## Path-based Matrix Decomposition Analysis: Theory and Application

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## Abstract.

Structural path analysis (SPA) (Defourny and Thorbecke, 1984) has recently been widely applied to studies in the field of industrial ecology (Lenzen, 2003; Peters and Hertwich, 2006; Strømman, Peters and Hertwich, 2009; Wood and Lenzen, 2009; Oshita, 2011). The utility provided by SPA is its ability to identify the individual paths and their impacts through the hierarchical sequences of sectors. Application of SPA enriches the interpretation in a life cycle-assessment study by decomposing a single numerical measure, i.e., the total impact, into the contribution of each path (Singh et al., 2011; Majeau-Bettez et al., 2011). In addition to environmental impacts embodied in inter-sector transactions, we can also track energy flows and material flows, which go through the individual paths, by applying SPA (Treloar, 1997; Suh, 2011). However, the results of a typical SPA are a huge number of paths, so that only a small portion of the results can be interpreted in detail, and the remaining portion is hardly studied. Therefore, there is a need for a method of giving structure to SPA results. With this background, this paper proposes a new method, named the Path-based Matrix Decomposition Analysis (PMDA), to additively decompose an input-output matrix into the contribution of each path based on SPA. The matrix to be decomposed is an input-output table that is induced by a given final demand based on the Leontief framework. The results obtained by the new decomposition method, PMDA, are also matrices of the same size, named path matrices, each of which corresponds to a path. The path matrices themselves provide a device for structural understanding of inter-sector transactions. Moreover, the PMDA can be applied to the analysis of linkage and key sectors, and effective visualization of inter-sector flows.