MONEY AND FINANCE IN AN INPUT-OUTPUT FRAMEWORK: A SURVEY

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THEME: INVESTIGATIONS INTO ECONOMIC STRUCTURAL CHANGE AND DYNAMICS

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I. INTRODUCTION

Leontief's inter-industry analysis inspired several 'activity-analysis' models, for example Linear Programming transportation models. One of the branches in Activity-analysis models focused on the role and nature of monetary circulation across and between different activities and 'actors' (sectors, entities like households, firms, enterprises etc). In a sense, monetary circulation arises on account of the sale/purchases or supply/demand based transactions, partly determined by the quantities required and partly by the money prices. Hence, the 'Real' flows are always implicitly complemented by their monetary or financial flows as counterparts of each other.

In a money wing economy the financial or monetary flows are relevant due to the special and unique nature of money. In addition to being a vehicle of transaction (i.e. facilitating greater and better exchanges) money also represents more convenient links between those who 'save' and those who 'invest' i.e. 'across time' and 'across agents'. This realization gave rise to the Flow of Funds approach as an analytical-cum-policy tool in economic research. We present here a summary of three seminal papers that succinctly illustrate the emergence and integration of moneyflows in an inter-industry or an activity-analysis framework.

II. MONEY AND FINANCE IN AN INTER-INDUSTRY / ACTIVITY-ANALYSIS FRAMEWORK

A. Tracing Copeland's 'Social Accounting for Moneyflows'[1]

The genesis of approaching moneyflows via social accounting can be traced to the pioneer work of Copeland in 1949. There was a visible laggard of social accounting in this field. The reason can be attributed to the fact that different social accounting approaches were needed for moneyflows vis-à-vis national income. Copeland's study further suggested an additional utility of this approach. In order to shun the existing quackery in this field and to identify the 'completely formulated and tested hypothesis', social accounting approach to moneyflows was considered to be an infallible tool by Copeland.

The title, 'Social accounting for Money flows' is duly justified at the commencement of his study itself. He considered unemployment to be the most important economic problem. He remarks that as we all dwell in a monetary economy, an in depth study of moneyflows will facilitate in grasping the problem of maintaining near-to-full employment. Copeland's study provides measures of moneyflows which attempt to answer questions such as[2]:

- When total purchases of our national product increase where does the money come from to finance them?
- When purchases of our national product decline, what becomes of the money that is not spent?
- Is it hoarded, or what?
- Who has and exercises discretion to increase or decrease expenditures on national product?

[1] Copeland, M. (1949) 'Social Accounting for Moneyflows'
[2] *Ibid*., p. 254

- What part do cash balances, other liquid assets holdings, and debts play in the cyclical expansion and contraction of moneyflow?

As moneyflows are generated in the process of transactions between two transactors, the social accounting to moneyflows approach doesn't evoke double-entry system but a quadruple-entry system. The underlying notion in his study is the use of social accounting approach to moneyflows and this approach is categorically contrary to the neo-classical hydraulic approach to moneyflows. Hence, he prescribed summarization of moneyflows in a financial statement format for which the economy would have to be divided into a number of sectors or group of transactors and a separate balance sheet for each sector to be prepared. A model would be needed to demonstrate each moneyflow as an inflow to a sector and an outflow from another sector.

The requirement for such a demonstration is one of the major distinctions between the social accounting approach to national income and that to moneyflows. Copeland's paper and the subsequent contributions facilitated development of the comprehensive framework of financial statement. This format was more aptly suited to moneyflows in conjunction with income flows. The four criteria and characteristics that Copeland's paper had invoked are found in the framework, viz.[3]

i. To reveal moneyflows, the statement should be on a cash basis. But for the sake of appropriateness, an important deviation from the cash basis was made and the statements developed in the study were based on moneyflow basis.
ii. For inter-relating and inter-sectoral analysis, adoption of a standard format for all sectors is a necessity. Such a standard format has compilation difficulties due to the presence of varied set of conventions for different sectors. The format ought to be like a greatest common denominator for the various accounting conventions.
iii. The format should be capable of bringing out not only the moneyflows but also the cash balances and all other balances which are related.
iv. Reporting of moneyflows as far as possible should be in gross and all moneyflows which have economic importance should be revealed.

Copeland acknowledges that it was a modification of the sources and application of funds statement. He christened it "Statement of Payments and Balances". Ordinary receipts and expenditures were to be detailed on the statements such that they comply with a standard classification of accounts and also aid in statistical estimation. This format proved to be useful in understanding the impact of one sector's economic activity on other sectors. In this scheme, Copeland suggests that there be thirteen main objects of ordinary receipt and expenditure – lines A through G, insurance benefits (lines H, J, and K) and lines K, S, U, V and W. Copeland had prepared the 'Statements of Payments and Balances' for the U.S. economy for the time period 1936-42. We have reproduced the 'Statements of Payments and Balances for Households'[4] from Copeland (1949).

[3] *Ibid*., p. 256
[4] *Ibid*., pp. 258-259

STATEMENT OF PAYMENTS AND BALANCES FOR HOUSEHOLDS

(Millions of Dollars)

		1935	1936	1937	1938	1939	1940	1941	1942	Source	
	PART ONE: MONEYFLOWS										
	ORDINARY RECEIPTS AND OTHER SOURCES OF MONEY										
A	Gross Cash Pay	—	41,000	45,200	42,100	45,100	48,900	60,400	79,100	HH 101 A	A
B	Cash Dividends	—	4,600	4,700	3,300	3,800	4,000	4,500	4,300	HH 101 D	B
C	Cash Interest	—	2,800	2,700	2,700	2,700	2,700	2,700	2,500	HH 101 G	C
D	Net Owner Take-Outs	—	8,100	10,000	8,800	9,000	10,200	13,000	15,700	HH 101 K	D
E	Receipts from Customers (for Secondhand Goods)	—	120	140	120	140	160	180	100	HH 101 P	E
F	Net Payments for Real Estate Transfers	—	800	800	800	800	800	800	800	HH 101 Q	F
G	Tax Refunds	—	10	20	20	20	30	30	30	HH 101 V	G
H	Unemployment Compensation Benefits	—	0	[3]	400	440	540	360	360	HH 102 A	H
J	Benefits from Other Social Insurance Funds	—	280	340	440	480	540	620	740	HH 102 G	J
K	All Other Insurance Benefits	—	2,500	2,600	2,700	2,800	2,800	2,800	2,800	HH 102 T	K
L	Public Purpose Payments	—	3,460	1,320	1,480	1,560	1,620	1,600	1,560	HH 103 I	L
M	Total Ordinary Receipts	—	63,700	67,800	62,800	66,800	72,300	87,000	107,900	A thru L	M
N	Net Funds Obtained through Financing	—	0	0	400	0	0	0	0	−p when p<zero	N
P	Total Sources of Money	—	63,700	67,800	64,200	67,500	72,300	87,400	110,800	M+N[1]	P
	ORDINARY EXPENDITURES AND OTHER DISPOSITIONS OF MONEY										
Q	Gross Cash Pay (Chiefly to Domestic Servants)	—	760	880	760	860	940	960	1,100	HH 201 C	Q
R	Cash Interest	—	1,360	1,380	1,320	1,320	1,400	1,540	1,300	HH 201 D	R
S	Gross Rents	—	3,300	3,600	3,800	3,900	4,000	4,300	4,500	HH 201 E	S
T	Spent by Households as Customers	—	47,400	51,000	48,500	51,200	55,100	63,700	69,700	HH 201 F	T
U	Instalments to Contractors	—	660	800	800	1,040	1,200	1,420	800	HH 201 P	U
V	Taxes Collected	—	2,600	3,700	3,500	3,200	3,500	4,300	7,100	HH 202 X	V
W	Insurance Premiums	—	4,200	4,300	4,300	4,300	4,600	4,900	5,200	HH 203 N	W
	Public Purpose Payments:										
X	To Private Agencies and Institutions	—	960	1,080	980	960	1,060	1,040	1,260	HH 203 Q	X
Y	Personal Remittances	—	180	180	160	140	180	140	100	HH 203 R	Y
Z	To Governments	—	30	30	30	30	30	30	40	HH 203 P	Z
a	Total Ordinary Expenditures	—	61,400	66,900	64,200	67,000	72,000	82,300	91,200	Q through Z	a
b	Net Funds Advanced to Finance Others	—	2,200	500	0	400	100	5,100	19,600	p when p>zero	b
c	Total Dispositions of Money	—	63,700	67,800	64,200	67,500	72,300	87,400	110,800	a+b[2]	c

STATEMENT OF PAYMENTS AND BALANCES FOR HOUSEHOLDS

(Millions of Dollars)

		1935	*1936*	*1937*	*1938*	*1939*	*1940*	*1941*	*1942*	*Source*	
	PART TWO: LOAN FUNDS										
	ASSETS, DECEMBER 31										
d	Currency and Deposits	29,000	31,800	32,200	32,400	34,900	36,600	40,200	47,200	HH 301 H	d
e	Federal Obligations	9,300	9,800	10,400	10,000	10,200	10,700	13,600	22,300	HH 301 I	e
f	Other Loans and Securities	110,700	110,500	110,500	110,300	109,400	108,900	108,600	108,900	HH 301 J	f
g	Total Loan Funds Receivable	149,000	152,000	153,100	152,700	154,500	156,100	162,300	178,400	d+e+f	g
	LIABILITIES, DECEMBER 31										
h	Accounts Payable	2,600	3,100	3,400	3,400	3,600	3,900	4,200	3,200	HH 301 O	h
i	Other Debts Payable	21,900	21,900	21,600	21,100	21,800	22,500	22,800	19,700	HH 302 R	i
j	Total Loan Funds Payable	24,500	24,900	25,000	24,500	25,400	26,400	27,000	22,900	h+i	j
	COMPUTATION OF LOAN FUND FINANCING										
k	Net Loan Fund Balance Receivable, December 31	124,400	127,100	128,100	128,200	129,100	129,700	135,300	155,500	g minus j	k
m	Increment in Loan Fund Balance Receivable	—	2,700	1,000	100	900	600	5,600	20,100	Increment in k	m
n	Net Gains a/c Forgiven Debts	—	500	500	500	500	500	500	500	HH 302 S	n
p	Funds Advanced Less Funds Obtained	—	2,200	500	−400	400	100	5,100	19,600	m minus n	p
	[1] Includes net sources of funds not accounted for as follows		0	0	1,000	700	0	500	3,000		
	[2] Includes net uses of funds not accounted for as follows		100	400	0	0	200	0	0		

[3] Less than $10 million.

Note: Due to rounding, figures for various lines calculated by formulas given in the source column may differ slightly from the entries shown.

Our study has now furnished Copeland's explanation for a lucid understanding of the above table. Copeland's statement had the following components[5]:

- Part One – was on a moneyflow basis. Except for E, T and U, rest 11 to be reported on a cash basis. Transaction and also the cash sales included in E and T entered at the time of settlement. Credit sales in E and T and instalments to contractors (line U) reported on a book credit basis. This isn't a cash account.
- Part Two – Partial balance sheet. The asset items are called loan-funds receivable and the liability items loan-funds payable. Line K is the net loan-fund balance receivable. The revaluations of forgiven debts are the only ones affecting the household sector. Line p on the household statement is when the effects of book revaluations from the increment in net loan-funds receivable are excluded. This is seen in Part One as the net money obtained through financing (line N) or net money advanced or returned (line b).
- Implication of the above point is that the changes in a sector's financial structure bear a relation with its non-financial transactions.

The statement for rest of the world found in Copeland's statements is a modified version of the balance of international payments statement. Federal government's statement is helpful in measurements of moneyflows and also it's a demonstration of the position of federal finances. There was a pressing need for a comprehensive federal financial statement which could be capable of summarizing the major aspects of fiscal policy that have an effect on the business activity. Copeland's Statement of Payments and Balances addresses this very particular need. His study further states four propositions that follow from his set of statements. They have been enlisted in Copeland (1949) as under[6]:

i. For any sector and any period, total sources equal total dispositions of money.
ii. A source of money for one sector means a disposition of money by some other sector, and vice versa.
iii. A loan fund receivable for one sector means a loan fund payable by some other sector and vice versa.
iv. An increase in a loan fund balance receivable is a disposition of money by the holder; for some other sector it means a decrease in the loan fund balance receivable or an increase in payables, and vice versa.

These statements are not an 'accounting summary' but only statistical approximation. The four propositions stated above assert equality between debits and credits; however, this may not hold true due to statistical discrepancies.

There was a prevalent misconception about moneyflows and the nature of inflation. According to Copeland, this misunderstanding was advanced further when banks were spoken of as "deposit creating" sector. He opined that banks should not be spoken of "deposit creating" sector because

[5] We have reproduced the explanation which was furnished by Copeland (1949) in order to retain the exact meaning and implication of his 'Statement of Payments and Balances'.
[6] *Ibid*. 1, p. 257

what merely takes place is a change in the *composition* of the net loan-fund balance receivable and not a change in the *amount* of the net balance. The social accounting approach disregards the view that 'moneyflows expand in such a manner that they ultimately lead to inflation'. Copeland reformulates propositions ii and iv for the sake of breaking down complexities. He gives the next two propositions:[7]

v. Total ordinary receipts equal total ordinary expenditures
vi. (Money advanced or returned) – (Money obtained through financial channels) = 0

Dishoarding of loan-fund balances and expansion of ordinary expenses by some sectors cause cyclical expansions in the total moneyflows in an economy. Copeland labelled transactors who initiated expansions as "bulls". Copeland explains this behaviour in a social accounting framework as follows[8]:

- *For bulls*

The excess of ordinary expenditures over ordinary receipts equals money obtained through financing. Total sources equal total dispositions of money.

- *For all others together*

The excess of ordinary receipts over ordinary expenditures equals money advanced or returned. Total sources equal total dispositions of money.

- *For all transactors*

Total ordinary receipts equal total ordinary expenditures.

Expansion in moneyflows and in business activity is triggered when bulls decide to increase their ordinary expenses. Similar is the contraction process in moneyflows and such transactors who curtail ordinary expenses were called "bears" by Copeland. Transactors who are mere passive followers of either bulls or bears were referred to as "sheep". A noteworthy finding of Copeland's study is that the moneyflows of bulls, bears and sheep have distinct patterns which are manifested in the 'statements of payments and balances' by their tendency to create expansion, contraction or remain passive.

As aforesaid, Copeland did not consider banks as "deposit creating". However, he gives them credit for being very active in the moneyflows that pass through financial channels. For the sake of simplicity, his study assumed no statistical discrepancies and no deviations from accounting uniformity. The only three deviations which Copeland's study has accounted for have been quoted as follows[9]

i. Those due to differences in the timing of entries into the accounts.
ii. Those due to differences in the valuation of loan fund balances.
iii. Those due to differences in the classification of accounts.

[7] *Ibid*., p 260
[8] *Ibid*., p 261
[9] *Ibid*., p 262

There have been many cases wherein Copeland's study has diverged from a standard classification of accounts. For example: 'Monetary gold stock' is treated as an asset of banks and U. S. monetary funds. The sale of gold by domestic producers gets the same treatment in the statement as does the sale of copper or zinc. Due to this deviation, a minor part of the monetary gold stock lacks a corresponding loan fund payable item on the statements of payments and balances for other transactors; same is the case with 'Treasury currency'.

Copeland's study brings out peculiar characteristics of 'Social accounting for Moneyflows'. The statement of payments and balances is remarkably different from the regular financial statement. However, the advantage of Copeland's statement is that if the regular financial statement is annexed with balance sheet schedules and details then Copeland's statement can be derived. His study candidly admits that part one of the 'statement of payments and balances' excludes a sizeable volume of data. This is because the money obtained or advanced through financial channels is shown on a net basis. Copeland labelled such netted transactions as 'technical transactions' and remarked that such transactions do not exert a significant economic impact.

Copeland lastly talks about an item known as 'unfinished business'; i.e. the difference between the social accounting approach to national income and wealth vis-à-vis the social accounting approach to moneyflows. He had prescribed quadruple entry system for moneyflows and the necessity to demonstrate moneyflows through separate sector statements. For a national income statement, the economy must be divided into:

i. productive enterprises
ii. households and quasi-households

Statements of payments and balances, Copeland believes, are not simply consolidated income statements but are combined statements wherein intra-sectoral transactions are shown; with exceptions of statements for banks and U.S. Monetary Funds and the rest of the world. The summation of moneyflow transactions in the eleven sectors comes to about three times more than the gross national product. On the same note, it is important to mention that there are some transactions including some accrual and imputed items which are specified in the gross national product account but they don't happen to be moneyflows.

Thus, the two approaches of social accounting are distinct in their perspective. The *social accounting approach to national income* presents a consolidated report on the economic activities of productive sectors on an accrual and imputation basis. The *social accounting approach to moneyflows* (i.e. Copeland's Statements of payments and balances) presents a set of combined sector statements of moneyflows and loan funds. His study acknowledges the significance of both the perspectives for a sound understanding of the economy. The underpinning of this study has been an attempt to bring together the two seemingly opposite perspectives. Copeland, in his concluding part of the study quotes that *"social accounting has already produced a good many findings that are very important for public policy and that it is likely to produce a great many more".*[10]

[10] *Ibid*., p. 264

B. Tracing Tsujimura and Mizoshita's 'Asset-Liability-Matrix Analysis derived from the Flow-of-Funds Accounts'[11]

There are 2 purposes to Tsujimura and Mizoshita (2003) study (hereby referred to as T-M). First, to demonstrate the compilation procedure of the Asset-Liability Matrix (ALM) from the Flow-of-Funds (FOF) accounts in the balance sheet format. Incorporating ALM into the framework of FOF analysis facilitates in utilizing the concepts of input-output analysis, especially the Leontief inverse. Second purpose is to present the application of ALM to the examination of the quantitative monetary policy which was adopted by the Bank of Japan (BOJ) in March 2001.

FOF analysis was born in Copeland (1949). Since then, it has taken the form of an 'accounting system' describing the intersectoral financial transactions between the institutional sectors. FOF accounts came to be included in the System of National Accounts in 1968, along with National Income accounts, National Balance sheets, Balance of International Payments accounts and Input-Output tables.[12]

FOF account comprises balance sheets of the institutional sectors in their classic tabulation practice. Since the beginning of FOF development, there have been attempts to inculcate FOF accounts into practical economic analysis. Analytical framework of FOF can be broadly classified into three groups[13]:

- Powelson (1960) and Mathews (1962) made complete use of the quadruple entry system proposed by Copeland (1949). However, it was Alford (1986) who brought this technique employed by Powelson and Mathews to completion. This technique is disadvantageous for the purpose of policy examination due to the existence of multiple (i.e. non unique) solutions.
- Dawson (1958), Cohen (1963), Hamada (1969), Tobin (1969) and Alho (1991) developed general equilibrium models based on FOF accounts. The problem with this kind is that it tries to include all the economic activities at once, making it complicated to study the effects of detailed changes in an economic policy.
- Stone (1966) and Klein (1983) proposed converting the balance sheets of FOF accounts into a square matrix, referred to as the ALM, with help of the supply-and-use method that is popularly employed in the compilation of the System of National Accounts. Square matrix, in such a case is useful for the applying the Leontief Inverse to the FOF data. ALM makes it feasible to examine the effects of a certain economic policy on separate sectors from the context of inter-sectoral financial transactions.

T-M (2003) is an expansion of the ideas found in Stone (1966) and Klein (1983). T-M study has essentially attempted to present the compilation procedure of ALM for an understanding of Stone (1966) and Klein (1983). Second purpose of this study has been to present the application of ALM in the examination of the quantitative monetary policy introduced by the BOJ in March 2001 and which

[11] Tsujimura. K and Masako Mizoshita, (2003), 'Asset-Liability-Matrix Analysis Derived from the Flow-of-Funds Accounts: The Bank of Japan's Quantitative Monetary Policy Examined'
[12] *Ibid*., p. 51
[13] *Ibid*., pp. 51-52

was revised accordingly in line with other central banks after the terrorist attacks on September 11, 2001.

Introduction of ALM into the FOF analysis capacitates the use of one of the most valuable tools of Input-Output (I-O) analysis, i.e. the Leontief Inverse. Nevertheless, there are some elementary differences between the ALM analysis and I-O analysis. T-M study remarks that "ALM is an I-O matrix that is descriptive of the flow of funds"[14] [15].

In the Y-table of ALM (i.e. the sector-by-sector matrix), column denotes the portfolio of fundraising, and row denotes the portfolio of fund employment of a particular institutional sector. Thus, ALM is an I-O matrix describing the FOF. An extension to this can be made by constructing an I-O matrix which will be a statement of the flow of financial instruments instead of FOF. Column of this Y* table denotes the portfolio of fund-employment and the row denotes the portfolio of fund raising; i.e. Y* is a transposition of matrix Y.

The simultaneous equations denoted by the Y tables, depict the relationship between the surplus in liabilities and the controlled total, whereas those represented by the Y* table represent the correspondence between the surplus in financial assets and the controlled total. T-M quote "the coexistence of these two systems of equations gives two alternative Leontief inverses to the structure of ALM".[16] T-M demonstrate this unique characteristic in what they call the second purpose of the study undertaken; i.e. investigation of the quantitative monetary policy adopted by the BOJ in the spring of 2001.

T-M study recalls the announcements made by the BOJ in 2001 wherein it quotes *"BOJ announced that it will shift the target of money market operation from the interbank interest rate to the balance of current accounts held by the financial institutions at the central bank. Simultaneously, BoJ proclaimed it was to increase the balance of current accounts by one trillion yen, while adding the same amount of Japanese Government Bonds (JGB) on its assets"*[17].

T-M study considered that the demand for funds needs to be financed by the gross induced savings (GIS), whereas the supply of funds brings gross induced investments (GII) in due course. The quantitative monetary policy has a role to play here as it requires the central bank to choose one item in assets and another in liabilities simultaneously. The aforestated action changes GII on the one hand and GIS on the other. A difference in these two, gives net induced investment (NII). Herein, the sign and the amount of NII indicate the effect of the monetary policy on the object economy. A monetary policy that yields a positive NII leads to an expansion in the economy and vice versa.

T-M constructed an ALM for the Japanese economy based on the preliminary Financial Assets and Liabilities tables of the FOF for the first quarter of 2001.[18] First, assets and liabilities vectors were

[14] *Ibid*., p. 52
[15] In the subsequent part, we have now reproduced tables, equations and explanations from the T-M (2003) for retaining the meaning which the authors wanted to suggest.
[16] *Ibid*. 23, p. 52
[17] *Ibid*
[18] We have now reproduced tables, equations, and explanation as they occur in T-M (2003).

picked out separately from the balance sheets of the FOF so as to construct matrices E and R. E is a matrix that shows the portfolio of fund-employment of each institutional sector, ε and t^E are vectors that represent excess liabilities and the sum of each row, respectively. Similarly, R is a matrix showing each institutional sector's fundraising-portfolio. ρ and t^R are vectors representing excess assets and the sum of each row, respectively. t is made up of the sum of assets or liabilities, whichever is larger. n is the number of financial instruments and m the number of institutional sectors. E and R tables extracted from T-M (2003) have been given below:

Figure1: E table

e_{11}	e_{12}	⋯	e_{1m}	t_1^E
e_{21}	e_{22}	⋯	e_{2m}	t_2^E
⋮	⋮	⋱	⋮	⋮
e_{n1}	e_{n2}	⋯	e_{nm}	t_n^E
ε_1	ε_2	⋯	ε_m	
t_1	t_2	⋯	t_m	

Figure 2: R table

r_{11}	r_{12}	⋯	r_{1m}	t_1^R
r_{21}	r_{22}	⋯	r_{2m}	t_2^R
⋮	⋮	⋱	⋮	⋮
r_{n1}	r_{n2}	⋯	r_{nm}	t_n^R
ρ_1	ρ_2	⋯	ρ_m	
t_1	t_2	⋯	t_m	

Y stands for the ALM based on the fund-raising portfolio, whereas Y* for the ALM based on the fund-employment assumption in T-M study. For the purpose of compiling the Y table in accordance with the fund-raising portfolio, matrix R is to be substituted for matrix U (commodity-by-sector) and the transposed matrix E' is substituted for V (sector-by-commodity)[19]:

U ≡ R … (1)

V ≡ E' … (2)

In the case of Y* table,

U* ≡ E … (3)

V* ≡ R' … (4)

The coefficient matrices, B and B*, are constructed by U, U* and t by dividing the cells in each column of U and U* by the column sums t[20].

$b_{ij} = u_{ij} / t_j$ … (5)

$b^*_{ij} = u^*_{ij} / t_j$ … (6)

Similarly, coefficient matrices D and D* corresponding to V and V* are defined as:

$d_{ij} = v_{ij} / t^E_j$ …(7)

$d_{ij}^* = v_{ij}^* / t^R_j$ …(8)

wherein;

t^E_j is the sum of liabilities for financial instrument j. Therefore, d_{ij} is regarded as institutional sector i's share of assets for financial instrument j, while d_{ij}^* is institutional sector i's share of liabilities for financial instrument j. The (m * m) coefficient matrices C and C* corresponding to Y and Y* have

[19] *Ibid*. 23, p. 54

[20] *Ibid*

been estimated in T-M using the institutional sector portfolio assumption, which is akin to the industry technology assumption which is found in the I–O analysis framework.

C = DB … (9)
C* = D*B* … (10)

Y and Y* tables as they appear in T-M (2003) as below:[21]

Figure 3: Y table

y_{11}	y_{12}	⋯	y_{1m}	ε_1^Y	t_1^Y
y_{21}	y_{22}	⋯	y_{2m}	ε_2^Y	t_2^Y
⋮	⋮	⋱	⋮	⋮	⋮
y_{m1}	y_{m2}	⋯	y_{mm}	ε_m^Y	t_m^Y
ρ_1^Y	ρ_2^Y	⋯	ρ_m^Y		
t_1^Y	t_2^Y	⋯	t_m^Y		

Figure 4: Y* table

y_{11}^*	y_{12}^*	⋯	y_{1m}^*	ρ_1^Y	t_1^Y
y_{21}^*	y_{22}^*	⋯	y_{2m}^*	ρ_2^Y	t_2^Y
⋮	⋮	⋱	⋮	⋮	⋮
y_{m1}^*	y_{m2}^*	⋯	y_{mm}^*	ρ_m^Y	t_m^Y
ε_1^Y	ε_2^Y	⋯	ε_m^Y		
t_1^Y	t_2^Y	⋯	t_m^Y		

With the help of these matrices, C and C*, the study obtained the transaction quantity matrices Y and Y* as follows:
$y_{ij} = c_{ij} t_j$ … (11)
$y_{ij}^* = c_{ij}^* t_j$ … (12)

wherein;
y_{ij} is the amount of funds provided from institutional sector i to institutional sector j, and y_{ij}^* is the amount of financial instruments supplied from institutional sector i to institutional sector j as shown in the aforementioned figures. T-M point out that Y and Y* matrices are "two sides of the same coin".
The crux of T-M study is that it is *"trying to analyse the effect of monetary policy by estimating the induced amount of demand and supply of funds through the intersectoral financial transactions represented by the Leontief inverse"*[22]*.* T-M obtained matrix C_{BOJ} from matrix C by removing the row and column containing elements concerning BOJ. Similarly, C^*_{BOJ} is the matrix obtained from matrix C* when the row and column containing BOJ elements are removed. T-M have sufficed equations and tables in justification of their second purpose which can be found in the T-M paper.

We will now provide the policy implications of T-M (2003) study. This study arrives at a conclusion *'that there was a tactical error in the quantitative monetary policy adopted by BOJ in terms of the combination of money market operations'.* Especially the combination of fund raising through the current account held by commercial banks etc, and the fund employment in Japanese Government Bonds, could be catastrophic. T-M suggests that such a combination of fund raising and employment be selected that it results into a positive NII which will then be a remedy in a recession. The study further also points out that *"the best mixture of money market operation is to raise funds through*

[21] *Ibid*., p. 55
[22] *Ibid*., p. 55

bank notes and employ these funds in the form of bill buying operations". In a nutshell, the study carried out by T-M (2003) brings to fore that the *"ALM framework of FOF analysis is a powerful and practical device to fulfil this kind of examination".*

C. Tracing Brody's 'Monetary Multiplier'[23]

Input-Output theory stood the test of time and amply manifested its need and usefulness. However, it has endowed research with an analytical tool which has popularly been used; i.e. the famous Leontief Inverse multiplier, $Q = (I - A)^{-1} = I + A + A^2 + \ldots + A^n + \ldots$

Brody (2000) is a sequel to Leontief and Brody (1993) wherein a different interpretation of monetary circulation has been considered. Brody considered a different interpretation of monetary circulation which he generalized as a finite (or countably infinite) dimensional discrete or continuous Markov-chain. Such a generalization yields a new and disaggregated form of the classical Kahn-Keynes' monetary multiplier yet distinct as it doesn't "converge in the usual sense"[24].

Brody pointed out that in any economy money actually travels in "leaps and bounds"[25] and doesn't confine to coordinated groups of monetary streams. His model, with a different interpretation of monetary circulation, sets up a Markov chain embedded in discrete or continuous time frame. In Brody's model, a unit of money, after staying in state 'i' (say, it is with an agent, enterprise or a bank) for a time period t_{ik}, jumps suddenly into state 'k' with probability a_{ik}. Time interval t_{ik} is also probabilistic in nature and can be considered a random variable distributed exponentially with an expected value t_{ik}. Although Brody prescribed a model for finite number of states, it holds the capacity to expand itself to a countably infinite set of states.

He illustrates his theory with the help of a simple model for demonstrating the characteristics, where all the intervals t_{ik} are of unit magnitude.[26] The model assumes that the $\{a_{ik}\}$ matrix of the transition probabilities is stochastic. It resembles a closed Leontief-type technology matrix $A = \{a_{ik}\}$. However, unlike Leontief type A, the column (or row) sums of A strictly add up to one. Thus, such a matrix A describes an ergodic Markov process implying that whatever initial state it might be in, the Markov process converges to a limiting distribution. Brody's model proposes the equations as cited below. He points out that transition can follow the steps of discrete chain:

$m_{k+1} = Am_k$ …(1)

or it can follow its continuous form:
$\Delta m/\Delta t = (A - I)m$ …(2)

[23] Brody, A. (2000) 'The Monetary Multiplier'
[24] *Ibid*., p. 215
[25] *Ibid*., p. 215
[26] We have now reproduced Brody's Monetary Multiplier model; with equations and explanations being retained as in the original

Here, m_k denotes the vector of the amount of money which is in possession of the 'states' (as defined in his model at the beginning) at time k. Therefore, after further mathematical treatment, the model arrives at an ergodic matrix A*, wherein all its powers are equal to the original matrix, and

$A^*A = AA^* = A^*$

Further in this study, Brody evoked the idea found in David Hume's illustrious essay 'Of money' (1752) when Brody posed to observe the impact when some additional money is injected. Hume, while demonstrating the multiplier process, also opined that the stimulating effect of an increase in money supply can only be transitory and not perpetual. The quantity theory of money and Hume's analysis acutely explains that if the equilibrium portions of money are injected everywhere, it leads to an erosion of purchasing power. In the beginning, per se every injection is non-equilibrium in nature, but the surge depends on the point of origin. Brody's study points out that this is identical to constructing a monetary multiplier, nonetheless not free from theoretical problem. Additional money which has been injected tends to persist forever and hence the multiplier doesn't converge. *The resultant geometric series has a quotient equal to 1 and goes to infinity*[27]. However, non-equilibrium nature of the surge will dwindle out after some lapse of time. But before dwindling out, Brody points out that it will leave an irreversible influence on the economic system.

By considering the non-equilibrium part of the new money injection, the deviation D will be as follows:

$D = A – A^* \ldots (3)$

This D will further be multiplied by matrix A in every step, therefore:

$D_2 = AD = A(A – A^*) = DD \ldots (4)$

Thus,

$\sum = I + D + D^2 + \ldots + D^n + \ldots = (I – D)^{-1} \ldots (5)$

As matrix D possesses all the lesser eigenvalues of A, Brody remarks that the above series will definitely converge. The series will thus represent sectoral benefits. Columns lucidly show the separation of branches which benefited and those which were harmed from an injection of additional money.

Brody further generalized the results for a pragmatic understanding of the economy. If equilibrium growth was taking place, its matrix can be formulated to a stochastic form, wherein B is the Leontief stock matrix and λ is the equilibrium rate of growth:

$S = A + \lambda B \ldots (6)$

In addition, Brody also considered the different velocities of money circulation which are present in different cells. These velocities are the consequences of varied 'financial arrangements'.

[27] *Ibid*., p. 217

$B = \{a_{ik}\ t_{ik}\}$: This matrix was proposed by Brody[28]. It demonstrates the density of moneyflows from branch i to k[29]. The column sums of B are the "branch-specific average turnover times" expressing the sector-specific density of the outflow of payments. Brody's study labelled this diagonal matrix as T and its elements are:

$T_i = \sum_k a_{ik}\, t_{ik}$

Brody's study sets up differential equation for circulation process as follows:

$dm/dt = (B - T)\, m$ … (7*)

This equation shows the change in monetary holdings which consists of the payments received (Bm) minus the payments made (Tm).

Recalling the difference-equation equivalent as given by Brody[30]:

$m_{k+1} = (I + B - T)\, m_k$ … (7)

This equation interprets that the new vector of money can be derived from the old, by adding the change which has taken place at a point of time. This change comprises of the influx minus the outflow of money. In this way, Brody states that the mathematics expressed in Leontief and Brody (1993) remains valid with only its interpretation becoming generic.

Brody's study cites the same small model of Hungarian economy which was used previously in Leontief and Brody (1993). We produce below the example from Brody (2000) and its subsequent explanation.

Matrix A[31] $=$

$$\begin{pmatrix} 0.21 & 0.18 & 0.11 & 0.15 & 0.05 \\ 0.20 & 0.24 & 0.51 & 0.17 & 0.37 \\ 0.1 & 0.27 & 0.12 & 0.25 & 0.15 \\ 0.22 & 0.12 & 0.09 & 0 & 0.33 \\ 0.23 & 0.19 & 0.17 & 0.43 & 0.10 \end{pmatrix}$$

and the turnover times t[32] $= (0.1 \quad 0.05 \quad 0.07 \quad 0.4 \quad 0.08)$ lead to a monetary multiplier,
M[33] $= (I - D)^{-1} - I$, shown in the ensuing table.

[28] *Ibid*., p. 218
[29] Brody (2000) mentions that this matrix was first defined by Lange (1952). Brody quotes that "it is a stock matrix of the system if the turnover times are inserted for the physical quantities instead of the monetary ones".
[30] *Ibid*. 11, p. 218
[31] *Ibid*
[32] *Ibid*
[33] *Ibid*

Table 1: Monetary Multipliers

	E	M	C	H	G
Extraction	+9.51	21.02	21.23	+0.23	21.41
Manufactur	27.82	+7.19	25.36	24.90	26.12
Commerce	22.13	22.85	+9.48	+0.28	22.36
Households	+0.36	20.25	20.23	+1.56	+0.41
Government	+0.08	23.07	22.66	+2.83	+9.48

Source: Brody, A (2000), 'The Monetary Multiplier', p. 219

The coefficients do show the sizeable impact and also its spread and unevenness. There is a strong positive diagonal. Its economic interpretation is that maximal benefits belong to the branch which initiates the spending. This study opines that when money is injected in the Government sector, it results into a small spill-over effect on Households but it is disadvantageous to all the other sectors except for the Government itself.

Brody's conclusion mentions that though his study is based on preliminary data, its validity will persist even when the same model is put to test for complex and large scale data. In a nutshell, what Brody's study has done is that it has managed to demonstrate that *"the classical multiplier can be broken down into a detailed picture that describes the process both in space (among branches) and through time (if solving the equation for the time path)"*[34].

III. CONCLUSION

This survey paper revisited the classic works pertaining to the less traversed field of 'Application of Input-Output technique in Finance'. These three seminal papers, we believe, throw a limelight on the course of development which has taken place in this field. By evoking the ideas and explanations presented in these noteworthy works, we have attempted to bring forth the scope of using Input-Output increasingly in the field of Finance.

[34] *Ibid*., p. 219

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