

AN INTERTEMPORAL LINEAR PRICE MODEL WITH EXTRACTIONS

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Abstract

The problem of lack of competitiveness has become one of the main concerns of European governments. This is reflected through the Europe 2020 Strategy that includes as key priority the promotion and efficient and productive use of inputs. Differently to other “well-behaved” European “neighbours”, in Spain productivity growth closely connected to competitiveness improvements has been remarkably slow during the last decade. Some analysts consider that the bad evolution of Spanish competitiveness levels is basically due to the increase in labour costs during these years. Consequently, this paper pursues to shed some light on the possible main reasons that explains the lack of competitiveness in the Spanish economy. In doing so, we use a multi-sectoral approach employing yearly Input-Output data for this economy that covers the 2000-2007 time-frame. This is the empirical contribution of our paper. In terms of methodology, to the best of our knowledge, the contribution of this paper relating to the Hypothetical Extraction Method (HEM) is two-fold. Differently to what is common practise, the first contribution has to do with evaluating endogenous price impacts using the HEM. Expanding the application of the original approach first proposed by Leontief (1949), the second contribution consists in introducing an inter-temporal analysis within the HEM. This helps in the analysis of the evolution and the main determinants of the rise in price levels that has generated a decline in Spanish competitiveness levels.

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1. INTRODUCTION

The theoretical structure of price models for linear input-output economies is well known from the contribution of Atsumi (1985). In an empirical vein, and rather surprisingly, the price model has been used less often than its sibling quantity model, despite the fact that they share the same theoretical basis and their viability is guaranteed by the very same Hawkins-Simon (1949) condition, or the Perron-Frobenius eigenvalue condition. Some applications of the price model include McElroy *et al* (1982) who study general price formation for the US; Catsambas (1982), in turn, uses a price model to evaluate the incidence of an excise gasoline tax on income distribution also in the US. In a related study, Hugues (1986) explores in a price model the distributive role of a fuel tax using data for Thailand. Derrick and Scott (1993) examine the role of the sales tax in prices whereas Roland-Holst and Sancho (1995) generalize the price model to the SAM framework and study and decompose its cost multipliers. More recently and for the Spanish case, Cardenete & Sancho (2002) develop a model of regional prices with taxes and Cardenete *et al* (2007) exploit the structure of a regional model to assess the impact of a fuel tax. Llop and Pié (2008), in turn, use in quite an innovative way a Leontief price model to study environmental issues in Catalonia, whereas Sancho (2010) proposes a methodological way to separate visible and non-visible price effects induced by the different indirect taxation instruments. All these applications seek to elicit and understand the empirical workings of the price formation mechanism using the linear paradigm as the basis of analysis. Clearly, the advantage of the linear approach, both in the quantity and the price versions, results from its operational simplicity and its ability to combine theory with structural, disaggregated data.

The quantity input-output model has been extensively used for the determination of so-called key or strategic sectors. When a sector receives an exogenous stimulus, the productive response to that stimulus involves the receiving sector as well as the remaining economic sectors that must adjust their production to fulfil, in a first stage, the needs of the receiving sector and they do so by supplying input deliveries to the triggering sector. Any such change activates, in second and posterior stages, new productive adjustments, which cease when the original stimulus has been fully absorbed by all sectors in the economy and an overall new balance is achieved. Any sectoral

stimulus can therefore be globally evaluated by the increased economic output that ensures it. A sector is termed as a key sector, therefore, if facing the same stimulus (usually unitary to facilitate comparisons) is capable of pulling production in all sectors above some economic average. In this case, and because of its pulling capacity, such a sector is denominated as a key backward sector. Forward key sectors have also been introduced to measure the pushing capacity of a sector but either because their interpretation is rather awkward (i.e. requires simultaneous identical increases in all sectors) or because they are based in the alternative input-output Ghosh model (i.e. often criticized in terms of its alleged implausibility), key forward sectors are not as commonly used in the empirical literature.

A competing approach to determine key sectors is based upon the hypothetical extraction method (HEM). Instead of measuring the pulling output capacity of a sector following an exogenous injection, the HEM investigates the role of a sector by way of simulating its absence in the economy. The absence is modelled setting relevant input-output coefficients to zero. The thus modified technology matrix is used to calculate the hypothetical new equilibrium in quantities. Since technical coefficients are now hypothetically lower, the new quantity equilibrium will also be lower. This can easily be seen to be a consequence of the series expansion of the Leontief inverse. The fall in output that would follow the extraction of a sector, even if hypothetical, indicates the hidden productive role of that sector in the interconnected economy. And the larger the output fall, the more relevant the sector would be in terms of its “key” contribution to the overall output of the economy. Check Miller and Lahr (2001) for an excellent and very complete discussion of the HEM in input-output economics.

The widespread use of the HEM to elicit key “productive” sectors has been restricted, to the best of our knowledge, to the quantity model of Leontief. The price model, however, could also be used to study key “cost” sectors in a fully dual manner to the formal procedures used for the quantity model. The detection and quantification of cost linkages would be informationally relevant for the design of tax policies or the implementation of primary factors policy stimulus. Sectors with high cost linkages would be prone, for instance, to exert larger inflationary pressures in response to exogenous increases in prices, as it is for example an increase in social contributions paid by employers or in wages. An evaluation of how these exogenous shocks travel and propagate through the economy would provide authorities with significant information for price containment policies.

In this paper we therefore propose to implement the HEM in the Leontief price model to evaluate hidden cost linkages. Furthermore, we also explore the inter-temporal dimension of these cost linkages by using SDA (structural decomposition analysis) to Spanish input-output data for the years 2000 to 2007. In Section 2 we provide the required technical procedural details of the analysis. In Section 3 data is presented and some empirical results are discussed. Finally, Section 4 concludes with a summary.

2. AN INTERTEMPORAL PRICE MODEL WITH EXTRACTIONS

We consider an economy composed by n productive sectors. For each sector, denoted by $j=1, \dots, n$, production takes place using a Leontief production function that models technology as a fixed combination of k primary inputs, $v_{kj} = [V]_{kj}$ and n non-primary or intermediate inputs, $z_{ij} = [Z]_{ij}$:

$$X_j = \min \left[\frac{z_{1j}}{a_{1j}}, \dots, \frac{z_{nj}}{a_{nj}}, \frac{v_{1j}}{l_{1j}}, \dots, \frac{v_{kj}}{l_{kj}} \right] \quad \forall j, i = 1, \dots, n \quad (1)$$

where $a_{ij} = [A]_{ij}$ refers to the well-known structural direct input-output technical coefficients while $l_{kj} = [L]_{kj}$ are the direct requirements of the k -th primary input per unit of gross output $x_j = [X]_j$. We therefore posit a standard fixed coefficient production process with constant returns to scale. This technology can be defined as a set of matrices, i.e. (A, L) , with each column of them specifying the combined amount of direct inputs per unit of output.

Because of the inherent budget constraint for each productive sector, the total value of all outlays for primary and non-primary inputs in the j sector must be equal to the value of the total gross output generated in this sector of the economy:

$$p_j x_j = \sum_{i=1}^n p_i z_{ij} + \sum_{k=1}^h w_k v_{kj} \quad \forall j = 1, \dots, n \quad (2)$$

with $p_j = [P]_j$ being the equilibrium price per unit of output in sector $j=1, 2, \dots, n$.

Consequently, equilibrium prices can be defined as a function of the technology (A , L) and the costs of primary inputs¹ i.e. $w_k = [W]_k$:

$$p_j = \sum_{i=1}^n p_i a_{ji} + \sum_{k=1}^h w_k l_{kj} \quad \forall j = 1, \dots, n \quad (3)$$

or in matrix notation:

$$P' = P' A + W' L \quad (4)$$

It is well known that if the non-negative matrix A has a dominant eigenvalue $\lambda \in (0,1)$, i.e. in economic terms, matrix A is productive. Then, the system of equations in (4) can be solved in the following way:

$$P' = W' L (I - A)^{-1} \quad (5)$$

We can transpose the model solution in (5) and express it in terms of column vectors rather than row vectors; then expression (5) would become:

$$P = (I - A')^{-1} L' W \quad (6)$$

For the purposes of this analysis, the n production units in the economy are split in two groups of sectors or block of industries, namely block 1 that contains h sectors and block 2 that is formed by the remaining $n-h$ sectors. Taking into account this subdivision of the n production units, we can express (6) accordingly in partitioned form as:

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} I - A'_{11} & A'_{21} \\ A'_{12} & I - A'_{22} \end{bmatrix}^{-1} \begin{bmatrix} L'_1 W \\ L'_2 W \end{bmatrix} \quad (7)$$

¹ As is common practise in general equilibrium models, we assume that there is a unique price for each primary input since there is perfect factor mobility in the economy between sectors.

where:

$$\begin{bmatrix} I - A'_{11} & A'_{21} \\ A'_{12} & I - A'_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \Lambda' & \Lambda' A'_{21} (I - A'_{22})^{-1} \\ (I - A'_{22})^{-1} A'_{12} \Lambda' & (I + (I - A'_{22})^{-1} A'_{12} \Lambda' A'_{21}) (I - A'_{22})^{-1} \end{bmatrix} \quad (8)$$

with $\Lambda' = (I - A'_{11} - A'_{21} (I - A'_{22})^{-1} A'_{12})$

This partitioned representation of the well-known Leontief price model makes possible to evaluate and quantified sectoral “hypothetical extraction” linkage measures (Miller and Lahr, 2001) not in terms of its economy-wide effects over gross output but rather in terms of its impact on sectors’ costs structure or final price composition, what we have called “the price linkage measure”. A question might arise now and it is how we proceed to model the extraction of an industry or groups of industries in order to obtain a comprehensive indicator that provides useful and quantifiable information about this proposed “price linkage measure”.

Several types of extractions have been suggested in the literature to quantify the average direct and indirect stimuli generated by one sector in the economy (Miller and Lahr, 2001; Miller and Blair, 2010) and each of them has been designed accordingly to the tasks of the analysis in question. For our analysis’ purposes, we have modelled the extraction of a sector by way of nullifying all the direct coefficients where that sector has an influence (either as a supplier to or as a demander of inputs), including self-supply deliveries. If the “hypothetically extracted” group of sectors refer to those that pertain to block 1 then the “new” technical coefficient matrix \bar{A} would become:

$$\alpha^{(-1)} a_{ij} = \bar{a}_{ij} = [\bar{A}]_{ij} \quad (9)$$

where $\alpha^{(-1)}$ is an auxiliary binary scalar that equals 1 if $i=1$ or $j=1$ and equals zero otherwise². Consequently, after applying the “full” extraction of block 1 and assuming that both primary inputs prices and technology remained unchanged, the “new” (in hypothetical terms) price equilibrium in the economy would be determined by:

² Similar and symmetrical considerations would apply to the extraction of the block of sectors 2. We omit the details here for simplicity.

$$\begin{bmatrix} \bar{P}_1 \\ \bar{P}_2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & (I - A'_{22})^{-1} \end{bmatrix} \begin{bmatrix} L'_1 W \\ L'_2 W \end{bmatrix} \quad (10)$$

If we now calculate the difference between the pre-extraction equilibrium reflected in expression (7) and the post-extraction equilibrium shown in (10):

$$\begin{bmatrix} \Delta P_{1(-1)} \\ \Delta P_{2(-1)} \end{bmatrix} = \begin{bmatrix} P_1 - \bar{P}_1 \\ P_2 - \bar{P}_2 \end{bmatrix} = \begin{bmatrix} I - \Lambda' & \Lambda' A'_{21} (I - A'_{22})^{-1} \\ (I - A'_{22})^{-1} A'_{12} \Lambda' & (I - A'_{22})^{-1} A'_{12} \Lambda' A'_{21} (I - A'_{22})^{-1} \end{bmatrix} \begin{bmatrix} L'_1 W \\ L'_2 W \end{bmatrix} \quad (11a)$$

or in simpler matrix notation:

$$\begin{bmatrix} \Delta P_{1(-1)} \\ \Delta P_{2(-1)} \end{bmatrix} = \begin{bmatrix} P_1 - \bar{P}_1 \\ P_2 - \bar{P}_2 \end{bmatrix} = \begin{bmatrix} H & C \\ G & U \end{bmatrix} \begin{bmatrix} L'_1 W \\ L'_2 W \end{bmatrix} \quad (11b)$$

Expressions (11) show the decline in all prices in both blocks after the simulated extraction of the cost linkages associated to block 1. This method of extraction was first proposed by Paelinck *et al* (1965) and then used by Strassert (1968), Schultz (1976, 1977) and has been widely used later on by Heimler (1991), Dietzenbacher and van der Linden (1997), and Temurshoev (2010), among others. The endogenous decline in unitary prices evaluated through expressions (11) above, i.e. $\Delta P_{1(-1)}$ and after simulating the removal of overall intermediate deliveries of block 1 is, in our view, an appropriate approximate indicator to the role played by the direct and indirect sectoral cost interdependencies originated by block 1 in determining the final price structure and thus competitiveness levels in the economy.

We now move to show how the inter-temporal dimension is incorporated in our approach. The idea here is to identify not only which sector is “key” in determining the unitary cost structure for a specific period but also how and why the “price linkage indicator” has varied within periods in an economy. In doing so, we adopt and implement the structural decomposition technique first proposed by Carter (1970) for the input-output methodology. In fact, the analysis of changes in technical coefficients across periods can provide useful information about actual and potential sources of

efficiency (Gowdy and Miller, 1987; Rose and Chen, 1991; Casler and Hadlock, 1997; Oosterhaven and Van der Linden, 1997). This constitutes indeed our main interest since the endogenous impact we aim to evaluate relates to unitary prices, which capture the underlying technologically efficient use of inputs and its value translation. We would like to stress, however, that we leave aside dynamic considerations in our input-output price model, such as those used in previous research (Leontief, 1970; Liew, C.K, 1977; Liew, C.J., 2000; Leontief and Duchin, 1986; Los, 2001).

If the objective is then to decompose the total inter-temporal change of our proposed price linkage indicator defined through expression (11) within s periods, from period t to period $t+s$, if we make use of the simpler version in (11b) the discrete approximation to the total differential of this expression can be seen to be given by:

$$\begin{aligned}\Delta P_{1(-1)}^s &\approx [\Delta H^s L_1' W + H \Delta L_1^{s'} W + H L_1' \Delta W^s] + [\Delta C^s L_2' W + C \Delta L_2^{s'} W + C L_2' \Delta W^s] \\ \Delta P_{2(-1)}^s &\approx [\Delta G^s L_1' W + G \Delta L_1^{s'} L_1^{s'} W + G L_1' \Delta W^s] + [\Delta U^s L_2' W + U \Delta L_2^{s'} W + U L_2' \Delta W^s]\end{aligned}\quad (12)$$

Rearranging the right-hand side terms presented in (12):

$$\begin{aligned}\Delta P_{1(-1)}^s &\approx [\Delta H^s L_1' + \Delta C^s L_2'] W + [H \Delta L_1^{s'} + C \Delta L_2^{s'}] W + [H L_1' + C L_2'] \Delta W^s \\ \Delta P_{2(-1)}^s &\approx [\Delta G^s L_1' + \Delta U^s L_2'] W + [G \Delta L_1^{s'} + U \Delta L_2^{s'}] W + [G L_1' + U L_2'] \Delta W^s\end{aligned}\quad (13)$$

Using expression (13) we see that the inter-temporal variation of the price linkage indicator ensuing the extraction of block 1, $\Delta P_{1(-1)}^s$ and $\Delta P_{2(-1)}^s$, can be grouped in three components: firstly, the variation that relates to changes in the weight of direct and indirect sectoral interdependencies, i.e. $(\Delta H^s, \Delta C^s, \Delta G^s, \Delta U^s)$; secondly, the variation due to the changes in the direct primary input requirements, i.e. $(\Delta L_1^{s'}, \Delta L_2^{s'})$, and thirdly the variation following the changes taking place in their payments, i.e. (ΔW^s) . In the empirical research that uses this kind of methodology, commonly known as the structural decomposition analysis, to compute inter-temporal changes in specific variables, the discrete approximation in the decomposition of expressions (12) and (13) are often based on initial period weights (ω_t), last period weights (ω_{t+s}) or mid-point

weights($\omega_{t+s/2}$). As an illustrative example, the discrete approximation of the changes in the contribution of block 1 in determining its own final prices $\Delta P_{1(-1)}^s$ would alternatively be given by:

$$\Delta_{\omega_t} P_{1(-1)}^s = \left[\Delta H^s L_1^t + \Delta C^s L_2^t \right] W^t + \left[H^t \Delta L_1^s + C^t \Delta L_2^s \right] W^t + \left[H^t L_1^t + C^t L_2^t \right] \Delta W^s + \varphi^t \quad (14)$$

$$\Delta_{\omega_{t+s}} P_{1(-1)}^s = \left[\Delta H^s L_1^{t+s} + \Delta C^s L_2^{t+s} \right] W^t + \left[H^{t+s} \Delta L_1^s + C^{t+s} \Delta L_2^s \right] W^{t+s} + \left[H^{t+s} L_1^{t+s} + C^{t+s} L_2^{t+s} \right] \Delta W^s + \varphi^{t+s} \quad (15)$$

$$\Delta_{\omega_{t+s/2}} P_{1(-1)}^s = \left[\Delta H^s L_1^{t+s/2} + \Delta C^s L_2^{t+s/2} \right] W^t + \left[H^{t+s/2} \Delta L_1^s + C^{t+s/2} \Delta L_2^s \right] W^{t+s/2} + \left[H^{t+s/2} L_1^{t+s/2} + C^{t+s/2} L_2^{t+s/2} \right] \Delta W^s + \varphi^{t+s/2} \quad (16)$$

In these expressions the coefficients φ represent a residual term that captures the interaction of simultaneous effects. Expressions (14)-(16) are in fact the three most widely used types of weighted decompositions. It is precisely because of the non-uniqueness of the solution to the structural decomposition problem, that there are large differences between them when computing the contribution of each determinant. Additionally, Dietzenbacher and Los (1998) argue that some of the ad-hoc solutions, i.e. taking the average of (14) and (15) or using (16) appear unsatisfactory. Consequently and coherently, these authors suggest computing average parameter contributions over all possible decompositions reporting their range and variance. For our empirical application, we will follow this lead.

3. DATA SOURCES AND SOME EMPIRICAL RESULTS FOR SPAIN

We use data contained in the input-output tables for the Spanish economy compiled by the National Statistics Institute (INE) for the years 2000 to 2007. The INE is a public organism whose charter mission is to provide the government and the public at large with all kind of official economic, demographic, and territorial data. The

Spanish input-output tables follow the SEC-95 methodology to comply with European Union data harmonization.

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