Computable Production Prices with Fixed Capital as a Joint Product and Technical Progress — A simple case*

Ariel Luis Wirkierman†

Abstract The aim of this paper is to devise a scheme of computable production prices in the spirit of Sraffa's (1960) system of production, in order to identify stylised forms of technical progress (in the sense of Schefold 1976) in actual economies.

In particular, by connecting fixed capital replacement procedures (Gossling 1974) with the time consuming character of production (see Lager 1997, Lager 2000), an empirical treatment of fixed capital as a joint product for the simplest case of constant efficiency and exogenously given length of life of capital goods is advanced. Complementarily, the method of growing subsystems (Pasinetti 1988) is applied in order to separate growth from technical change in empirically observed structures, allowing to perform analyses of comparative dynamics.

In this way, by computing shifts in wage-profit schedules implicit in different techniques in use, changes in the distributive possibilities of actual systems due to technical progress may be assessed. Moreover, alternative price systems may be used to aggregate changes in physical inputs and outputs to obtain industry-level indicators of surplus generating capacity.

An empirical application to the case of Italy for the period 1999-2007 is presented, and compared with traditional measures of economy-wide profitability (i.e. productivity-*cum*-exchange) like TFP Growth (TFPG). In order to correctly separate prices from volume growth, all magnitudes are computed directly from the set of commodity × activity Supply-Use Tables (SUT) of the System of National Accounts (SNA), avoiding the use of particular Input-Output technology assumptions which result in square matrices that include statistical price structures in the derived coefficients.

Keywords Prices of Production, Fixed Capital, Dynamic Input-Output Analysis, Growing Subsystems, Wage-Profit Curves, TFP Growth.

JEL classification C67, B51, D24, O14, O41

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†Facoltà di Economia, Università Cattolica SC, Milano, Italy.
Author's correspondence: `ariwirkierman@gmail.com`

1 Introduction

The aim of this paper is to devise a scheme of computable production prices in the spirit of Sraffa's (1960) system of production, in which fixed capital is treated as a joint product.

The paper is organised as follows. Section 2 discusses the treatment of fixed capital by Sraffa (1960) for the simple case of constant efficiency, given technical durability of capital inputs, and more than one fixed asset employed in each industry.

In Section 3, through the 'algebra of replica replacement' of fixed capital and acknowledging that current inputs are met from past outputs, an explicit connection between observable Input-Output matrices and technical coefficient matrices is established.

Section 4 proceeds to formulate alternative computable systems of price indexes, dealing with the complications introduced by imported commodities, government taxation and data availability in nominal terms. Growing subsystems are introduced into the price equations to separate growth from technical change in empirically given structures. Then, changes in total labour productivity, shifts in wr-schedules and growth rates of industry surplus are suggested to assess productivity dynamics, actual distributional possibilities due to technical progress and surplus generating capacity by industry in value terms, respectively.

Section 5 reports the results of empirical computations for the case of Italy during period 1999-2007.[1] Some final remarks in Section 6 close the paper.

2 Sraffa and Fixed Capital of constant efficiency

The concept of *physical real cost* — at the basis of classical analysis — aims to establish a definite contrast with the notion of cost as the inducement to overcome the sacrifice involved in rendering resources available for their productive use. In this sense, the true contrast between both notions and

[1] In Appendix B, a Statistical Companion includes supplementary tables reporting additional computations.

the understanding of why the former represents an objective foundation for a system of exchange ratios comes to the fore when explicitly considering durable means of production, i.e. fixed capital.

In a system with only circulating capital "the process of value transfer to the product and the physical 'destruction' of the input are one and the same thing" (Kurz & Salvadori 2005b, p. 415). This is not so in the presence of machines and other durable means of production, lasting for several periods. The process of value transfer and physical exhaustion of a machine become separated, and the problem regarding the form that the profit and depreciation components take in the price equation needs to be solved.

In traditional dynamic input-output models, profits are strictly connected to growth, so that circulating capital inputs constitute material costs on which no profit factor is applied, while profits are computed only on the matrix of stocks of fixed capital, as indicated by the 'received' price theory of 'competitive valuation' (Solow 1959, p. 30).[2]

But precisely one of the most relevant points emerging from Sraffa (1960) has been the clarification of the conceptual origin behind the 'received' theory of competitive valuation. To be fully consequent with Sraffa's (1960) account, one should do without *the idea behind* the computation of 'profits on stocks', as its motivation lies on the inducement not to withdraw funds tied up in the production process (an inducement to 'invest').[3]

Instead, in Sraffa's (1960) system of production, profits are also computed on circulating capital and it is through the method of joint products that it is possible to correctly establish the depreciation *and* profit component for each machine that is being priced, according to its effective use as an input and its availability as a product of one more period of age.[4]

Hence, with durable instruments of production, it is the whole stream of outputs generated with them that needs to be considered when allocating physical real costs to obtain a set of exchange ratios allowing for reproduction, given a rule of distribution of the surplus.

[2]This 'received' price theory states that "the price of each commodity must cover its current costs plus interest on the value of the capital equipment required per unit of output" (Solow 1959, p. 30).

[3]See the discussion in Kurz & Salvadori (2005a).

[4]For details, see Sraffa (1960, Chapter X).

Consider fixed capital of constant efficiency, with known and given average length of life of capital inputs, abstracting from the problem of choice of technique. While the case of one type of durable input in each industry has been explicitly dealt with by Sraffa (1960, p. 65-6), the generalisation to the case of more than one machine employed in each activity has been the object of debate in the literature.[5]

To illustrate with an example the set of price equations corresponding to this case, consider a system that utilises k circulating capital inputs, quantities of labour, and machines of only two different types, M and N, with a length of life of 3 and 2 production periods (called 'years'), respectively. Take commodity g, and formulate a set equations representing different processes in which the two machines take part at different ages of their lives, in order to reproduce the observed quantities of commodity g, $G_{(g)}$:

$$(M_0 p_{m_0} + N_0 p_{n_0} + A_g p_a + \ldots + K_g p_k)(1+r) + L_g w = G_{(g)} p_g + M_1 p_{m_1} + N_1 p_{n_1}$$
$$(M_1 p_{m_1} + N_1 p_{n_1} + A_g p_a + \ldots + K_g p_k)(1+r) + L_g w = G_{(g)} p_g + M_2 p_{m_2}$$
$$(M_2 p_{m_2} + N_0 p_{n_0} + A_g p_a + \ldots + K_g p_k)(1+r) + L_g w = G_{(g)} p_g + N_1 p_{n_1}$$
$$(M_0 p_{m_0} + N_1 p_{n_1} + A_g p_a + \ldots + K_g p_k)(1+r) + L_g w = G_{(g)} p_g + M_1 p_{m_1}$$
$$(M_1 p_{m_1} + N_0 p_{n_0} + A_g p_a + \ldots + K_g p_k)(1+r) + L_g w = G_{(g)} p_g + M_2 p_{m_2} + N_1 p_{n_1}$$
$$(M_2 p_{m_2} + N_1 p_{n_1} + A_g p_a + \ldots + K_g p_k)(1+r) + L_g w = G_{(g)} p_g$$

This equation set should *not* be thought of as establishing a sequence of processes in historical time.[6] It is clear that the complete stream of outputs to which the use of machines of different ages needs to be allocated will not be reproduced during the current year; though the essential point is being able to assess different profiles of inputs and outputs giving rise to quantity $G_{(g)}$ under the current technique in use. In fact, these processes may be

[5] See for example Roncaglia (1971), Varri (1980), Baldone (1980), Schefold (1980), Kurz & Salvadori (1995, Chapter 9) and Lager (1997).

[6] If this were the case, when a machine ends its life a new machine of the same type would replace it in the following period in order to exactly reconstitute productive capacity, the system being in a stationary state: "In fact, the main difficulty in this case is not the system of prices but the very concept of stationary state at the basis of the particular theory of prices of production which we are examining here. We need to be able to identify a regularly repeating period in the uninterrupted succession of productive operations if the notion of stationary equilibrium is to be used" (Varri 1980, p. 85).

thought of as running parallel to each other:

> These processes need not be separate in ownership or in operation, and will indeed often be run side by side in the same shed; all that is necessary is that the amounts of means of production and labour employed by each should be separately ascertainable by the use of measures of quantity, without need of knowing the values — so that an independent production equation can be set up for each.
>
> (Sraffa 1960, p. 64)

By multiplying each equation by $(1+r)^5, (1+r)^4, \ldots, (1+r), 1$, respectively, summing over and operating, it is possible to obtain:[7]

$$M_0 p_{m_0} \frac{\sum_{n=0}^{1}(1+r)^{3n+3}}{\sum_{n=0}^{5}(1+r)^n} + N_0 p_{n_0} \frac{\sum_{n=0}^{2}(1+r)^{2n+2}}{\sum_{n=0}^{5}(1+r)^n} = G_{(g)}p_g - (A_g p_a + \ldots + K_g p_k)(1+r) - L_g w$$

from where the price equation for commodity g can be written as:[8]

$$M_0 p_{m_0} \frac{r(1+r)^3}{(1+r)^3-1} + N_0 p_{n_0} \frac{r(1+r)^2}{(1+r)^2-1} + (A_g p_a + \ldots + K_g p_k)(1+r) + L_g w = G_{(g)}p_g$$

Notice that prices of 'old' machines $(p_{m_1}, p_{m_2}, p_{n_1})$ have disappeared from the integrated equation, and as a consequence of the presence of fixed capital inputs the price equation is a non-linear polynomial in r.

The example above may be generalised to a given square economy with n commodities and industries, reproducing circulating (denoted by matrix $\mathbf{A}$) and fixed capital inputs of different lengths of life (denoted by matrices $\mathbf{M}_{0(2)}, \ldots, \mathbf{M}_{0(T)}$, where T stands for the maximum technical durability), and using quantities of labour (denoted by row vector $\mathbf{l}^T$).[9] The equation set

[7]The operation of applying discounting factors to flow equations of different processes — to arrive at a single equation for each commodity reproduced — has been called 'vertical integration in a temporal sense' (Baldone 1980, p. 96), and the associated equation represents an 'integrated process'.

[8]For a (related) derivation of a similar result, see Roncaglia (1971, pp. 240-243).

[9]All throughout the analysis upper case boldface letters denote matrices (e.g. $\mathbf{X}$), lower case boldface letters indicate vectors (e.g. $\mathbf{x}$); all vectors are intended as column vectors unless explicitly transposed (e.g. $\mathbf{x}^T$); and a vector with a hat stands for a diagonal matrix with the vector elements on its main diagonal (e.g. $\widehat{\mathbf{x}}$).

representing the system of production is given by:[10]

$$\mathbf{p}^T\mathbf{A}(1+r)+\mathbf{p}^T\mathbf{M}_{0(2)}\frac{\sum_{k=0}^{T!/2}(1+r)^{2k+2}}{\sum_{k=0}^{T!-1}(1+r)^k}+\ldots+\mathbf{p}^T\mathbf{M}_{0(T)}\frac{\sum_{k=0}^{T!/T}(1+r)^{Tk+T}}{\sum_{k=0}^{T!-1}(1+r)^k}+w\mathbf{l}^T=\mathbf{p}^T$$

which can be written as:

$$\mathbf{p}^T\mathbf{A}(1+r)+\mathbf{p}^T\sum_{\mu=2}^{T}\mathbf{M}_{0(\mu)}\frac{r(1+r)^\mu}{(1+r)^\mu-1}+w\mathbf{l}^T=\mathbf{p}^T \tag{2.1}$$

where μ is the length of life of the *new* fixed capital inputs in matrix $\mathbf{M}_{0(\mu)}$.

Equation set (2.1) represents Sraffa's (1960) system of production for the case of constant efficiency of fixed capital inputs in a given *square* economy, having abstracted from the problem of choice of technique. Basically, with data on the technique in use — as represented by $(\mathbf{A}, \{\mathbf{M}_{0(\mu)}\}_{\mu=2}^{T}, \mathbf{l}^T)$ — one of the distributive variables r or w, and after choosing a standard of value, it is possible to obtain a set of exchange ratios that allow the system to reproduce itself.

Note that for the (limiting) case in which $r \to 0$, the profit and depreciation factor multiplying fixed capital inputs in price equations (2.1) becomes:

$$\lim_{r\to 0}\frac{r(1+r)^\mu}{(1+r)^\mu-1}=\frac{1}{\mu} \tag{2.2}$$

which corresponds to the typical proportional depreciation scheme. Moreover, note that for the (limiting) case in which $\mu \to \infty$, the profit factor becomes:

$$\lim_{\mu\to\infty}\frac{r(1+r)^\mu}{(1+r)^\mu-1}=r \tag{2.3}$$

which corresponds to the typical notion of 'profits on stocks' associated to

[10]Notice in the equation set below that $\sum_{k=0}^{T!-1}(1+r)^k = ((1+r)^{T!}-1)/r$, where $T!$ stands for T factorial. In the present context, $T!$ is (proportional to) the smallest common multiple of the different lengths of life of fixed capital inputs, and indicates the (maximum) number of equations (each representing a different elementary process) that have to be set up to obtain each new commodity as the only output of the corresponding integrated process. As has been noted by Schefold (1980, p. 149): "If activity levels were such that at some time all integrated processes with turnover periods $T_1 \ldots T_f$ could start simultaneously, they would all simultaneously end again after a time equal to the smallest common multiple of $T_1 \ldots T_f$".

fixed capital inputs of infinite length of life in traditional dynamic Input-Output models.

The (very) simplified theoretical setting represented by (2.1) has been adopted *mainly* because of its empirical tractability. It is one of the simplest cases in which fixed capital is treated as a joint product, though still being able to actually carrying out empirical computations.

However, this methodological choice does not imply that computable prices are immediate to obtain. At least two further issues need to be addressed.

In the first place, according to the chosen setting in historical time of equation set (2.1), different empirical structures may be used to compute $\mathbf{p}^T$. This amounts to providing an explicit connection between Sraffa's (1960) system of production and empirical magnitudes, which cannot be said to be unique in the literature. Two examples may suffice to see the multiplicity of perspectives:

> This is the point of view chosen by Sraffa. The labour/consumption relation is kept in the background. Final demand is taken as given, or assumed to be unchanged, so as to cause a minimum of complications. The technique of production itself is taken as given or is supposed to be unchanged. The analysis *must* be carried out either as referring to one single period of time, considered in isolation, or as referring to a (stationary) economic system that exactly reproduces itself without any change from one period to another.
>
> (Pasinetti 1986, p. 11, our italics)

> Sraffa's system of production cannot be properly said to be set in a "particular year" (the *actual* magnitudes corresponding to it, would, if anything, be a moving average calculated over *several* years).
>
> (Garegnani 1988, p. 256)

Sraffa confined his analysis to "such properties of an economic system as do not depend on changes in the scale of production" (Sraffa 1960, p. v). This statement has sometimes been interpreted as the description of a system in stationary equilibrium, though this is not necessarily so, as Sraffa is only assuming his system to satisfy self-replacing conditions (see Sraffa 1960, p. 5, n. 1).

Nevertheless, how can the presence of a physical surplus be compatible with no changes in output proportions? Different uses of the surplus in all probability imply differing input and output profiles in the forthcoming periods. The idea of a set of equations representing 'one single period of time, considered in isolation' allows to abstract from the uses of the surplus and its consequences.[11]

In fact, with a given technique in use, as represented by the record of transactions on input and output flows of a given historical 'year', it is possible to derive exchange ratios that — for a given distributive configuration — allow the system to reproduce itself. For each historical 'year' there will be a set of production prices, obtained from an associated system of production that is recovered from the set of actually executed transactions.[12] In this sense, the preceding argument suggests that an empirical correspondence between Sraffa's system of production and empirical magnitudes should be established at the year-by-year level, and not by averaging medleys of techniques in use of different historical years.

This interpretation also makes clear that prices of production need not have attached a predictive content as centre of gravitation for market prices, to have a most useful role in empirical applications. These prices represent one of the (many) possible set of aggregators for physical quantities, satisfying given technical and distributive conditions, associated to the current technique in use, and need not be involved in the mechanism that may (or may not) bring them into being in the long-run of actual economies. These prices can be interpreted as a *norm*, which is present in the short-run as well as in the long-run (see Pasinetti 1981, p. 127, n. 1).

A second issue still unresolved involves the empirical availability of matrices $\{\mathbf{M}_{0(\mu)}\}_{\mu=2}^{T}$ of fixed capital inputs entering into price equations (2.1). Each matrix $\mathbf{M}_{0(\mu)}$ represents the quantity of *new* durable means of production of length of life μ employed in each process. In fact, for Sraffa:

[11]Note, moreover, that projections onto the past/future when setting-up equations for integrated processes always occur in *logical* time, not in historical time.

[12]See also the discussion on the interpretation of production prices as 'out of time mathematical structures' or 'stationary equilibrium' solutions in Zalai (1997, pp. 280-2), where the notion of average (as opposed to marginal) magnitudes is distinguished as essential for classical analysis.

> The quantity of machines of a given type that are required to produce annually $G_{(g)}$ (a quantity of a commodity) will be denoted by M_0 when they are new[.]
>
> (Sraffa 1960, p. 65)

However, the *total* quantity of new machines of a given type required to produce annually a commodity will not coincide with the physical counterpart of gross fixed capital formation (as registered by National Accounts).

The demand for gross investment translates (with a production lag) into new machines put into use by different processes which, however, also utilise previous vintages of fixed assets mixed up in the (average) technique in use. Nevertheless, information on the age profiles (current age and remaining technical average lifetime) of these previous vintages of fixed capital assets participating in production is not available.

Thus, a connection must be established between the current vintage of gross investment, whose composition and technical durability is known, and the total quantity of new machines that *would* be required to reproduce current outputs.

A possible path to follow is to assume that steady growth at given rate(s) has taken place, and compute the *total* quantity of new machines — of types corresponding to the currently observed gross investment vintage — that would have to be made available in order to comply with the (logical) time profile of outputs. In this sense, *growth* comes in when there is a need to know how many machines and buildings have to be priced.

3 Fixed Capital Replacements in Dynamic Input-Output

Throughout the analysis it is assumed a gestation period for fixed capital assets of one accounting year, which moreover coincides with the production lag required to render available new buildings and machines for productive use. Moreover, the length of life (denoted by μ) of a durable instrument is between 2 and T years.

Consider the flow of gross investment F_t in a one commodity system, which may be separated into expansion and replacement requirements according to:

$$F_t = gS_t + R_t \tag{3.1}$$

where g is the growth rate, S_t the stock of machines at the beginning of the period, and R_t are replacement requirements.

As has been early noted by Eisner (1952, p. 823), under steady growth at rate g and constant efficiency until the moment of 'sudden death', current replacement requirements corresponding to gross investment undertaken μ years ago is given by:

$$R_t = (1+g)^{-\mu} F_t \tag{3.2}$$

By combining (3.1) and (3.2) it is possible to express R_t in terms of S_t as:

$$R_t = \frac{gS_t}{(1+g)^{\mu} - 1} \tag{3.3}$$

Hence, by introducing (3.3) in (3.1) and solving for F_t, we obtain:

$$F_t = \frac{g(1+g)^{\mu}}{(1+g)^{\mu} - 1} S_t \tag{3.4}$$

which establishes that gross investment can be unambiguously determined by the stock S_t at the beginning of the current year, the length of life μ of the capital input, and the steady growth rate g. However, (3.4) may also be expressed as:

$$S_t = \frac{(1+g)^{\mu} - 1}{g(1+g)^{\mu}} F_t \tag{3.5}$$

where the stock S_t at the beginning of the current period is *implied by* the current gross investment flow F_t, technical durability μ and growth rate g. Hence, with this set of data (F_t, μ, g) it is possible to obtain the 'total quantity of machines' employed in production.

Note that when g approaches zero, the factor multiplying F_t in (3.5) becomes:

$$\lim_{g \to 0} \frac{(1+g)^{\mu} - 1}{g(1+g)^{\mu}} = \mu \tag{3.6}$$

Hence, *only* in the absence of growth, $S_t = \mu F_t$, so that $F_t = (1/\mu) S_t$, i.e. the flow of replacement investment is a constant proportion of the stock of machines.

However, this precise correspondence between S_t and F_t is achieved under the stringent assumptions of steady growth and 'sudden death'. In empirically given structures the relation is more intricate and, with all probability, the *actual* flow of gross investment and the *actual* stock of fixed capital inputs will not be functionally related by (3.4) or (3.5).

In studying fixed capital replacements in the dynamic model of Stone & Brown (1962), Gossling (1974) formulates an equation system aimed at solving for industries' intensity of operation, given actual (statistically compiled) circulating and fixed capital stock matrices, as well as data on gestation periods and technical durability of each capital input, i.e. for each durable instrument of a different vintage. In his system, the flow of gross investment turned out to be a derived magnitude, obtained by expanding actual stocks at a given (uniform) steady growth rate and expressing replacements in terms of stocks available at the beginning of the production period, as in (3.3). In multisectoral terms, the implied gross fixed capital formation is given by:[13]

$$\tilde{\mathbf{F}}_k = g\mathbf{S}_k + \sum_{\mu=2}^{T} \mathbf{S}_{k(\mu)} \frac{g}{(1+g)^{\mu} - 1} \tag{3.7}$$

$$\mathbf{S}_k = \sum_{\mu=2}^{T} \mathbf{S}_{k(\mu)} \tag{3.8}$$

which, by introducing (3.8) in (3.7), can be written as:[14]

$$\tilde{\mathbf{F}}_k = \sum_{\mu=2}^{T} \mathbf{S}_{k(\mu)} \frac{g(1+g)^{\mu}}{(1+g)^{\mu} - 1} \tag{3.9}$$

In empirically given systems, matrix $\tilde{\mathbf{F}}_k$ will not coincide with $\mathbf{F}_k$, the actual matrix of gross investment. This is the rationale behind solving for activity levels in a model of this kind.

Recalling now the open problem of section 2, i.e. determining the total

[13] C.f. Gossling (1974, p. 527), in particular equation (5) for $\gamma = 1$ (one year gestation period) and no inventory expansion ($r\mathbf{C} = 0$).

[14] Note that in (3.7) for each length of life μ there are specific replacement requirements, while expansion takes place for the whole stock of machines of different ages $\mathbf{S}_k$ (i.e of different vintages) at uniform rate g.

quantity of new machines of each type that *would* be required to reproduce current output flows, and the algebra of replacements for a one commodity economy described above, a suggested route could be to turn upside-down the approach giving rise to (3.9). Thus, instead of computing the the 'flow' implied by the 'stock' (as Gossling (1974) did), depart from an empirically given flow of gross investment (the current vintage of reproduced new machines) $\mathbf{F}_k$, and compute the 'stock' implied by the 'flow', $\tilde{\mathbf{S}}_k$, according to:

$$\mathbf{F}_k = g\tilde{\mathbf{S}}_k + \sum_{\mu=2}^{T} \mathbf{F}_{k(\mu)}(1+g)^{-\mu} \tag{3.10}$$

$$\mathbf{F}_k = \sum_{\mu=2}^{T} \mathbf{F}_{k(\mu)} \tag{3.11}$$

$$\tilde{\mathbf{S}}_k = \sum_{\mu=2}^{T} \mathbf{F}_{k(\mu)} \frac{(1+g)^{\mu}-1}{g(1+g)^{\mu}} \tag{3.12}$$

It should be borne in mind that the implied 'stock' matrix $\tilde{\mathbf{S}}_k$ does not aim at providing an accurate estimate for $\mathbf{S}_k$, it is simply a counter-factual magnitude that answers the following question: provided that the economy has been undergoing steady growth at rate g, there is a one-period production lag (equal to the gestation period) and the technical durability of each type of fixed asset of the *current* vintage is known, how many new machines would have been necessary to accumulate to satisfy the production requirements of the time profile of outputs? This quantity of new machines could enter price equations (2.1).

In the traditional discrete dynamic Input-Output model due to Leontief (1970), current inputs are met from current outputs. Hence, intensity ratios of (flow) inputs per unit of (flow) output represent technical coefficients. However, as has been noted by Lager (2000, p. 249), there exist inhomogeneities with respect to time in actual data. These are essentially due to the fact that production takes time (there are production lags and gestation periods), which means that observed Input-Output matrices implicitly contain growth (or decay) rates. Moreover, industries reproduce commodities with technical durability greater than the accounting period of one 'year', but an analytical decomposition of each industry into as many elementary processes as there are 'years' in the technical lifetime of its capital inputs cannot be achieved in *historical* time.[15]

Hence, to empirically obtain production prices it is first necessary to compute 'technical coefficients' from observed period-by-period data. As has been shown by Lager (2000, p. 250), if current inputs and final demand are met from past outputs, with a one-period production lag, and assuming uniform steady growth at rate g, we have:

$$\mathbf{A} = \mathbf{U}(1+g)^{-1} \tag{3.13}$$

with $\mathbf{U}$ being an observable 'Use' matrix for domestic output, and $\mathbf{A}$ the (implied) input matrix of circulating capital entering price equations (2.1).

As has been argued above, by setting $\sum_{\mu=2}^{T} \mathbf{M}_{0(\mu)}$ in (2.1) equal to $\tilde{\mathbf{S}}_k$ in (3.12) and, additionally, allowing for pure joint production (by introducing make matrix $\mathbf{V}$), the price equations can be set up as:

$$\mathbf{p}^T\mathbf{U}\frac{1+r}{1+g} + \mathbf{p}^T\sum_{\mu=2}^{T}\mathbf{F}_{k(\mu)}\frac{(1+g)^\mu - 1}{g(1+g)^\mu}\frac{r(1+r)^\mu}{(1+r)^\mu - 1} + w\mathbf{l}^T = \mathbf{p}^T\mathbf{V} \tag{3.14}$$

For a closed economy without government, no changes in inventories and valuables, with matrices $(\mathbf{U}, \{\mathbf{F}_{k(\mu)}\}_{\mu=2}^{T}, \mathbf{V})$ available in physical terms, from (3.14) it is possible to derive a rule of computation for production prices. However, the fact that actual empirical structures contain imported commodities, changes in inventories and valuables, taxation by the government and, most importantly, $(\mathbf{U}, \{\mathbf{F}_{k(\mu)}\}_{\mu=2}^{T}, \mathbf{V})$ are only available at current basic (statistical) prices, suggests that there is still a gap between (3.14) and empirical computations. This gap is closed in the next section.

[15] For a detailed treatment of the relationship between observed and technical coefficients in Input-Output schemes, see Lager (2000, section 7).

4 Computable Systems

4.1 Accounting framework for expenditure and value added

Consider, on the one hand, the following expenditure side of an empirically given *square* Supply-Use scheme:

$$\mathbf{Ve} \equiv \mathbf{Ue} + \mathbf{F}_k\mathbf{e} + \mathbf{f}_{vk} + \mathbf{f}_c \tag{4.1}$$

$$\mathbf{f}_c \equiv \mathbf{f}_{c_p} + \mathbf{f}_g + \mathbf{f}_x \tag{4.2}$$

with $\mathbf{V}$ a commodity × industry Supply matrix, $\mathbf{U}$ a commodity × industry Use matrix for domestic output, $\mathbf{F}_k$ a commodity × industry matrix of gross fixed capital formation, $\mathbf{f}_{vk}$ a vector of changes in inventories and valuables by commodity, and $\mathbf{f}_c$ a vector of final uses by commodity, comprising final private and public consumption ($\mathbf{f}_{c_p}$ and $\mathbf{f}_g$, respectively), as well as exports ($\mathbf{f}_x$).[16] All magnitudes refer to domestically produced commodities valued at current basic (statistical) prices.

In absence of additional assumptions, vector $\mathbf{e}$ in (4.1) can be interpreted as the observed operation intensities of each industry, equal to one. Given that there is insufficient data to obtain a commodity × industry matrix of changes in inventories and valuables, gross outputs, means of production and labour inputs can be re-proportioned by adopting a new set of activity levels that disregards $\mathbf{f}_{vk}$ in (4.1), according to:

$$\mathbf{V}\boldsymbol{\lambda}_x = \mathbf{U}\boldsymbol{\lambda}_x + \mathbf{F}_k\boldsymbol{\lambda}_x + \mathbf{f}_c \tag{4.3}$$

$$\boldsymbol{\lambda}_x = (\mathbf{V} - \mathbf{U} - \mathbf{F}_k)^{-1}\mathbf{f}_c \tag{4.4}$$

$$L = \mathbf{l}^T\boldsymbol{\lambda}_x \tag{4.5}$$

Note that $\boldsymbol{\lambda}_x$ is a vector of activity level indexes, each of its components measuring the deviation from observed unitary intensities.

[16] Note that the distinction between final uses and means of production does not depend on the nature of the commodity itself but on the use to which it is put. All produced inputs (including gross investment) that re-enter the circular flow are considered means of production, as they alter the productive capacity of the system. On the contrary, for example, an exported new machine is considered a final commodity, even if it is a durable good to be used in production (abroad).

Moreover, note that nothing has been said about the conditions to obtain non-negative solutions for $\boldsymbol{\lambda}_x$ in (4.4). A sufficient condition would be that matrix $(\mathbf{V} - \mathbf{U} - \mathbf{F}_k)^{-1}$ is non-negative, as $\mathbf{f}_c \geq \mathbf{0}$. For the case of single product systems (i.e. $\mathbf{V} = \widehat{\mathbf{z}}$ a diagonal matrix), $(\widehat{\mathbf{z}} - \mathbf{U} - \mathbf{F}_k)$ is necessarily a real matrix with non-positive off-diagonal entries, therefore being an M-matrix that admits a non-negative inverse.[17] However, in the presence of pure-joint products, the non-diagonal character of $\mathbf{V}$ does not allow to assure *a priori* that $\boldsymbol{\lambda}_x \geq \mathbf{0}$ (even though in the empirical computations performed below it actually is).[18]

Consider, on the other hand, the value added side of an empirically given *square* Supply-Use scheme:

$$\mathbf{e}^T\mathbf{V} \equiv \mathbf{e}^T\mathbf{U} + \mathbf{e}^T\mathbf{U}^m + \boldsymbol{\tau}^T + \mathbf{w}^T + \boldsymbol{\pi}^T \tag{4.6}$$

with $\mathbf{U}^m$ a commodity $\times$ industry matrix of imported intermediate consumption, $\boldsymbol{\tau}^T$ a (row) vector of taxes (net of subsidies) on products, production and labour by industry, $\mathbf{w}^T$ a (row) vector of wages by industry, and $\boldsymbol{\pi}^T$ a (row) vector of gross operating surplus by industry.

In order to deal with imported commodities in the production price equations, it is assumed that the (observed) price ratios of imported to domestic intermediates by product (denoted by $\boldsymbol{\epsilon}$) is given, so that if there is a uniform statistical *basic* price per commodity, total intermediate consumption matrix $\mathbf{U}^*$ can be defined according to:

$$\mathbf{U}^* := \mathbf{U}^*(\widehat{\boldsymbol{\epsilon}}) = \mathbf{U} + \mathbf{U}^m = \widehat{\mathbf{p}}_s(\mathbf{U}_q + \widehat{\boldsymbol{\epsilon}}\mathbf{U}_q^m) \tag{4.7}$$

$$\widehat{\boldsymbol{\epsilon}} = \widehat{\mathbf{p}}_s^m(\widehat{\mathbf{p}}_s)^{-1} \tag{4.8}$$

with $\mathbf{U}_q$ and $\mathbf{U}_q^m$ being domestic and imported intermediate consumption matrices in physical units (subscript q stands for 'quantities'), $\mathbf{p}_s$ and $\mathbf{p}_s^m$ being statistical basic prices for domestic and imported commodities in local currency, respectively. In a sense, $\boldsymbol{\epsilon}$ can be interpreted as a 'terms of trade' vector by commodity, which parametrically enters the production price equa-

[17] See, for example, Meyer (2000, p. 639).

[18] Table 10 in Appendix B reports the empirical values for $\boldsymbol{\lambda}_x$.

tions in each period.[19]

A second step towards a system of computable prices is dealing with net taxes. Taxation constitutes a purely institutional feature of the value added side of an Input-Output scheme.[20] Instead, the present analysis intends to compute production price systems that depend exclusively on the technical conditions of reproduction and the rule of distribution of the surplus between wages and profits. Hence, rather than evaluating inputs and outputs in (4.6) in terms of statistical basic prices, these can be evaluated by means of a price system that disregards $\boldsymbol{\tau}^T$. To do so, compute:

$$\boldsymbol{\lambda}_\tau^T \mathbf{V} = \boldsymbol{\lambda}_\tau^T \mathbf{U}^* + \mathbf{w}^T + \boldsymbol{\pi}^T \tag{4.9}$$

$$\boldsymbol{\lambda}_\tau^T = (\mathbf{w}^T + \boldsymbol{\pi}^T)(\mathbf{V} - \mathbf{U}^*)^{-1} \tag{4.10}$$

Note that $\boldsymbol{\lambda}_\tau^T$ represents a price *index* vector by commodity, measuring the deviation between statistical prices excluding and including net taxes.

This last point is of a general character: given that all observable magnitudes are in current prices, it is not possible to compute price systems, but systems of price *indexes*. In fact, for any magnitude of dimension commodity × industry, e.g. matrix $\mathbf{V}$, multiplying by a price *index* vector changes the prices with which physical quantities are being evaluated: $\boldsymbol{\lambda}_\tau^T \mathbf{V} = \mathbf{p}_\tau^T \widehat{\mathbf{p}}_s^{-1} \mathbf{V} = \mathbf{p}_\tau^T \widehat{\mathbf{p}}_s^{-1} \widehat{\mathbf{p}}_s \mathbf{V}_q = \mathbf{p}_\tau^T \mathbf{V}_q$. This point is crucial, as systems of computable price indexes can be specified using input and output matrices in nominal terms.[21]

A similar point to that made for $\boldsymbol{\lambda}_x$ in (4.4) is in place. The presence of pure joint-products means that it cannot be assumed that $(\mathbf{V} - \mathbf{U}^*)$ in (4.10) is a real Z-matrix (i.e. a real matrix with non-positive off-diagonal entries), so its inverse may, in principle, contain some negative elements.

Much of the analysis that follows aims at computing systems of price

[19] Hence, it is relevant to quantify the extent to which changes in $\boldsymbol{\epsilon}$ may be influencing computable prices. Table 19 in Appendix B reports the dynamics of 'terms of trade' by commodity for the whole period of analysis.

[20] To quantify the importance of net taxes per (monetary) unit of gross output in each industry, see Table 20 on Appendix B.

[21] This does not exclude that the standard of value in which prices are measured needs to be expressed in prices of a different year than that of the physical quantities composing the *numéraire* composite commodity.

indexes that can be used as aggregators of inputs and outputs in order to measure the surplus generating capacity of an economy, as well as changes in actual distributional possibilities due to technical change. In all cases, price index vector $\boldsymbol{\lambda}_{\tau}^{T}$ in (4.10) will be the reference system of prices.

4.2 Multiple systems of price indexes

4.2.1 Static Input-Output prices

The first price (index) system to be considered corresponds to a static Input-Output scheme where circulating capital is reproducible but fixed capital stocks are considered as a non-produced (primary) factor, on the same ground as labour inputs. These prices are found in traditional Input-Output measurement of TFP growth.[22] Price (index) equations are given by:

$$\boldsymbol{\lambda}_{\tilde{p}}^{T}\mathbf{V} = \boldsymbol{\lambda}_{\tilde{p}}^{T}\mathbf{U}^{*} + w_o\mathbf{l}^{T} + r_o\mathbf{k}^{T} \tag{4.11}$$

$$\mathbf{k}^{T} = \mathbf{e}^{T}(\mathbf{U}^{*} + \mathbf{S}_{k}^{*}) \tag{4.12}$$

$$w_o = \frac{\mathbf{w}^{T}\mathbf{e}}{\mathbf{l}_{w}^{T}\mathbf{e}} \tag{4.13}$$

$$r_o = \frac{\Pi}{S^{*}} = \frac{(\boldsymbol{\pi}^{T} - w_o(\mathbf{l}^{T} - \mathbf{l}_{w}^{T}))\mathbf{e}}{\mathbf{k}^{T}\mathbf{e}} \tag{4.14}$$

with $\mathbf{k}^{T}$ a vector of circulating and fixed capital stocks by industry, $\mathbf{S}_{k}^{*}$ a commodity × industry matrix of total (domestically produced and imported) gross fixed capital stocks at the beginning of the production period, and $\mathbf{l}_{w}^{T}$ a (row) vector of wage-labour by industry.

There are, however, two subtle differences with the standard treatment.

First, given that not all employment is wage-labour, though the wage-bill corresponds only to employees, the average (uniform) money wage rate (w_o) in (4.13) is computed according to the wages-to-employees ratio. As a consequence, profits (Π) in (4.14) are given by gross operating surplus ($\boldsymbol{\pi}^{T}$) net of 'hypothetical' wages corresponding to those employment units beyond wage-labour, $w_o(\mathbf{l}^{T} - \mathbf{l}_{w}^{T})$. This is done because the interest of the analysis lies in studying distributional possibilities between wages and profits, rather than between workers and capitalists.

[22] See, for example, Wolff (1985, p. 269, especially footnote 2).

The second difference is that r_o is computed on *both* fixed and circulating capital, S^* in (4.14). In this way, $r = r_o$ can be parametrically used in production price equations where a profit factor is also applied on circulating capital inputs.

Solving for $\boldsymbol{\lambda}_{\tilde{p}}^T$ in (4.11) gives:

$$\boldsymbol{\lambda}_{\tilde{p}}^T = (w_o \mathbf{l}^T + r_o \mathbf{k}^T)(\mathbf{V} - \mathbf{U}^*)^{-1} \tag{4.15}$$

Given that $\boldsymbol{\lambda}_{\tilde{p}}^T = \tilde{\mathbf{p}}^T \widehat{\mathbf{p}}_s^{-1}$ is a price index with respect to (statistical) basic prices, pos-multiplying by $\widehat{\boldsymbol{\lambda}}_\tau^{-1}$ allows to correct for the effect of net taxes:

$$\boldsymbol{\lambda}_{\tilde{p},\tau}^T = \boldsymbol{\lambda}_{\tilde{p}}^T \widehat{\boldsymbol{\lambda}}_\tau^{-1} = (\tilde{\mathbf{p}}^T \widehat{\mathbf{p}}_s^{-1})(\widehat{\mathbf{p}}_\tau \widehat{\mathbf{p}}_s^{-1})^{-1} = \tilde{\mathbf{p}}^T \widehat{\mathbf{p}}_s^{-1} \widehat{\mathbf{p}}_s \widehat{\mathbf{p}}_\tau^{-1} = \tilde{\mathbf{p}}^T \widehat{\mathbf{p}}_\tau^{-1} \tag{4.16}$$

Hence, a procedure like (4.16) is applied to all alternative computable price indexes obtained below.

4.2.2 Production prices with only circulating capital

A second system of price indexes considered is the system of production prices involving only circulating capital, often found in the literature on measurement of shifts in wage-profit (wr-)schedules in actual economies.[23] These studies are almost exclusively based on single product systems, empirically represented by square Input-Output tables.[24] These square matrices of an industry × industry or commodity × commodity type, are built from commodity × industry Supply-Use tables by means of an Input-Output technology assumption or 'transformation model'.[25] However, it can be shown[26] that any of the four main technology assumptions implies a resulting model in which either: (a) each input coefficient depends on statistical *relative* prices (so that simply deflating the tables will not neutralise the nominal influence), or (b) contemplate the possibility of having negative elements in *direct* input

[23]For a pioneering work, see Marzi & Varri (1977).
[24]For a recent example, see Degasperi & Fredholm (2010).
[25]For a review of the four main transformation models (i.e. product technology, industry technology, fixed industry sales structure and fixed product sales structure) see EUROSTAT (2008, chapter 11).
[26]See Wirkierman (2011, Appendix B).

requirement matrices; making it therefore impossible to guarantee an economically meaningful separation between prices and proportional changes in inputs and outputs, essential for the measurement of technical change.

Thus, a theoretically sound alternative consists in directly working with Supply-Use tables. This route is very recent indeed[27] and is taken here. The price index equations in this case are given by:

$$\boldsymbol{\lambda}_{p_c}^T \mathbf{V} = \boldsymbol{\lambda}_{p_c}^T \mathbf{U}^*(1+r) + w\mathbf{l}^T \tag{4.17}$$

$$\boldsymbol{\lambda}_{p_c}^T \overline{\mathbf{f}}_{c_p} = 1 \tag{4.18}$$

where $\overline{\mathbf{f}}_{c_p}$ is the *numéraire* commodity (the standard of value), equal to the average (per-capita) final private consumption domestically produced of a given year.

By solving for w in (4.17), pos-multiplying by $\overline{\mathbf{f}}_{c_p}$, and using (4.18), it is possible to obtain a solution for the real wage rate $w(r)$ and production price indexes $\boldsymbol{\lambda}_{p_c}^T(r)$:

$$w(r) = \frac{1}{\mathbf{l}^T(\mathbf{V} - \mathbf{U}^*(1+r))^{-1}\overline{\mathbf{f}}_{c_p}} \tag{4.19}$$

$$\boldsymbol{\lambda}_{p_c}^T(r) = w(r)\mathbf{l}^T(\mathbf{V} - \mathbf{U}^*(1+r))^{-1} \tag{4.20}$$

Note that both $w(r)$ in (4.19) and $\boldsymbol{\lambda}_{p_c}^T(r)$ in (4.20) parametrically depend on r, the rate of profits. Moreover, with respect to (4.15), the choice of a standard of value is explicit and, most importantly, w and r cannot change *independently* of one another.

Nothing has been said so far about the range of r for which there are meaningful (non-negative) solutions for $\boldsymbol{\lambda}_{p_c}^T(r)$, and for which $w(r)$ is 'well-behaved' (i.e. there is an inverse monotonic relation between w and r). Given the presence of pure-joint products, the system *a priori* not being r-*all productive* (see Schefold 1989, p. 87), it is not possible to guarantee that the desirable properties of single product systems hold for all feasible values of r, and for all *numéraire* commodities.[28]

[27] See, for example, Soklis (2011).

[28] In fact, "[w]hen commodities are produced jointly, it need not be the case that $dw/dr < 0$ irrespective of the standard of value that is chosen" (Kurz 2011, p. 39n). See also the

In fact, by setting $w = 0$ in (4.17), it is possible to obtain the following eigenequation:

$$\boldsymbol{\lambda}_{p_c}^T \mathbf{V} = \boldsymbol{\lambda}_{p_c}^T \mathbf{U}^*(1 + R_c) \tag{4.21}$$

If eigensystem (4.21) has a positive dominant eigenvalue, associated to non-negative left and right eigenvectors, then R_c given by:[29]

$$R_c = \frac{1}{\varrho(\mathbf{U}^*(\mathbf{V} - \mathbf{U}^*)^{-1})} \tag{4.22}$$

can be considered an 'empirically meaningful' maximum rate of profits.[30]

In this case, $w(r)$ and $\boldsymbol{\lambda}_{p_c}^T(r)$ can be computed according to (4.19) and (4.20), respectively, for $0 \leq r < R_c$. What is empirically relevant is that if $\boldsymbol{\lambda}_{p_c}^T(r) \geq \mathbf{0}^T$ within $0 \leq r < R_c$, production prices may be used as meaningful aggregators for inputs and outputs; even if $w(r)$ is not strictly decreasing for all *numéraire* commodities.

Finally, to correct for the effect of net taxes, compute:

$$\boldsymbol{\lambda}_{p_c,\tau}^T(r) = \boldsymbol{\lambda}_{p_c}^T(r)\widehat{\boldsymbol{\lambda}}_\tau^{-1} \tag{4.23}$$

4.2.3 A necessary *detour*: growing subsystems

In order to consider the third system of price indexes, a necessary *detour* consists in discussing the notion of growing subsystems.

The crucial idea behind the subsystem — as introduced by Sraffa (1960) — is its degree of autonomy. By repartitioning the whole *row* vector of gross outputs and matrix of intermediate uses by industry into as many logical parts as there components in the *column* vector of final uses by commodity, all means of production, labour and outputs are redistributed into each of these parts, according to their contribution as a supporting industry to the activity which produces the final commodity.

The redistribution of commodities in association to others, as an alternative to the aggregation of industries, is thoroughly discussed by Leontief

example provided by Soklis (2011, pp. 554-5) for the Finnish economy.

[29]Note that $\varrho(\mathbf{X})$ stands for the spectral radius of matrix $\mathbf{X}$.

[30]See the discussion in Soklis (2011, pp. 552-3).

(1967), who noticed that aggregation and reduction were two strategies to deal with too detailed empirical structures:

> Aggregation, i.e. summation of essentially heterogeneous quantities, is one of the two devices that the economist uses to limit the number of variables and functional relationships in terms of which he describes what he observes. The other is reduction, that is, elimination of certain goods and processes.
>
> (Leontief 1967, p. 419)

The strategy of reduction has been explicitly adopted by Pasinetti (1963) in his scheme for structural analysis (see Pasinetti 1963, p. 49). Pasinetti (1973) established explicit connections between the subsystem and the logical device of vertical integration, i.e. the reduction of some commodities in terms of others. By introducing a compact algebraic representation of a self-replacing subsystem, as the result of vertically integrating co-existing total employment and capital goods, it became possible to work with alternative representations of the same technique in use, either in direct terms (direct labour and direct productive capacity) or in vertically integrated terms (vertically integrated labour and productive capacity).[31]

But even though it dealt with the case of balanced growth at a uniform rate, the vertically integrated sector in Pasinetti (1973) remained an essentially static construct, in the sense of representing only self-replacing subsystems. New investments were still included in the net output, so part of the physical surplus of industries producing capital goods still needed to be exchanged between (or redistributed among) these self-replacing sectors, in order for each of them to expand their *commodity-specific* productive capacity. This clearly posed difficulties to the degree of autonomy of the self-replacing subsystem.[32]

Thus, in the context of a dynamic economy, Pasinetti (1988) introduced the logical device of vertical hyper-integration in explicit association to the notion of a *growing* subsystem.[33] The key difference is that *gross* invest-

[31]See Kurz & Salvadori (1995, chapter 6) for a discussion.

[32]The idea on this subtle but essential point is due to Garbellini (2010, pp. 48-51). The reader is referred to this source for a clear exposition and discussion.

[33]The mathematical specification of total gross output requirements to satisfy demand for *each* consumption commodity exponentially growing at a different steady rate can be already found in Mathur's (1964, pp. 74-5) discussion of Stone & Brown's (1962) dynamic

ment to self-replace and expand commodity-specific productive capacities is redistributed among industries according to their reproduction requirements (which now includes expansion/contraction), when the reduction process is performed. Therefore, investment becomes fully induced by the growth of effective demand for final uses.

When dealing with an empirically given economy, instead of applying (future) growth rates of final consumption demand to given fixed capital stocks, it is preferable to use current gross investment flows of fixed *and* circulating capital to build final commodity subsystems, so that one is not restricted to assuming the explicit form of the current growth path of the economy:

> in each year, the gross investment undertaken by each industry represents the flow of capital goods required to maintain the industry on its current growth path.
>
> (Peterson 1979, p. 220)

In this way, in a system with n final commodities domestically produced, whose input matrices already (implicitly) contain growth (or decay) rates, final commodity subsystems can be formulated as:

$$\mathbf{V}\boldsymbol{\lambda}_x^{(i)} = \mathbf{U}\boldsymbol{\lambda}_x^{(i)} + \mathbf{F}_k\boldsymbol{\lambda}_x^{(i)} + \mathbf{f}_c^{(i)} \tag{4.24}$$

$$\mathbf{f}_c^{(i)} = \widehat{\mathbf{f}}_c\mathbf{e}_i = \mathbf{e}_i f_{c_i}; \qquad \mathbf{f}_c = \sum_{i=1}^{n} \mathbf{f}_c^{(i)} \tag{4.25}$$

Note that summing over final consumption by subsystem ($\mathbf{f}_c^{(i)}$) adds up to the total ($\mathbf{f}_c$). The solution for subsystem-specific indexes of activity levels by industry $\boldsymbol{\lambda}_x^{(i)}$ is given by:

$$\boldsymbol{\lambda}_x^{(i)} = (\mathbf{V} - \mathbf{U} - \mathbf{F}_k)^{-1}\mathbf{f}_c^{(i)}; \qquad \boldsymbol{\lambda}_x = \sum_{i=1}^{n} \boldsymbol{\lambda}_x^{(i)} \tag{4.26}$$

Input-Output model. Its explicit association to the notion of a growing subsystem has been noticed by Gossling (1972, pp. 141-2). In Pasinetti (1988) it is possible to find an explicit formulation with general joint production, and the application of the method of growing subsystems to find units of measurement for capital goods in terms of expanding (or contracting) commodity-specific capacities. See also Pasinetti (1989) for additional remarks.

Note that, because of linearity, subsystem-specific activity levels add up to activity level indexes $\boldsymbol{\lambda}_x$, as specified in (4.4).

The growing or hyper- subsystem also has total labour requirements associated to it. These correspond not only to direct and indirect labour inputs required to reproduce final goods and to self-replace existing capacity, but also to those *hyper*-indirect labour requirements to expand (or contract) productive capacity. Total labour requirements (denoted by $\boldsymbol{\eta}^T$) can be computed as:[34]

$$\boldsymbol{\eta}^T = \mathbf{l}^T(\mathbf{V} - \mathbf{U} - \mathbf{F}_k)^{-1} \tag{4.27}$$

By pre-multiplying $\boldsymbol{\lambda}_x^{(i)}$ in (4.26) by $\mathbf{l}^T$, and using (4.25) and (4.27), the scalar total labour requirements to reproduce current final consumption demand of commodity i is given by:

$$L^{(i)} = \mathbf{l}^T\boldsymbol{\lambda}_x^{(i)} = \boldsymbol{\eta}^T\mathbf{f}_c^{(i)} = \eta_i f_{c_i}, \qquad i = 1, \ldots, n \tag{4.28}$$

Note the change in dimensions, as total labour requirements by commodity ($L^{(i)}$) can be decomposed into the product between a scalar quantity of labour input requirements (η_i) and a scalar quantity of commodities (f_{c_i}). This will be crucial to devise a consistent measure of subsystem-specific total labour productivity changes below.

4.2.4 Production prices including fixed capital as a joint product

At this point, reconsider the original price equations (3.14). However, as has been noted by Gilibert (2003, p. 36): "While a common growth rate, in our decentralized economy, requires a common profit rate [...] the reverse is not true".

Hence, differently from (3.14), it is possible to assume commodity-specific steady growth (or decay) rates — complemented by information on technical durabilities of fixed capital inputs of the current gross investment vintage — with which it is possible to extract technical coefficients from observed Supply-Use tables. These derived matrices of inputs and outputs enter the

[34]Note that $\boldsymbol{\eta}^T$ is an index with respect to statistical basic prices, but if the interest of the analysis is on measuring *changes* in labour requirements, by using Supply-Use matrices in constant prices, changes in $\boldsymbol{\eta}^T$ will only reflect changes in labour content.

price equations of a joint product system where fixed capital is assumed to have constant efficiency.

In this case, the system of computable production price indexes is given by:[35]

$$\boldsymbol{\lambda}_p^T \sum_{i=1}^{n} \mathbf{V}\widehat{\boldsymbol{\lambda}}_x^{(i)} = w \sum_{i=1}^{n} \mathbf{l}^T \widehat{\boldsymbol{\lambda}}_x^{(i)} + \boldsymbol{\lambda}_p^T \sum_{i=1}^{n} \mathbf{U}^* \frac{1+r}{1+g_i} \widehat{\boldsymbol{\lambda}}_x^{(i)} + \\ + \boldsymbol{\lambda}_p^T \sum_{i=1}^{n} \sum_{\mu=2}^{T} \mathbf{M}_{k(\mu)}^{*(i)} \widehat{\boldsymbol{\lambda}}_x^{(i)} \frac{r(1+r)^{\mu}}{(1+r)^{\mu}-1} \tag{4.29}$$

$$\boldsymbol{\lambda}_p^T \overline{\mathbf{f}}_{c_p} = 1 \tag{4.30}$$

where:

$$\mathbf{M}_{k(\mu)}^{*(i)} := \begin{cases} \mathbf{F}_{k(\mu)}^* \dfrac{(1+g_i)^{\mu}-1}{(1+g_i)^{\mu} g_i} & \text{for} \quad g_i > 0, \mu = 2 \dots T \\ \mathbf{F}_{k(\mu)}^* \dfrac{(1+g_i)^{\mu}-1}{(1+g_i)^{-1} g_i} & \text{for} \quad g_i < 0, \mu = 2 \dots T \end{cases} \tag{4.31}$$

for $i = 1, \dots, n$ and $\mu = 2, \dots, T$.

Several points may be noticed:

1. Growth takes place at the final commodity subsystem level, and not at the industry level. This is because fixed capital inputs by subsystem are *non-linear* functions of g_i, therefore not allowing to find an equivalent expression in terms of industry averages of subsystem growth rates (weighted by the industry participation in each subsystem).[36]

2. Growth rates $\{g_i\}_{i=1}^{n}$ adopted to compute (4.29) are 'trend' growth rates of final uses (private-public consumption and exports), for the whole period of analysis.[37]

3. The possibility of decay rates ($g_i < 0$) is contemplated in (4.31). If final

[35]C.f. Gossling (1975, p. 104), in particular equations (35) and (38).

[36]C.f. Gossling (1975, pp. 72-3), especially equations (15) and (16).

[37]Trend growth rates have been estimated from a linear probability model like: $\ln f_{c_i,t} = \ln f_{c_i,0} + g_i t + \epsilon_t$, where the point estimate of $\partial E(\ln f_{c_i,t})/\partial t$ is the value adopted for g_i in (4.29).

demand for a commodity is diminishing (in its trend), the subsystem is contracting, so the fixed capital component of productive capacity should be progressively reduced.[38]

4. From (4.29) it can be inferred that there has been trend growth up to the current period, which allows to estimate the quantity of new machines that need to be priced and the quantity of circulating capital allowing for system expansion, even though redistribution of outputs, means of production and labour by subsystem is performed according to current subsystem activity level indexes, reflecting current (and not 'trend') growth paths.

5. Circulating ($\mathbf{U}^*/(1+g_i)$) and fixed capital ($\{\mathbf{M}^{*(i)}_{k(\mu)}\}^T_{\mu=2}$) input matrices include domestically produced as well as imported commodities.

6. As in the case with only circulating capital (4.17)-(4.18), average (per-capita) final private consumption domestically produced of a fixed year is chosen as *numéraire* commodity, as can be read from (4.30).

The general idea behind (4.29) is that prices are applied on technical coefficient matrices that have been extracted from observed data, coming from a system assumed to have been following steady growth within each final commodity subsystem. This is why physical magnitudes need to be redistributed across unbalanced growing subsystems before applying a uniform profit factor on circulating capital inputs and obtaining a profit and (endogenous) depreciation component for fixed capital inputs. A crucial consequence of (4.29) is that its solution will parametrically depend on r *and* $\{g_i\}^n_{i=1}$, i.e. to solve for prices we need a (given) trend of accumulation for each final commodity subsystem.

[38]See Gossling (1975, Appendix 5.I) for a description of the 'algebra of replica replacement of (fixed) capital'.

In order to simply notation, define:

$$\mathbf{C}^*(r) = \sum_{i=1}^{n} \mathbf{U}^* \frac{1+r}{1+g_i} \widehat{\boldsymbol{\lambda}}_x^{(i)} \tag{4.32}$$

$$\mathbf{C}_k^*(r) = \sum_{i=1}^{n} \sum_{\mu=2}^{T} \mathbf{M}_{k(\mu)}^{*(i)} \widehat{\boldsymbol{\lambda}}_x^{(i)} \frac{r(1+r)^\mu}{(1+r)^\mu - 1} \tag{4.33}$$

which correspond to the circulating and fixed capital components in price equations (4.29), respectively.

By solving for w in (4.29), pos-multiplying by $\overline{\mathbf{f}}_{c_p}$, and using (4.30), it is possible to obtain a solution for the real wage rate $w(r)$ and production price indexes $\boldsymbol{\lambda}_p^T(r)$:

$$w(r) = \frac{1}{\mathbf{l}^T(\mathbf{V} - \mathbf{C}^*(r) - \mathbf{C}_k^*(r))^{-1}\overline{\mathbf{f}}_{c_p}} \tag{4.34}$$

$$\boldsymbol{\lambda}_p^T(r) = w(r)\mathbf{l}^T(\mathbf{V} - \mathbf{C}^*(r) - \mathbf{C}_k^*(r))^{-1} \tag{4.35}$$

One of the most delicate issues in this context is computing the maximum rate of profits and standard proportions (in the sense of Sraffa 1960). Usually, empirical treatments of fixed capital assume exogenously given depreciation matrices of initial stocks and single product industries. In this way, to compute a maximum growth or profit rate reduces to solving an eigenproblem involving a stock matrix (stocks set the upper 'limit' to potential expansion). However, the joint product treatment advanced above aims at both disregarding the notion of *given* stock matrices as well as allowing for pure joint products, making it difficult to apply standard results of matrix theory.

In fact, by setting $w = 0$ in (4.29), and recalling definitions (4.32) and (4.33), the following system is obtained:

$$\boldsymbol{\lambda}_p^T \mathbf{V} \widehat{\boldsymbol{\lambda}}_x = \boldsymbol{\lambda}_p^T \mathbf{C}^*(R) + \boldsymbol{\lambda}_p^T \mathbf{C}_k^*(R) \tag{4.36}$$

Solving for $\boldsymbol{\lambda}_p^T$ and R such that $R > 0$ and $\boldsymbol{\lambda}_p^T \geq \mathbf{0}^T$ in (4.36) involves finding conditions for the non-negative solution(s) of a non-linear system in R (with given $\{g_i\}_{i=1}^n$). To do this, note that (4.36) can be equivalently expressed as:

$$\delta \boldsymbol{\lambda}_p^T = \boldsymbol{\lambda}_p^T \left[(\mathbf{C}^*(R) + \boldsymbol{\lambda}_p^T \mathbf{C}_k^*(R))(\mathbf{V}\widehat{\boldsymbol{\lambda}}_x)^{-1} \right] \tag{4.37}$$

$$\delta = 1 \tag{4.38}$$

Define $\mathbf{P}(R) := (\mathbf{C}^*(R) + \boldsymbol{\lambda}_p^T \mathbf{C}_k^*(R))(\mathbf{V}\widehat{\boldsymbol{\lambda}}_x)^{-1}$, expressing the system as:

$$\delta \boldsymbol{\lambda}_p^T = \boldsymbol{\lambda}_p^T \mathbf{P}(R) \tag{4.39}$$

$$\delta = 1 \tag{4.40}$$

If $\mathbf{P}(R)$ is a real non-negative matrix (for different values of R), finding the conditions for which $\delta = 1$ is the dominant eigenvalue of system (4.39) leads to identifying the set of non-negative prices $\boldsymbol{\lambda}_p^T$ and the associated value for R, the maximum rate of profits. To do so, it would be necessary to find a connection between δ and R.

In fact, in the case where δ is a continuous strictly increasing function of R, i.e. $\delta(R), \delta'(R) > 0$, it is possible to proceed as follows: (i) For a given $R_m = R_0$, compute the associated maximal eigenvalue δ_m in (4.39), i.e. $\delta_m = \varrho(\mathbf{P}(R_m))$, (ii) Adjust R_m according to the rule: $R_{m+1} = R_m + \alpha(1 - \delta_m)$, with α a positive (arbitrary) constant, and compute again $\delta_{m+1} = \varrho(\mathbf{P}(R_{m+1}))$. As argued by Aberg & Persson (1981, p. 451), it is possible to iterate (ii) in a convergent way, until $\delta_m(R_m) \to 1$, such that, in the limit, $R = R_m$ and the left eigenvector $\boldsymbol{\lambda}_p^T(R)$ — associated to this limiting δ_m — is the solution for (4.36).

In this way, for $0 \leq r < R$ and given $\{g_i\}_{i=1}^n$, it is possible to compute $w(r)$ and $\boldsymbol{\lambda}_p^T(r)$, according to (4.34) and (4.35), respectively. Finally, to correct for the effect of net taxes, compute:

$$\boldsymbol{\lambda}_{p,\tau}^T(r) = \boldsymbol{\lambda}_p^T(r) \widehat{\boldsymbol{\lambda}}_\tau^{-1} \tag{4.41}$$

4.2.5 Production prices when profits equal gross investment

One more computable system of price indexes is considered. Note that by assuming $g = r$ in (3.14) prices would be the value counterpart of a material balance equation in which all wages correspond to final uses of the system:

$$\mathbf{p}^T\mathbf{U} + \mathbf{p}^T\mathbf{F}_k + w\mathbf{l}^T = \mathbf{p}^T\mathbf{V} \tag{4.42}$$

While the assumption of $g = r$ implies that the value of profits equals that of gross investment, it may be interesting to directly assume that profits equal gross investment, even in conditions of unbalanced growth (and, probably, heterogeneous profit rates). The price (index) equations in this case could be formulated as:

$$\boldsymbol{\lambda}_\eta^T \sum_{i=1}^{n} \mathbf{V}\widehat{\boldsymbol{\lambda}}_x^{(i)} = w \sum_{i=1}^{n} \mathbf{l}^T \widehat{\boldsymbol{\lambda}}_x^{(i)} + \boldsymbol{\lambda}_\eta^T \sum_{i=1}^{n} \mathbf{U}^* \widehat{\boldsymbol{\lambda}}_x^{(i)} + \boldsymbol{\lambda}_\eta^T \sum_{i=1}^{n} \mathbf{F}_k^* \widehat{\boldsymbol{\lambda}}_x^{(i)} \tag{4.43}$$

$$\boldsymbol{\lambda}_\eta^T \overline{\mathbf{f}}_{c_p} = 1 \tag{4.44}$$

Note that (4.43) includes domestically produced and imported commodities, while it is not strictly required that total wages equal total private final consumption. Hence, even in this case there is not full empirical duality between the expenditure and value added side of the economy.

Given linearity properties of (4.43), price equations may be expressed as:

$$\boldsymbol{\lambda}_\eta^T \mathbf{V}\widehat{\boldsymbol{\lambda}}_x = w\mathbf{l}^T \widehat{\boldsymbol{\lambda}}_x + \boldsymbol{\lambda}_\eta^T \mathbf{U}^* \widehat{\boldsymbol{\lambda}}_x + \boldsymbol{\lambda}_\eta^T \mathbf{F}_k^* \widehat{\boldsymbol{\lambda}}_x \tag{4.45}$$

so that pos-multiplying by $\widehat{\boldsymbol{\lambda}}_x^{-1}$ and separating domestically produced from imported commodities, gives:

$$\boldsymbol{\lambda}_\eta^T \mathbf{V} = w\mathbf{l}^T + \boldsymbol{\lambda}_\eta^T \mathbf{U} + \boldsymbol{\lambda}_\eta^T \mathbf{F}_k + \boldsymbol{\lambda}_\eta^T \mathbf{U}^m + \boldsymbol{\lambda}_\eta^T \mathbf{F}_k^m \tag{4.46}$$

By using (4.27), (4.46) may be equivalently written as:

$$\boldsymbol{\lambda}_\eta^T = w\boldsymbol{\eta}^T + \boldsymbol{\lambda}_\eta^T (\mathbf{U}^m + \mathbf{F}_k^m)(\mathbf{V} - \mathbf{U} - \mathbf{F}_k)^{-1} \tag{4.47}$$

form where it is immediate to see that prices are given by the sum of vertically hyper-integrated labour costs ($w\boldsymbol{\eta}^T$) and vertically hyper-integrated import requirements (as imported commodities are not re-produced within the economy).[39] Hence, after fixing a standard of value — as is done in (4.44) — the real wage rate w_η and the system of price indexes $\boldsymbol{\lambda}_\eta^T$ may be computed as:

$$w_\eta = \frac{1}{\boldsymbol{\eta}^T \left(\mathbf{I} - (\mathbf{U}^m + \mathbf{F}_k^m)(\mathbf{V} - \mathbf{U} - \mathbf{F}_k)^{-1}\right)^{-1} \overline{\mathbf{f}}_{c_p}} \tag{4.48}$$

$$\boldsymbol{\lambda}_\eta^T = w_\eta \boldsymbol{\eta}^T \left(\mathbf{I} - (\mathbf{U}^m + \mathbf{F}_k^m)(\mathbf{V} - \mathbf{U} - \mathbf{F}_k)^{-1}\right)^{-1} \tag{4.49}$$

Finally, to correct for the effect of net taxes, compute:

$$\boldsymbol{\lambda}_{\eta,\tau}^T = \boldsymbol{\lambda}_\eta^T \widehat{\boldsymbol{\lambda}}_\tau^{-1} \tag{4.50}$$

With systems of price indexes (4.23), (4.41), (4.50), wr-schedules (4.19), (4.34) and total subsystem labour requirements (4.28), different measures aimed at analysing technical progress from a Classical perspective — as opposed to traditional growth accounting procedures — are devised in the following section.

4.3 Technical progress: hyper-integrated labour productivity changes, wr-schedules and surplus generating capacity

By examining (4.28), it is possible to define a measure of total labour productivity (intended as a physical output/input ratio) for each final commodity i as:

$$\rho_i := \frac{1}{\eta_i} = \frac{f_{c_i}}{L^{(i)}}, \qquad i = 1, \ldots, n \tag{4.51}$$

Coefficient ρ_i is the reciprocal of total labour content of final commodity i, expressed as a deviation from basic statistical prices. Given that $L^{(i)}$ is independent of prices, expressing f_{c_i} in constant prices and evaluating changes in ρ_i provides an adequate disaggregated (the growing subsystem being the unit of analysis) physical measure of productivity changes.

As has been argued by Pasinetti (1981, p. 214), any aggregate measure of technical change will depend on the changing physical commodity composition of final uses through time. Hence, it cannot be given a strictly technical

[39] Recall that $\mathbf{U}^m + \mathbf{F}_k^m = \mathbf{p}_s^T \widehat{\epsilon}(\mathbf{U}_q^m + \mathbf{F}_{q_k}^m)$, so that imported input matrices parametrically depend on the 'terms of trade' vector $\epsilon = \widehat{\mathbf{p}}_s^{-1}\mathbf{p}_s^m$.

interpretation (though still retaining a purely physical character). An aggregate indicator of total productivity changes is given by Pasinetti's (1981, pp. 101-4) 'standard rate of productivity growth', which may be computed as:[40]

$$\rho^* = \frac{\sum_{i=1}^{n}(d\ln\rho_i)L^{(i)}}{\sum_{i=1}^{n}L^{(i)}} \tag{4.52}$$

However, while (4.51) and (4.52) focus on physical productivity changes as measured from a set of commodity balances (the physical counterpart to the expenditure side of a Supply-Use scheme), this paper intends mainly to assess: (a) the change in distributional possibilities of actual systems due to technical change by means of shifts in wr-schedules, and (b) the surplus generating capacity of an economy in terms of industry equations aggregating (physical) output net of input changes. Both (a) and (b) are obtained by looking at price and distribution relations, departing from the value added side of the system.

The use of wr-schedules for assessing potential changes in actual distributional possibilities as a consequence of technical change is due to Schefold (1976). According to the pattern with which wr-curves like (4.19) and (4.34) shift, different 'stylised forms' of technical progress (or *regress*) may be identified. The main three forms correspond to: (a) saving of labour, (b) saving of raw materials and (c) mechanisation.[41]

Complementarily, the construction of surplus equations under different price (index) systems may be traced back to Steedman's (1983) article on measuring and aggregation (across *industries*) of productivity differences,[42] where the discussion is always kept at a theoretical level of general joint-production.

For the case of production price (index) system (4.41), by totally differentiating (4.29) and reordering terms, it is possible to define a vector of

[40] For a discussion of this formulation and other aspects of changes in hyper-integrated labour productivity, see Garbellini & Wirkierman (2011).

[41] For further explanations, see Schefold (1979).

[42] Steedman (1983) makes always clear that exercises of this sort, performed on alternative production price systems, are of a 'comparative dynamics' type. Hence, instead of talking about productivity 'changes' (which has clear *historical* time connotations), he refers to productivity 'differences' between two unrelated systems differing only in one of the set of givens.

r-Productivity growth rates by industry $\boldsymbol{\rho}_g(r)$, for parametrically given r (as well as $\{g_i\}_{i=1}^n$), to be computed according to:

$$\boldsymbol{\lambda}_{p,\tau}^T(r)\mathbf{V}\widehat{\boldsymbol{\lambda}}_x\widehat{\boldsymbol{\rho}}_g(r) = \boldsymbol{\lambda}_{p,\tau}^T(r)\left[\mathrm{d}(\mathbf{V}\widehat{\boldsymbol{\lambda}}_x) - \mathrm{d}\mathbf{C}^*(r) - \mathrm{d}\mathbf{C}_k^*(r)\right] - w(r)\,\mathrm{d}(\mathbf{l}^T\widehat{\boldsymbol{\lambda}}_x) \tag{4.53}$$

Rates of r-Productivity difference measure, for a given distributive configuration and growth path, the value of the surplus obtained per unit of gross output. Aggregating inputs (including labour) and outputs with production prices $\boldsymbol{\lambda}_{p,\tau}^T(r)$ and wage rate $w(r)$, gives an industry measure of surplus generating capacity (which may be read as a profitability indicator).

Proceeding similarly for price (index) system (4.50), by totally differentiating (4.43) and reordering terms, a vector of industry surplus growth rates, $\boldsymbol{\rho}_{g,\eta}$, may be defined according to:

$$\boldsymbol{\lambda}_{\eta,\tau}^T\mathbf{V}\widehat{\boldsymbol{\lambda}}_x\widehat{\boldsymbol{\rho}}_{g,\eta} = \boldsymbol{\lambda}_{\eta,\tau}^T\left[\mathrm{d}(\mathbf{V}\widehat{\boldsymbol{\lambda}}_x) - \mathrm{d}(\mathbf{U}^*\widehat{\boldsymbol{\lambda}}_x) - \mathrm{d}(\mathbf{F}_k^*\widehat{\boldsymbol{\lambda}}_x)\right] - w_\eta\,\mathrm{d}(\mathbf{l}^T\widehat{\boldsymbol{\lambda}}_x) \tag{4.54}$$

Rates of surplus growth by industry measure the value of surplus per unit of gross output, when gross profits equal gross investment, describing the additional real (in terms of *numéraire* commodity $\overline{\mathbf{f}}_{c_p}$) purchasing power created in each *industry* due to changes in physical inputs (including labour) and outputs.

Empirical computations involving the standard rate of productivity growth — ρ^* in (4.52), wr-curves — $w(r)$ in (4.19) and (4.34), and r-Productivity and surplus growth rates by industry — $\boldsymbol{\rho}_g(r)$ in (4.53) and $\boldsymbol{\rho}_{g,\eta}$ in (4.54), respectively — are reported in the next section.

5 An empirical exploration: Italy 1999-2007

Computations have been performed for the case of Italy (1999-2007), at a disaggregation level of 30 commodities/industries. As regards dataset characteristics and data preparation procedures, please refer to Appendix A.

5.1 A birds' eye view

Tables 1 and 2 report levels and rates of change between 1999 and 2007 of some relevant aggregate variables.

Table 1: Selected Aggregate Level Variables, Italy (1999-2007)

variable	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
W/L	20.26	20.86	21.59	22.15	22.86	23.64	24.45	25.22	25.82	22.98
Π/S^*	5.75	5.84	5.90	5.78	5.67	5.62	5.36	5.15	5.16	5.58
Π/S_c^*	34.94	33.94	34.13	34.24	34.20	34.21	33.03	31.32	31.42	33.49
R	17.05	16.10	16.08	15.54	15.96	15.83	15.44	14.29	14.61	15.66
R_c	86.07	82.43	82.87	83.58	82.90	81.16	80.20	76.88	76.19	81.37
Ω_W	46.34	45.90	45.82	45.86	46.11	46.02	46.47	47.20	46.73	46.27
Ω_{L_w}	32.20	31.92	32.02	32.23	32.27	32.18	32.95	33.58	33.42	32.53
S^*/W	14.28	14.48	14.40	14.58	14.65	14.93	15.17	15.22	15.51	14.80

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT
References: W/L - Money Wage Rate (mln.MU/th.ULE), Π/S^* - Rate of Profits on Capital (%), Π/S_c^* - Rate of Profits on Circulating Capital (%), R - Maximum Rate of Profits (%), R_c - Maximum Rate of Profits on Circulating Capital (%), Ω_W - Wage Share (%), Ω_{L_w} - Wage labour Share (%), S^*/W - Value of Capital to Wages Ratio

Table 2: Selected Aggregate Dynamics, Italy (1999-2007)

variable	units	99-00	00-01	01-02	02-03	03-04	04-05	05-06	06-07	mean
$\Delta\%(w/c_{p,-1})$	(pp)	-0.23	0.11	-0.47	-0.70	1.09	1.24	1.21	0.06	0.29
$\Delta\%w_{\eta,c_p}$	(pp)	1.14	-0.57	-2.20	-1.87	-0.07	-0.81	-0.73	-0.60	-0.71
$\Delta\%w_{c_p}(0.05)$	(pp)	-0.62	-0.04	-3.70	-1.17	-0.74	-1.89	-5.93	4.00	-1.26
$\Delta\%w_{c_p}(0.06)$	(pp)	-1.25	-0.06	-4.20	-0.98	-1.00	-2.38	-7.21	4.43	-1.58
$\Delta\%TFP$	(pp)	2.82	-0.32	-1.43	-2.11	1.06	0.22	0.44	0.61	0.16
ρ^*	(pp)	2.15	-0.04	-1.46	-0.42	1.72	0.82	1.33	0.97	0.63

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

While functional income distribution between wages and profits (Ω_W) or between workers and capitalists (Ω_{L_w}) has not seen dramatic changes, the rate of profits on fixed and circulating capital ($r_o = \Pi/S^*$) has had a declining overall trend (the exception being sub-period 1999-2001). This is in line with the generally increasing trend of the capital-wages ratio (S^*/W), for if $\Pi/W = r_o(S^*/W)$, a quasi-stationary value of Π/W with increasing S^*/W implies a decreasing profit rate. Moreover, the maximum rate of profits on only circulating (R_c) as well as that on circulating and fixed capital (R) have both followed a decreasing trend throughout the period.[43]

As to Table 2, for some variables — $\Delta\%(w/c_{p,-1})$, $\Delta\% TFP$, and ρ^* computed according to (4.52) — the whole period may be approximately divided into three sub-periods: 1999-2000, 2000-2003 and 2003-2007. While 1999-2000 shows relatively high hyper-integrated labour productivity growth as well as TFP growth (though with decreasing market average real wage rate $\Delta\%(w/c_{p,-1}) < 0$), the 2000-2003 period shows an overall negative performance, which is reverted in the final 2003-2007 sub-period.

The picture is markedly different when looking at $\Delta\% w_{\eta,c_p}$ — computed according to (4.48) — and $\Delta\% w_{c_p}(r)$ — computed according to (4.34) — for $r = \{0.05, 0.06\}$, i.e. the interval containing actual average $r_o = \Pi/S^*$, as reported in Table 1. The trend is negative for the whole period.[44] Hence, this clearly indicates that actual distributive possibilities due to technical change have descreased throughout the period, suggesting that in Italy — according to real wage rate computations — there has been technical *regress* between 1999 and 2007 (see the analysis of wr-curves below).

Note that, on the contrary, both ρ^* and $\Delta\% TFP$, though mildly, suggest an increase in physical total labour productivity and aggregate surplus from the value added side, respectively.[45] Moreover, a comparison between ρ^* and $\Delta\%(w/c_{p,-1})$ (0.63 p.p. and 0.29 p.p. at an average yearly basis, respectively) suggests that productivity increases have not fully accrued to real wages, throughout the 1999-2007 period.

All in all, Italy presents mild total labour productivity increases with lower average market real wage rate growth, a decreasing rate of profits, a nearly constant functional income distribution, where actual distributional possibilities due to technical change have actually decreased.

[43] Note that working with only circulating capital implies an artificially high (average and maximum) rate of profits (Π/S_c^* and R_c), which makes it difficult to give results an order of magnitude with (potentially) related economic variables, like the rate of interest on time deposits.

[44] The only exception being 2006-2007 for $\Delta\% w_{c_p}(r)$.

[45] Even the dynamics of the average market real wage rate $\Delta\%(w/c_{p,-1})$ shows an overall increase (with an average yearly growth of 0.29 p.p.).

5.2 Shape of wr-curves

Table 3 reports the year-by-year computations of $w(r)$ for $0 \leq r < R$, according to (4.34), and Figure 1 displays the resulting wr-curves for 1999 and 2007.

Table 3: Real wage rate growth, Italy - $\Delta\% w_{c_p}(r)$ (pp)

r	99-00	00-01	01-02	02-03	03-04	04-05	05-06	06-07	mean
0	1.13	−0.04	−2.31	−1.51	0.31	−0.29	−2.25	2.66	−0.29
0.01	0.91	−0.03	−2.48	−1.49	0.13	−0.51	−2.76	2.86	−0.42
0.02	0.64	−0.02	−2.70	−1.45	−0.06	−0.78	−3.35	3.08	−0.58
0.03	0.30	−0.02	−2.96	−1.39	−0.27	−1.10	−4.06	3.34	−0.77
0.04	−0.11	−0.03	−3.29	−1.31	−0.50	−1.47	−4.91	3.64	−1.00
0.05	−0.62	−0.04	−3.70	−1.17	−0.74	−1.89	−5.93	4.00	−1.26
0.06	−1.25	−0.06	−4.20	−0.98	−1.00	−2.38	−7.21	4.43	−1.58
0.07	−2.04	−0.08	−4.83	−0.69	−1.28	−2.96	−8.81	4.97	−1.97
0.08	−3.05	−0.12	−5.62	−0.29	−1.59	−3.67	−10.90	5.67	−2.45
0.09	−4.37	−0.17	−6.64	0.30	−1.95	−4.55	−13.72	6.60	−3.06
0.1	−6.13	−0.23	−8.02	1.16	−2.39	−5.69	−17.74	7.94	−3.89
0.11	−8.58	−0.31	−9.97	2.46	−2.94	−7.29	−23.89	10.03	−5.06
0.12	−12.18	−0.44	−12.93	4.58	−3.71	−9.71	−34.50	13.79	−6.89
0.13	−17.90	−0.65	−17.99	8.41	−4.93	−13.90	−57.34	22.68	−10.20
0.14	−28.31	−1.04	−28.71	16.90	−7.31	−23.13	−152.51	73.30	−18.85
0.15	−53.03	−2.13	−67.56	49.69	−14.76	−62.54			−25.06
0.16	−224.35	−27.65							−126.00

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Results from Table 3 suggest that the wage-profit schedule has experienced and inward shift between 1999 and 2007 (see Figure 1), the only exceptions being 1999-2000 and 2003-2004 (but for very low values for r) and 2006-2007 (for all feasible values of r).

Hence, instead of classifying stylised forms of technical progress, there is a need to explain technical *regress*. Given that the ordinate (related to labour requirements) has shifted much less than the abscissa (representing the maximum rate of profits R), this outcome may have come about by a mechanisation trend together with a *decreasing* efficiency in the use of raw materials, leading to overall technical regress.

So far nothing has been said about the sensibility of the results to the set of growth rates $\{g_i\}_{i=1}^{n}$ adopted to construct growing final commodity subsystems in price equations (4.29). If instead of computing production prices by assuming 'trend' growth rates, computations were performed using year-by-year (next period) growth rates, the variability of the resulting price

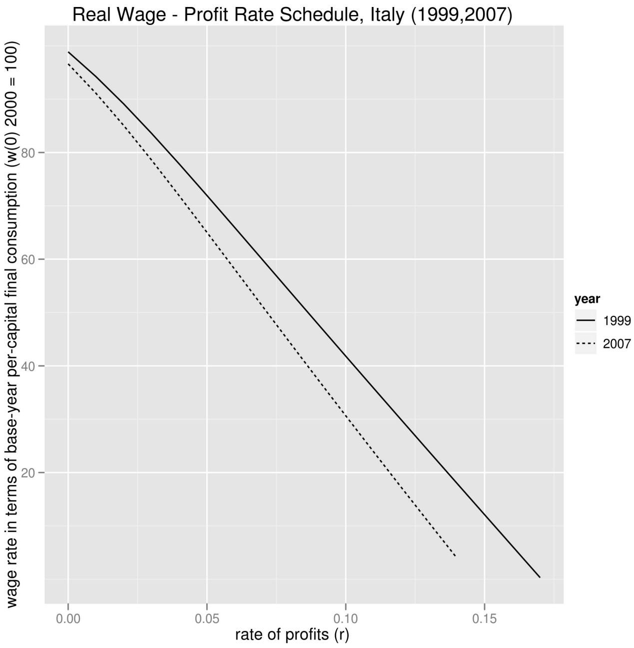

Figure 1: wr-schedule using fixed and circulating capital inputs

indexes and real wage rate would have drastically increased.[46] Figure 2 displays the wr-curve for 1999 and 2007. In fact, the inward shift of the schedule is of greater magnitude, showing acute technical regress. It is clear that mainly via its influence on the determination of the quantity of new machines (because of the compound nature of growth factors) the sensibility of the results to the assumed growth rates remains of utmost importance.

The comparison with the case of only circulating capital suggests that adopting such a simplified framework may produce misleading results. Figure 3 displays the wr-curve for 1999 and 2007, as emerges from the computations of $w(r)$ for $0 \leq r < R_c$, according to (4.19).

Given that the two wr-curves cross to the left of the observed average rate of profits on circulating capital ($\Pi/S_c^* = 0.33$), it is not possible to affirm that there is unambiguous technical progress. However, for a range of values for r there is a clear stylised form of mechanisation, leading to increasing potential

[46]Table 9 in Appendix B reports year-by-year growth rates by final commodity, the standard deviation (σ_x) across years, average ($\overline{x}$) and trend growth rate (g). Volatility is a persistent feature both at the aggregate and disaggregated levels.

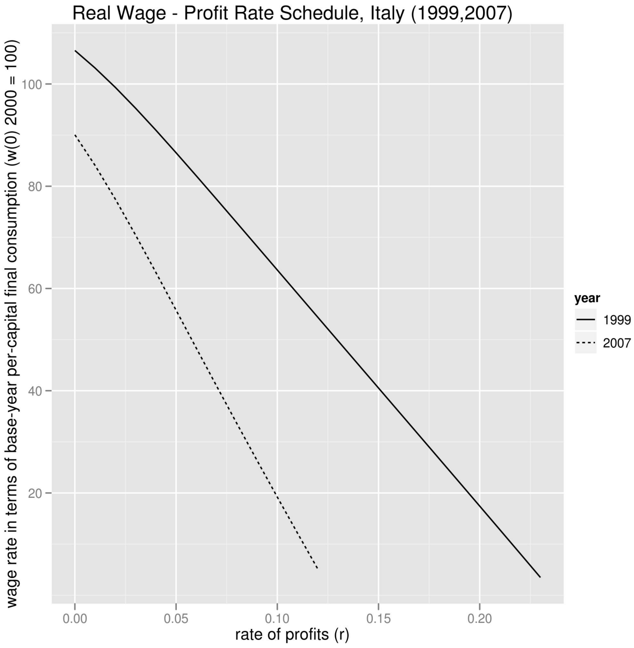

Figure 2: wr-schedule with year-by-year growth rates

Figure 3: wr-schedule using circulating capital inputs

distributional possibilities in the economy. This result is in sharp opposition to the unambiguous technical *regress* emerging from Figure 1, based on the price (index) system including both fixed and circulating capital inputs.

5.3 TFP growth, r-Productivity differences and surplus growth rates by industry

Table 4 reports a comparison between (traditional) industry TFP growth, industry surplus growth rates $\boldsymbol{\rho}_{g,\eta}$ — computed according to (4.54) — and rates of r-Productivity difference by industry $\boldsymbol{\rho}_g(r)$ — computed according to (4.53).[47]

Table 4: TFPG, Industry Surplus Growth and r-Productivity Growth Rates, Italy (1999-2007)
(Average yearly growth rates in percentage points)
In decreasing order according to column (3) $\overline{x}$

Activity	(1)$\Delta\%TFP$		(2)$\rho_{g,\eta}$		(3)$\rho_g(0.05)$		(4)$\rho_g(0.06)$		Correlations		
	$\overline{x}$	σ_x	$\overline{x}$	σ_x	$\overline{x}$	σ_x	$\overline{x}$	σ_x	(1),(2)	(1),(3)	(2),(3)
JJ:Finance	3.26	4.10	1.41	2.24	1.29	2.46	1.26	2.54	0.736	0.643	0.984
DK:Machinery n.e.c.	1.40	2.73	1.19	1.57	1.18	1.64	1.18	1.66	0.895	0.867	0.993
DD:Wood	0.88	4.03	0.94	2.24	0.98	2.28	0.99	2.32	0.880	0.856	0.998
DL:Electr. Machinery	0.58	3.17	1.03	1.15	0.95	1.18	0.92	1.24	0.564	0.236	0.920
MM:Education	-0.46	3.71	0.66	1.18	0.88	1.60	0.96	1.80	0.074	0.141	0.932
II:Transport-Comm.	2.18	2.54	1.22	1.49	0.80	1.43	0.67	1.46	0.942	0.901	0.939
DA:Food-Tobacco	-0.07	4.33	0.74	1.60	0.69	1.62	0.67	1.63	0.789	0.782	0.999
NN:Health	0.18	1.86	0.77	0.55	0.60	0.69	0.53	0.76	-0.385	-0.515	0.955
DG:Chemicals	0.68	2.16	0.49	0.83	0.54	0.98	0.55	1.02	-0.191	-0.173	0.994
DH:Plastics	-0.07	3.57	0.46	1.25	0.43	1.29	0.42	1.31	0.775	0.714	0.990
DC:Leather	0.65	4.88	0.50	2.28	0.40	2.22	0.37	2.23	0.888	0.873	0.996
DI:Non-met. minerals	-0.36	2.46	0.43	1.69	0.38	1.95	0.35	2.07	0.474	0.323	0.983
AA:Agriculture	-0.27	5.06	0.50	2.99	0.33	2.76	0.25	2.77	0.901	0.824	0.976
DE:Paper-Printing	-0.09	2.74	0.38	1.45	0.31	1.86	0.28	1.99	-0.187	-0.262	0.991
DN:Manufacture n.e.c.	0.21	3.57	0.27	1.37	0.30	1.26	0.30	1.26	0.707	0.525	0.966
DB:Textiles	0.28	3.91	0.34	1.72	0.29	1.56	0.27	1.54	0.903	0.857	0.991
DJ:Metals	0.76	0.85	0.08	1.39	0.02	1.53	-0.02	1.61	-0.188	-0.270	0.978
FF:Construction	-1.10	1.61	-0.10	0.74	-0.13	0.98	-0.15	1.08	0.299	0.086	0.930
DM:Transport Equip.	0.43	4.45	-0.11	1.75	-0.16	1.72	-0.18	1.75	0.869	0.886	0.991
GG:Trade	-0.21	2.37	0.01	1.25	-0.24	1.62	-0.32	1.75	0.716	0.563	0.954
CB:Mining non-energy	-0.13	5.62	-0.36	4.53	-0.29	5.07	-0.30	5.27	0.570	0.485	0.993
HH:Hotel-Restaurant	-2.07	4.21	-0.65	2.43	-0.71	2.45	-0.73	2.48	0.946	0.891	0.977
EE:Energy	1.08	3.00	-0.92	4.37	-1.06	4.90	-1.10	5.03	0.401	0.374	0.997
OO:Personal Services	-3.88	4.79	-1.15	3.41	-1.07	3.63	-1.05	3.70	0.958	0.934	0.995
BB:Fishing	-2.72	10.07	-1.12	8.34	-1.24	8.60	-1.29	8.73	0.948	0.939	0.997
KK:Business Services	-1.38	1.90	-1.42	1.15	-1.29	1.47	-1.27	1.55	0.332	0.442	0.921
DF:Coke-Petroleum	-7.98	14.46	-3.50	8.97	-3.78	9.45	-3.82	9.54	0.825	0.809	0.999
CA:Mining energy	-9.12	10.63	-18.62	22.56	-10.34	36.73	-10.75	37.68	0.638	0.515	0.237

Source: Own computation based on Supply-Use Tables (SUT), ISTAT

By inspecting Table 4 it emerges that:

[47] Detailed year-by-year results can be found in Tables 22, 23 and 24 of Appendix B.

1. there is a positive correlation higher than 0.9 (with the only exception of industry CA) between $\rho_{g,\eta}$ and $\rho_g(0.05)$, pointing to the intimate relation between the price (index) systems used as aggregators.

2. the correlation between $\Delta\% TFP$ and $\rho_{g,\eta}$ is higher (but always of the same sign) than that between $\Delta\% TFP$ and $\rho_g(r)$, pointing to a sharp contrast between r-Productivity growth rates based on production prices and traditional TFPG, departing from a traditional value added accounting identity.

3. under all three columns (1)-(3), Finance (JJ) and Mechanical machinery (DK) are among those industries with highest surplus growth, industry DK being only surpassed in columns (1)-(2) by the Transport-Telecomm. industry (II).

4. generally, $\Delta\% TFP$ shows higher standard deviation (column σ_x) across years within industries, pointing to a higher volatility with respect to both $\rho_{g,\eta}$ and $\rho_g(r)$.

5.4 Alternative systems of price indexes

A final exercise, not strictly related to the assessment of technical progress, consist in the comparison of alternative price (index) systems. Table 5 reports summary statistics for:[48]

1. $\boldsymbol{\lambda}^T_{\tilde{p},\tau}$ (Static Input-Output price indexes), computed according to (4.16)

2. $\boldsymbol{\lambda}^T_{p_c,\tau}(r)$ (Production price indexes with only circulating capital inputs), computed according to (4.23)

3. $\boldsymbol{\lambda}^T_{p,\tau}(r)$ (Production price indexes), computed according to (4.41)

4. $\boldsymbol{\lambda}^T_{\eta,\tau}$ (Production price indexes with profits equal to gross investment), computed according to (4.50)

By inspecting Table 5 it emerges that:

[48] Detailed year-by-year results are reported in Tables 12, 13, 14 and 15 of Appendix B.

Table 5: Alternative computable price systems, Italy (1999-2007)
(Price index with respect to statistical basic prices without net taxes)
In increasing order according to column (3) $\overline{x}$

Activity	(1)$\boldsymbol{\lambda}^T_{\tilde{p},\tau}$		(2)$\boldsymbol{\lambda}^T_{p_c,\tau}(r)$		(3)$\boldsymbol{\lambda}^T_{p,\tau}(r)$		(4)$\boldsymbol{\lambda}^T_{\eta,\tau}$		Correlations			
(across years)	$\overline{x}$	σ_x	$\overline{x}$	σ_x	$\overline{x}$	σ_x	$\overline{x}$	σ_x	(1),(2)	(1),(3)	(2),(3)	(3),(4)
JJ:Finance	0.605	0.028	0.465	0.020	0.553	0.027	0.413	0.017	0.964	0.941	0.921	0.935
CA:Mining energy	0.260	0.135	0.131	0.029	0.593	0.446	0.368	0.252	0.883	0.891	0.703	0.913
DF:Coke-Petroleum	0.451	0.096	0.304	0.039	0.679	0.356	0.411	0.190	0.864	0.844	0.584	0.912
EE:Energy	0.765	0.023	0.405	0.019	0.737	0.133	0.459	0.078	0.499	0.009	0.520	0.888
NN:Health	0.817	0.013	0.771	0.015	0.804	0.009	0.681	0.011	0.956	0.645	0.488	0.906
CB:Mining non-energy	0.926	0.030	0.684	0.043	0.815	0.060	0.585	0.043	0.875	0.786	0.800	0.972
KK:Business Services	0.969	0.027	0.386	0.011	0.838	0.019	0.464	0.017	-0.622	0.205	0.219	0.375
DG:Chemicals	0.950	0.027	0.957	0.040	0.853	0.050	0.590	0.039	0.937	0.722	0.746	0.928
GG:Trade	0.877	0.029	0.760	0.022	0.864	0.044	0.642	0.032	0.986	0.938	0.927	0.937
DL:Electr. Machinery	0.925	0.011	0.951	0.014	0.889	0.018	0.665	0.015	0.943	-0.152	-0.393	0.456
DK:Machinery n.e.c.	0.981	0.018	0.992	0.027	0.905	0.022	0.673	0.020	0.860	0.640	0.651	0.772
DE:Paper-Printing	0.940	0.023	0.875	0.017	0.905	0.028	0.649	0.025	0.871	0.699	0.441	0.830
DI:Non-met. minerals	0.929	0.016	0.815	0.016	0.908	0.045	0.647	0.032	0.769	0.609	0.560	0.899
FF:Construction	0.938	0.013	0.937	0.016	0.927	0.016	0.724	0.011	0.869	0.151	-0.261	0.411
II:Transport-Comm.	0.883	0.018	0.731	0.016	0.938	0.038	0.654	0.027	0.905	0.146	-0.226	0.856
MM:Education	0.965	0.012	0.842	0.016	0.939	0.028	0.847	0.016	0.904	0.886	0.834	0.897
DH:Plastics	1.015	0.030	0.984	0.036	0.943	0.038	0.673	0.028	0.953	0.816	0.737	0.892
DJ:Metals	1.011	0.016	0.977	0.033	0.957	0.023	0.707	0.017	0.526	0.109	0.284	0.661
OO:Personal Services	1.007	0.036	0.821	0.031	0.999	0.039	0.759	0.040	0.990	0.887	0.851	0.905
DM:Transport Equip.	1.081	0.028	1.156	0.043	1.013	0.041	0.721	0.033	0.946	0.832	0.834	0.892
DN:Manufacture n.e.c.	1.063	0.019	1.092	0.027	1.015	0.021	0.745	0.021	0.961	0.374	0.261	0.764
HH:Hotel-Restaurant	1.054	0.033	0.996	0.024	1.034	0.047	0.806	0.038	0.936	0.888	0.719	0.977
DC:Leather	1.058	0.030	1.229	0.045	1.034	0.024	0.739	0.021	0.875	0.816	0.723	0.846
DB:Textiles	1.115	0.033	1.131	0.032	1.050	0.034	0.776	0.031	0.769	0.868	0.501	0.909
DD:Wood	1.161	0.038	1.108	0.043	1.084	0.039	0.820	0.034	0.948	0.876	0.767	0.951
DA:Food-Tobacco	1.194	0.029	1.167	0.017	1.105	0.048	0.835	0.038	0.913	0.938	0.808	0.961
LL:Public Admin.	1.316	0.072	0.694	0.043	1.125	0.062	0.760	0.035	0.990	0.952	0.950	0.937
BB:Fishing	1.160	0.050	1.018	0.047	1.151	0.046	0.988	0.043	0.995	0.744	0.737	0.934
AA:Agriculture	1.561	0.069	1.164	0.041	1.387	0.083	1.138	0.063	0.988	0.959	0.932	0.982
PP:Household Services	1.836	0.066	1.836	0.066	1.836	0.066	1.836	0.066	1.000	1.000	1.000	1.000
Year (across activities)												
1999	0.984	0.281	0.874	0.325	0.907	0.271	0.699	0.277	0.817	0.979	0.882	0.959
2000	0.969	0.307	0.853	0.330	0.916	0.277	0.684	0.285	0.833	0.978	0.890	0.958
2001	0.975	0.306	0.861	0.326	0.915	0.287	0.688	0.284	0.839	0.979	0.889	0.948
2002	0.983	0.296	0.875	0.326	0.946	0.271	0.704	0.282	0.838	0.978	0.884	0.955
2003	0.998	0.281	0.887	0.333	0.948	0.245	0.717	0.267	0.847	0.965	0.866	0.958
2004	1.003	0.282	0.896	0.335	0.975	0.228	0.735	0.261	0.847	0.893	0.782	0.959
2005	1.012	0.285	0.896	0.339	0.985	0.233	0.753	0.264	0.859	0.900	0.787	0.963
2006	1.008	0.287	0.888	0.340	1.062	0.242	0.770	0.267	0.865	0.583	0.465	0.812
2007	1.013	0.277	0.885	0.336	1.009	0.228	0.783	0.257	0.847	0.825	0.681	0.962

Columns (2) and (3) are evaluated at their respective actual average r of the corresponding year. See Table 1 for details.
Source: Own computation based on Supply-Use Tables (SUT), ISTAT

1. there is high (greater than 0.8) and positive correlation between columns (3)-(4) for almost all commodities, exceptions being the Machinery complex (composed of industries DJ, DK, DL), as well as Business and Personal Services (industries KK and OO).

2. for price (index) systems (1)-(3) there is a clear pattern of under- and over- valuation by the market. Agri-food complex (industries AA, BB, DA), Wood (industry DD) and the Dressing complex (industries DB, DC), together with Public Administration and Household Services (industries LL and PP), are among those activities mostly under-valued by the market. On the contrary, the Mining-Energy complex (industries CA, CB, DF, EE), Finance (industry JJ), Health and Business Services (industries HH and KK) are among those activities mostly over-valued by the market. Wage and profit rate differentials may help to explain this phenomenon.[49]

3. there is a high degree of stability in the deviation from statistical prices across activities within years, as can be read from the lower section of the Table.

6 Concluding remarks

Starting from an abstract theoretical description of Sraffa's (1960) system of production, it has been shown that, for the simple case of constant efficiency, it is possible to give a concrete empirical treatment to fixed capital as a joint product, setting up computable systems of production price indexes which reflect technical conditions of reproduction and a given rule of distribution of the surplus.

The importance of correctly accounting for replacements in Dynamic Input-Output models has been highlighted. If fixed capital is infinitely durable there is no need to define replacements, all fixed capital formation is new investment. When this is not the case, the advantage of working with gross investment matrices rather than with given endowments of fixed capital

[49]Tables 17 and 18 of Appendix B report wage and profit rate differentials with respect to average market rates.

stocks is that in a stock matrix there are fixed assets of different ages; thus not being able to identify the different vintages contained in it. The investment matrix consists of a single vintage of fixed capital goods, that can be transformed into the total quantity of new machines required to reproduce gross outputs, by assuming a growth path and correct replacements.

As a consequence, in computable systems of production prices dealing with fixed capital flows there is a need to perform an artificial separation between growth and technical change. This is due to the fact that growth is implicit in the observed matrices, and pricing is specified on the basis of technical coefficients. Once this fact is acknowledged, alternative value bases are sketched and utilised to asses (a) potential changes in actual distributional possibilities due to technical change (through the measurement of shifts in wr-curves), and (b) the surplus generating capacity of each industry by using alternative price systems as aggregators of physical inputs (including labour) and outputs.

Appendices

A Dataset Methodology for Italy

Computations have been performed by using two sources of data: Supply-Use and Input-Output Tables (SUIOT) database, February 2011 Edition, together with the Annual National Accounts by Industry (1970-2009), August 2010 Edition, both published by the Italian National Institute of Statistics (ISTAT).[50]

The SUIOT Database contains commodity × activity Supply and Use tables at basic and purchasers' prices, for total (domestically produced plus imported) as well as imported transactions, at current prices for the years 1995-2007.

Moreover, it also renders available a series of Tables at past-year-prices for the period 2000-2007. As the aim of the analysis is that of separating price from volume *growth*, data valued at past-year-prices has been essential, restricting the coverage of the exercise to the 1999-2007 period (as the 2000 Table is valued at 1999 prices).

Supply-Use Tables are presented in two formats: 59 × 59 and 30 × 30. Given that vectors for total gross fixed capital formation by industry of destination are available only at the 30 industry level, the computations have been performed under this aggregation scheme. However, 59 × 1 vectors of gross fixed capital formation have been used to estimate capital flow matrices (see below). Table 6 reports the 30 commodity × industry classification used for the computations.

Matrices of Gross Fixed Capital Formation

ISTAT renders available data on total (domestically produced *plus* imported) gross fixed capital formation by industry of destination (at a 30 activity level) for 8 categories of fixed capital inputs, valued at purchasers' prices. Thus, we have a 8 (commodity) × 30 (activity) matrix that needs to be expanded to a 30 (commodity) × 30 (activity) matrix.

[50] Datasets can be obtained from: `http://www.istat.it/`

Table 6: CPA/NACE 30 × 30 Commodity/Industry Classification

CPA/NACE	Short Desc.	Description
AA	Agriculture	Agriculture, hunting and forestry
BB	Fishing	Fishing
CA	Mining energy	Mining and quarrying of energy producing materials
CB	Mining non-energy	Mining and quarrying except energy producing materials
DA	Food-Tobacco	Manufacture of food products; beverages and tobacco
DB	Textiles	Manufacture of textiles and textile products
DC	Leather	Manufacture of leather and leather products
DD	Wood	Manufacture of wood and wood products
DE	Paper-Printing	Manufacture of pulp, paper and paper products; publishing and printing
DF	Coke-Petroleum	Manufacture of coke, refined petroleum products and nuclear fuel
DG	Chemicals	Manufacture of chemicals, chemical products and man-made fibres
DH	Plastics	Manufacture of rubber and plastic products
DI	Non-met. minerals	Manufacture of other non-metallic mineral products
DJ	Metals	Manufacture of basic metals and fabricated metal products
DK	Machinery n.e.c.	Manufacture of machinery and equipment n.e.c.
DL	Electr. Machinery	Manufacture of electrical and optical equipment
DM	Transport Equip.	Manufacture of transport equipment
DN	Manufacture n.e.c.	Manufacturing n.e.c.
EE	Energy	Electricity, gas and water supply
FF	Construction	Construction
GG	Trade	Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods
HH	Hotel-Restaurant	Hotels and restaurants
II	Transport-Comm.	Transport, storage and communication
JJ	Finance	Financial intermediation
KK	Business Services	Real estate, renting and business activities
LL	Public Admin.	Public administration and defence; compulsory social security
MM	Education	Education
NN	Health	Health and social work
OO	Personal Services	Other community, social, personal service activities
PP	Household Services	Activities of households

Source: Own elaboration based on ISTAT

Three steps are needed to obtain a matrix $\mathbf{F}_k$ of gross fixed capital formation domestically produced valued at basic prices: (a) expand the 8 × 30 matrix into a 30 × 30 matrix, (b) separate trade and transport margins as well as net taxes on products to transform it into a matrix at basic prices, and (c) separate domestically produced from imported commodities.

As regards step (a), it should be borne in mind that not all 30 commodity categories contain products demanded as fixed capital inputs. In fact, Table 7 reports the correspondence between each of the 8 fixed asset categories and the products of the 30 and 59 CPA commodity classification.

From Table 7 it emerges that 5 out of 8 categories (K2, K3, K5, K6, K7) have a direct one-to-one correspondence with a commodity group either at the 30 or 59 level of CPA. Therefore, the disaggregation of column vectors of fixed capital formation by industry of origin into different sources of demand amounts to separating categories K1+K4 (Machinery and Equipment + Furniture) and K8 (Other services and non-tangible assets).

As to category K8, given the absence of any complementary data, gross investment for products 50, 70, 74 and 92 has been distributed between demanding industries in proportion to their category total, i.e. the 1×30 vector of investment demand by industry for category K8 has been proportionally divided into 4 vectors whose row sum coincides with the entries in $\mathbf{f}_k^*$ corresponding to commodities 50, 70, 74 and 92.

In contradistinction, and given its crucial weight in total gross fixed capital formation, row vector 1×30 of investment demand by activity for category K1+K4 has been distributed among its constituent products more carefully.

In 1995, OECD has published its first Input-Output database, with 1985 data for Italy. Among the tables provided, the database included a square 35 × 35 matrix of gross fixed capital formation, valued at current producers' prices (in Billions of lire). This Table has been used to obtain a matrix of distribution quotas by industry of destination for each commodity of category K1+K4 (see Table 7), i.e. a 12 (commodity) × 35 (industry) matrix. This has mainly been done to see which industries demand each capital good. In this way, biproportional methods could be later applied to update this structure to more recent years.[51]

[51] Note that using a matrix of distribution quotas implies reducing as much as possible

With the matrix of distribution quotas for all capital inputs of category K1+K4 at the 59 CPA level (i.e. 12 commodity groups), together with the row vectors of gross investment by demanding activity for those commodities of categories K2, K3, K5, K6, K7, K8 of Table 7, and the 59 $\times$ 1 vector of fixed capital formation by commodity of the Use Table, a matrix balancing procedure has been applied to obtain full 59 $\times$ 30 matrices of total (domestic plus imported) fixed capital formation at purchasers' prices for current as well as past-year-prices for each year.

The second step consisted in separating trade and transport margins as well as net taxes on products to obtain full matrices at basic prices. This has been done by computing the difference between the column vector of gross fixed capital formation of each Use Table at basic and purchasers' prices ($\Delta\mathbf{f}^*_{k(p-b)} = \mathbf{f}^*_{k(p)} - \mathbf{f}^*_{k(b)}$, where (p) stands for purchasers' prices and (b) for basic prices), rescaling the row distribution of each matrix at purchasers' prices to the column total valued at basic prices, and distributing total trade and transport margins (which consist in the negative entries of $\Delta\mathbf{f}^*_{k(p-b)}$ that occur for commodities CPA 50-52 and 60-62) among industries, according to the row distribution of total investment by industry of destination.

The third step consisted in separating domestically produced from imported gross investment demand at basic prices. Given the absence of imported row vectors of gross investment by industry of destination, the proportions between domestically produced and imported gross investment demand by commodity at the 59 CPA level have been used. This amounts to assuming that the *proportion* of imported to domestic demand by all industries, for *each* commodity, coincides. In matrix terms: $\widehat{\boldsymbol{\theta}} = \widehat{\mathbf{f}}(\widehat{\mathbf{f}}^*_k)^{-1}$ (where $\mathbf{f}_k$ and $\mathbf{f}^*_k$ are column vectors of domestic and total gross fixed capital formation, respectively), and then, $\mathbf{F}_k = \widehat{\boldsymbol{\theta}}\mathbf{F}^*_k$.

Finally, the 59$\times$30 matrices have been aggregated to a square 30 $\times$ 30 format.

the influence of prices in the proportions obtained, as it is rows that sum to one, instead of columns.

Table 7: Fixed Capital Inputs: Correspondence with CPA

ISTAT Category	CPA (30 comm.)	CPA (59 comm.)
K1:Machinery and Equipment	AA:Agriculture	01:Products of agriculture, hunting and related services
	BB:Fishing	05:Fish and other fishing products
	DB:Textiles	17:Textiles
	DC:Leather	19:Leather and leather products
	DD:Wood	20:Wood and products of wood and cork (except furniture)
	DE:Paper-Printing	22:Printed matter and recorded media
	DH:Plastics	25:Rubber and plastic products
	DI:Non-met. minerals	26:Other non-metallic mineral products
	DJ:Metals	27:Basic metals 28:Fabricated metal products, except machinery and equipment
	DK:Machinery n.e.c.	29:Machinery and equipment n.e.c.
	DL:Electr. Machinery	31:Electrical machinery and apparatus n.e.c. 33:Medical, precision and optical instruments, watches and clocks
	DN:Manufacture n.e.c.	36:Furniture; Other manufactured goods n.e.c.
K2:Office Machinery	DL:Electr. Machinery	30:Office machinery and computers
K3:Communication Equipment	DL:Electr. Machinery	32:Radio, television and communication equipment and apparatus
K4:Furniture	DN:Manufacture n.e.c.	36:Furniture; Other manufactured goods n.e.c.
K5:Transport Equipment	DM:Transport Equip.	34:Motor vehicles, trailers and semi-trailers 35:Other transport equipment
K6:Construction	FF:Construction	45:Construction work
K7:Software	KK:Business Services	72:Computer and related services
K8:Other services and non-tangible assets	GG:Trade	50:Trade, maintenance and repair services of motor vehicles
	KK:Business Services	70:Real estate services 74:Other business services
	OO:Personal Services	92:Recreational, cultural and sporting services

Source: Own elaboration based on ISTAT National Accounts Data.

Length of Life of Capital Inputs

The length of life of fixed capital inputs plays an important role in the computations. However, the 30 commodity × activity aggregation scheme — with only 16 commodity categories involving durable means of production — implies that in some cases a choice has to be made as to the technical durability of the aggregate, even if its constituent products have differing length of life. Table 8 reports the average length of life for 16 fixed capital input categories as documented by ISTAT (2006), in correspondence to the aggregation scheme used.

The following choices were made to obtain a unique category-aggregate figure in presence of further available information:

1. Category DL (Manufacture of electrical and optical equipment) includes CPA 30 (Office machinery and computers) and 32 (Radio, television and communication equipment and apparatus) with an average technical life of 7 years, as well as CPA 31 (Electrical machinery and apparatus n.e.c.) and 33 (Medical, precision and optical instruments, watches and clocks) with an average length of life of 18 years: 18 years has been adopted.

2. Category DM (Manufacture of transport equipment) includes CPA 34 (Motor vehicles, trailers and semi-trailers) with an average technical life of 10 years and CPA 35 (Other transport equipment) with an average length of life of 18 years: 10 years has been adopted.

3. Category DN (Manufacturing n.e.c.) includes CPA 36 (Furniture, sport equipment, toys, etc.) with an average technical life of 12/16 (according to the industry of destination) or 18 years: 12/16 years has been adopted.

4. Category KK (Real estate, renting and business activities) includes CPA 70 (Real estate services) and 74 (Other business services) with an average technical life of 34 years, as well as CPA 72 (Computer and related services) with an average length of life of 5 years: 5 years has been adopted.

Table 8: Length of Life of Fixed Capital Inputs
(Activity and commodity codes correspond to those reported in Table 6)

Destination	Commodity of Origin															
	AA	BB	DB	DC	DD	DE	DH	DI	DJ	DK	DL	DM	DN	FF	KK	OO
AA	18	18	18	18	18	18	18	18	18	18	18	10	16	51	5	34
BB	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
CA	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
CB	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DA	18	18	18	18	18	18	18	18	18	18	18	10	16	36	5	34
DB	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DC	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DD	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DE	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DF	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DG	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DH	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DI	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DJ	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DK	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DL	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DM	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
DN	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
EE	18	18	18	18	18	18	18	18	18	18	18	10	16	40	5	34
FF	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
GG	18	18	18	18	18	18	18	18	18	18	18	10	12	65	5	34
HH	18	18	18	18	18	18	18	18	18	18	18	10	12	65	5	34
II	18	18	18	18	18	18	18	18	18	18	18	10	16	50	5	34
JJ	18	18	18	18	18	18	18	18	18	18	18	10	16	65	5	34
KK	18	18	18	18	18	18	18	18	18	18	18	10	16	79	5	34
LL	18	18	18	18	18	18	18	18	18	18	18	10	16	60	5	34
MM	18	18	18	18	18	18	18	18	18	18	18	10	16	57	5	34
NN	18	18	18	18	18	18	18	18	18	18	18	10	16	35	5	34
OO	18	18	18	18	18	18	18	18	18	18	18	10	16	56	5	34

Source: Own elaboration based on ISTAT National Accounts Data.

Labour inputs

Given data availability from ISTAT, the measure of labour inputs used throughout the paper has been: 'units of full time labour equivalent for total employment, measured in thousand of man-years'. This concept quantifies the volume of homogeneous employment by activity, correcting part-time labour by means of industry coefficients which transform part-time jobs into full-time equivalent units, on the basis of the ratio between the hours effectively worked in a part-time position and the corresponding hours in a full-time position, *within* the same activity.

B Statistical Companion

Table 9: Growth Rates of Demand for Final Uses, Italy
(Growth rates in percetange points)
In decreasing order according to column g

Activity	99-00	00-01	01-02	02-03	03-04	04-05	05-06	06-07	07-08	σ_x	$\overline{x}$	g
CA:Mining energy	18.98	-28.70	23.98	-62.90	156.00	-68.72	-11.43	419.59	-17.88	154.45	47.66	33.36
DJ:Metals	12.46	3.87	-0.21	3.34	12.13	3.18	12.66	4.93	-2.84	5.69	5.50	6.11
DK:Machinery n.e.c.	8.11	4.40	-2.50	1.67	5.33	-0.46	8.53	7.11	-3.19	4.51	3.22	3.25
DG:Chemicals	10.78	6.77	5.32	-1.17	-0.81	4.57	2.70	-0.56	-4.18	4.71	2.60	2.94
II:Transport-Comm.	6.28	-0.21	1.74	3.95	4.43	2.03	2.46	2.59	-4.25	3.01	2.12	2.82
PP:Household Services	1.77	4.39	1.82	-0.22	4.22	3.68	3.36	3.95	1.69	1.55	2.74	2.78
NN:Health	2.87	3.09	3.24	2.06	3.64	3.25	1.03	1.01	0.82	1.12	2.34	2.67
JJ:Finance	5.14	6.45	-4.75	-3.86	0.46	8.03	7.12	9.05	3.09	5.09	3.41	2.45
DH:Plastics	8.38	1.23	2.59	0.55	5.59	-0.52	3.17	0.26	-6.85	4.25	1.60	2.42
DA:Food-Tobacco	3.63	-1.04	3.87	-0.05	2.35	4.20	2.01	1.96	-4.32	2.77	1.40	2.08
EE:Energy	-2.68	1.52	4.59	4.22	6.14	2.63	-3.60	-5.20	8.73	4.74	1.82	2.04
CB:Mining non-energy	11.81	-0.95	3.55	-7.23	8.86	1.93	1.48	-2.28	-4.91	6.15	1.36	1.79
DL:Electr. Machinery	12.84	4.86	-8.49	-1.31	3.70	3.71	4.71	1.52	-2.32	5.90	2.14	1.76
DE:Paper-Printing	5.08	1.95	-1.49	3.01	-1.36	2.88	1.78	2.31	-1.73	2.38	1.38	1.41
OO:Personal Services	2.45	1.69	0.34	-0.38	5.19	-1.56	2.75	1.21	1.28	1.95	1.44	1.38
KK:Business Services	2.35	2.09	0.80	0.91	0.96	2.51	0.72	0.92	0.21	0.82	1.27	1.35
LL:Public Admin.	1.44	2.13	1.93	1.87	1.20	1.05	-0.21	0.52	-0.02	0.85	1.10	1.31
HH:Hotel-Restaurant	8.39	2.44	-1.68	-0.73	1.05	0.69	3.02	2.20	-0.21	2.95	1.69	1.25
GG:Trade	2.91	-0.05	-1.69	-0.83	1.49	1.54	4.26	1.09	-0.28	1.87	0.94	0.85
AA:Agriculture	2.52	3.17	-1.82	-9.06	9.41	4.04	-2.26	-0.53	-2.53	5.21	0.33	0.63
MM:Education	-0.09	1.83	1.60	1.76	-1.52	0.36	-0.13	0.77	0.12	1.10	0.52	0.58
DM:Transport Equip.	10.92	-13.12	2.09	-6.07	3.53	-2.78	8.80	10.65	-8.70	8.78	0.59	0.41
DI:Non-met. minerals	6.78	0.56	-1.20	-5.26	0.64	-4.18	0.56	2.07	-8.89	4.58	-0.99	-0.85
FF:Construction	2.09	0.65	-3.34	-1.03	-4.20	-2.26	2.74	0.22	-1.31	2.36	-0.71	-1.16
DB:Textiles	5.92	3.98	-4.01	-2.85	-2.88	-4.02	-0.28	-0.98	-2.66	3.55	-0.87	-1.40
DF:Coke-Petroleum	-4.27	5.77	-6.00	6.64	-6.04	0.82	-13.67	5.91	-2.85	6.87	-1.52	-1.49
DD:Wood	9.48	7.77	-7.26	-7.60	1.13	-8.13	2.18	1.45	-7.29	6.89	-0.92	-1.50
BB:Fishing	13.24	-21.35	-2.11	2.57	0.63	-3.90	2.57	-3.00	-12.25	9.76	-2.62	-1.88
DN:Manufacture n.e.c.	6.38	-3.64	-2.49	-6.22	-0.69	-4.35	0.99	-1.08	-7.17	4.10	-2.03	-2.13
DC:Leather	8.71	1.47	-9.54	-5.98	-1.41	-6.23	1.04	-1.53	-8.69	5.81	-2.46	-2.83
Aggregate	4.55	1.60	-0.14	0.09	1.87	1.38	2.38	1.95	-1.33	1.70	1.37	1.43

Source: Own computation based on Supply-Use Tables (SUT), ISTAT

Table 10: Activity Level Indexes, Italy - $\boldsymbol{\lambda}_x$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	0.985	0.999	1.001	1.009	1.005	0.976	1.009	1.000	0.990	0.997
BB:Fishing	0.986	0.987	0.985	0.983	0.983	0.992	0.990	0.976	0.976	0.984
CA:Mining energy	1.068	0.707	1.118	0.813	1.039	1.012	1.046	0.630	1.132	0.952
CB:Mining non-energy	0.989	1.017	1.022	1.022	1.007	1.009	1.011	1.015	1.013	1.012
DA:Food-Tobacco	0.992	1.004	1.003	1.013	0.999	0.994	1.009	1.006	0.994	1.002
DB:Textiles	1.015	0.978	0.978	0.996	1.002	0.999	1.000	0.996	0.983	0.994
DC:Leather	1.005	1.023	0.988	1.001	0.989	1.002	0.990	0.984	0.973	0.995
DD:Wood	0.977	0.992	0.970	0.975	0.965	0.955	0.982	0.968	0.974	0.973
DE:Paper-Printing	0.998	1.002	1.007	0.997	1.001	1.012	1.011	0.992	0.996	1.002
DF:Coke-Petroleum	1.012	0.993	1.029	0.982	1.022	0.992	1.015	0.990	0.995	1.003
DG:Chemicals	0.999	1.011	1.012	0.996	0.997	1.007	1.002	1.010	0.993	1.003
DH:Plastics	0.995	0.999	0.996	0.995	0.996	1.001	0.999	0.991	0.989	0.996
DI:Non-met. minerals	0.991	0.998	0.985	0.995	0.994	0.991	1.007	1.015	1.000	0.997
DJ:Metals	0.989	0.997	0.991	0.992	0.984	1.005	0.997	0.971	0.973	0.989
DK:Machinery n.e.c.	0.994	0.999	1.011	1.002	0.994	0.994	1.002	0.994	0.991	0.998
DL:Electr. Machinery	0.992	1.007	1.005	1.012	0.996	0.995	1.014	0.995	0.991	1.001
DM:Transport Equip.	0.971	1.009	0.964	0.999	0.988	0.994	1.014	0.980	1.000	0.991
DN:Manufacture n.e.c.	0.953	0.979	0.962	0.971	0.957	0.965	0.970	0.966	0.966	0.965
EE:Energy	0.997	0.999	0.998	0.999	0.998	0.998	1.002	0.998	0.996	0.998
FF:Construction	0.997	1.000	0.998	1.000	0.998	0.998	1.001	0.998	0.996	0.998
GG:Trade	0.998	1.000	0.999	1.000	0.998	0.999	1.000	0.998	0.997	0.999
HH:Hotel-Restaurant	0.999	1.000	1.000	1.000	0.999	1.000	1.000	0.999	0.999	1.000
II:Transport-Comm.	0.995	0.997	0.997	0.998	0.996	0.997	0.999	0.996	0.995	0.997
JJ:Finance	0.997	0.999	0.999	0.999	0.998	0.999	1.000	0.998	0.997	0.998
KK:Business Services	0.998	1.000	0.998	1.000	0.998	0.999	1.000	0.998	0.997	0.999
LL:Public Admin.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
MM:Education	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
NN:Health	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
OO:Personal Services	0.998	0.999	0.999	0.999	0.999	0.999	1.000	0.998	0.998	0.999
PP:Household Services	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 11: Statistical Basic Price Indexes without Net Taxes, Italy - $\boldsymbol{\lambda}_\tau^T$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	0.951	0.954	0.947	0.951	0.941	0.939	0.956	0.970	0.960	0.952
BB:Fishing	0.880	0.886	0.861	0.869	0.871	0.868	0.859	0.865	0.863	0.869
CA:Mining energy	0.919	0.938	0.903	0.868	0.906	0.923	0.916	0.900	0.889	0.907
CB:Mining non-energy	0.818	0.819	0.824	0.820	0.812	0.809	0.808	0.809	0.808	0.814
DA:Food-Tobacco	0.880	0.885	0.884	0.881	0.878	0.878	0.873	0.868	0.865	0.877
DB:Textiles	0.811	0.818	0.818	0.812	0.809	0.806	0.802	0.799	0.800	0.808
DC:Leather	0.823	0.829	0.837	0.831	0.829	0.830	0.827	0.824	0.826	0.828
DD:Wood	0.828	0.832	0.841	0.828	0.822	0.827	0.818	0.820	0.818	0.826
DE:Paper-Printing	0.808	0.815	0.817	0.818	0.813	0.813	0.807	0.805	0.807	0.812
DF:Coke-Petroleum	0.832	0.877	0.849	0.817	0.851	0.888	0.885	0.873	0.871	0.860
DG:Chemicals	0.790	0.798	0.794	0.797	0.787	0.794	0.784	0.778	0.782	0.789
DH:Plastics	0.801	0.804	0.801	0.804	0.796	0.797	0.791	0.788	0.793	0.797
DI:Non-met. minerals	0.787	0.797	0.802	0.805	0.797	0.802	0.792	0.789	0.792	0.796
DJ:Metals	0.799	0.805	0.804	0.802	0.801	0.803	0.797	0.799	0.803	0.801
DK:Machinery n.e.c.	0.791	0.802	0.799	0.794	0.793	0.795	0.789	0.791	0.795	0.794
DL:Electr. Machinery	0.797	0.807	0.807	0.803	0.800	0.803	0.796	0.797	0.798	0.801
DM:Transport Equip.	0.790	0.800	0.792	0.792	0.791	0.787	0.778	0.783	0.784	0.789
DN:Manufacture n.e.c.	0.817	0.824	0.823	0.819	0.815	0.819	0.812	0.813	0.815	0.817
EE:Energy	0.801	0.822	0.826	0.820	0.828	0.837	0.832	0.834	0.831	0.826
FF:Construction	0.819	0.827	0.832	0.829	0.825	0.830	0.825	0.822	0.822	0.826
GG:Trade	0.856	0.861	0.867	0.859	0.858	0.857	0.851	0.846	0.844	0.856
HH:Hotel-Restaurant	0.895	0.898	0.895	0.889	0.884	0.885	0.881	0.877	0.874	0.886
II:Transport-Comm.	0.802	0.818	0.830	0.833	0.832	0.836	0.830	0.825	0.827	0.826
JJ:Finance	0.737	0.756	0.755	0.768	0.775	0.778	0.775	0.756	0.773	0.764
KK:Business Services	0.894	0.896	0.894	0.891	0.894	0.894	0.895	0.895	0.894	0.894
LL:Public Admin.	0.718	0.722	0.726	0.727	0.725	0.730	0.734	0.732	0.734	0.728
MM:Education	0.699	0.703	0.712	0.710	0.717	0.722	0.719	0.720	0.721	0.714
NN:Health	0.769	0.772	0.773	0.771	0.774	0.777	0.775	0.768	0.770	0.772
OO:Personal Services	0.846	0.845	0.846	0.841	0.842	0.837	0.833	0.829	0.830	0.839
PP:Household Services	0.931	0.938	0.947	0.910	0.895	0.902	0.902	0.902	0.901	0.914

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 12: Static Input-Output Price Indexes, Italy - $\boldsymbol{\lambda}_{\tilde{p},\tau}^{T}$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	1.444	1.497	1.550	1.551	1.539	1.546	1.635	1.647	1.645	1.561
BB:Fishing	1.218	1.159	1.229	1.158	1.067	1.141	1.152	1.119	1.194	1.160
CA:Mining energy	0.197	0.068	0.108	0.169	0.283	0.290	0.372	0.389	0.466	0.260
CB:Mining non-energy	0.887	0.926	0.893	0.895	0.935	0.981	0.946	0.931	0.942	0.926
DA:Food-Tobacco	1.163	1.167	1.189	1.170	1.178	1.188	1.225	1.238	1.231	1.194
DB:Textiles	1.090	1.068	1.073	1.099	1.135	1.149	1.151	1.140	1.132	1.115
DC:Leather	1.114	1.093	1.022	1.041	1.064	1.055	1.058	1.044	1.028	1.058
DD:Wood	1.160	1.147	1.071	1.145	1.190	1.182	1.198	1.183	1.171	1.161
DE:Paper-Printing	0.921	0.921	0.924	0.904	0.946	0.950	0.963	0.968	0.967	0.940
DF:Coke-Petroleum	0.424	0.289	0.337	0.400	0.484	0.494	0.523	0.528	0.580	0.451
DG:Chemicals	0.903	0.909	0.947	0.944	0.964	0.963	0.977	0.970	0.970	0.950
DH:Plastics	0.962	0.980	1.011	0.994	1.037	1.044	1.041	1.038	1.031	1.015
DI:Non-met. minerals	0.929	0.918	0.915	0.898	0.932	0.935	0.941	0.940	0.951	0.929
DJ:Metals	0.997	0.995	1.011	1.024	1.025	1.030	1.026	1.007	0.988	1.011
DK:Machinery n.e.c.	0.973	0.952	0.972	0.995	1.004	0.996	0.996	0.977	0.963	0.981
DL:Electr. Machinery	0.931	0.913	0.914	0.929	0.949	0.927	0.926	0.920	0.917	0.925
DM:Transport Equip.	1.044	1.034	1.077	1.098	1.101	1.105	1.119	1.081	1.071	1.081
DN:Manufacture n.e.c.	1.052	1.038	1.042	1.058	1.088	1.085	1.085	1.064	1.053	1.063
EE:Energy	0.795	0.737	0.731	0.765	0.784	0.784	0.773	0.739	0.776	0.765
FF:Construction	0.963	0.952	0.943	0.938	0.938	0.924	0.927	0.926	0.932	0.938
GG:Trade	0.844	0.845	0.845	0.867	0.880	0.884	0.892	0.910	0.925	0.877
HH:Hotel-Restaurant	1.010	1.009	1.019	1.051	1.077	1.082	1.084	1.080	1.075	1.054
II:Transport-Comm.	0.916	0.898	0.879	0.856	0.871	0.866	0.876	0.891	0.896	0.883
JJ:Finance	0.655	0.620	0.610	0.620	0.586	0.595	0.588	0.612	0.554	0.605
KK:Business Services	1.004	0.997	1.006	0.978	0.954	0.958	0.946	0.934	0.949	0.969
LL:Public Admin.	1.402	1.406	1.381	1.356	1.304	1.273	1.243	1.229	1.246	1.316
MM:Education	0.974	0.986	0.974	0.965	0.945	0.970	0.960	0.958	0.953	0.965
NN:Health	0.842	0.814	0.813	0.820	0.828	0.817	0.804	0.797	0.817	0.817
OO:Personal Services	0.946	0.968	0.981	1.003	1.009	1.026	1.032	1.049	1.050	1.007
PP:Household Services	1.759	1.752	1.778	1.809	1.835	1.861	1.891	1.930	1.905	1.836

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 13: Circulating Capital Production Price Indexes, Italy - $\boldsymbol{\lambda}^{T}_{p_c,\tau}(r)$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	1.095	1.123	1.171	1.162	1.142	1.152	1.207	1.217	1.208	1.164
BB:Fishing	1.080	1.016	1.076	1.016	0.929	1.000	1.011	0.987	1.048	1.018
CA:Mining energy	0.123	0.069	0.103	0.139	0.139	0.143	0.154	0.148	0.164	0.131
CB:Mining non-energy	0.625	0.642	0.641	0.665	0.703	0.745	0.736	0.707	0.692	0.684
DA:Food-Tobacco	1.158	1.145	1.173	1.147	1.152	1.165	1.192	1.188	1.179	1.167
DB:Textiles	1.129	1.087	1.088	1.138	1.170	1.170	1.158	1.125	1.111	1.131
DC:Leather	1.303	1.271	1.201	1.236	1.263	1.231	1.211	1.185	1.161	1.229
DD:Wood	1.129	1.099	1.010	1.083	1.150	1.144	1.133	1.114	1.112	1.108
DE:Paper-Printing	0.877	0.868	0.860	0.839	0.883	0.889	0.888	0.889	0.886	0.875
DF:Coke-Petroleum	0.308	0.220	0.262	0.309	0.317	0.346	0.333	0.313	0.325	0.304
DG:Chemicals	0.911	0.893	0.926	0.943	0.974	0.993	0.992	0.993	0.991	0.957
DH:Plastics	0.936	0.933	0.961	0.967	1.011	1.032	1.013	1.008	0.995	0.984
DI:Non-met. minerals	0.815	0.792	0.797	0.798	0.828	0.833	0.832	0.820	0.820	0.815
DJ:Metals	0.943	0.927	0.946	0.970	0.982	1.017	1.012	1.006	0.989	0.977
DK:Machinery n.e.c.	0.980	0.942	0.966	0.994	1.012	1.026	1.020	1.000	0.987	0.992
DL:Electr. Machinery	0.967	0.933	0.936	0.953	0.974	0.961	0.953	0.944	0.941	0.951
DM:Transport Equip.	1.100	1.079	1.134	1.171	1.189	1.199	1.200	1.165	1.171	1.156
DN:Manufacture n.e.c.	1.090	1.060	1.053	1.081	1.122	1.129	1.119	1.090	1.084	1.092
EE:Energy	0.384	0.371	0.391	0.414	0.426	0.418	0.420	0.402	0.420	0.405
FF:Construction	0.974	0.948	0.932	0.932	0.938	0.934	0.931	0.921	0.921	0.937
GG:Trade	0.737	0.732	0.733	0.755	0.769	0.770	0.771	0.784	0.793	0.760
HH:Hotel-Restaurant	0.969	0.962	0.970	0.998	1.020	1.024	1.022	1.004	0.995	0.996
II:Transport-Comm.	0.765	0.745	0.731	0.712	0.726	0.715	0.724	0.730	0.727	0.731
JJ:Finance	0.503	0.468	0.464	0.471	0.448	0.460	0.458	0.480	0.430	0.465
KK:Business Services	0.367	0.366	0.392	0.394	0.385	0.391	0.391	0.393	0.395	0.386
LL:Public Admin.	0.751	0.741	0.730	0.718	0.694	0.668	0.656	0.645	0.641	0.694
MM:Education	0.864	0.867	0.843	0.832	0.817	0.849	0.834	0.842	0.832	0.842
NN:Health	0.799	0.763	0.764	0.777	0.782	0.779	0.762	0.749	0.765	0.771
OO:Personal Services	0.776	0.780	0.796	0.820	0.826	0.840	0.841	0.857	0.855	0.821
PP:Household Services	1.759	1.752	1.778	1.809	1.835	1.861	1.891	1.930	1.905	1.836

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 14: Production Price Indexes, Italy - $\boldsymbol{\lambda}_{p,\tau}^{T}(r)$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	1.251	1.314	1.338	1.369	1.362	1.408	1.468	1.514	1.457	1.387
BB:Fishing	1.160	1.104	1.206	1.199	1.065	1.151	1.135	1.153	1.189	1.151
CA:Mining energy	0.190	0.185	0.154	0.280	0.479	0.767	0.827	1.402	1.057	0.593
CB:Mining non-energy	0.777	0.788	0.723	0.775	0.772	0.884	0.877	0.879	0.862	0.815
DA:Food-Tobacco	1.056	1.065	1.076	1.078	1.075	1.112	1.135	1.201	1.148	1.105
DB:Textiles	1.006	1.006	1.015	1.050	1.063	1.062	1.069	1.103	1.072	1.050
DC:Leather	1.065	1.062	0.994	1.045	1.034	1.029	1.023	1.048	1.006	1.034
DD:Wood	1.092	1.078	0.994	1.086	1.099	1.072	1.107	1.137	1.089	1.084
DE:Paper-Printing	0.867	0.916	0.893	0.882	0.896	0.891	0.909	0.962	0.931	0.905
DF:Coke-Petroleum	0.378	0.339	0.320	0.432	0.594	0.825	0.852	1.333	1.041	0.679
DG:Chemicals	0.802	0.801	0.823	0.846	0.848	0.835	0.863	0.953	0.909	0.853
DH:Plastics	0.878	0.909	0.935	0.936	0.964	0.947	0.948	1.015	0.960	0.943
DI:Non-met. minerals	0.863	0.856	0.863	0.897	0.915	0.919	0.914	0.997	0.946	0.908
DJ:Metals	0.921	0.943	0.961	0.989	0.950	0.951	0.937	0.992	0.965	0.957
DK:Machinery n.e.c.	0.874	0.877	0.889	0.935	0.914	0.912	0.911	0.933	0.900	0.905
DL:Electr. Machinery	0.849	0.894	0.895	0.897	0.899	0.885	0.881	0.913	0.888	0.889
DM:Transport Equip.	0.941	0.959	1.003	1.037	1.009	1.030	1.060	1.061	1.014	1.013
DN:Manufacture n.e.c.	0.988	1.003	0.993	1.030	1.021	1.009	1.024	1.058	1.014	1.015
EE:Energy	0.617	0.602	0.599	0.674	0.736	0.760	0.780	0.994	0.873	0.737
FF:Construction	0.916	0.935	0.935	0.949	0.924	0.900	0.908	0.944	0.928	0.927
GG:Trade	0.797	0.814	0.829	0.877	0.865	0.869	0.882	0.927	0.917	0.864
HH:Hotel-Restaurant	0.981	0.985	0.981	1.013	1.039	1.055	1.067	1.116	1.067	1.034
II:Transport-Comm.	0.905	0.921	0.898	0.908	0.925	0.940	0.950	1.012	0.980	0.938
JJ:Finance	0.587	0.567	0.559	0.578	0.526	0.537	0.534	0.574	0.509	0.553
KK:Business Services	0.817	0.853	0.861	0.854	0.814	0.814	0.829	0.852	0.847	0.838
LL:Public Admin.	1.168	1.194	1.189	1.184	1.122	1.106	1.054	1.065	1.040	1.125
MM:Education	0.966	0.995	0.943	0.950	0.906	0.925	0.918	0.933	0.918	0.939
NN:Health	0.822	0.806	0.801	0.806	0.806	0.793	0.792	0.806	0.807	0.804
OO:Personal Services	0.923	0.967	0.998	1.018	0.986	0.998	1.006	1.055	1.039	0.999
PP:Household Services	1.759	1.752	1.778	1.809	1.835	1.861	1.891	1.930	1.905	1.836

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 15: Production Price Indexes ($\Pi = I$), Italy - $\boldsymbol{\lambda}_{\eta,\tau}^{T}$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	1.049	1.081	1.106	1.117	1.112	1.133	1.202	1.232	1.209	1.138
BB:Fishing	1.013	0.949	1.031	1.005	0.905	0.982	0.985	0.975	1.044	0.988
CA:Mining energy	0.141	0.085	0.140	0.179	0.328	0.497	0.555	0.596	0.795	0.368
CB:Mining non-energy	0.558	0.557	0.522	0.551	0.561	0.632	0.630	0.613	0.638	0.585
DA:Food-Tobacco	0.813	0.801	0.808	0.804	0.809	0.831	0.864	0.899	0.884	0.835
DB:Textiles	0.758	0.732	0.733	0.760	0.787	0.790	0.803	0.809	0.808	0.776
DC:Leather	0.780	0.754	0.700	0.730	0.739	0.736	0.739	0.742	0.731	0.739
DD:Wood	0.827	0.802	0.744	0.812	0.831	0.817	0.851	0.856	0.842	0.820
DE:Paper-Printing	0.634	0.637	0.626	0.618	0.642	0.642	0.665	0.686	0.689	0.649
DF:Coke-Petroleum	0.258	0.187	0.231	0.268	0.383	0.510	0.545	0.587	0.731	0.411
DG:Chemicals	0.560	0.542	0.557	0.572	0.587	0.583	0.613	0.643	0.654	0.590
DH:Plastics	0.637	0.639	0.656	0.653	0.685	0.677	0.689	0.713	0.707	0.673
DI:Non-met. minerals	0.631	0.607	0.610	0.623	0.650	0.651	0.663	0.689	0.698	0.647
DJ:Metals	0.690	0.682	0.694	0.714	0.707	0.709	0.711	0.730	0.730	0.707
DK:Machinery n.e.c.	0.658	0.639	0.648	0.677	0.679	0.679	0.691	0.695	0.688	0.673
DL:Electr. Machinery	0.648	0.653	0.649	0.655	0.675	0.664	0.675	0.684	0.682	0.665
DM:Transport Equip.	0.684	0.673	0.697	0.720	0.714	0.733	0.771	0.754	0.745	0.721
DN:Manufacture n.e.c.	0.731	0.718	0.716	0.740	0.752	0.747	0.767	0.774	0.763	0.745
EE:Energy	0.402	0.364	0.377	0.410	0.460	0.477	0.509	0.531	0.599	0.459
FF:Construction	0.727	0.720	0.718	0.721	0.720	0.707	0.722	0.736	0.744	0.724
GG:Trade	0.611	0.607	0.608	0.632	0.639	0.642	0.660	0.684	0.696	0.642
HH:Hotel-Restaurant	0.768	0.760	0.759	0.787	0.812	0.824	0.841	0.857	0.845	0.806
II:Transport-Comm.	0.660	0.643	0.624	0.623	0.638	0.644	0.664	0.689	0.699	0.654
JJ:Finance	0.445	0.417	0.411	0.421	0.395	0.403	0.405	0.429	0.391	0.413
KK:Business Services	0.445	0.450	0.459	0.459	0.452	0.456	0.473	0.486	0.495	0.464
LL:Public Admin.	0.808	0.801	0.789	0.780	0.751	0.735	0.721	0.725	0.726	0.760
MM:Education	0.866	0.874	0.845	0.843	0.820	0.844	0.838	0.853	0.843	0.847
NN:Health	0.708	0.682	0.677	0.679	0.684	0.671	0.671	0.673	0.686	0.681
OO:Personal Services	0.703	0.719	0.735	0.751	0.746	0.763	0.781	0.815	0.817	0.759
PP:Household Services	1.759	1.752	1.778	1.809	1.835	1.861	1.891	1.930	1.905	1.836

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 16: Relative Standard Prices, Italy - $\boldsymbol{\lambda}_p^T(R)$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	1.032	1.103	1.111	1.077	0.964	0.982	1.002	0.774	0.845	0.988
BB:Fishing	0.712	0.686	0.791	0.773	0.585	0.586	0.526	0.428	0.471	0.618
CA:Mining energy	0.331	0.401	0.262	0.393	0.721	1.205	1.254	1.593	1.377	0.837
CB:Mining non-energy	0.996	0.971	0.876	0.850	0.722	0.826	0.824	0.613	0.695	0.819
DA:Food-Tobacco	1.130	1.139	1.164	1.061	0.937	0.936	0.922	0.730	0.800	0.980
DB:Textiles	0.977	1.003	1.054	0.952	0.813	0.755	0.749	0.594	0.668	0.841
DC:Leather	1.083	1.095	1.067	1.013	0.850	0.803	0.785	0.608	0.677	0.887
DD:Wood	1.127	1.085	1.001	0.959	0.840	0.758	0.778	0.609	0.674	0.870
DE:Paper-Printing	1.043	1.161	1.128	0.978	0.854	0.792	0.788	0.628	0.707	0.898
DF:Coke-Petroleum	0.603	0.638	0.523	0.596	0.848	1.264	1.286	1.538	1.382	0.964
DG:Chemicals	1.119	1.101	1.135	1.024	0.891	0.822	0.829	0.713	0.786	0.936
DH:Plastics	1.102	1.131	1.174	1.047	0.938	0.861	0.838	0.691	0.751	0.948
DI:Non-met. minerals	1.001	0.995	1.035	0.993	0.880	0.845	0.801	0.682	0.734	0.885
DJ:Metals	1.043	1.094	1.129	1.025	0.825	0.788	0.747	0.614	0.713	0.886
DK:Machinery n.e.c.	0.972	0.994	1.012	0.947	0.782	0.741	0.710	0.551	0.628	0.815
DL:Electr. Machinery	0.886	1.004	1.027	0.882	0.743	0.700	0.664	0.522	0.597	0.780
DM:Transport Equip.	1.109	1.155	1.231	1.134	0.954	0.911	0.901	0.678	0.752	0.980
DN:Manufacture n.e.c.	1.046	1.088	1.072	0.989	0.822	0.755	0.755	0.603	0.667	0.866
EE:Energy	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
FF:Construction	0.842	0.887	0.914	0.839	0.683	0.616	0.603	0.479	0.540	0.712
GG:Trade	0.834	0.861	0.933	0.906	0.756	0.720	0.714	0.555	0.634	0.768
HH:Hotel-Restaurant	0.981	0.962	0.957	0.860	0.783	0.755	0.752	0.615	0.658	0.814
II:Transport-Comm.	1.075	1.137	1.153	1.054	0.963	0.945	0.929	0.735	0.825	0.980
JJ:Finance	0.555	0.559	0.557	0.533	0.405	0.392	0.385	0.304	0.311	0.444
KK:Business Services	1.618	1.635	1.649	1.444	1.197	1.107	1.125	0.849	0.981	1.290
LL:Public Admin.	1.305	1.325	1.377	1.241	1.028	0.968	0.891	0.663	0.749	1.061
MM:Education	0.351	0.396	0.333	0.322	0.236	0.210	0.210	0.154	0.176	0.265
NN:Health	0.482	0.480	0.488	0.440	0.386	0.365	0.371	0.289	0.333	0.404
OO:Personal Services	0.979	1.022	1.097	0.982	0.798	0.735	0.713	0.547	0.630	0.834
PP:Household Services	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 17: Money Wage Rate Differentials, Italy - $(W_j/L_j)/(W/L)$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	0.645	0.627	0.608	0.602	0.607	0.592	0.599	0.588	0.594	0.607
BB:Fishing	0.635	0.616	0.597	0.590	0.592	0.577	0.587	0.577	0.583	0.595
CA:Mining energy	1.912	1.963	1.829	1.868	1.839	1.841	1.831	1.807	1.826	1.857
CB:Mining non-energy	0.957	0.954	0.954	0.957	0.952	0.940	0.932	0.934	0.943	0.947
DA:Food-Tobacco	1.005	0.998	0.992	0.989	0.981	0.975	0.972	0.968	0.978	0.984
DB:Textiles	0.767	0.772	0.766	0.774	0.764	0.764	0.768	0.775	0.784	0.770
DC:Leather	0.715	0.728	0.732	0.738	0.735	0.745	0.747	0.753	0.763	0.740
DD:Wood	0.734	0.731	0.730	0.736	0.732	0.736	0.744	0.740	0.752	0.737
DE:Paper-Printing	1.131	1.122	1.115	1.113	1.103	1.111	1.105	1.100	1.119	1.113
DF:Coke-Petroleum	1.568	1.613	1.592	1.559	1.536	1.543	1.534	1.531	1.521	1.555
DG:Chemicals	1.453	1.448	1.435	1.440	1.451	1.452	1.441	1.441	1.466	1.447
DH:Plastics	0.969	0.968	0.956	0.959	0.960	0.964	0.965	0.968	0.981	0.966
DI:Non-met. minerals	0.990	0.987	0.977	0.970	0.967	0.974	0.972	0.976	0.984	0.977
DJ:Metals	0.931	0.932	0.932	0.926	0.921	0.924	0.914	0.916	0.918	0.924
DK:Machinery n.e.c.	1.104	1.103	1.108	1.101	1.099	1.105	1.094	1.097	1.110	1.102
DL:Electr. Machinery	1.098	1.099	1.098	1.097	1.089	1.089	1.076	1.071	1.074	1.088
DM:Transport Equip.	1.122	1.122	1.115	1.115	1.112	1.112	1.088	1.093	1.095	1.108
DN:Manufacture n.e.c.	0.792	0.792	0.791	0.792	0.786	0.789	0.789	0.797	0.809	0.793
EE:Energy	1.444	1.428	1.436	1.453	1.431	1.407	1.387	1.381	1.397	1.418
FF:Construction	0.814	0.808	0.802	0.797	0.792	0.794	0.786	0.782	0.786	0.796
GG:Trade	0.920	0.926	0.926	0.923	0.913	0.912	0.917	0.906	0.907	0.917
HH:Hotel-Restaurant	0.875	0.872	0.868	0.846	0.829	0.835	0.826	0.818	0.822	0.844
II:Transport-Comm.	1.152	1.132	1.117	1.107	1.101	1.091	1.080	1.078	1.077	1.104
JJ:Finance	1.799	1.806	1.786	1.775	1.769	1.750	1.768	1.817	1.875	1.794
KK:Business Services	1.028	1.042	1.051	1.052	1.038	1.029	1.035	1.041	1.045	1.040
LL:Public Admin.	1.052	1.059	1.103	1.132	1.212	1.264	1.285	1.291	1.285	1.187
MM:Education	1.049	1.041	1.053	1.068	1.073	1.023	1.050	1.047	1.058	1.051
NN:Health	1.141	1.181	1.193	1.192	1.168	1.209	1.211	1.242	1.177	1.190
OO:Personal Services	0.860	0.847	0.830	0.831	0.825	0.820	0.822	0.807	0.800	0.827
PP:Household Services	0.568	0.571	0.563	0.553	0.545	0.537	0.529	0.518	0.525	0.545

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 18: Profit Rate Differentials, Italy - $(\Pi_j/S_j^*)/(\Pi/S^*)$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	0.614	0.555	0.517	0.526	0.532	0.545	0.429	0.430	0.419	0.507
BB:Fishing	1.156	1.378	1.209	1.567	1.940	1.691	1.644	1.915	1.545	1.561
CA:Mining energy	5.899	6.047	4.801	4.702	3.868	3.323	2.981	2.725	2.102	4.050
CB:Mining non-energy	1.266	1.079	1.200	1.211	1.083	0.935	1.044	1.077	1.042	1.104
DA:Food-Tobacco	0.933	0.962	0.943	1.012	0.945	0.905	0.828	0.771	0.779	0.898
DB:Textiles	0.936	0.971	0.947	0.838	0.733	0.649	0.631	0.650	0.674	0.781
DC:Leather	0.910	0.959	1.464	1.261	1.173	1.217	1.204	1.308	1.387	1.209
DD:Wood	0.817	0.828	1.090	0.834	0.709	0.729	0.691	0.747	0.740	0.798
DE:Paper-Printing	1.067	1.046	1.048	1.164	0.955	0.921	0.848	0.810	0.827	0.965
DF:Coke-Petroleum	1.117	0.992	0.874	0.572	0.708	0.782	0.977	0.888	1.075	0.887
DG:Chemicals	0.944	0.884	0.793	0.803	0.733	0.712	0.629	0.625	0.631	0.750
DH:Plastics	0.985	0.852	0.764	0.846	0.667	0.642	0.661	0.635	0.663	0.746
DI:Non-met. minerals	1.098	1.085	1.112	1.242	1.087	1.079	1.006	0.971	0.947	1.070
DJ:Metals	0.939	0.904	0.844	0.804	0.815	0.794	0.803	0.879	0.974	0.862
DK:Machinery n.e.c.	0.833	0.936	0.842	0.725	0.696	0.741	0.742	0.821	0.869	0.800
DL:Electr. Machinery	1.018	1.111	1.121	1.041	0.947	1.079	1.106	1.163	1.185	1.086
DM:Transport Equip.	0.535	0.550	0.415	0.350	0.392	0.341	0.276	0.363	0.386	0.401
DN:Manufacture n.e.c.	0.985	1.023	0.907	0.896	0.797	0.817	0.817	0.925	0.953	0.902
EE:Energy	1.025	0.907	0.957	0.993	0.988	1.007	1.008	1.069	1.067	1.002
FF:Construction	1.616	1.658	1.788	1.855	1.967	2.110	2.164	2.213	2.127	1.944
GG:Trade	2.314	2.178	2.168	1.965	1.847	1.822	1.737	1.632	1.518	1.909
HH:Hotel-Restaurant	1.709	1.701	1.665	1.448	1.313	1.268	1.335	1.381	1.383	1.467
II:Transport-Comm.	1.018	1.057	1.184	1.319	1.261	1.286	1.240	1.149	1.137	1.183
JJ:Finance	1.708	2.085	2.214	2.153	2.539	2.479	2.578	2.259	2.909	2.325
KK:Business Services	0.965	0.965	0.945	0.986	1.033	1.029	1.045	1.067	1.036	1.008
LL:Public Admin.	0.324	0.318	0.317	0.323	0.332	0.338	0.355	0.369	0.370	0.338
MM:Education	0.782	0.711	0.782	0.724	0.879	0.970	0.877	0.902	0.885	0.835
NN:Health	1.819	1.947	1.935	1.873	1.952	1.923	2.158	2.102	2.170	1.987
OO:Personal Services	1.580	1.442	1.401	1.252	1.221	1.157	1.115	1.082	1.102	1.261
PP:Household Services	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 19: Rate of Change of Terms of Trade, Italy - $\Delta\%\varepsilon_i$ (pp)

act	99-00	00-01	01-02	02-03	03-04	04-05	05-06	06-07	mean
AA:Agriculture	−0.31	−7.62	−2.37	−5.75	4.68	3.88	2.57	1.12	−0.48
BB:Fishing	8.70	−10.82	−11.78	−5.79	−2.55	−4.50	1.83	2.16	−2.84
CA:Mining energy	56.65	−2.43	−3.87	4.86	8.32	25.34	14.38	21.27	15.56
CB:Mining non-energy	8.47	1.49	−5.86	−9.49	5.31	13.25	2.31	0.71	2.02
DA:Food-Tobacco	−0.54	−1.59	−3.18	−2.28	1.06	1.90	−2.26	−4.49	−1.42
DB:Textiles	−0.13	1.70	0.55	−1.39	−0.21	0.33	−0.77	−1.62	−0.19
DC:Leather	4.63	4.43	4.53	−5.89	0.64	3.94	2.24	1.90	2.05
DD:Wood	0.74	−3.57	−0.85	−2.45	−2.67	−0.54	1.28	1.30	−0.84
DE:Paper-Printing	−1.84	−3.56	−7.59	−4.60	−3.11	−0.01	0.95	−0.83	−2.57
DF:Coke-Petroleum	−13.21	3.98	11.32	−1.21	−29.64	−15.12	8.45	−2.71	−4.77
DG:Chemicals	−5.56	−0.24	−1.89	−2.28	1.32	2.12	−0.24	0.04	−0.84
DH:Plastics	−3.68	0.36	−1.90	−0.95	0.30	−0.50	0.36	2.55	−0.43
DI:Non-met. minerals	−0.22	0.38	−3.22	−2.26	−0.93	0.14	0.44	2.25	−0.43
DJ:Metals	6.70	−1.86	−1.06	−1.90	4.52	5.13	9.22	5.08	3.23
DK:Machinery n.e.c.	2.87	0.67	1.65	−2.43	−2.07	−0.82	1.51	−3.83	−0.31
DL:Electr. Machinery	4.93	−0.27	−0.62	−5.99	−4.76	−3.21	−3.75	−0.50	−1.77
DM:Transport Equip.	−4.21	3.59	3.16	2.57	−2.75	0.03	−0.49	0.40	0.29
DN:Manufacture n.e.c.	−1.50	0.04	−2.01	−4.50	−0.98	1.52	−0.61	−2.25	−1.29
EE:Energy	−17.35	−7.56	0.09	−11.59	13.25	−1.30	11.01	5.80	−0.96
FF:Construction	4.87	0.65	−3.52	−6.49	−4.43	1.02	1.15	−3.96	−1.34
GG:Trade	4.72	−1.08	0.45	−4.61	−1.71	2.14	−1.89	−1.48	−0.43
HH:Hotel-Restaurant	7.02	−0.82	−3.10	−7.72	−2.58	3.62	1.65	−1.68	−0.45
II:Transport-Comm.	4.88	1.04	1.00	−2.77	−1.85	3.49	−1.12	−0.67	0.50
JJ:Finance	6.44	−0.62	−4.19	−7.09	1.22	2.93	7.30	−1.51	0.56
KK:Business Services	6.47	0.93	0.67	−5.61	−2.03	−0.09	2.55	−1.32	0.20
MM:Education	4.03	−0.36	0.01	−8.30	−1.60	−0.96	0.86	−4.56	−1.36
OO:Personal Services	7.08	−1.61	−2.16	−7.83	1.16	1.44	0.16	−0.42	−0.27

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 20: Net Taxes per monetary unit of Industry Output, Italy - $\boldsymbol{\tau}^T$ (%)

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	0.69	0.47	0.93	0.45	1.29	1.58	−0.26	−1.69	−0.98	0.28
BB:Fishing	6.28	6.19	8.69	8.20	8.26	8.48	9.20	8.47	8.58	8.04
CA:Mining energy	5.89	4.78	7.11	9.97	7.44	5.87	6.47	7.77	8.38	7.07
CB:Mining non-energy	9.47	9.22	8.58	8.69	8.93	8.94	8.65	8.61	8.95	8.89
DA:Food-Tobacco	3.54	3.31	3.18	3.45	3.30	3.27	3.79	4.35	4.25	3.60
DB:Textiles	6.91	6.61	6.55	6.76	6.94	7.20	7.31	7.36	7.35	7.00
DC:Leather	5.16	4.95	4.83	4.99	5.01	5.10	5.08	5.02	4.98	5.01
DD:Wood	6.08	5.98	5.99	6.61	6.51	6.33	6.87	6.68	6.54	6.40
DE:Paper-Printing	7.50	6.87	7.03	7.18	7.26	7.28	7.52	7.44	7.22	7.25
DF:Coke-Petroleum	8.45	5.23	5.70	6.26	5.51	2.60	2.41	2.45	2.04	4.52
DG:Chemicals	6.80	6.41	6.60	6.47	6.89	6.54	6.87	6.75	6.55	6.65
DH:Plastics	7.01	6.91	7.10	6.85	7.04	6.93	7.15	6.99	6.87	6.98
DI:Non-met. minerals	9.60	8.95	8.63	8.28	8.71	8.51	9.02	8.95	8.86	8.83
DJ:Metals	7.89	7.45	7.58	7.67	7.74	7.34	7.40	7.01	6.76	7.43
DK:Machinery n.e.c.	7.63	7.20	7.41	7.75	7.76	7.62	7.86	7.62	7.37	7.58
DL:Electr. Machinery	7.66	7.19	7.23	7.58	7.85	7.77	8.11	7.86	7.73	7.66
DM:Transport Equip.	6.02	5.41	5.69	5.42	5.34	5.86	6.15	5.73	5.59	5.69
DN:Manufacture n.e.c.	5.58	5.33	5.65	5.75	5.89	5.77	5.95	5.82	5.64	5.71
EE:Energy	12.31	10.08	8.98	8.92	8.74	8.61	8.48	7.66	7.61	9.04
FF:Construction	6.67	6.40	6.38	6.73	7.18	6.96	7.08	7.30	7.45	6.91
GG:Trade	6.43	6.08	5.67	6.07	6.17	6.24	6.70	6.88	6.88	6.35
HH:Hotel-Restaurant	4.23	4.17	4.40	4.74	4.93	4.98	5.30	5.53	5.65	4.88
II:Transport-Comm.	9.97	8.63	7.81	7.75	7.82	7.73	7.86	8.02	7.91	8.17
JJ:Finance	17.78	16.48	16.59	15.38	15.15	14.66	14.62	15.61	14.93	15.69
KK:Business Services	6.55	6.49	6.45	6.69	6.59	6.55	6.37	6.30	6.40	6.49
LL:Public Admin.	23.83	23.59	23.19	23.01	23.21	22.74	22.19	22.59	22.54	22.99
MM:Education	27.61	27.12	26.38	26.65	26.07	25.28	26.00	25.93	25.79	26.31
NN:Health	17.63	17.70	17.52	17.59	17.26	16.93	16.99	17.55	17.24	17.38
OO:Personal Services	8.15	8.38	8.22	8.40	8.38	8.83	9.26	9.50	9.36	8.72
PP:Household Services	6.85	6.21	5.34	9.02	10.49	9.81	9.77	9.76	9.88	8.57

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 21: Price Index for Private Consumption (2000=1), Italy - $\boldsymbol{\xi}^T$

act	1999	2000	2001	2002	2003	2004	2005	2006	2007	mean
AA:Agriculture	0.994	1.000	1.023	1.062	1.133	0.957	0.882	0.890	0.933	0.986
BB:Fishing	1.067	1.000	1.116	1.253	1.318	1.371	1.489	1.511	1.479	1.289
CA:Mining energy	1.398	1.000	1.141	1.103	1.177	1.430	1.385	0.752	1.395	1.198
CB:Mining non-energy	0.997	1.000	1.021	1.061	1.096	1.136	1.162	1.204	1.247	1.103
DA:Food-Tobacco	1.004	1.000	1.047	1.056	1.089	1.077	1.071	1.035	1.041	1.047
DB:Textiles	1.023	1.000	1.019	1.045	1.063	1.071	1.091	1.051	1.068	1.048
DC:Leather	1.020	1.000	1.033	1.069	1.079	1.080	1.086	1.041	1.039	1.050
DD:Wood	1.015	1.000	1.023	1.023	1.059	1.093	1.163	1.023	0.970	1.041
DE:Paper-Printing	1.012	1.000	1.015	1.066	1.084	1.096	1.120	1.094	1.103	1.066
DF:Coke-Petroleum	0.774	1.000	0.912	0.894	0.918	0.955	1.099	1.287	1.205	1.005
DG:Chemicals	1.019	1.000	1.027	1.045	1.052	1.061	1.068	0.991	0.966	1.025
DH:Plastics	1.044	1.000	1.024	1.036	1.058	1.092	1.152	1.056	1.099	1.062
DI:Non-met. minerals	1.027	1.000	1.031	1.078	1.104	1.128	1.134	1.016	1.072	1.066
DJ:Metals	1.036	1.000	1.040	1.075	1.097	1.105	1.133	0.721	0.767	0.997
DK:Machinery n.e.c.	1.051	1.000	1.014	1.027	1.028	1.037	1.053	1.016	1.037	1.029
DL:Electr. Machinery	1.080	1.000	1.020	1.037	1.076	1.090	1.105	0.823	0.930	1.018
DM:Transport Equip.	1.021	1.000	1.019	1.037	1.046	1.065	1.083	1.035	1.020	1.036
DN:Manufacture n.e.c.	1.003	1.000	1.020	1.038	1.053	1.071	1.093	1.051	1.086	1.046
EE:Energy	0.933	1.000	1.053	1.018	1.049	1.054	1.112	1.232	1.273	1.080
FF:Construction	0.987	1.000	1.027	1.051	1.090	1.127	1.116	1.154	1.184	1.082
GG:Trade	0.982	1.000	1.028	1.058	1.113	1.153	1.152	1.163	1.182	1.092
HH:Hotel-Restaurant	0.971	1.000	1.041	1.087	1.133	1.174	1.195	1.220	1.250	1.119
II:Transport-Comm.	1.005	1.000	1.019	1.025	1.043	1.077	1.086	1.107	1.115	1.053
JJ:Finance	0.867	1.000	1.080	1.099	1.155	1.124	1.202	1.325	1.433	1.143
KK:Business Services	0.948	1.000	1.049	1.115	1.164	1.241	1.278	1.325	1.377	1.166
LL:Public Admin.	0.996	1.000	1.001	1.000	1.001	1.000	1.009	1.010	1.190	1.023
MM:Education	0.978	1.000	1.025	1.057	1.089	1.136	1.169	1.196	1.220	1.097
NN:Health	0.972	1.000	1.025	1.056	1.089	1.107	1.135	1.158	1.186	1.081
OO:Personal Services	0.915	1.000	1.070	1.117	1.190	1.091	1.184	1.198	1.225	1.110
PP:Household Services	0.975	1.000	1.008	1.058	1.094	1.109	1.129	1.141	1.185	1.077

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 22: Industry Surplus Growth ($\Pi = I$), Italy - $\rho_{g,\eta}$ (pp)

act	99-00	00-01	01-02	02-03	03-04	04-05	05-06	06-07	mean
AA:Agriculture	−2.38	−0.87	−0.38	−0.91	6.84	0.13	−1.31	2.91	0.50
BB:Fishing	12.64	−13.22	−6.10	7.34	−7.07	−3.40	2.57	−1.73	−1.12
CA:Mining energy	14.39	−26.52	−2.38	−55.49	−39.73	−13.57	−1.70	−23.97	−18.62
CB:Mining non-energy	1.28	5.56	−3.86	−0.39	−8.54	2.39	3.37	−2.66	−0.36
DA:Food-Tobacco	3.85	−1.24	0.98	0.10	−1.11	1.30	1.18	0.88	0.74
DB:Textiles	3.53	−0.43	−1.59	−0.92	0.66	−0.28	2.21	−0.51	0.34
DC:Leather	2.73	1.99	−4.29	−0.88	1.80	0.70	2.02	−0.08	0.50
DD:Wood	3.06	4.30	−2.89	−0.38	1.27	−0.64	1.36	1.43	0.94
DE:Paper-Printing	−2.38	2.39	0.91	−0.75	1.50	0.45	0.78	0.13	0.38
DF:Coke-Petroleum	−11.43	11.04	3.83	−7.23	−16.62	−8.03	1.02	−0.59	−3.50
DG:Chemicals	0.58	0.48	−0.54	1.24	2.07	0.06	0.14	−0.15	0.49
DH:Plastics	−0.01	−0.20	2.24	−1.06	2.41	0.45	−0.57	0.44	0.46
DI:Non-met. minerals	3.00	−0.85	−1.53	−1.20	1.59	2.30	−0.24	0.38	0.43
DJ:Metals	0.85	0.93	0.50	1.80	−2.45	0.86	−1.16	−0.65	0.08
DK:Machinery n.e.c.	2.99	0.16	−1.53	1.50	2.57	0.17	2.79	0.87	1.19
DL:Electr. Machinery	0.52	1.27	1.37	−0.85	3.28	1.26	0.61	0.78	1.03
DM:Transport Equip.	0.15	−0.61	−0.96	−0.32	−0.12	−3.27	2.59	1.65	−0.11
DN:Manufacture n.e.c.	1.72	−1.03	−1.35	−0.73	2.00	−0.63	1.59	0.58	0.27
EE:Energy	−3.72	−4.67	−0.21	−4.43	8.43	1.78	−2.64	−1.91	−0.92
FF:Construction	−0.24	0.62	−0.22	0.60	0.60	−1.29	0.17	−1.00	−0.10
GG:Trade	0.70	−0.71	−2.25	−0.59	0.91	1.86	−0.22	0.35	0.01
HH:Hotel-Restaurant	0.76	0.02	−4.62	−3.84	−1.48	0.64	1.72	1.62	−0.65
II:Transport-Comm.	3.16	3.24	1.82	0.31	−0.11	1.82	−0.62	0.17	1.22
JJ:Finance	3.82	1.26	−2.67	0.94	0.51	2.27	0.55	4.61	1.41
KK:Business Services	−1.84	−2.09	−0.30	0.47	−2.57	−2.83	−0.77	−1.41	−1.42
LL:Public Admin.	0.45	0.28	0.92	0.63	1.51	2.71	1.76	1.28	1.19
MM:Education	−0.78	2.36	1.91	1.56	0.12	−0.39	−0.32	0.80	0.66
NN:Health	0.91	1.59	0.18	0.12	0.17	0.92	1.14	1.09	0.77
OO:Personal Services	−3.51	−4.05	−2.70	−1.59	6.53	−3.19	−0.82	0.09	−1.15
PP:Household Services	0.21	0.55	0.04	−0.05	0.35	0.31	0.31	0.37	0.26

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 23: Industry r-Productivity Growth, Italy - $\rho_{g,j}(r)$ (pp)(1999-2003)

	99-00				**00-01**				**01-02**				**02-03**			
	r=0	r=0.05	r=0.06	r=0.14	r=0	r=0.05	r=0.06	r=0.14	r=0	r=0.05	r=0.06	r=0.14	r=0	r=0.05	r=0.06	r=0.14
AA:Agriculture	−2.08	−2.75	−2.92	−4.85	−1.03	−0.37	−0.21	1.41	−0.01	−1.02	−1.28	−4.35	−0.53	−1.74	−2.06	−5.76
BB:Fishing	12.65	12.39	12.32	11.49	−12.59	−14.26	−14.75	−22.55	−5.66	−7.37	−7.84	−14.39	7.37	7.54	7.60	8.63
CA:Mining energy	3.72	1.32	0.90	−4.23	−3.35	5.87	7.48	15.88	−14.74	−14.67	−14.62	−13.54	−31.41	−45.24	−47.88	−63.17
CB:Mining non-energy	1.25	2.18	2.37	3.78	5.25	6.50	6.76	8.49	−3.62	−4.49	−4.68	−5.97	−0.74	−0.18	−0.06	1.00
DA:Food-Tobacco	3.87	3.77	3.75	3.63	−1.21	−1.37	−1.41	−1.78	1.03	1.02	1.02	0.90	0.09	0.00	−0.02	−0.23
DB:Textiles	3.39	3.02	2.93	2.11	−0.20	−0.70	−0.82	−1.96	−1.61	−1.42	−1.38	−0.95	−1.02	−0.53	−0.41	0.66
DC:Leather	2.78	2.47	2.40	1.79	1.89	1.61	1.55	1.00	−4.06	−4.48	−4.58	−5.50	−1.05	−0.58	−0.47	0.57
DD:Wood	2.95	3.15	3.20	3.72	4.08	4.45	4.54	5.28	−2.77	−2.71	−2.70	−2.68	−0.43	−0.50	−0.51	−0.51
DE:Paper-Printing	−1.81	−3.55	−3.93	−6.88	2.28	2.77	2.87	3.66	0.69	0.93	0.98	1.41	−0.69	−0.59	−0.57	−0.38
DF:Coke-Petroleum	−11.46	−12.70	−12.94	−14.52	10.83	11.14	11.19	11.52	3.39	4.19	4.36	5.02	−6.40	−7.41	−7.62	−8.32
DG:Chemicals	0.51	0.80	0.85	1.36	0.40	0.38	0.38	0.45	−0.87	−0.51	−0.44	0.10	1.39	1.36	1.36	1.22
DH:Plastics	0.00	−0.09	−0.11	−0.20	−0.22	−0.26	−0.27	−0.32	2.32	1.95	1.87	1.29	−0.86	−1.14	−1.21	−1.68
DI:Non-met. minerals	2.95	2.99	2.99	3.10	−0.71	−1.34	−1.48	−2.59	−0.97	−2.24	−2.52	−4.79	−1.01	−1.01	−1.02	−1.13
DJ:Metals	1.13	0.41	0.25	−1.02	0.98	0.85	0.82	0.56	0.46	0.52	0.54	0.63	1.59	2.27	2.43	3.73
DK:Machinery n.e.c.	3.06	2.76	2.69	2.15	0.19	0.28	0.31	0.57	−1.53	−1.90	−1.99	−2.64	1.36	1.71	1.78	2.43
DL:Electr. Machinery	1.23	−0.49	−0.88	−4.30	1.27	1.05	1.01	0.67	1.07	1.75	1.90	3.26	−0.96	−0.46	−0.34	0.50
DM:Transport Equip.	0.52	−0.08	−0.21	−1.18	−0.77	−1.06	−1.12	−1.59	−0.62	−0.84	−0.89	−1.28	−0.33	−0.49	−0.52	−0.75
DN:Manufacture n.e.c.	2.04	1.14	0.94	−0.76	−1.18	−0.67	−0.55	0.41	−1.15	−1.38	−1.44	−1.95	−0.97	−0.31	−0.16	1.22
EE:Energy	−4.02	−4.40	−4.47	−4.63	−4.66	−5.43	−5.56	−6.34	0.14	−0.77	−0.94	−1.95	−3.91	−5.07	−5.28	−6.49
FF:Construction	−0.07	−0.79	−0.96	−2.66	0.81	0.39	0.28	−0.68	−0.17	−0.68	−0.81	−1.86	0.50	1.15	1.31	2.79
GG:Trade	0.75	0.33	0.24	−0.64	−0.29	−1.76	−2.11	−4.88	−1.93	−3.33	−3.66	−6.33	−0.69	−0.12	0.00	1.06
HH:Hotel-Restaurant	0.62	1.05	1.16	2.08	−0.02	0.22	0.27	0.78	−4.83	−4.23	−4.09	−2.73	−3.61	−4.33	−4.52	−6.27
II:Transport-Comm.	3.14	2.24	2.04	0.60	3.66	2.97	2.82	1.98	1.78	1.02	0.86	−0.33	0.87	−0.76	−1.12	−3.45
JJ:Finance	3.92	3.31	3.17	1.93	1.36	1.24	1.22	0.97	−2.55	−3.28	−3.46	−4.93	0.75	1.69	1.92	3.71
KK:Business Services	−1.91	−1.92	−1.92	−1.95	−2.43	−0.89	−0.65	0.40	−0.63	0.11	0.22	0.72	0.62	0.70	0.71	0.72
LL:Public Admin.	0.47	0.09	0.01	−0.51	0.62	−0.26	−0.45	−1.77	1.01	0.44	0.33	−0.49	1.00	0.27	0.12	−1.04
MM:Education	−0.61	−1.42	−1.66	−5.13	2.02	3.63	4.10	11.03	2.04	1.34	1.12	−2.54	1.34	2.40	2.72	8.27
NN:Health	0.86	0.79	0.78	0.61	1.66	1.55	1.53	1.19	0.21	−0.02	−0.09	−0.93	0.26	−0.13	−0.24	−1.62
OO:Personal Services	−3.54	−3.74	−3.79	−4.27	−3.72	−4.62	−4.83	−6.68	−2.76	−2.72	−2.72	−2.72	−1.72	−1.02	−0.85	0.63
PP:Household Services	0.21	0.21	0.21	0.21	0.55	0.55	0.55	0.55	0.04	0.04	0.04	0.04	−0.05	−0.05	−0.05	−0.05

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

Table 24: Industry r-Productivity Growth, Italy - $\rho_{g,j}(r)$ (pp)(2003-2007)

	03-04				**04-05**				**05-06**				**06-07**			
	r=0	r=0.05	r=0.06	r=0.14	r=0	r=0.05	r=0.06	r=0.14	r=0	r=0.05	r=0.06	r=0.14	r=0	r=0.05	r=0.06	r=0.14
AA:Agriculture	6.82	5.66	5.38	2.37	0.44	0.53	0.54	0.62	−1.46	−0.79	−0.62	1.09	2.77	3.09	3.18	4.12
BB:Fishing	−7.26	−6.84	−6.72	−4.93	−3.86	−2.72	−2.41	1.63	2.55	2.58	2.59	2.68	−1.82	−1.29	−1.14	0.94
CA:Mining energy	−38.01	−46.41	−47.87	−57.81	−11.29	−13.42	−13.79	−15.31	−31.60	−36.21	−36.95	−41.46	61.23	66.00	66.73	70.57
CB:Mining non-energy	−7.66	−9.36	−9.72	−12.65	2.65	1.45	1.20	−0.58	3.07	4.14	4.36	5.93	−2.37	−2.56	−2.60	−3.00
DA:Food-Tobacco	−1.05	−1.19	−1.22	−1.52	1.31	1.29	1.28	1.30	1.32	1.16	1.11	0.69	0.82	0.85	0.85	0.91
DB:Textiles	0.60	0.74	0.77	1.04	−0.30	−0.40	−0.43	−0.64	2.28	2.19	2.17	1.95	−0.47	−0.60	−0.63	−0.91
DC:Leather	1.81	1.75	1.74	1.56	0.63	0.51	0.48	0.28	2.03	1.86	1.82	1.51	−0.03	0.03	0.03	0.10
DD:Wood	1.09	1.56	1.68	2.70	−0.34	−0.82	−0.94	−2.11	1.32	1.29	1.29	1.30	1.49	1.42	1.40	1.24
DE:Paper-Printing	1.56	1.64	1.66	1.79	0.36	0.37	0.37	0.37	0.73	0.77	0.78	0.82	0.26	0.14	0.11	−0.05
DF:Coke-Petroleum	−17.18	−17.86	−17.98	−18.40	−8.18	−7.77	−7.69	−7.25	1.10	1.05	1.04	1.03	−0.49	−0.89	−0.95	−1.26
DG:Chemicals	1.86	2.47	2.60	3.46	0.07	0.01	0.00	−0.10	0.61	0.10	0.00	−0.84	−0.22	−0.29	−0.31	−0.36
DH:Plastics	2.41	2.57	2.61	2.89	0.30	0.52	0.57	0.95	−0.35	−0.76	−0.85	−1.53	0.27	0.68	0.77	1.39
DI:Non-met. minerals	1.45	1.42	1.42	1.50	2.04	2.94	3.13	4.53	0.25	−0.26	−0.37	−1.31	0.14	0.57	0.66	1.38
DJ:Metals	−2.15	−2.31	−2.34	−2.57	0.64	1.05	1.15	1.88	−0.92	−1.58	−1.73	−2.96	−0.31	−1.07	−1.24	−2.59
DK:Machinery n.e.c.	2.67	2.56	2.53	2.25	0.08	0.30	0.35	0.72	2.74	2.93	2.97	3.31	1.03	0.82	0.78	0.43
DL:Electr. Machinery	3.43	3.04	2.95	2.12	1.23	1.56	1.63	2.22	0.52	0.63	0.65	0.84	0.95	0.55	0.47	−0.24
DM:Transport Equip.	−0.02	−0.03	−0.04	−0.05	−3.09	−3.07	−3.07	−3.15	2.28	2.45	2.49	2.85	1.62	1.86	1.91	2.36
DN:Manufacture n.e.c.	1.79	2.23	2.34	3.26	−0.46	−0.82	−0.90	−1.63	1.81	1.31	1.19	0.16	0.46	0.86	0.96	1.79
EE:Energy	8.02	9.17	9.36	10.48	1.54	2.65	2.85	3.74	−2.41	−2.60	−2.63	−2.75	−1.48	−2.02	−2.10	−2.45
FF:Construction	0.37	1.10	1.28	2.93	−1.25	−1.41	−1.45	−1.81	0.25	0.24	0.23	0.17	−1.01	−1.05	−1.06	−1.10
GG:Trade	1.00	0.75	0.69	0.18	1.80	1.94	1.97	2.19	−0.21	0.04	0.09	0.46	0.37	0.26	0.23	−0.03
HH:Hotel-Restaurant	−1.43	−1.59	−1.63	−2.05	0.73	0.26	0.14	−0.88	2.10	0.74	0.40	−2.63	1.44	2.22	2.42	3.96
II:Transport-Comm.	0.20	−0.74	−0.93	−2.22	1.67	1.86	1.90	2.10	−0.49	−0.19	−0.13	0.25	0.21	−0.03	−0.08	−0.43
JJ:Finance	0.65	0.17	0.07	−0.72	2.33	2.09	2.03	1.54	0.79	0.11	−0.04	−1.22	4.51	5.01	5.13	6.10
KK:Business Services	−2.51	−2.62	−2.63	−2.71	−2.51	−3.83	−4.03	−5.08	−0.64	−0.65	−0.66	−0.70	−1.44	−1.24	−1.21	−1.10
LL:Public Admin.	1.92	0.42	0.11	−1.81	2.31	3.33	3.54	4.82	1.88	1.86	1.86	1.93	1.19	1.82	1.96	2.77
MM:Education	0.00	0.60	0.79	4.16	−0.40	−0.47	−0.50	−1.00	−0.38	0.05	0.19	2.79	0.79	0.87	0.90	1.71
NN:Health	0.34	−0.21	−0.37	−2.38	1.07	0.36	0.17	−2.20	1.22	1.19	1.19	1.13	1.06	1.26	1.31	2.07
OO:Personal Services	6.45	6.97	7.11	8.14	−3.38	−2.93	−2.83	−1.93	−0.82	−0.40	−0.30	0.62	0.19	−0.11	−0.19	−0.85
PP:Household Services	0.35	0.35	0.35	0.35	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.37	0.37	0.37	0.37

Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT

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