

# AN INPUT-OUTPUT MULTI-OBJECTIVE LINEAR MODEL INCORPORATING GHG EMISSIONS APPLIED TO THE BRAZILIAN ECONOMY

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In this study the development of a hybrid Input-Output (IO) framework is presented, which leads to a multi-objective linear programming (MOLP) model. The goal is jointly assessing the trade-offs between economic, energy, environmental and social aspects in the Brazilian economic system. Firstly, the National Accounts System is reorganized to allocate endogenously the National Energy Balance, creating a hybrid IO framework in which the activity level of each sector is associated with its energy demand. This IO system is extended externally to assess greenhouse gases (GHG) emissions caused by fossil fuels combustion. The MOLP model structure (decision variables, constraints and objective functions) is developed using this hybrid IO framework. The objective functions explicitly considered are the maximization of GDP and employment levels, as well as the minimization of energy consumption and Global Warming Potential (GWP). Preliminary illustrative results of this model suggest a positive correlation between GDP growth and employment levels, as well as energy consumption and GWP. Additionally, the maximization of GDP and the employment levels have resulted in an increased energy consumption and GWP, while the minimization of GWP, and energy consumption caused negative impacts on GDP and employment.

## METHODOLOGY

The IO Analysis has been used in economic analysis to study the inter-relationship among different sectors in the economic system, describing the relationships between the inputs used and the outputs produced (Miller and Blair, 1985). The IO model assumes that each sector consumes outputs of various other sectors in fixed ratios in order to produce its own unique and distinct output (Tan et al., 2008). The basic IO relationship is:

$$\mathbf{x} = \mathbf{Ax} + \mathbf{y} \tag{1}$$

in which  $\mathbf{x}$  is a vector of total output,  $\mathbf{y}$  is the vector of the final demand<sup>1</sup> and  $\mathbf{A}$  is a matrix of inter-sectoral direct requirements (or technical coefficients matrix) where each element  $a_{ij}$  represents the value of input required from sector  $i$  to produce one monetary unit worth output of sector  $j$  ( $i = 1...n$ , and  $j = 1...n$ ).

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<sup>1</sup> This vector represents the sum of the vectors with the components of final demand: exports **exp**, public consumption **g**, consumption of residents **cr** (households and nonprofit institutions serving households (NPISH)), gross fixed capital formation **gfcf** and stock changes **sc**.

In order to assess the energy consumption, the 2005 Brazilian IO table (IBGE, 2008) is rearranged in this study to turn the energy uses endogenous into the technical coefficients. Thus, the production and consumption of the energy products in the Brazilian National Energy Balance (MME, 2009) are incorporated into the IO table through artificial sectors. For this purpose, the IO table is adjusted and new rows and columns are included in the transaction matrix to allocate the energy sectors (or commodities). Then, the rows with energy sectors (or commodities) are transformed from monetary to physical units in order to allocate the energy flows into the inter-sector transaction matrix and final demand vector. Thus, a new technical coefficient matrix and new vectors of total output and final demand with hybrid units are generated, in which the energy flows are considered in physical quantities of energy (tons of oil equivalent, toe) and all non-energy sector flows are measured in monetary units (Hilgemberg, 2004; Oliveira and Antunes, 2004, 2011).

The adjustments performed in the IO framework provide: a square matrix with 109 activity sectors (51 economic sectors, 6 energy producing sectors, 5 artificial sectors used for distributing the energy consumed by each transportation mean and 47 artificial sectors for energy commodities); 6 column vectors with the final demand items; 1 column vector for competitive imports (considered only for energy commodities); and 6 row vectors for the primary inputs (wages, gross mixed income, gross operating surplus, other production taxes and other production subsidies).

The IO framework is extended to assess GHG emissions from energy combustion. In this step the level of activity in each sector is associated with its energy demand (by fuel source). Thus, specific GHG emission factors per unit of each fuel consumed from IPCC (2006) are applied to the total energy consumed in each sector, in order to obtain the total emissions of each activity sector and the whole economy (Oliveira and Antunes, 2004).

Finally, the IO framework is utilized to formulate a MOLP model. In this step, the MOLP models proposed by Oliveira and Antunes (2004, 2011) are adapted for the Brazilian economic system (Oliveira and Antunes, 2004, 2011). The model includes a coherence constraint, several economic constraints (such as GDP, disposable income of residents, public debt, public deficit, among others) and environmental constraints.

The coherence constraint establishes that the intermediate consumption ( $\mathbf{Ax}$ ) and the final demand (multiplied by a technical coefficients matrix  $\mathbf{FD}$ ) shall not exceed the total amount available from national production ( $\mathbf{x}$ ) and competitive imports  $\mathbf{imp}^c$ :

$$\mathbf{Ax} + \mathbf{FDy} \leq \mathbf{x} + \mathbf{imp}^c \quad (2)$$

The GDP is estimated by the expense (3) and income (4) approaches. The GDP in the expense approach  $gdp_{exp}$  is estimated by the summation of the final demand items less imports at FOB (free on board) prices<sup>2</sup>:

$$gdp_{exp} = exp + g + cr + gfcf \pm sc - impfob \quad (3)$$

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<sup>2</sup> The model also formalizes and establishes some assumptions for several consumption relations (households' consumption in the territory, residents' consumption, resident households' consumption in the territory and the consumption of non-residents), exports and imports (considering constant or current FOB and CIF prices).

The GDP in the income approach  $gdp_{inc}$  is estimated by the sum of gross value added  $gva$  (obtained by technical coefficients applied to the primary input vectors) and the total of taxes less subsidies on products that are not included in the production  $ts$  (obtained by vectors and matrices of technical coefficients applied to intermediate and final demand):

$$gdp_{inc} = gva + ts \quad (4)$$

The GDP at current prices is estimated by the items of the GDP at constant prices in the expense approach and specific deflators. The residents' disposable income at current prices is estimated by subtracting the public administration and (non-financial and financial) corporations' disposable incomes from the National Disposable Income. Public administration's global balance is computed by subtracting the public administration's expenditures from the public administration's revenues. Public debt ( $debt$ ) is given by the summation of the debt in the previous period ( $debt_{.1}$ ) with the symmetrical value of the public administration global balance ( $gbg^+ - gbg^-$ ) and an adjustment variable ( $dat$ ):

$$debt = debt_{.1} - (gbg^+ - gbg^-) + dat \quad (5)$$

The employment level is obtained by using labour gross productivity coefficients for each sector:

$$emp = \mathbf{lp} \mathbf{x} \quad (6)$$

The total energy consumption is obtained from the sum of national and imported energy excluding the energy consumed for non-energy purposes. Specific technical coefficients are applied to the intermediary consumption and final demand:

$$cfe = (\mathbf{A}_E \mathbf{x} - \mathbf{A}_{NE} \mathbf{x}) + (\mathbf{FD}_E \mathbf{y} - \mathbf{FD}_{NE} \mathbf{y}) \quad (7)$$

Emissions from fossil energy combustion include emissions from the intermediary sectors (energy industries; trade and services; agriculture, forestry, fishing and livestock; manufacturing industries; transports and construction) and final demand (excluding exports and GFCF).

$$ec_w = (\mathbf{EF}_w)^T \{ \mathbf{FC}_{TJ} [(\mathbf{A}_E \mathbf{x} - \mathbf{A}_{NE} \mathbf{x}) + (\mathbf{FD}_E \mathbf{y} - \mathbf{FD}_{NE} \mathbf{y})] \} (10^{-9}) \quad (8)$$

The total emissions of CO<sub>2</sub> ( $teco$ ), CH<sub>4</sub> ( $tech$ ) and NO<sub>2</sub> ( $teno$ ) from each sector and final demand are summed up and used to estimate the Global Warming Potential taking into account the respective CO<sub>2</sub>e weighting factors:

$$gwp = teco + 21 (tech) + 310 (teno) \quad (9)$$

According to the objectives of this study, which is assessing the interactions between the economic growth, fossil energy use, GHG emissions and employment level in Brazil for a 2015 scenario, the model herein proposed considers 4 different objective functions: maximization of GDP, minimization of energy consumed for energy purpose, minimization of the GWP and maximization of employment level:

$$\text{Max } z_1 = gdp_{exp} \quad (10)$$

$$\text{Min } z_2 = cfe \quad (11)$$

$$\text{Min } z_3 = gwp \quad (12)$$

## RESULTS AND DISCUSSION

The MOLP model has been supplied with realistic data gathered from several Brazilian sources (MMA, 2009; IBGE, 2008a; MCT, 2010). The IO framework has been compiled in multiple inter-related workbooks using commercial spreadsheet software providing the model data for the multi-objective optimization which has been performed using the Risk Solver Platform (RSP). The RSP is fully adapted to run within a spreadsheet framework, which avoids cumbersome data exporting processes as required by other optimization solvers. Each objective function was optimized individually, resulting in 4 non-dominated solutions (see table 1). Those solutions provide an overview of the range of variation of the objective values within the non-dominated region, which are briefly described as follows:

**Solution 1:** The optimal level for GDP reached the upper limit established in our projections. This GDP level leads to the lowest value of public debt (corresponding to 64.3% of GDP). The highest level for the residents' disposable income also leads to the highest private consumption, which is the main item contributing to the GDP in the expense approach. On the other hand, to achieve this GDP more energy (fossil and renewable) have to be consumed ( $333,086 \times 10^3$  toe), which resulted in the higher GWP ( $549,290$  Gg  $\text{CO}_2\text{e}$ ). Energy and trade and private services sectors were the sectors that have improved more the output.

**Table 1 - Illustrative Results**

Notation	Main Variables		Max gdp	Min cfe	Min gwp	Max emp
gdpepx	GDP in the expense approach (constant prices - 2005)	R\$ x $10^9$	<b>3,475,589</b>	3,317,176	3,316,452	3,475,589
cfe	Total Energy Consumption	toe x $10^3$	333,086	<b>298,430</b>	298,510	322,504
gwp	Global warming potential	Gg $\text{CO}_2\text{e}$	549,290	496,077	<b>495,859</b>	546,870
emp	Employment	Employees x $10^3$	58,733	57,581	56,531	<b>60,180</b>
cr	Consumption of residents (constant prices - 2005)	R\$ x $10^9$	2,363,845	2,241,717	2,240,150	2,225,456
g	Public administration consumption (constant prices - 2005)	R\$ x $10^9$	584,928	631,143	631,143	631,143
sc	Stock variations (constant prices - 2005)	R\$ x $10^9$	5,713	0	0	0
gfef	Gross fixed capital formation (constant prices - 2005)	R\$ x $10^9$	629,163	629,163	629,163	691,593
impfob	Imports including tourism (constant prices - 2005)	R\$ x $10^9$	444,306	435,549	434,706	468,695
exp	Exports including the tourism (constant prices - 2005)	R\$ x $10^9$	336,246	250,702	250,702	396,092
gva	Gross value added (constant prices - 2005)	R\$ x $10^9$	2,953,564	2,820,532	2,821,344	2,961,409
itsub	Total indirect taxes less subsidies (constant prices - 2005)	R\$ x $10^9$	522,025	496,644	495,108	514,180
gdpcurr	GDP current (current prices - 2015)	R\$ x $10^9$	7,477,258	7,064,373	7,061,147	7,112,750
ydcorr	Household and NPISHs disposable income (current prices - 2015)	R\$ x $10^9$	4,500,299	4,267,790	4,264,808	4,236,832
gbg+ - gbg-	Public administration global balance (current prices - 2015)	R\$ x $10^9$	-136,919	-152,257	-152,257	-69,868
debt	Public debt (current prices - 2015)	R\$ x $10^9$	4,889,546	4,904,884	4,904,884	4,822,495
cfe_f	Fossil energy consumption	toe x $10^3$	176,540	159,114	159,050	175,244

**Solution 2:** The optimal energy consumption ( $298,430 \times 10^3$  toe) has not led to the lowest total fossil energy consumption ( $159,114 \times 10^3$  toe) (slightly higher than in solution 3). Furthermore, the public consumption is also raised causing negative pressures on the public global balance and public debt levels (corresponding to 2.16% and 69.4% of GDP, respectively).

**Solution 3:** The results in this optimization are quite similar (and correlated) to the results in solution 2. The best value for GWP ( $495,859$  Gg  $\text{CO}_2\text{e}$ ) led to lowest GDP and employment

levels. The energy intensive sectors (such as: extractive industry, petroleum refining and coke, chemicals and cement) are the main sectors the output of which was negatively affected in both 2 and 3 solutions.

**Solution 4:** The optimization of the employment level is highly related to the optimization of GDP. Both solutions reached the upper limit computed for GDP. This solution leads to a higher level of GFCF, which suggests that investments have an important role in the job creation process. In this solution, the agriculture and fishing, extractive, chemical and building sectors are the main sectors that reached higher output levels.

Other more balanced solutions, in the sense of establishing more acceptable compromises between the competing objective functions, have been computed using scalarizing procedures based on weighted-sums and minimization of a distance to the ideal solution (the one that would optimize simultaneously all objective functions, whose values are displayed in bold in Table 1), thus embodying more compensatory and non-compensatory approaches to the problem. The computation of these solutions, after having a first overview provided by the non-dominated solutions that optimize individually each objective function, the characteristics of which are briefly described above, would enable to grasp the trade-offs at stake between the objective functions in different parts of the feasible region.

## CONCLUSION

In this paper an integrated approach developed by Oliveira and Antunes (2004, 2011) has been applied to the Brazilian economic system. In this model the Brazilian 2005 IO table is rearranged to allocate endogenously the Brazilian Energy Balance, and extended externally to assess GHG emissions. This framework leads to the construction of a MOLP model, explicitly including 4 objective functions and considering several activity sectors and a scenario for 2015. The solutions from this model provide useful insights about the inter-relationships between those axes of evaluation of the merits of different policies and the trade-offs at stake. The results suggest positive correlations between GDP growth and employment level, as well as for energy consumption and GWP. In addition, divergences between axes of evaluation are also identified, such as maximizing GDP and employment levels leading to higher energy consumption and GWP, and minimizing GWP and energy consumption resulting in negative impacts on GDP and employment level. These conclusions are in concordance with economic and environmental premises, which can afford the robustness of the model.

On the other hand, there are some limitations inherent to this type of models. Since IO models comprise sectors rather than simple processes and sectors may be too heterogeneous, the results cannot correctly reflect the real conditions in each sector. In addition, due to the lack of statistical information, the IO 2005 system was utilized. Since the economic system in Brazil possibly will be different in 2015, the inter-relationships among sectors can also be different leading to a bias in the estimates. Additionally, the emissions estimates and economic projections are another source of uncertainty, which can result in over or sub estimation of the variables. Nevertheless, the model provides an important tool to assess the economy-energy-environment interactions in Brazil for prospective scenarios. The treatment

of uncertainty and other GHG emissions will be incorporated into the model in a further study.

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