As only one household sector is incorporated in our semi-closed model, the decomposition results show that the changes in many factors such as the domestic input coefficient matrix have an identical effect on all sectors. Nevertheless, if the household sector is further disaggregated according to different characteristics such as income levels, the effects of these factors on different sectors could be different.

Table 1 The correlation coefficients between the changes in some input-output coefficients (1997-2007)

	$\Delta\tilde{a}$	Δm	Δb	$\Delta\tilde{v}$
$\Delta\tilde{a}$	1			
Δm	-0.25	1		
Δb	-0.22	-0.28	1	
$\Delta\tilde{v}$	-0.73	-0.12	-0.29	1

Note: $\Delta\tilde{a}$ denotes the change in the column sum of the domestic input coefficient matrix; Δm denotes the change in the import coefficient; Δb denotes the change in the labor compensation coefficient; $\Delta\tilde{v}$ denotes the change in the other value added coefficient.

Table 2 Decomposition results for gross output growth and labor compensation growth

(open model)

Sector code	Gross output growth	Labor compensation growth	E $\mathbf{A}$	E $\overline{\mathbf{c}}$	E $\overline{\mathbf{g}}$	E α_c	E $\hat{\mathbf{B}}$
1	1.93	1.81	0.93	0.68	0.95	0.84	0.94
2	4.33	2.93	1.11	1.03	0.96	1.03	0.68
3	5.84	5.71	1.56	1.04	0.90	1.04	0.98
4	5.15	4.07	1.06	1.01	1.14	1.11	0.79
5	2.61	2.12	0.77	0.97	0.83	1.11	0.81
6	3.03	3.11	1.19	0.87	0.93	0.82	1.03
7	2.72	1.82	0.89	0.93	0.83	1.04	0.67
8	2.97	1.86	1.10	0.97	0.76	0.96	0.63
9	4.91	3.80	1.14	0.94	1.13	1.06	0.77
10	3.38	1.86	0.98	1.00	0.88	1.03	0.55
11	6.80	7.64	1.70	1.06	0.95	1.04	1.12
12	4.08	2.59	1.09	0.97	1.00	1.01	0.63
13	2.59	1.91	0.75	0.96	0.83	1.13	0.74
14	7.86	4.17	1.59	1.01	1.15	1.11	0.53
15	3.55	2.31	0.89	0.98	0.97	1.10	0.65
16	4.80	2.97	1.02	1.01	1.08	1.13	0.62
17	6.21	4.50	1.14	1.07	1.24	1.07	0.72
18	4.88	2.59	0.97	0.99	1.25	1.07	0.53
19	8.41	4.75	1.02	1.01	1.92	1.12	0.57
20	5.88	3.28	0.92	1.02	1.44	1.13	0.56
21	2.68	2.19	0.67	1.04	1.00	1.00	0.82
22	8.17	-	1.80	1.00	1.09	1.09	-
23	8.06	5.01	2.01	1.09	0.96	0.99	0.62
24	7.94	3.28	1.78	1.23	1.09	0.88	0.41
25	3.08	3.44	0.81	1.14	0.94	0.93	1.12
26	3.61	2.20	0.96	1.02	0.81	1.18	0.61
27	6.26	3.80	1.39	1.04	1.11	1.02	0.61
28	3.65	4.36	1.43	0.71	1.00	0.94	1.19
29	5.70	6.74	0.92	1.43	1.15	0.99	1.18
30	2.61	2.91	0.68	1.09	0.95	0.97	1.12
31	4.88	4.09	1.10	1.30	0.98	0.91	0.84
32	5.42	5.31	1.24	1.17	1.05	0.94	0.98
33	7.96	6.04	1.13	1.70	1.27	0.85	0.76
34	5.33	3.97	1.02	1.10	1.23	1.01	0.75
35	4.97	4.29	1.61	1.01	0.71	1.13	0.86
36	5.76	4.39	1.83	0.99	0.75	1.11	0.76
37	1.49	1.13	0.78	0.98	0.48	1.07	0.75
38	4.38	3.50	0.85	1.48	1.01	0.89	0.80
39	5.68	4.89	1.03	1.37	1.04	1.01	0.86
40	6.19	5.07	1.09	1.37	1.16	0.94	0.82
41	3.51	2.81	1.18	0.94	0.84	0.98	0.80
42	3.57	4.54	1.01	1.00	0.77	1.20	1.27

Note:
1. The scale effect is 3.82
2. the labor compensation of sector 22 in 1997 is 0, so we do not report the result for sector 22.

Table 3 Decomposition results for gross output growth and labor compensation growth (semi-closed model)

Sector code	Gross output growth	Labor compensation growth	E $\mathbf{A}$	E $\bar{\mathbf{c}}$	E $\overline{\mathbf{g}}$	E r	E $\hat{\mathbf{B}}$ *	E $\hat{\mathbf{B}}$
1	1.93	1.81	0.94	0.64	0.90	0.97	0.80	0.75
2	4.33	2.93	1.12	1.01	0.94	0.98	0.91	0.61
3	5.84	5.71	1.57	1.02	0.89	0.99	0.92	0.89
4	5.15	4.07	1.06	0.99	1.13	0.99	0.95	0.75
5	2.61	2.12	0.77	0.96	0.82	0.99	0.95	0.77
6	3.03	3.11	1.20	0.82	0.89	0.96	0.79	0.81
7	2.72	1.82	0.89	0.90	0.81	0.99	0.92	0.61
8	2.97	1.86	1.11	0.94	0.74	0.98	0.87	0.54
9	4.91	3.80	1.14	0.92	1.12	0.99	0.92	0.72
10	3.38	1.86	0.99	0.97	0.86	0.98	0.91	0.50
11	6.80	7.64	1.70	1.03	0.94	0.99	0.91	1.03
12	4.08	2.59	1.09	0.95	0.98	0.98	0.90	0.57
13	2.59	1.91	0.75	0.95	0.82	0.99	0.96	0.71
14	7.86	4.17	1.59	1.00	1.14	0.99	0.95	0.51
15	3.55	2.31	0.89	0.97	0.95	0.99	0.95	0.61
16	4.80	2.97	1.02	1.00	1.07	0.99	0.96	0.60
17	6.21	4.50	1.14	1.05	1.23	0.99	0.93	0.68
18	4.88	2.59	0.97	0.97	1.23	0.99	0.93	0.49
19	8.41	4.75	1.02	0.99	1.90	0.99	0.96	0.54
20	5.88	3.28	0.92	1.01	1.43	0.99	0.97	0.54
21	2.68	2.19	0.68	1.01	0.97	0.98	0.89	0.73
22	8.17	-	1.81	0.98	1.08	0.99	0.94	-
23	8.06	5.01	2.02	1.06	0.94	0.98	0.89	0.55
24	7.94	3.28	1.79	1.17	1.04	0.97	0.82	0.34
25	3.08	3.44	0.81	1.09	0.91	0.98	0.85	0.95
26	3.61	2.20	0.96	1.02	0.81	1.00	0.99	0.61
27	6.26	3.80	1.40	1.01	1.09	0.98	0.91	0.55
28	3.65	4.36	1.44	0.68	0.97	0.97	0.86	1.02
29	5.70	6.74	0.92	1.38	1.12	0.98	0.88	1.05
30	2.61	2.91	0.68	1.05	0.93	0.98	0.87	0.98
31	4.88	4.09	1.11	1.24	0.95	0.97	0.84	0.71
32	5.42	5.31	1.24	1.12	1.02	0.98	0.86	0.84
33	7.96	6.04	1.14	1.62	1.21	0.97	0.80	0.61
34	5.33	3.97	1.03	1.06	1.20	0.98	0.90	0.67
35	4.97	4.29	1.61	1.00	0.70	0.99	0.96	0.83
36	5.76	4.39	1.84	0.97	0.74	0.99	0.95	0.73
37	1.49	1.13	0.78	0.97	0.47	0.99	0.93	0.70
38	4.38	3.50	0.86	1.41	0.98	0.97	0.83	0.66
39	5.68	4.89	1.03	1.34	1.01	0.98	0.90	0.77
40	6.19	5.07	1.09	1.32	1.12	0.98	0.86	0.70
41	3.51	2.81	1.19	0.91	0.82	0.98	0.88	0.71
42	3.57	4.54	1.01	1.00	0.77	1.00	1.00	1.27

Note:

1. The scale effect is 4.58
2. The labor compensation of sector 22 in 1997 is 0, so we do not report the result for sector 22.
3. E $\hat{\mathbf{B}}$ * denotes the effect of $\hat{\mathbf{B}}$ on the gross output growth; E $\hat{\mathbf{B}}$ denotes the effect of $\hat{\mathbf{B}}$ on the labor compensation growth.

Table 4 Decomposition results comparison for other countries and regions

Countries and regions	r	$E\mathbf{A}$		$E\overline{\mathbf{c}}$		$E\overline{\mathbf{g}}$		$E\hat{\mathbf{B}}$	
		Mean	*Std*	*Mean*	*Std*	*Mean*	*Std*	*Mean*	*Std*
Australia	-0.41	0.34	0.15	0.38	0.17	1.56	0.68	1.25	0.54
Austria	-0.19	0.37	0.29	0.09	0.07	1.55	1.23	6.84	5.41
Belgium	-0.26	0.18	0.19	0.38	0.39	0.46	0.48	2.27	2.36
Bulgaria	0.06								
Brazil	0.18	0.48	0.19	2.81	1.09	2.01	0.81	5.81	2.29
Canada	0.26	2.93	1.90	0.92	0.58	0.10	0.15	3.09	2.01
China	-0.27	0.82	0.48	1.76	1.06	2.03	1.19	11.70	6.99
Chinese Taipei	-0.75	0.85	0.64	1.16	0.88	5.42	4.02	9.37	7.00
Cyprus	-0.27								
Czech Republic	0.28	2.24	1.60	0.27	0.19	3.62	2.57	2.20	1.57
Germany	-0.31	0.96	0.59	0.10	0.07	1.93	1.21	7.43	4.67
Denmark	0.36	0.74	0.59	0.03	0.04	2.25	1.83	1.73	1.41
Spain	-0.10	0.06	0.03	0.38	0.19	0.28	0.14	9.74	4.74
Estonia	-0.06	0.86	0.72	0.42	0.34	0.14	0.12	0.04	0.04
Finland	-0.22	0.21	0.15	0.06	0.04	0.80	0.57	5.21	3.70
France	-0.11	1.13	0.71	0.03	0.03	0.81	0.51	3.74	2.35
United Kindom	-0.50	0.11	0.06	0.04	0.04	0.56	0.34	1.29	0.71
Greece	0.05								
Hungary	-0.17	2.12	1.41	0.09	0.06	3.35	2.25	0.37	0.25
Indonesia	-0.20	1.99	1.04	0.66	0.44	3.27	1.82	1.92	1.10
India	-0.67	4.30	1.97	0.39	0.19	3.73	1.71	12.53	5.84
Ireland	-0.48	1.47	1.14	0.61	0.47	1.72	1.29	3.93	3.04
Italy	-0.10	1.89	0.94	1.13	0.56	0.82	0.41	7.31	3.67
Japan	-0.15	0.30	0.16	1.67	0.92	1.01	0.56	3.68	2.02
Korea	-0.17	1.47	0.96	0.65	0.43	1.41	0.93	12.23	7.99
Lithuania	-0.14	0.49	0.42	0.51	0.56	0.95	0.85	2.58	2.43
Luxembourg	-0.27	0.50	0.70	0.11	0.15	2.05	3.08	5.04	7.29
Latvia	-0.32	2.73	2.05	0.55	0.44	0.60	0.46	6.70	5.17
Mexico	0.24	0.56	0.31	1.08	0.62	3.51	1.99	6.71	3.85
Malta	-0.11	3.11	1.94	0.51	0.36	1.25	0.97	6.83	4.41
Netherlands	-0.14	0.19	0.19	0.03	0.03	0.33	0.32	1.48	1.42
Poland	0.01	3.89	2.21	2.56	1.62	2.69	1.52	21.39	11.54
Portugal	-0.41	1.53	0.97	1.33	0.83	0.44	0.29	0.59	0.36
Romania	-0.03								
Russia	-0.63	2.51	1.24	0.11	0.07	0.79	0.40	3.75	1.85
Slovak Republic	0.07	4.47	3.39	0.47	0.34	2.50	1.97	0.16	0.16
Slovenia	0.09	2.79	2.25	0.27	0.23	1.39	1.15	3.85	3.20
Sweden	0.05	0.62	0.48	0.68	0.53	0.54	0.42	2.86	2.23
Turkey	-0.35	7.81	3.21	4.58	1.87	12.20	5.09	19.31	8.04
USA	-0.72	0.89	0.39	0.06	0.03	0.18	0.08	1.00	0.43

Note: 1. r denote the correlation coefficient between the change in the column sum of the domestic input coefficient matrix and the change in the corresponding labor compensation coefficient during 1997-2007.
2. *Mean* and *Std* denote the mean and standard deviation of the ARDs (%) across all sectors, respectively.

Table 5 The decomposition results for sectoral household consumption growth

Sector code	consumption growth	E $\bar{\mathbf{c}}$
1	1.05	0.36
2	2.26	0.77
6	2.19	0.75
7	0.69	0.24
8	2.46	0.84
9	1.32	0.45
10	1.61	0.55
11	15.73	5.37
12	2.25	0.77
13	0.53	0.18
15	1.50	0.51
16	1.38	0.47
17	4.31	1.47
18	1.92	0.66
19	2.41	0.82
20	2.11	0.72
21	3.14	1.07
23	5.60	1.91
24	4.12	1.40
25	3.72	1.27
27	3.72	1.27
28	0.55	0.19
29	12.57	4.29
30	3.92	1.34
31	5.01	1.71
32	4.51	1.54
33	7.03	2.40
34	4.30	1.47
37	2.11	0.72
38	7.32	2.50
39	8.13	2.77
40	5.47	1.87
41	1.99	0.68

Note:

1.The scale effect is 4.58. The effect of $\mathbf{A}$, r, $\hat{\mathbf{B}}$ and $\bar{\mathbf{g}}$ are 1.01,0.95,0.72 and 0.94, respectively.

2. We only report the results for sectors that provide consumption products.

Reference

Batey, Peter, Moss Madden, and M.J.Weeks, "Household Income and Expenditure in Extended Input-Output Models: A Comparative Theoretical and Empirical Analysis," *Journal of Regional Science*, 27, 341-356, 1987.

Batey, Peter, and Adam Z. Rose, "Extended Input-Output Models: Progress and Potential," *International Regional Science Review*, 13, 27-49, 1990.

Batey, Peter, Moss Madden, and Graham Scholefield, "Social-economic Impact Assessment of Large-Scale Projects Using Input-Output Analysis: a CaseStudy of an Airport ," *Regional Studies*, 27, 179-191, 1993.

Bai, Chongen and Qian Zhenjie, "The Factor Income Distribution in China: 1978-2007," *China Economic Review*, 21, 650-670, 2010.

Cloutier, Martin L. and Paul J. Thomasin, "Closing the Canadian Input-Output model: Homogenous vs Non-homogenous Household Sector Specifications," *Economic Systems Research*, 6, 397-414, 1994.

De Haan, Mark, "A structural Decomposition Analysis of Pollution in the Netherlands," *Economic Systems Research*, 13, 181-196, 2001.

Dietzenbacher, Erik and Bart Los, "Structural Decomposition Techniques: Sense and Sensitivity," *Economic Systems Research*, 10, 307-3244, 1998.

Dietzenbacher, Erik and Bart Los, "Structural Decomposition Analysis with Dependent Determinants," *Economic Systems Research*, 12, 497-514, 2000.

Dietzenbacher, Erik, Alex R. Hoen and Bart Los, "Labor Productivity in Western Europe 1975-1985: An Intercountry, Interindustry Analysis," *Journal of Regional Science*, 40, 425-452, 2000.

Dietzenbacher, Erik and Gülay Günlük-Şenesen, "Demand-Pull and Cost-Push Effects on Labor Income in Turkey, 1973-90," *Environment and Planning A*, 35, 1785-1807, 2003.

Dixon, Peter B. and B.R. Parmenter, "Computable General Equilibrium Modeling for Policy Analysis and Forecasting," In H. M. Amman, D.A. Kendrick & J. Rust (Ed.), *Handbook of computational Economics*. Amsterdam: Elsevier, 1996.

Leontief, Wassily W., "Input and Output Relations in the Economic Systems of the United States," *Review of Economics and Statistics*,18, 105-125, 1936.

Fujikawa, Kioshi. and Carlo Milana, "Input-Output Decomposition Analysis of Sectoral Price Gaps between Japan and China," *Economic Systems Research*, 14, 59-79, 2002.

Hewings, Geoffrey J.D. and M.C. Romanos, "Simulating less developed Regional Economies under Conditions of Limiting Information," *Geographical Analysis*, 13, 373-390, 1981.

Jensen-Butler, Chris and Bjarne Madsen, "Decomposition Analysis: an Extended Theoretical Foundation and Its Application to the Study of Regional Growth in Denmark," *Environment and Planning A*, 37, 2189-2208, 2005.

Kagawa, Shigemi, Yuki Kudoh, Keisuke Nansai and Tomohiro Tasaki, "The Economic and Environmental Consequences of Automobile Lifetime Extension and Fuel Economy Improvement: Japan's Case," *Economic Systems Research*, 20, 3-28, 2008.

McGregor, Peter G., J. Kim Swales and Ya Ping Yin, "Spillover and Feedback Effects in General Equilibrium Interregional Models of the National Economy: A Requiem for Interregional Input-Output? " In G. J.D., Hewings, M. Sonis, M. Madden, and Y. Kimura (Ed.) , *Understanding and Interpreting Economic Structure*, Berlin: Springer Press, 1999.

Magnus, Jan R. and Neudecker, Heinz, Matrix Differential Calculus with Applications in Statistics and Econometrics, John Wiley & Sons Ltd, Chichester, England, 2007.

Miller, Ronald E. and Peter D. Blair, Input-Output Analysis: Foundation and Extension, Cambridge University Press, New York, U.S.A., 2009.

Pyatt, Graham, "A SAM Approach to Modeling," *Journal of policy Modeling*, 10, 327-352, 1988.

Rose, Adam and Stephen Casler, "Input-Output Structural Decomposition Analysis: A Critical Appraisal," *Economic Systems Research*, 8, 33-62, 1996.

Trigg Andrew B. and Moss Madden, “Using a Demand System to Estimate Extended Input-Output Multipliers,” *Economic Systems Research*, 6, 385-395, 1994.

Schumann, Jochen, “Does It Make Sense to Apply the Static Open Input-Output Model for Imputation and Structural Decomposition?” *Economic Systems Research*, 6, 171-178, 1994.

Wolff, Edward N., “The Growth of Information Workers in the US Economy, 1950-2000: the Role of Technological Change, Computerization, and Structural Change,” *Economic Systems Research*, 18, 221-255, 2006.

Yamakawa Asuka and Glen P. Peters, “Structural Decomposition Analysis of Greenhouse Gas Emissions in Norway 1990-2002,” *Economic Systems Research*, 23, 303-318, 2011.

Yang Cuihong, Xikang Chen and Jian Xu, “A Method to Optimize Gross Fixed Capital Investments for Water Conservancy in China,” *Economic Systems Research*, 20, 151-172, 2008.

Appendix The sector classification for Chinese input-output tables

Sector code	Sector
1	Agriculture
2	Coal mining, washing and processing
3	Crude petroleum and natural gas products
4	Metal ore mining
5	Non-ferrous mineral mining
6	Manufacture of food products and tobacco processing
7	Textile goods
8	Wearing apparel, leather, furs, down and related products
9	Sawmills and furniture
10	Paper and products, printing and record medium reproduction
11	Petroleum processing, coking and nuclear fuel processing
12	Chemicals
13	Nonmetal mineral products
14	Metals smelting and pressing
15	Metal products
16	Common and special equipment
17	Transport equipment
18	Electric equipment and machinery
19	Telecommunication equipment, computer and other electronic equipment
20	Instruments, meters, cultural and office machinery
21	Art and craft and other manufacturing products
22	Scrap and waste
23	Electricity and heating power production and supply
24	Gas production and supply
25	Water production and supply
26	Construction
27	Transport and warehousing
28	Post
29	Information communication, computer service and software
30	Wholesale and retail trade
31	Accommodation, eating and drinking places
32	Finance and insurance
33	Real estate
34	Renting and commercial service
35	Research and development
36	General technical services
37	Water conservancy, environment, and public accommodation management
38	Household service and other social services
39	Education
40	Health service, social guarantee and social welfare
41	Culture, sports and amusements
42	Public management and social administration