ENVIRONMENTAL INPUT-OUTPUT MODELING OF THE PRICING WITH NON-LINEAR INTERSECTORAL LINKS

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Abstract

The problem of including the ecological constituent in the price and the necessity of construction input-output models of pricing taking into account the terms of ecologic-economy equilibrium are analyzed. The issue of pricing based on the Leontief-Ford input-output model is investigated. This model is an important tool for the practical development of a number of predictions and quantitative estimates of the pricing process in terms of ecological and economic balance of a region or country.

1. Introduction

Since the main source of pollution is the production that is pollution is the result of the economic activity so this result must be reflected in the models of industrial and economical systems, particularly in the nonlinear input-output model. This nonlinear model can more fully identify the features of pricing activity and on the basis of it to forecast changes in price indices by changing certain elements of input-output balance.

The problem of the forecasting of the prices in ecological economy can be realized on the basis of the nonlinear input-output model of interagency

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environmental and economic balance, reflecting the simultaneous operation of two plants: main (the branches of material production) and secondary (the industry of destruction of pollutants). On this basis it is possible to realize the process of pricing in the ecological economy.

2. The environmental input-output model with nonlinear intersectoral links

Consider a variant of the environmental input-output model with nonlinear intersectoral links in the form:

$$\begin{cases} x_i^{(1)} = \sum_{j=1}^{n} \varphi_{ij}^{(11)}\left(x_j^{(1)}\right) + \sum_{s=1}^{m} \varphi_{is}^{(12)}\left(x_s^{(2)}\right) + y_i^{(1)}, \ i = \overline{1,\ n}, \\ x_l^{(2)} = \sum_{j=1}^{n} \varphi_{lj}^{(21)}\left(x_j^{(1)}\right) + \sum_{s=1}^{m} \varphi_{ls}^{(22)}\left(x_s^{(2)}\right) - y_l^{(2)}, \ l = \overline{1,\ m}, \end{cases} \tag{1}$$

where $x^{(1)} \in \mathbb{R}_+^n$ – vector of total output of main production ($\mathbb{R}_+^n$ – positive orthant of n-dimensional vector space);

$x^{(2)} \in \mathbb{R}_+^m$ – vector of total destroyed industrial contaminant (that total output vector of support sector);

$y^{(1)} \in \mathbb{R}_+^n$ – vector of final output;

$y^{(2)} \in \mathbb{R}_+^m$ – vector of undestroyed industrial contaminant;

$\varphi_{ij}^{(11)}\left(x_j^{(1)}\right)$ – nonlinear function of spending of the good i for producing the good j in number $x_j^{(1)}$;

$\varphi_{is}^{(12)}\left(x_s^{(2)}\right)$ – nonlinear function of spending of the good i for destroying the contaminant s in number $x_s^{(2)}$;

$\varphi_{lj}^{(21)}\left(x_j^{(1)}\right)$ – nonlinear function of production of the contaminant l during the production process of the good $x_j^{(1)}$ in number j;

$\varphi_{ls}^{(22)}\left(x_s^{(2)}\right)$ – nonlinear function of production of the contaminant l during the destroying process of the contaminant s.

Model (1) is also present in vector form:

$$\begin{cases} x^{(1)} = \Phi^{(11)}\left(x^{(1)}\right) + \Phi^{(12)}\left(x^{(2)}\right) + y^{(1)}, \\ x^{(2)} = \Phi^{(21)}\left(x^{(1)}\right) + \Phi^{(22)}\left(x^{(2)}\right) - y^{(2)}, \end{cases} \tag{2}$$

where $\Phi_i^{(11)}\left(x^{(1)}\right) = \sum_{j=1}^{n} \varphi_{ij}^{(11)}\left(x_j^{(1)}\right),\ i = \overline{1, n}$; $\Phi_i^{(12)}\left(x^{(2)}\right) = \sum_{s=1}^{m} \varphi_{is}^{(12)}\left(x_s^{(2)}\right),\ i = \overline{1, n}$;

$\Phi_l^{(21)}\left(x^{(1)}\right) = \sum_{j=1}^{n} \varphi_{lj}^{(21)}\left(x_j^{(1)}\right),\ l = \overline{1, m}$; $\Phi_l^{(22)}\left(x^{(2)}\right) = \sum_{s=1}^{m} \varphi_{ls}^{(22)}\left(x_s^{(2)}\right),\ l = \overline{1, m}$;

$\Phi^{(11)}\left(x^{(1)}\right) = \left(\Phi_1^{(11)}\left(x^{(1)}\right), \ldots, \Phi_n^{(11)}\left(x^{(1)}\right)\right)^T$; $\Phi^{(12)}\left(x^{(2)}\right) = \left(\Phi_1^{(12)}\left(x^{(2)}\right), \ldots, \Phi_n^{(12)}\left(x^{(2)}\right)\right)^T$;

$\Phi^{(21)}\left(x^{(1)}\right) = \left(\Phi_1^{(21)}\left(x^{(1)}\right), \ldots, \Phi_m^{(21)}\left(x^{(1)}\right)\right)^T$; $\Phi^{(22)}\left(x^{(2)}\right) = \left(\Phi_1^{(22)}\left(x^{(2)}\right), \ldots, \Phi_m^{(22)}\left(x^{(2)}\right)\right)^T$;

T – transpose.

Model (2) (or (1)) is often called the direct model the Leontief-Ford [1, 2].

3. The model of the pricing with non-linear intersectoral links

If we denote by $\tilde{x}_j^{(1)}$ – the cost of main good from sector j, $\tilde{x}_s^{(2)}$ – the cost of the destroyed contaminant s, $\tilde{z}_j^{(1)}$ – the cost of net output from the main sector j, $\tilde{z}_s^{(2)}$ – the cost of net output from the support sector s, and by $\tilde{\varphi}_{ij}^{(11)}\left(x_j^{(1)}\right)$, $\tilde{\varphi}_{is}^{(12)}\left(x_s^{(2)}\right)$ and by $\tilde{\varphi}_{lj}^{(21)}\left(x_j^{(1)}\right)$, $\tilde{\varphi}_{ls}^{(22)}\left(x_s^{(2)}\right)$ – appropriate valuable analogues of the above costs

$\varphi_{ij}^{(11)}\left(x_j^{(1)}\right)$, $\varphi_{is}^{(12)}\left(x_s^{(2)}\right)$ and outputs $\varphi_{lj}^{(21)}\left(x_j^{(1)}\right)$, $\varphi_{ls}^{(22)}\left(x_s^{(2)}\right)$, than in analogue with [3] we can consider the scheme of environmental input-output model with nonlinear intersectoral links in the valuable form "by columns" (that to consider the first and the third quadrants) and receive the system of relations:

$$\begin{cases} \tilde{x}_j^{(1)} = \sum_{i=1}^{n} \tilde{\varphi}_{ij}^{(11)}\left(x_j^{(1)}\right) + \sum_{l=1}^{m} \tilde{\varphi}_{lj}^{(21)}\left(x_j^{(1)}\right) + \tilde{z}_j^{(1)}, \ j=\overline{1,n}, \\ \tilde{x}_s^{(2)} = \sum_{i=1}^{n} \tilde{\varphi}_{is}^{(12)}\left(x_s^{(2)}\right) + \sum_{l=1}^{m} \tilde{\varphi}_{ls}^{(22)}\left(x_s^{(2)}\right) + \tilde{z}_s^{(2)}, \ s=\overline{1,m}. \end{cases} \tag{3}$$

Assume $p_j^{(1)}$ – the price of one good j, $p_s^{(2)}$ – the price of destroying of contaminant s in number 1, $k_j^{(1)}$ – coefficient of net output at one good j, $\left(\tilde{z}_j^{(1)} = k_j^{(1)} x_j^{(1)}\right)$ or relative price of main product in amount 1 that included in net output from sector j of main production (if $\hat{k}_j^{(1)}$ – portion of good j that included in net output than $k_j^{(1)} = \hat{k}_j^{(1)} p_j^{(1)}$ and $\tilde{z}_j^{(1)} = k_j^{(1)} x_j^{(1)} = p_j^{(1)}\left(\hat{k}_j^{(1)} x_j^{(1)}\right)\Big)$; $k_s^{(2)}$ coefficient of net output from support sector s $\left(\tilde{z}_s^{(2)} = k_s^{(2)} x_s^{(2)}\right)$ or relative price of destroying contaminant s that included in net output from sector s of support production (if $\hat{k}_s^{(2)}$ – portion of the destroyed contaminant that included in net output than $k_s^{(2)} = \hat{k}_s^{(2)} p_s^{(2)}$ і $\tilde{z}_s^{(2)} = k_s^{(2)} x_s^{(2)} = p_s^{(2)}\left(\hat{k}_s^{(2)} x_s^{(2)}\right)\Big)$.

Taking into consideration above sings we can write equality in form:

$$\begin{cases} p_j^{(1)} x_j^{(1)} = \sum_{i=1}^{n} p_i^{(1)} \varphi_{ij}^{(11)}\left(x_j^{(1)}\right) + \sum_{l=1}^{m} p_l^{(2)} \varphi_{lj}^{(21)}\left(x_j^{(1)}\right) + k_j^{(1)} x_j^{(1)}, \ j=\overline{1,n}, \\ p_s^{(2)} x_s^{(2)} = \sum_{i=1}^{n} p_i^{(1)} \varphi_{is}^{(12)}\left(x_s^{(2)}\right) + \sum_{l=1}^{m} p_l^{(2)} \varphi_{ls}^{(22)}\left(x_s^{(2)}\right) + k_s^{(2)} x_s^{(2)}, \ s=\overline{1,m}. \end{cases}, \tag{4}$$

Dividing the first n equalities of the system (4) appropriately into $x_j^{(1)} > 0$ and the next m equalities into $x_s^{(2)} > 0$, we will get the system:

$$\begin{cases} p_j^{(1)} = \sum_{i=1}^{n} p_i^{(1)} \left[\varphi_{ij}^{(11)} \left(x_j^{(1)} \right) / \left(x_j^{(1)} \right) \right] + \sum_{l=1}^{m} p_l^{(2)} \left[\varphi_{lj}^{(21)} \left(x_j^{(1)} \right) / x_j^{(1)} \right] + k_j^{(1)}, \ j = \overline{1, n}, \\ p_s^{(2)} = \sum_{i=1}^{n} p_i^{(1)} \left[\varphi_{is}^{(12)} \left(x_s^{(2)} \right) / \left(x_s^{(2)} \right) \right] + \sum_{l=1}^{m} p_l^{(2)} \left[\varphi_{ls}^{(22)} \left(x_s^{(2)} \right) / x_s^{(2)} \right] + k_s^{(2)}, \ s = \overline{1, m}, \end{cases} \tag{5}$$

that in vector-matrix form can be written as follows:

$$\begin{cases} p^{(1)} = \left(F^{(11)} \left(x^{(1)} \right) \right)^T p^{(1)} + \left(F^{(21)} \left(x^{(1)} \right) \right)^T p^{(2)} + k^{(1)}, \\ p^{(2)} = \left(F^{(12)} \left(x^{(2)} \right) \right)^T p^{(1)} + \left(F^{(22)} \left(x^{(2)} \right) \right)^T p^{(2)} + k^{(2)}, \end{cases} \tag{6}$$

where $F^{(11)}\left(x^{(1)}\right) = \left(\varphi_{ij}^{(11)} \left(x_j^{(1)} \right) / x_j^{(1)} \right)_{i,j=1}^{n}$, $F^{(12)}\left(x^{(2)}\right) = \left(\varphi_{is}^{(12)} \left(x_s^{(2)} \right) / x_s^{(2)} \right)_{i,s=1}^{n,\,m}$, $F^{(21)}\left(x^{(1)}\right) = \left(\varphi_{lj}^{(21)} \left(x_j^{(1)} \right) / x_j^{(1)} \right)_{l,j=1}^{m,\,n}$, $F^{(22)}\left(x^{(2)}\right) = \left(\varphi_{ls}^{(22)} \left(x_s^{(2)} \right) / x_s^{(2)} \right)_{l,s=1}^{m}$, – appropriate functional matrixes.

The system of relations (6) (or (5)) is the double-natured model relatively to prices in comparison with model (2). The model is nonlinear because the elements of matrixes $F^{(11)}\left(x^{(1)}\right)$, $F^{(21)}\left(x^{(1)}\right)$, $F^{(12)}\left(x^{(2)}\right)$, $F^{(22)}\left(x^{(2)}\right)$ are nonlinear functions. The main problem of the model (6) (as the direct model (2)) is the problem of productivity. For model (6) the problem productivity is the existence of positive vector of prices $p^{(1)}$, $p^{(2)}$ with set matrixes $F^{(11)}\left(x^{(1)}\right)$, $F^{(21)}\left(x^{(1)}\right)$, $F^{(12)}\left(x^{(2)}\right)$, $F^{(22)}\left(x^{(2)}\right)$ and vectors $k^{(1)}$, $k^{(2)}$. In partial case, when the elements of above matrixes are linear, the model (6) transforms into linear the double-natured model of Leontief-Ford relatively to prices [3], for which the problem of productivity is the same as the problem of productivity for Leontief model [4], moreover the model (6) in this case does not depend on $x^{(1)}$ and $x^{(2)}$, therefore the process of its solving does not depend on the solving of the direct model (2).

There is completely different situation if the models (2) and (6) are nonlinear as in our case. Then the solving of the model (6) (if it exists) depends on $x^{(1)}$ and $x^{(2)}$, therefore $p^{(1)} = p^{(1)}\left(x^{(1)}, x^{(2)}\right)$, $p^{(2)} = p^{(2)}\left(x^{(1)}, x^{(2)}\right)$. Obviously, for determining the solving of model (6), which together with model (2) shows the processes of balances within ecological and economy system, we should for the first determine $x^{(1)*}$ and $x^{(2)*}$ of model (2) with set matrixes $\Phi^{(11)}\left(x^{(1)}\right)$, $\Phi^{(12)}\left(x^{(2)}\right)$, $\Phi^{(21)}\left(x^{(1)}\right)$, $\Phi^{(22)}\left(x^{(2)}\right)$ and vectors $y^{(1)}$, $y^{(2)}$. Thereafter we must place $x^{(1)*}$ and $x^{(2)*}$ into model (6) and determine the solution $p^{(1)*} = p^{(1)}\left(x^{(1)*}, x^{(2)*}\right)$, $p^{(2)*} = p^{(2)}\left(x^{(1)*}, x^{(2)*}\right)$. We should note that model (6) can be written in form of the system of equations

$$p = F^{T}\left(x\right)p + k, \tag{7}$$

where $p = \left(p^{(1)},\ p^{(2)}\right)^{T}$, $k = \left(k^{(1)},\ k^{(2)}\right)^{T}$,

$$F^{T}\left(x\right) = \begin{pmatrix} F^{(11)}\left(x^{(1)}\right)^{T} & F^{(21)}\left(x^{(1)}\right)^{T} \\ F^{(12)}\left(x^{(2)}\right)^{T} & F^{(22)}\left(x^{(2)}\right)^{T} \end{pmatrix},$$

$x = \left(x^{(1)},\ x^{(2)}\right)^{T}$.

If $x^{*} = \left(x^{(1)*},\ x^{(2)*}\right)^{T}$ – the solution of the model (2), than by $x = x^{*}$ the system (7) will be the system of linear equations, therefore its solution $p^{*} \geq 0$ $\left(0 \in \mathbb{R}^{n+m}\right)$ will exist if the known features of Leontief model productivity are performed [4].

Consider the example of the environmental input-output model with nonlinear intersectoral links. Assume that the nonlinear functions of output and costs are quadratic:

$$\varphi_{ij}^{(11)}\left(x_j^{(1)}\right)=a_{ij}^{(11)}\left(x_j^{(1)}\right)^2,\ i,\ j=\overline{1,\ n};$$

$$\varphi_{is}^{(12)}\left(x_s^{(2)}\right)=a_{is}^{(12)}\left(x_s^{(2)}\right)^2,\ i=\overline{1,\ n};\ s=\overline{1,\ m};$$

$$\varphi_{lj}^{(21)}\left(x_j^{(1)}\right)=a_{lj}^{(21)}\left(x_j^{(1)}\right)^2,\ l=\overline{1,\ m};\ j=\overline{1,\ n};$$

$$\varphi_{ls}^{(22)}\left(x_s^{(2)}\right)=a_{ls}^{(22)}\left(x_s^{(2)}\right)^2,\ l,\ s=\overline{1,\ m},$$

where $a_{ij}^{(11)}$, $a_{is}^{(12)}$, $a_{lj}^{(21)}$, $a_{ls}^{(22)}$ – number coefficients.

In the most easy case by $n=2$, $m=1$ (there are two main sectors and one support sector) the model (1) can be written as

$$\begin{cases} x_1^{(1)}=a_{11}^{(11)}\left(x_1^{(1)}\right)^2+a_{12}^{(11)}\left(x_2^{(1)}\right)^2+a_{11}^{(12)}\left(x_1^{(2)}\right)^2+y_1^{(1)}, \\ x_2^{(1)}=a_{21}^{(11)}\left(x_1^{(1)}\right)^2+a_{22}^{(11)}\left(x_2^{(1)}\right)^2+a_{21}^{(12)}\left(x_1^{(2)}\right)^2+y_2^{(1)}, \\ x_1^{(2)}=a_{11}^{(21)}\left(x_1^{(1)}\right)^2+a_{12}^{(21)}\left(x_2^{(1)}\right)^2+a_{11}^{(22)}\left(x_1^{(2)}\right)^2-y_1^{(2)}, \end{cases}$$

and the double-natured to prices, namely model (5) – as

$$\begin{cases} p_1^{(1)}=p_1^{(1)}a_{11}^{(11)}x_1^{(1)}+p_2^{(1)}a_{21}^{(11)}x_1^{(1)}+p_1^{(2)}a_{11}^{(21)}x_1^{(1)}+k_1^{(1)}, \\ p_2^{(1)}=p_1^{(1)}a_{12}^{(11)}x_2^{(1)}+p_2^{(1)}a_{22}^{(11)}x_2^{(1)}+p_1^{(2)}a_{12}^{(21)}x_2^{(1)}+k_2^{(1)}, \\ p_1^{(2)}=p_1^{(1)}a_{11}^{(12)}x_1^{(2)}+p_2^{(1)}a_{21}^{(12)}x_1^{(2)}+p_1^{(2)}a_{11}^{(22)}x_1^{(2)}+k_1^{(2)}. \end{cases}$$

Taking, for example, $k_1^{(1)} = k_2^{(1)} = k_1^{(2)} = 0{,}3$ and rewriting the last system as $A(x)p = b$, where $p = \left(p_1^{(1)},\, p_2^{(1)},\, p_1^{(2)}\right)^T$, $b = (0{,}3\ \ 0{,}3\ \ 0{,}3)^T$, $x = \left(x_1^{(1)},\, x_2^{(1)},\, x_1^{(2)}\right)^T$,

$$A(x) = \begin{pmatrix} 1 - a_{11}^{(11)} x_1^{(1)} & -a_{21}^{(11)} x_1^{(1)} & -a_{11}^{(21)} x_1^{(1)} \\ -a_{12}^{(11)} x_2^{(1)} & 1 - a_{22}^{(11)} x_2^{(1)} & -a_{12}^{(21)} x_2^{(1)} \\ -a_{11}^{(12)} x_1^{(2)} & -a_{21}^{(12)} x_1^{(2)} & 1 - a_{11}^{(22)} x_1^{(2)} \end{pmatrix},$$

we can determine the solution:

$$p_1^{(1)} = p_1^{(1)}\left(x_1^{(1)},\, x_2^{(1)},\, x_1^{(2)}\right) = \frac{0{,}3\left(1 + 0{,}4x_1^{(1)} - 0{,}2x_2^{(1)} - 0{,}2x_1^{(2)}\right)}{1 - 0{,}2\left(x_1^{(1)} + x_2^{(1)} + x_1^{(2)}\right)},$$

$$p_2^{(1)} = p_2^{(1)}\left(x_1^{(1)},\, x_2^{(1)},\, x_1^{(2)}\right) = \frac{0{,}3\left(1 + 0{,}4x_2^{(1)} - 0{,}2x_1^{(1)} - 0{,}2x_1^{(2)}\right)}{1 - 0{,}2\left(x_1^{(1)} + x_2^{(1)} + x_1^{(2)}\right)},$$

$$p_1^{(2)} = p_1^{(2)}\left(x_1^{(1)},\, x_2^{(1)},\, x_1^{(2)}\right) = \frac{0{,}3\left(1 + 0{,}4x_1^{(2)} - 0{,}2x_1^{(1)} - 0{,}2x_2^{(1)}\right)}{1 - 0{,}2\left(x_1^{(1)} + x_2^{(1)} + x_1^{(2)}\right)}.$$

It is obvious that the solved functions of balanced prices have appropriate domains of and the concrete values of prices can be solved by the concrete values of the components $x_1^{(1)},\, x_2^{(1)},\, x_1^{(2)}$. In particular, substituting in these formulas the solving of the direct environmental input-output model with nonlinear intersectoral links, we will determine the balanced prices.

Now consider the problem of pricing, which can be sold on the base of model (6) (or (7)) using some approaches. Almost all the approaches are based on the assumptions concerning the methods of calculations of the net output, namely concerning the structure of the net output vectors. Denoting by $\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)} \in \mathbb{R}_+^n$ and $\alpha^{(2)}, \beta^{(2)}, \gamma^{(2)} \in \mathbb{R}_+^m$ – appropriate vectors of such coefficients as amortization, salaries and additional product of main and support sectors $\left(k^{(1)} = \alpha^{(1)} + \beta^{(1)} + \gamma^{(1)},\ k^{(2)} = \alpha^{(2)} + \beta^{(2)} + \gamma^{(2)}\right)$ and assuming, that additional product is proportional to salaries with coefficient υ (υ – the only rule of additional product), we will go from the system (6) to system

$$\begin{cases} p^{(1)} = \left(F^{(11)}\left(x^{(1)}\right)\right)^T p^{(1)} + \left(F^{(21)}\left(x^{(1)}\right)\right)^T p^{(2)} + \alpha^{(1)} + (1+\upsilon)\beta^{(1)}, \\ p^{(2)} = \left(F^{(12)}\left(x^{(2)}\right)\right)^T p^{(1)} + \left(F^{(22)}\left(x^{(2)}\right)\right)^T p^{(2)} + \alpha^{(2)} + (1+\upsilon)\beta^{(2)}. \end{cases}$$

If to assume that $\bar{\upsilon}$ – the only rule of additional product in one number of cost, namely

$$\gamma^{(1)} = \bar{v}\left(\left(F^{(11)}\left(x^{(1)}\right)\right)^T p^{(1)} + \left(F^{(21)}\left(x^{(1)}\right)\right)^T p^{(2)} + \alpha^{(1)} + \beta^{(1)}\right),$$

$$\gamma^{(2)} = \bar{v}\left(\left(F^{(12)}\left(x^{(2)}\right)\right)^T p^{(1)} + \left(F^{(22)}\left(x^{(2)}\right)\right)^T p^{(2)} + \alpha^{(2)} + \beta^{(2)}\right),$$

than the model (6) can be written as

$$\begin{cases} p^{(1)} = (1+\bar{v})\left(\left(F^{(11)}\left(x^{(1)}\right)\right)^T p^{(1)} + \left(F^{(21)}\left(x^{(1)}\right)\right)^T p^{(2)} + \alpha^{(1)} + \beta^{(1)}\right), \\ p^{(2)} = (1+\bar{v})\left(\left(F^{(12)}\left(x^{(2)}\right)\right)^T p^{(1)} + \left(F^{(22)}\left(x^{(2)}\right)\right)^T p^{(2)} + \alpha^{(2)} + \beta^{(2)}\right). \end{cases} \quad (9)$$

The systems of equations (8) and (9) allow to determine the complex of balanced prices $p^{(1)} = p^{(1)*}\left(x^{(1)}, x^{(2)}\right)$, $p^{(2)} = p^{(2)*}\left(x^{(1)}, x^{(2)}\right)$, that are the solutions of these systems. We should note that in the case of system (8) we talk about so-called formula of "cost", and in the case of (9) – about "average cost", that can be used during solving the problem of pricing. Of course, on the base of model (6) it is possible to propose many other its modifications, that can be used for modeling of pricing in ecological-economy system with nonlinear intersectoral links.

The problem of the pricing is not only difficult and essential for any changes in economy (or ecological-economy) system, as the determining, for example on the base of models (8), (9), balanced prices does not mean that these prices will be really in practice. Besides, the changes at least in one component of the price vector $p^{(1)}$ or $p^{(2)}$ break the balancing in the set of prices. In order to access from one set of balanced prices to another set it is necessary to use the price indexes. It is proven that the problem of determining the price indexes can be solved using the double-natured model.

For determining the price indexes let us back to question of the structure of net output vectors. It is clear that except amortization, salaries and additional product in this structure can be present also other components. Without writing its economic sense purpose that

$$\tilde{z}_j^{(1)} = \sum_{r=1}^{R} \tilde{\upsilon}_{rj}^{(1)} = \sum_{r=1}^{R} \upsilon_{rj}^{(1)} p_j^{(1)}, \ j = \overline{1, n}, \tag{10}$$

$$\tilde{z}_s^{(2)} = \sum_{q=1}^{Q} \tilde{\upsilon}_{qs}^{(2)} = \sum_{q=1}^{Q} \upsilon_{qs}^{(2)} p_s^{(2)}, \ s = \overline{1, m}, \tag{11}$$

where $\tilde{\upsilon}_{rj}^{(1)}$ – the cost of net output with type r from sector j of main industry;

$\upsilon_{rj}^{(1)}$ – the amount of net output with type r from sector j of main industry in kind;

$\tilde{\upsilon}_{qs}^{(2)}$ – the cost of net output with type q from sector s of support industry;

$\upsilon_{qs}^{(2)}$ – the cost of net output with type q from sector s of support industry in kind.

Considering (3), (4), (10), (11) and written above markings, get the system of balanced interrelations

$$\begin{cases} p_j^{(1)} x_j^{(1)} = \sum_{i=1}^{n} p_i^{(1)} \varphi_{ij}^{(11)}\left(x_j^{(1)}\right) + \sum_{l=1}^{m} p_i^{(2)} \varphi_{lj}^{(21)}\left(x_j^{(1)}\right) + \sum_{r=1}^{R} \upsilon_{rj}^{(1)} p_j^{(1)},\ j = \overline{1, n}, \\ p_s^{(2)} x_s^{(2)} = \sum_{i=1}^{n} p_i^{(1)} \varphi_{is}^{(12)}\left(x_s^{(2)}\right) + \sum_{l=1}^{m} p_l^{(2)} \varphi_{ls}^{(22)}\left(x_s^{(2)}\right) + \sum_{q=1}^{Q} \upsilon_{qs}^{(2)} p_s^{(2)},\ s = \overline{1, m}. \end{cases} \quad (12)$$

Denote by $\pi^{(1)} = \left(\pi_1^{(1)}, \dots, \pi_n^{(1)}\right)^T \in \mathbb{R}_+^n$, $\pi^{(2)} = \left(\pi_1^{(2)}, \dots, \pi_m^{(2)}\right)^T \in \mathbb{R}_+^m$ – appropriate vectors of price indexes for the next time of period $[t, t+1]$ in the comparison of previous time of period $[t-1, t]$. On that, for providing the balance of cost we ought to go from (12) to

$$\begin{cases} p_j^{(1)} \pi_j^{(1)} x_j^{(1)} = \sum_{i=1}^{n} p_i^{(1)} \pi_i^{(1)} \varphi_{ij}^{(11)}\left(x_j^{(1)}\right) + \sum_{l=1}^{m} p_l^{(2)} \pi_l^{(2)} \varphi_{lj}^{(21)}\left(x_j^{(1)}\right) + \sum_{r=1}^{R} \upsilon_{rj}^{(1)} p_j^{(1)} \pi_j^{(1)},\ j = \overline{1, n}, \\ p_s^{(2)} \pi_s^{(2)} x_s^{(2)} = \sum_{i=1}^{n} p_i^{(1)} \pi_i^{(1)} \varphi_{is}^{(12)}\left(x_s^{(2)}\right) + \sum_{l=1}^{m} p_l^{(2)} \pi_l^{(2)} \varphi_{ls}^{(22)}\left(x_s^{(2)}\right) + \sum_{q=1}^{Q} \upsilon_{qs}^{(2)} p_s^{(2)} \pi_s^{(2)},\ s = \overline{1, m}. \end{cases} \quad (13)$$

or to

$$\begin{cases} \tilde{x}_j^{(1)} \pi_j^{(1)} = \sum_{i=1}^{n} \tilde{\varphi}_{ij}^{(11)}\left(x_j^{(1)}\right) \pi_i^{(1)} + \sum_{l=1}^{m} \tilde{\varphi}_{lj}^{21}\left(\tilde{x}_j^{(1)}\right) \pi_l^{(2)} + \sum_{r=1}^{R} \tilde{\upsilon}_{rj}^{(1)} \pi_j^{(1)},\ j = \overline{1, n}, \\ \tilde{x}_s^{(2)} \pi_s^{(2)} = \sum_{i=1}^{n} \tilde{\varphi}_{is}^{(12)}\left(x_s^{(2)}\right) \pi_i^{(1)} + \sum_{l=1}^{m} \tilde{\varphi}_{ls}^{(22)}\left(x_s^{(2)}\right) \pi_l^{(2)} + \sum_{q=1}^{Q} \tilde{\upsilon}_{qs}^{(2)} \pi_s^{(2)},\ s = \overline{1, m}. \end{cases} \quad (14)$$

So, taking into the consideration that at period $[t, t+1]$ the structure of costs is constant in comparison with the previous period $[t-1, t]$, the system (14) (or (13)) can be used for determining the price indexes in multisectoral ecological-economy system. The determining of the price indexes lets control the prices balances and in time to react on changes in any constituent of net output. From the point of view of the decision making person it allows to optimize the process of pricing and its dynamics.

4. Empirical Analysis

The aim of empirical analysis is the prognostication of price indexes in Ukraine for 2012-2015 years on the base of built models. For this reason we design the application using Matlab software. Its structure is shown at the Fig. 1:

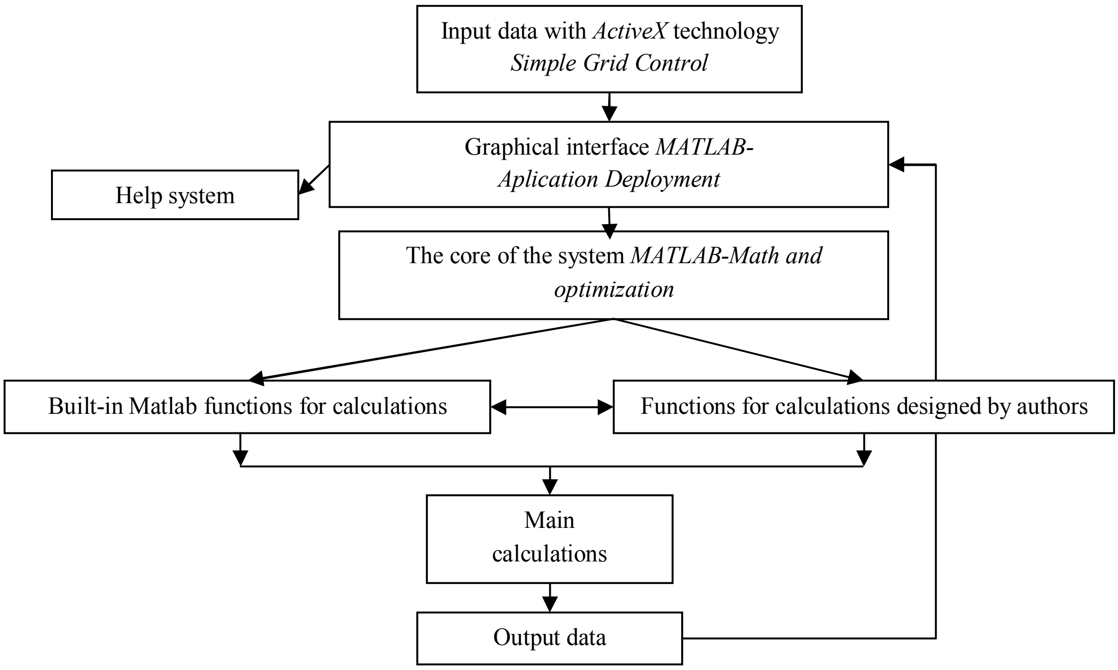

Fig. 1. The structure of the application

The empirical analysis of pricing in environmentally balanced economy was made for one supporting sector and such main sectors as (according to classification in Ukraine):

- agriculture, hunting, forestry;
- fishing, fish farming;
- mining industry;
- processing industry;
- production and distribution of electricity, gas and water;
- construction;
- trade, repair of motor vehicles, household goods and personal and household goods;

– hotels and restaurants;

– transport and communication;

– financial affairs;

– real estate, renting and services to individuals;

– public administration;

– education;

– health care and social assistance;

– communal and personal service activities in the field of culture and sport.

On the base of statistical data for the environmental input-output model with nonlinear intersectoral links it was get such results:

1. The vectors of main sectors $x^{(1)} \in \mathbb{R}_+^n$ and support and destroyed industrial contaminant $x^{(2)} \in \mathbb{R}_+^m$ are as follows:

Table 1

The total output of main production and total destroyed industrial contaminant (UAH)

The total output of main production	The total destroyed industrial contaminant
128257,32	1608,2729
1257,0561	
74178,722	
802288,8	
68339,831	
100115,3	
174268,02	
15779,222	
140590,59	
59054,469	
114523,74	
46485,135	
46711,98	
36367,618	
27167,807	

If to compare the data that shown in Table 1 with the data that is present in the official Ukrainian input-output tables we can make the conclusions that determined results correctly express the logic of the model (1) and designed software.

2. The prognostic values of net outputs coefficients from main and support industries are calculated according to econometric methods (table 2).

Table 2

The prognostic values of net outputs coefficients

The net outputs coefficients		Years			
		2012	2013	2014	2015
Main sectors	agriculture, hunting, forestry;	0,359	0,348	0,3339	0,329
	fishing, fish farming;	0,304	0,313	0,321	0,328
	mining industry;	0,545	0,577	0,609	0,641
	processing industry;	0,271	0,283	0,293	0,303
	production and distribution of electricity, gas and water;	0,368	0,354	0,286	0,16
	construction;	0,313	0,294	0,276	0,258
	trade, repair of motor vehicles, household goods and personal and household goods;	0,545	0,539	0,533	0,527
	hotels and restaurants;	0,551	0,54	0,552	0,564
	transport and communication;	0,492	0,473	0,455	0,437
	financial affairs;	0,756	0,769	0,789	0,809
	real estate, renting and services to individuals;	0,514	0,495	0,476	0,457
	public administration;	0,798	0,831	0,862	0,893
	education;	0,702	0,696	0,69	0,684
	health care and social assistance;	0,644	0,651	0,658	0,665
	communal and personal service activities in the field of culture and sport.	-0,366	-0,447	-0,527	-0,607
Support sector	$k_1^{(2)}$	0,913	0,9	0,876	0,829

3. According to obtained coefficients of net outputs and the double-natured Leontief-Ford model (7) we have got such price indexes (table 3):

Table 3

The prognostic values of price indexes

Price indexes		Years			
		2012	2013	2014	2015
Price indexes of goods of main production	agriculture, hunting, forestry;	0,359	0,348	0,3339	0,329
	fishing, fish farming;	0,304	0,313	0,321	0,328
	mining industry;	0,545	0,577	0,609	0,641
	processing industry;	0,271	0,283	0,293	0,303
	production and distribution of electricity, gas and water;	0,368	0,354	0,286	0,16
	construction;	0,313	0,294	0,276	0,258

	trade, repair of motor vehicles, household goods and personal and household goods;	0,545	0,539	0,533	0,527
	hotels and restaurants;	0,551	0,54	0,552	0,564
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	financial affairs;	0,756	0,769	0,789	0,809
	real estate, renting and services to individuals;	0,514	0,495	0,476	0,457
	public administration;	0,798	0,831	0,862	0,893
	education;	0,702	0,696	0,69	0,684
	health care and social assistance;	0,644	0,651	0,658	0,665
	communal and personal service activities in the field of culture and sport.	-0,366	-0,447	-0,527	-0,607
The price indexes of destroying the industrial contaminant		0,913	0,9	0,876	0,829

The analysis of the prices on the base of the environmental input-output model with nonlinear intersectoral links provides the construction of output and cost functions. According to the empirical calculations we have got such functions that demonstrate real economic situation:

$$\varphi_{ij}^{(11)}\left(x_j^{(1)}\right) = a_{ij}^{(11)} \ln\left(x_j^{(1)}\right)^2, \; i,\, j = \overline{1,\, n};$$

$$\varphi_{is}^{(12)}\left(x_s^{(2)}\right) = a_{is}^{(12)} \ln\left(x_s^{(2)}\right)^2, \; i = \overline{1,\, n};\; s = \overline{1,\, m};$$

$$\varphi_{lj}^{(21)}\left(x_j^{(1)}\right) = a_{lj}^{(21)} \ln\left(x_j^{(1)}\right)^2, \; l = \overline{1,\, m};\; j = \overline{1,\, n};$$

$$\varphi_{ls}^{(22)}\left(x_s^{(2)}\right) = a_{ls}^{(22)} \ln\left(x_s^{(2)}\right)^2, \; l,\, s = \overline{1,\, m},$$

where $a_{ij}^{(11)}$, $a_{is}^{(12)}$, $a_{lj}^{(21)}$, $a_{ls}^{(22)}$ – numerical coefficients. The concrete values of these coefficients allow determining price indexes with $x_1^{(1)}$, $x_2^{(1)}$, $x_1^{(2)}$.

5. Conclusions

The constructed models can be used for determining the price indexes in multisectoral ecological-economy system. The determining of the price indexes lets control the prices balances and in time to react on changes in any constituent of net output. From the point of view of the decision making person it allows to optimize the process of pricing and its dynamics in environmental economics.

The information system of uniting of pricing tasks in the ecologically balanced economy is created that contains the kit of visual resources for empirical analysis of developed economic-mathematical models and its usage in experimental researches and monitoring tasks.

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