

# Application of Dynamically Calculated Total Requirements Coefficients to CGE Simulation Analysis

Robert A. McDougall  
Badri Narayanan G.

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## Abstract

Total requirements coefficients are applicable to analysis of results from computable general equilibrium models. For example, in estimating the effect of a change in final demand on output of some commodity, it is the total rather than the direct requirements coefficients for the commodity that are relevant, especially if the commodity is intermediate-usage-oriented. In a CGE model, however, the total requirements coefficients themselves are not constant, but are liable to change through the simulation. We develop a method for calculating total requirements coefficients dynamically through a simulation, and employ them in analysing positive and welfare results.

To illustrate the application of the concept, we simulate the introduction of a production subsidy for textiles in India. A change in household purchases of apparel affects production of textiles, with the size of the production change depending on the total requirements coefficient for textiles in production of apparel. The change in purchases of apparel also affects allocative efficiency by affecting the level of a subsidized flow (textiles production), and again the magnitude of the effect depends on the total requirements coefficient.

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## 1 Introduction

We present here a program for calculating total requirements coefficients (*TRCs*) from the GTAP data base, and an extension to the GTAP model to update total requirements coefficients through the course of a simulation.

## 2 Framework

We begin with the idea of a single-region input-output (*I-O*) table, produced on a commodity-by-commodity basis, in which the cost of production of each commodity is accounted for in terms of market costs of various inputs, including domestic product and imports of individual commodities, primary factors, and various forms of tax; exactly which we leave unspecified at this stage.

With  $c$  commodities (*tradables*, in GTAP terminology),  $f$  primary factors (GTAP *endowments*), and  $t$  taxes, this framework accounts for each sector's costs as a sum of  $2c + f + t$  terms: basic value (*market value*) of domestic product and imports of each commodity, market value of employment of each primary factor, and the taxes.

### 2.1 Direct requirements coefficients

We define the *direct requirements coefficient* for input  $i$  in production of commodity  $j$  as the ratio of the quantity of  $i$  used in producing commodity  $j$  to the quantity of  $j$  produced. In general, we cannot calculate direct requirements coefficients from the I-O table alone, since that provides not quantity but money value data. But if we supplement the I-O table with prices, we can convert money values to quantities. We assumed above that money value data were provided as market values; if so, the prices we need to convert them to quantities are market prices. Thus we obtain:

$D_{ij}$ : coefficient for direct requirements of domestically produced commodity  $i$  in production of commodity  $j$ ,

$M_{ij}$ : coefficient for direct requirements of imports of commodity  $i$  in production of commodity  $j$ ,

$E_{ij}$ : coefficient for direct requirements of endowment  $i$  in production of commodity  $j$ ,

$T_{ij}$ : coefficient for direct requirements of tax type  $i$  in production of commodity  $j$ .

In this we treat taxes as just another kind of input. That raises the question, what meaning can be given to the quantity of a tax? For answer, we suppose that production requires, for legal not technical reasons, certain quantities of *tax chits* of various types. The price of a tax chit is the market price of the commodity to which the tax attaches; for an input tax, the price of the input, for an output tax, the price of the output.

This approach fits well with the common modeling assumption that all taxes are *ad valorem*. With *ad valorem* taxes, there are three sources of change in tax payable:

- Change in the direct requirements coefficient for the physical input is treated as a driver of the direct requirements coefficient for tax chits.
- Change in the price of the input or output to which the tax attaches is represented as change in the price of tax chits.
- Change in the *ad valorem* tax rate is treated as another driver of the direct requirements coefficient for tax chits.

Now with  $D_{ij}$  representing the quantity of commodity  $i$  required per unit output of  $j$ , the cost of usage of commodity  $i$  per unit output of  $j$  is  $P_{Di}D_{ij}$ , where  $P_{Di}$  represents the market cost of commodity  $i$  (note the assumption that that price is uniform across output commodity). Under the zero profits assumption, whereby operating surplus is accounted for as returns to capital, total unit cost is equal to price; so we have, for all  $j$  from 1 to  $c$ ,

$$\sum_{i=1}^c P_{Di}D_{ij} + \sum_{i=1}^c P_{Mi}M_{ij} + \sum_{i=1}^f P_{Ei}E_{ij} + \sum_{i=1}^t P_{Ti}T_{ij} = P_{Dj}, \quad (1)$$

where  $P_{Di}$ ,  $P_{Mi}$ ,  $P_{Ei}$ , and  $P_{Ti}$  denote market prices of domestic product, imports, and so on.

We have not yet discussed how price data are to be obtained. In some particular cases, we may have supplementary quantity data, for labor employment in person hours, for instance, or land employment in hectares, or energy usage in millions of tonnes of oil equivalent, from which prices can be inferred. More often, there are no natural quantity units, and we have to choose our own. A natural choice is to set quantity units so that for each input, the market price of one unit is one unit of currency. Then quantities are numerically equal to money values, and direct requirements coefficients are equal to shares in total costs, which we can readily calculate from the table. Furthermore, the cost exhaustion condition (1) simplifies to an adding-up condition:

$$\sum_{i=1}^c D_{ij} + \sum_{i=1}^c M_{ij} + \sum_{i=1}^f E_{ij} + \sum_{i=1}^t T_{ij} = 1.$$

## 2.2 Total requirements coefficients

In to the Leontief quantity model, the quantity of domestic product of commodity  $i$  used directly in the production of commodity  $j$ ,

$$x_{ij} = D_{ij}y_j,$$

where  $y_j$  denotes quantity produced of commodity  $j$ . So domestic production  $(y_1, \dots, y_c)' = \mathbf{y}$  of all commodities entails intermediate usage  $\sum_j D_{ij}y_j$  of domestic product of each commodity  $i$ , that is, usage  $\mathbf{D}\mathbf{y}$  of domestic product of all commodities. Then, to meet total final demand  $\mathbf{z}$  for domestic product, domestic production  $\mathbf{y}$  must cover both final demand and intermediate usage,

$$\mathbf{y} = \mathbf{D}\mathbf{y} + \mathbf{z}, \quad (2)$$

Solving for  $\mathbf{y}$ , we obtain

$$\mathbf{y} = (\mathbf{I} - \mathbf{D})^{-1} \mathbf{z} = \mathbf{D}^* \mathbf{z}, \quad (3)$$

where  $\mathbf{I}$  denotes a suitably sized identity matrix, and

$$\mathbf{D}^* = (\mathbf{I} - \mathbf{D})^{-1}. \quad (4)$$

Differentiating equation (3), holding requirements coefficients fixed, we obtain

$$d\mathbf{y} = \mathbf{D}^* d\mathbf{z}. \quad (5)$$

That is, an increase  $dz_j$  in final consumption of domestic product of commodity  $j$  entails an increase  $dy_i = D_{ij}^* dz_j$  in domestic production of commodity  $i$ ; so  $\mathbf{D}^*$  is a matrix of *total requirements coefficients for domestic product*.

Writing  $\mathbf{m}$  for the vector of imports required to support domestic production  $\mathbf{y}$  (not the vector of total imports, since it excludes imports for final consumption), we have

$$\mathbf{m} = \mathbf{M}\mathbf{y} = \mathbf{M}\mathbf{D}^* \mathbf{z} = \mathbf{M}^* \mathbf{z}, \quad (6)$$

where

$$\mathbf{M}^* = \mathbf{M}\mathbf{D}^* \quad (7)$$

is a matrix of *total requirements coefficients for imports*. Likewise we define matrices  $\mathbf{E}^* = \mathbf{E}\mathbf{D}^*$  and  $\mathbf{T}^* = \mathbf{T}\mathbf{D}^*$  of total requirements coefficients for endowments and taxes.

From the adding up condition (1) for direct requirements coefficients, we may derive cost exhaustion condition for total requirements coefficients: for all  $j$  from 1 to  $c$ ,

$$\sum_{i=1}^c P_{Mi} M_{ij}^* + \sum_{i=1}^f P_{Ei} E_{ij}^* + \sum_{i=1}^t P_{Ti} T_{ij}^* = P_j. \quad (8)$$

This amounts to the intuitive assertion that the value of domestic product is exhausted by the value of total requirements for imports, endowments, and taxes. For proof, post-multiply by  $D_{jk}^*$  and

(on the right hand side) use equation (4), in the form  $\mathbf{D}^* = \mathbf{I} + \mathbf{D}\mathbf{D}^*$ . With unit prices, the cost exhaustion condition simplifies to an adding-up condition:

$$\sum_{i=1}^c M_{ij}^* + \sum_{i=1}^f E_{ij}^* + \sum_{i=1}^t T_{ij}^* = 1.$$

### 2.3 Effects of changes in direct requirements coefficients

In the previous section, we obtained a formula (equation (5)) for change in output, holding requirements coefficients fixed. But in a CGE model, such as GTAP, requirements coefficients are not fixed; they change in response to changes in prices and, in some simulations, technology. We now consider changes in total requirements coefficients and in input usage when direct requirements coefficients vary.

We begin with coefficients for usage of domestic product. For domestic product, from equation (4), we have

$$\mathbf{D}^*(\mathbf{I} - \mathbf{D}) = \mathbf{I}.$$

Differentiating and rearranging, we obtain

$$(d\mathbf{D}^*)(\mathbf{I} - \mathbf{D}) = \mathbf{D}^*d\mathbf{D}.$$

Postmultiplying by  $(\mathbf{I} - \mathbf{D})^{-1}$ , and substituting from equation (4), we obtain a closed-form solution

$$d\mathbf{D}^* = \mathbf{D}^*(d\mathbf{D})\mathbf{D}^*. \quad (9)$$

Having obtained changes in total requirements coefficients for domestic product, we can use them to obtain changes in total requirements coefficients for other inputs; for imports, for instance, differentiating equation (7), we obtain

$$d\mathbf{M}^* = (d\mathbf{M})\mathbf{D}^* + \mathbf{M}d\mathbf{D}^*. \quad (10)$$

We now consider changes in output in the presence of changes in requirements coefficients. Differentiating equation (3), and substituting from equations (9) and (3), we obtain

$$d\mathbf{y} = \mathbf{D}^*(d\mathbf{D})\mathbf{y} + \mathbf{D}^*d\mathbf{z}. \quad (11)$$

This is readily interpretable. The second term on the right hand side is just the standard I-O effect of changes in final demand. The first term captures the effects of changes in direct requirements coefficients: a change  $dD_{jk}$  in the direct requirements coefficient for  $j$  in production of  $k$  raises direct requirements for  $j$  in production of  $k$  by  $dD_{jk}z_k$ ; that in turn raises total requirements for  $i$  by  $D_{ij}^*dD_{jk}z_k$ . Not only is the formula interpretable, it is well of interest in simulation analysis: it lets us trace sources of change in demand through changes in direct demand to changes in direct requirements coefficients in upstream industries, and to changes in final demand.

Similar decompositions are available for other variables. For imports for intermediate usage,

for example, differentiating equation (6) and substituting from equations (10), (9), (7), and (3), we obtain

$$d\mathbf{m} = d\mathbf{M}\mathbf{y} + \mathbf{M}^*(d\mathbf{D})\mathbf{y} + \mathbf{M}^*d\mathbf{z},$$

decomposing the total change into the effects of changes in direct requirements coefficients for imports, direct requirements coefficients for domestic product, and final demand. Similar decompositions can be obtained for endowments and taxes.

In simulation analysis, one need not rest content with direct requirements coefficient effects, but may decompose those effects further. One may for instance decompose changes in direct requirements coefficients into price and technology effects; one may then decompose the effects of changes in domestic product prices into the effects of changes in factor prices, border prices of imports, tax rates, and technology. In analyzing changes in tax payments, one may decompose the effects of changes in direct requirements coefficients for input tax chits into changes in tax rates and changes in direct requirements coefficients for physical inputs.

## 2.4 Decomposition of changes in output

In terms of elements of matrices and vectors, we can rewrite equation (11) as

$$dy_i = \sum_{j=1}^c \sum_{k=1}^c D_{ij}^* (dD_{jk}) y_k + \sum_{j=1}^c D_{ij}^* dz_j. \quad (12)$$

This brings out its character as a decomposition: change in output of any one commodity is decomposed into components attributable to  $c^2$  changes  $dD_{jk}$  in direct requirements coefficients, and  $c$  changes  $dz_j$  in final demand.

We can also use equation (2) to obtain a decomposition not involving total requirements coefficients. Differentiating equation (2), we obtain

$$d\mathbf{y} = (d\mathbf{D})\mathbf{y} + \mathbf{D}d\mathbf{y} + d\mathbf{z},$$

or, in scalar notation,

$$dy_i = \sum_{j=1}^c (dD_{ij}) y_j + \sum_{j=1}^c D_{ij} dy_j + dz_i. \quad (13)$$

This decomposes change in output of a commodity  $i$  into components attributable to  $c$  changes  $dD_{ij}$  in direct requirements coefficients,  $c$  changes  $dy_j$  in output, and one change  $dz_i$  in final demand. This is close to the natural decomposition obtainable from CGE simulation results, in which the terms  $(dD_{ij})y_j$  and  $D_{ij}dy_j$  taken together represent the component attributable to change in intermediate usage in production of  $j$ , and the component attributable to change  $dz_i$  in final demand is separated into individual final demand categories. For ease of reference, we may call equation 13 the *direct decomposition* and equation 12 the *indirect decomposition*.

The direct decomposition, as already remarked, is more readily calculated, since it does not involve total requirements coefficients. Another key difference is that the direct decomposition involves changes  $dy_j$  in output, while the indirect decomposition does not. Indeed, one motivation

for the indirect decomposition is a desire to move beyond output changes as explanatory factors. A change in output of textiles for example, might be explained in the direct decomposition by a change in output of clothing, but in the indirect decomposition, by a change in final demand for clothing. Or a change in output of coal might be explained in the direct decomposition by a change in output of electricity, but in the indirect decomposition by a change in direct requirements coefficients for electricity in production of various commodities.

In a CGE simulation analysis, one would probably not use the indirect decomposition in the form given above (equation (12)), but would elaborate it to a greater or lesser extent. Change in final demand, first of all, might be decomposed into changes in individual demand categories. These in turn might be further decomposed; for example, change in consumer demand might be decomposed into income, own-price, and cross-price effects. Changes in direct requirements coefficients might be decomposed into commodity substitution effects and Armington substitution effects.

### 3 Implementation in GTAP

Calculating total requirement coefficients in GTAP might mean either calculating them from a GTAP database, or updating them through a GTAP model simulation. We provide both facilities. In so doing, we encounter the following considerations:

- Since calculating the initial levels of the total requirements coefficients for domestic product involves solving a system of simultaneous equations (matrix equation (4) or its equivalent), it cannot be done in the model solution program, but must be done in advance, and the results must be stored in the database.
- Given the total requirements coefficients for domestic product, the total requirements coefficients for all other inputs can be calculated from formulae (e.g., equation (7)). So levels for those other total requirements coefficients need not be calculated in advance nor stored in the database.
- To calculate total requirements coefficients on a consistent basis from initial and updated databases, we need consistent prices, so prices must be stored in the database and updated through the simulation.

We provide then the following facilities:

- an extended GTAP model, including a database structure extended to hold total requirements coefficients for domestic product, and market prices for all inputs, and a solution program extended to calculate total requirements coefficients for all inputs;
- a program for calculating total requirements from an extended GTAP database, consistent with the prices stored therein, or with some other supplied set of prices; and
- a program for extending the GTAP database.

The last two facilities share a common core, a program that solves a system of simultaneous equations to calculate total requirements coefficients for domestic product.

We have assumed so far that for prices we use market prices, and that those prices are uniform across uses. For a static calculation that assumption is not strictly necessary, and may sometimes be undesirable. For example, if we have independent data for energy usage in money values and in energy units, they are unlikely to support that assumption. But if prices vary across uses, then changes in output, as calculated from the quantity data, are liable to differ from changes as calculated from the money value data, so the I-O and the CGE results are liable to be at odds. So we maintain our assumption that every good in the TRC system is priced uniformly across uses.

That imposes some constraints on our classification of inputs in the TRC calculation, specifically, on the classification of taxes and of endowments. In the GTAP theory, prices of sluggish endowments vary across industries. We can set them equal, if we please, in the base state, but in the course of a simulation they will diverge. To accommodate this, we must treat sluggish endowments employed by different industries as different inputs. Likewise, since we identify the price of a tax chit with the commodity to which the tax attaches, we must treat taxes on different commodities, and on domestic and imported varieties, as separate inputs; likewise, taxes on different endowments, on sluggish endowments employed in different industries, and on different outputs.

So the four matrices of direct requirements coefficients in the general theory become, in the GTAP context, nine arrays. Also, since the model incorporates not just one but many regions, each array acquires a region subscript. We have:

$D_{ikr}$	DRC for use of domestically produced tradable $i$ in production of tradable $k$ in region $r$
$M_{ikr}$	DRC for use of imported tradable $i$ in production of tradable $k$ in region $r$
$E_{Mikr}$	DRC for employment of tradable $i$ in production of tradable $k$ in region $r$
$E_{Sijkr}$	DRC for employment of tradable $i$ specific to production of %2 in production of tradable $k$ in region $r$
$T_{Dikr}$	DRC in production of tradable $k$ in region $r$ for taxes on domestically produced tradable $i$
$T_{Mikr}$	DRC in production of tradable $k$ in region $r$ for taxes on imported tradable $i$
$T_{EMikr}$	DRC in production of tradable $k$ in region $r$ for taxes on tradable $i$
$T_{ESijkr}$	DRC in production of tradable $k$ in region $r$ for taxes on tradable $i$ employed in production of %2
$T_{Ojkr}$	DRC in production of tradable $k$ in region $r$ for taxes on output of tradable $j$

For total requirements coefficients we use similar symbols with asterisk superscripts:  $D_{ikr}^*$ , for instance, for total requirements coefficients for domestic product.

In three of these arrays of direct requirements coefficients, one index is arguably redundant. In  $T_{Ojkr}$ , for instance, the subscript  $j$  is arguably redundant, since employment in the production of commodity  $k$  of tax chits for output of commodity  $j$  is zero for every  $j \neq k$ . The same is not true however for the array  $T_{Ojkr}^*$  of total requirements coefficients. For example, the direct requirements coefficient for use in the production of petroleum and coal products of tax chits for output of crude



Table 1: Array structure for data for market prices

Symbol	Header	Dimensions	Description
$P_{Dir}$	PDM	T*R	price of %1 produced in %2
$P_{Mir}$	PIM	T*R	price of %1 imported into %2
$P_{EMir}$	PEMM	EM*R	price of %1 in %2
$P_{ESijr}$	PESM	ES*T*R	price of %1 specific to production of %2 in %3

oil is necessarily zero, but the total requirements coefficient may be non-zero; indeed, in any oil producing country, a non-zero value would be likely; in Mexico, notably, it is as high as 0.36.

Similarly, in the array  $E_{Sijkr}$  of direct requirements coefficients for sluggish endowments, and the array  $T_{ESijkr}$  for taxes on such endowments, elements with  $j \neq k$  are necessarily zero, but the corresponding total requirements coefficients may be non-zero.

Before discussing the programs for calculating and using direct and total requirements coefficients, we describe the data structures read and written by them. In describing the contents of the arrays, we use the notation “% $n$ ” to refer to the entity indicated by the  $n$ th index of the array; for example, in an array of total requirements for domestic product, in which the first and second indices range over tradable commodities and the third over regions, “TRC for use of domestically produced %1 in production of %2 in %3” means “TRC for use of the domestically produced tradable indicated by the first index, in production of the tradable indicated by the second index, in the region indicated by the third index”. In stating the dimensions of the arrays, we use the abbreviations:

EM : ENDWM\_COMM : mobile endowment commodities  
ES : ENDWS\_COMM : sluggish endowment commodities  
R : REG : regions  
T : TRAD\_COMM : tradable commodities

To compute or update total requirements coefficients we need market price data (though *assumptions* or *conventions* might be a better word, since the empirical content is typically nil). We store these in a header array file as four arrays, as shown in table 1. Such arrays are contained in the extended GTAP data base, but may also be provided in a separate file to support a static calculation. Note that we provide no separate price information for tax chits; the assumption that they take their prices from the commodities to which they attach is fixed in our programs.

From prices and from I-O data in monetary units, we calculate direct requirements coefficients, and for these we use the array structure shown in table 2. A similar structure is used for total requirements coefficients (table 3). In static calculations of direct and total requirements coefficients, we generate the full nine-array structure; in the extended GTAP database, we include just total requirements coefficients for domestic product (header TRD of table 3).

Table 2: Array structure for direct requirements coefficients

Symbol	Header	Dimensions	Description
$D_{ikr}$	DRD	T*T*R	DRC for use of domestically produced %1 in production of %2 in %3
$M_{ikr}$	DRM	T*T*R	DRC for use of imported %1 in production of %2 in %3
$E_{Mikr}$	DREM	EM*T*R	DRC for employment of %1 in production of %2 in %3
$E_{Sijkr}$	DRES	ES*T*T*R	DRC for employment of %1 specific to production of %2 in production of %3 in %4
$T_{Dikr}$	DTD	T*T*R	DRC in production of %2 in %3 for taxes on domestically produced %1
$T_{Mikr}$	DTM	T*T*R	DRC in production of %2 in %3 for taxes on imported %1
$T_{EMikr}$	DTEM	EM*T*R	DRC in production of %2 in %3 for taxes on %1
$T_{ESijkr}$	DTES	ES*T*T*R	DRC in production of %3 in %4 for taxes on %1 employed in production of %2
$T_{Ojkr}$	DTO	T*T*R	DRC in production of %2 in %3 for taxes on output of %1

Table 3: Array structure for total requirements coefficients

Symbol	Header	Dimensions	Description
$D_{ikr}^*$	TRD	T*T*R	TRC for use of domestically produced %1 in production of %2 in %3
$M_{ikr}^*$	TRM	T*T*R	TRC for use of imported %1 in production of %2 in %3
$E_{Mikr}^*$	TREM	EM*T*R	TRC for employment of %1 in production of %2 in %3
$E_{Sijkr}^*$	TRES	ES*T*T*R	TRC for employment of %1 specific to production of %2 in production of %3 in %4
$T_{Dikr}^*$	TTD	T*T*R	TRC in production of %2 in %3 for taxes on domestically produced %1
$T_{Mikr}^*$	TTM	T*T*R	TRC in production of %2 in %3 for taxes on imported %1
$T_{EMikr}^*$	TTEM	EM*T*R	TRC in production of %2 in %3 for taxes on %1
$T_{ESijkr}^*$	TTES	ES*T*T*R	TRC in production of %3 in %4 for taxes on %1 employed in production of %2
$T_{Ojkr}^*$	TTO	T*T*R	TRC in production of %2 in %3 for taxes on output of %1

## 4 Total requirements coefficients in the GTAP model solution program

For any variable  $x$ , for the relative differential  $(dx)/x$  of  $x$ , we write  $\hat{x}$ , that is,

$$\hat{x} = \frac{dx}{x}.$$

For any two matrices  $\mathbf{X}$  and  $\mathbf{Y}$  of equal dimensions, for their Hadamard or elementwise product, we write

$$\mathbf{X} \circ \mathbf{Y}.$$

Then, for any matrix  $\mathbf{X}$ ,

$$d\mathbf{X} = \hat{\mathbf{X}} \circ \mathbf{X} = \mathbf{X} \circ \hat{\mathbf{X}}.$$

To obtain an equation for total requirements coefficients (in percentage changes) we return to equation 4. From this we obtain the pleasantly interpretable

$$\mathbf{D}^* = \mathbf{I} + \mathbf{D}^* \mathbf{D}.$$

The first term on the right represents the final demand component of total requirements; obviously, a unit of final demand for product  $k$  contributes one unit directly to total requirements for product  $k$ , but none to the requirements products  $i$  other than  $k$ . The second term represents the intermediate usage component: each unit of final demand for product  $k$  entails  $D_{jk}$  units of direct inputs of each product  $j$  into production of  $k$ , and those entail, directly and indirectly,  $\sum_j D_{ij}^* D_{jk}$  units of intermediate usage demand for product  $i$ . Differentiating, we obtain

$$d\mathbf{D}^* = (d\mathbf{D}^*) \mathbf{D} + \mathbf{D}^* d\mathbf{D},$$

that is,

$$\mathbf{D}^* \circ \hat{\mathbf{D}}^* = \mathbf{D}^* \mathbf{D} \circ \hat{\mathbf{D}} + \hat{\mathbf{D}}^* \circ \mathbf{D}^* \mathbf{D}.$$

## 5 Calculating total requirements coefficients from the GTAP database

The GTAP database incorporates  $r$  I-O tables, where  $r$  is the number of regions distinguished. We could split out the  $r$  I-O tables, and calculate total requirements for each one separately; and we should do so if necessary for computational tractability. We find however that we can solve readily for all regions simultaneously.

In broad outline, the procedure is:

1. Calculate direct requirements coefficients ( $\mathbf{D}$ ,  $\mathbf{M}$ ,  $\mathbf{E}$ ,  $\mathbf{T}_D$ ,  $\mathbf{T}_M$ ,  $\mathbf{T}_E$ ,  $\mathbf{T}_O$  in the notation of sections 2 and 3).
2. Solve for total requirements coefficients for domestic product ( $\mathbf{D}^*$ ), following equation (4).
3. Calculate total requirements coefficients for imports, endowments, and taxes ( $\mathbf{M}^*$ ,  $\mathbf{E}^*$ ,  $\mathbf{T}_D^*$ ,  $\mathbf{T}_M^*$ ,  $\mathbf{T}_E^*$ ,  $\mathbf{T}_O^*$ ), following equations (7) etc.

We perform the calculation with the Cygwin-for-GEMPACK tool set described in section 2 of the Cygpack manual ([1]). Required elements of the operating environment include Microsoft Windows, a GEMPACK-compatible Fortran compiler, GEMPACK, the Cygwin base and Make packages, and Cygpack.

The data inputs are a GTAP sets file ‘gsdset.har’ and state data file ‘gsddat.har’. The main data output is the file ‘trc.har’ of total requirements coefficients, but the file ‘drc.har’ of direct requirements coefficients may also be of interest.

To calculate direct requirements coefficients, we run a TABLO-generated program ‘drc\_calc’. That writes to ‘drc.har’ seven arrays:

DRD :  $\mathbf{D}$  : direct requirements coefficients for domestic product  
 DRM :  $\mathbf{M}$  : direct requirements coefficients for imports  
 DRE :  $\mathbf{E}$  : direct requirements coefficients for endowments  
 DRTD :  $\mathbf{T}_D$  : direct requirements coefficients for tax on domestic product  
 DRTM :  $\mathbf{T}_M$  : direct requirements coefficients for tax on imports  
 DRTE :  $\mathbf{T}_E$  : direct requirements coefficients for tax on endowments  
 DRTO :  $\mathbf{T}_O$  : direct requirements coefficients for tax on output

The program also checks the summation condition (1). That is, for all regions, for all sectors  $j$ , it calculates the expression

$$\sum_{i=1}^c D_{ij} + \sum_{i=1}^c M_{ij} + \sum_{i=1}^f E_{ij} + \sum_{i=1}^c T_{Dij} + \sum_{i=1}^c T_{Mij} + \sum_{i=1}^f T_{Eij} + \sum_{i=1}^c T_{Oij} - 1,$$

which by the summation condition should equate to zero, and writes it to an array “ERR” in a file ‘drc\_err.har’. Also, it exits unsuccessfully if any element of the array exceeds in absolute magnitude a tolerance TOL, calculated as  $2.0^{-\text{TOLPAR}}$ , where TOLPAR is read from a file ‘drc.stdin’. In the current release, TOLPAR is set to 16.0.

To calculate total requirements coefficients  $\mathbf{D}^*$  for domestic product, we run a TABLO-generated program ‘trcc\_solve’. This solves the equation system (4), rewritten as

$$\mathbf{D}^* = \mathbf{I} + \mathbf{D}^* \mathbf{D}. \quad (14)$$

So written it has an intuitive interpretation: to provide one unit of commodity  $k$  for final consumption, there is required of commodity  $i$ , first one unit for final consumption, if  $i = k$ . But also, production of one unit of  $k$  entails direct requirements of  $D_{jk}$  of each commodity  $j$ , and production of that entails total requirements  $D_{ij}^* D_{jk}$  of  $i$ .

In writing ‘trcc\_solve’ we abuse TABLO, which is designed to solve a linearized system of equations, in order to solve a linear system. There are just two variables, “trc”, representing the

array  $\mathbf{D}^*$  of total requirements coefficients, and “one”, a scalar which should always be exogenous and always receive a shock of 1.0. The solution method is of course Johansen.

Having obtained a solution for total requirements coefficients for domestic product as variable “trc” in file ‘trcc.sl4’, we use the GEMPACK utility SLTOHT to convert it to a header array file ‘trcc.har’.

Finally, we run a TABLO-generated program ‘trc\_comp1’ to calculate total requirements coefficients for imports, endowments, and taxes, according to equations (7) etc. To ‘trc.har’ it writes seven arrays:

TRD :  $\mathbf{D}^*$  : total requirements coefficients for domestic product  
 TRM :  $\mathbf{M}^*$  : total requirements coefficients for imports  
 TRE :  $\mathbf{E}^*$  : total requirements coefficients for endowments  
 TRTD :  $\mathbf{T}_D^*$  : total requirements coefficients for tax on domestic product  
 TRTM :  $\mathbf{T}_M^*$  : total requirements coefficients for tax on imports  
 TRTE :  $\mathbf{T}_E^*$  : total requirements coefficients for tax on endowments  
 TRTO :  $\mathbf{T}_O^*$  : total requirements coefficients for tax on output

The program also checks the summation condition (8). That is, for all regions, for all sectors  $j$ , it calculates the expression

$$\sum_{i=1}^c M_{ij}^* + \sum_{i=1}^f E_{ij}^* + \sum_{i=1}^c T_{Dij}^* + \sum_{i=1}^c T_{Mij}^* + \sum_{i=1}^f T_{Eij}^* + \sum_{i=1}^c T_{Oij}^* - 1,$$

and writes it to an array “ERR” in a file ‘trc\_err.har’. Also, it exits unsuccessfully if any element exceeds in absolute magnitude a tolerance TOL, calculated as  $2.0^{-\text{TOLPAR}}$ , where TOLPAR is read from a file ‘trc.stdin’. In the current release, TOLPAR is set to 16.0. In short, the error-checking arrangements parallel those for the direct requirements coefficients.

## 6 Future

Opportunities for future work include:

- Calculate total requirements coefficients for income taxes and import duties.
- Solve for total requirements coefficients region by region, for a user-defined subset of the database regions.

## References

- [1] Robert M<sup>c</sup>Dougall and Badri Narayanan. *CygpacK: Utilities for working with GEMPACK under Cygwin*. Center for Global Trade Analysis, Purdue University, 2013. Release 1.