Toward an Optimal Economic Development Strategy: 
 a Shannon Entropy Index Analysis of 
 Export Expansion and Import Substitution

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Abstract

Two important strategies of local economic development include export expansion and import substitution. We assume that the former causes the latter until there is an optimal combination of both. A social accounting model of an economy represented as the product of a Leontief inverse closed to include households and a diagonalized vector of final demand generates measures of both export base output by sectors summed down the column and gross output added along the rows. We assume that gross output reflect a measure, in part, of the size of import substitution while export base output reflects the quantity of export expansion.

With this data we can address the following questions. What is the difference in sector diversity between production for local consumption and for exports as the diversity of export production increases in a region? Is there a range in this difference in sector diversity that corresponds to the greatest positive effect in per capita income? To measure sector diversity, we applied a Shannon entropy index to the base and gross measures of output across twenty sectors for all the counties in North Carolina, USA (n=100). We checked for spatial dependency using a spatial error model and a Moran’s I-test.

For North Carolina, the standard interaction between base diversity and the range of income per capita observed in counties across the state reveals that income increases when a normalized Shannon index of export sector diversity ranges approximately from 0.65 to 0.70 and import substitution sector diversity leads by about 10 percent. Beyond this range, ceteris paribus, the greater the export sector diversity the greater the income per capita.

Our interpretation of these measures suggests that the optimal range of export expansion and import substitution lies between company town with high import substitution
diversity and low export diversity, on the one hand, and transfer-dependent towns where the diversity in sectors producing for local and export consumption is nearly equal, on the other. This result is consistent with the common sense notion that company towns need to increase export diversity to add stability to their economy, while transfer dependent regions need to increase their production for local consumption through import substitution to increase income. Now with parameters applied to common sense notion of economic development, not only can a community measure their progress but also knows when they have reached an optimal range of export and import substitution diversity to maximize stability and income.

**The Problem:**
A Proper Mix of Economic Development Strategies
There are three broad strategies for economic development: export enhancement, import substitution, and increasing productivity—whether technological, biological, or institutional. Cooke & Watson show that when competitive advantage is equal for either export enhancement or import substitution activity, the economic impact would be about the same for marginal changes. However import substitution has a better claim as a long run strategy, because it deepens the inter-industry trade and associated multipliers. “Thus, a discrete unit of import substitution creates unambiguously more economic activity in the local economy than a discrete unit of export enhancement, assuming the identical comparative advantage of both strategies” (2011).

On another front, economists have worked unsuccessfully to find a strong connection between economic diversity and income and employments. For example, Attaran states: “The results suggest that no strict assumptions should be made regarding a clear relationship
between economic diversity and growth and stability of unemployment, and per capita income-based measure of economic performance” (1986).

Measures of economic diversity have depended on the standard measures of output, employment, wages and value added by industry such as those reported by the US Bureau of Economic Analysis (USDC. BEA, 2014a, 2014b, 2014c, 2014d). We referred to this set of four measures collectively as gross contributions or gross measures—defined as each sectors’ observed portion of economic activity used, directed or generated in the process of meeting both foreign and domestic exports as well as local demand.

Waters, Weber and Holland demonstrated an approach that measures economic activity in relation to its support of the export base or simply “base” contributions of each sector (1999). Base contributions to economic activity are defined as both the observed and unobserved portion of economic output across all sectors—including indirect and induced effects—needed to produce a given sector’s direct effect of domestic and foreign exports. The Waters et al. approach consists of multiplying the Leontief inverse with a diagonalized matrix of exogenous final demand to estimate base output—from which employment, wages and value added can be determined as a proportion of output. The history of economic thought regarding base output derived from the relationship between the multiplier and exogenous final demand extends at least as far back as Keynes (Dimand, 1988).

We argue that part of the difficulty in finding a relationship between diversity and income and employment or economic development more generally hinges on the absence of base measures of economic activity with which to derive this relationship. In addition, we argue that the interaction of gross and base measures with each other and with income can lead to a
better understanding of the development process associated with the broad strategies of export expansion and import substitution as outlined by the seminal works in the literature (Heckscher, 1955; Krugman, 1991; Myrdal, 1957; Ohlin, 1967; Romer, 1986; Rostow, 1962; Solow, 1956). Both export expansion and import substitution strategies depend on productivity as the source of competitive advantage as a prerequisite before either strategy will succeed. For reasons we describe below in more detail but largely definitional in nature, we believe that the gross measure of economic activity provide a means to assess the role of import substitution. Conversely, base measures can be used to represent the extent to which export expansion drives an economy. Together export expansion and import substitution work in concert to produce a synergy called economic development. We will use Shannon diversity indexes of gross and base measures of economic activity across the 100 counties of North Carolina’s heterogeneous economic landscape in an attempt to find a combination that is most conductive to increasing income, employment and stability.

**Theory: Shannon Index of Gross & Base Economic Diversity Measures**

Diversity contains two elements: type and abundance—regardless of whether the diversity under consideration involves biology, information or physics. In economics, for example, the sectors of an economy represent an array of types such as the one through four digit NAICS codes associated with ever greater disaggregation of sectors into subsectors. Measures of output, employment, wage bill or value added denotes a sector’s abundance.

A confusing number of diversity indexes exist. However, Jost provides an insightful discussion of the unifying characteristics of several widely used diversity indexes including: species richness, Shannon entropy index, Simpson concentration index, Gini-Simpson index,
HCDT entropy index and the Renyi entropy index (2006). Jost describes the Shannon entropy index as “the most profound and useful of all the diversity indices (p. 364), in part because it weights each type in proportion to its abundance and because it can be derived from a generalized model of diversity (Shannon, 1948). Though often confused, the Shannon entropy index of diversity differs from the Shannon measure of diversity. The Shannon entropy index of diversity, when expressed in logarithms of base two, represents the average number of yes/no guesses needed to determine, for example, the sector in which a random worker works or output’s produced. See equations (1) and (2). The problem with using these entropy indices of diversity relates to the problem that similar entropy indices when the true diversity between regions is nearly identical “may mean they are moderately similar or may mean they are completely different” (Jost, 2006, p. 366).

The Shannon measure of diversity, when expressed in antilogarithms of indices in the base e, signifies the effective number of equivalents types relative to a maximum of N possible types. See equations (3) and (4). The effective number refers to abundance-corrected equivalent types expressed as the number of types (same or different) with equally common abundance (Jost, 2006, p. 364). For example, the concept of full-time equivalent employment (FTE) is the effective number of part-time employees after correcting for an equivalent forty hour work week over a year as expected from full-time employees—making possible a comparison, through equivalents, two different types of employment. In the case of a two-digit 20-sector economy the diversity measure will be expressed a number between 1 and 20. In this economic context, an effective number of one implies these regions have an equal (or equally unequal) abundance of output (employment, income, value added) in one sector (same or
different) relative to the other nineteen. Conversely, an effective number of twenty suggests that regions with this number have an equally equal distribution of output across all twenty sectors. Effective numbers on a continuum between one and twenty represent similar regions with unique combinations of equal distribution of output among the twenty sectors (same or different) and parts thereof.

Because the base measures in this study includes nine types of households by income, the number of base “sectors” (N=29) is always greater than that for the gross (M=20). Therefore we need a way to compare diversity measures that accounts for the differences in the number of types. Jost suggest a linear transformation that provides a measure of diversity that is always between zero and one. See equations (5) and (6).

**Shannon Entropy Index and Diversity Measure**

$H_B$; Shannon Entropy Index of Base or Export Base Diversity

\[ H_B \equiv -\sum \left( \frac{Q_{Bi}}{\sum_i Q_i} \log_2 \left( \frac{Q_{Bi}}{\sum_i Q_i} \right) \right), \quad \text{sector } i = 1 \ldots n. \]  \hspace{1cm} (1)

$H_G$; Shannon Entropy Index of Gross or Import Substitution Diversity

\[ H_G \equiv -\sum \left( \frac{Q_{Gj}}{\sum_j Q_j} \log_2 \left( \frac{Q_{Gj}}{\sum_j Q_j} \right) \right), \quad \text{sector } j = 1 \ldots m, \]  \hspace{1cm} (2)

(Shannon, 1948, p. 394).

$D(H_B)$; Shannon Measure of Base Diversity

\[ D(H_B) \equiv \exp \left( -\sum \left( \frac{Q_{Bi}}{\sum_i Q_i} \log_2 \left( \frac{Q_{Bi}}{\sum_i Q_i} \right) \right) \right) \cdot \ln(2). \]  \hspace{1cm} (3)

$D(H_G)$; Shannon Measure of Gross Diversity
\[
D(H_G) \equiv \exp \left( -\sum_j \left( \frac{Q_{Gj}}{\sum_j Q_j} \log_2 \left( \frac{Q_{Gj}}{\sum_j Q_j} \right) \right) * \ln(2) \right),
\]
(Jost, 2006, p. 365).

\[S_B: \text{Normalized Shannon Measure of Base Diversity } [0,1]\]
\[S_B \equiv (D(H_B)/N -1/N)/(1-1/N).\]
\[S_G: \text{Normalized Shannon Measure of Gross Diversity } [0,1]\]
\[S_G \equiv (D(H_G)/M -1/M)/(1-1/M),\]
(Jost, 2006, p. 367).
Data: Base and Gross Measures of Output

A region’s firms use intermediate inputs (other sectors’ goods and services to produce their own) as well as labor, capital, government services and imports to supply their goods and services. Regional production goes to meet the consumption demands of local, domestic and foreign institutions including households, investors, governments, and for export. In addition, economic data shows the transfer of payments made from the value added by labor, capital and taxes among households, investors, and governments. This comprehensive accounting approach of production, consumption plus transfers represents a social accounting matrix (SAM) of the local economy.

The (I-A) SAM matrix for a region’s economy is shown in eq. 7, where a is the factor share, x is total output, and d is exogenous final demand across sectors 1 through n.

\[
\begin{bmatrix}
-(1-a_{11}) & -a_{12} & \ldots & -a_{1n} \\
-a_{21} & (1-a_{22}) & \ldots & -a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
-a_{n1} & -a_{n2} & \ldots & (1-a_{nn})
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix}
= 
\begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
d_n
\end{bmatrix}
\]

\( (7) \)

Representing the matrix format with single terms in which matrices are expressed in compact notation as capital letters and vectors are in lower case.

\[
(I - A)x = d
\]

Solving the vector of output (x) in terms of the matrix of coefficients (I-A) and final demand (d):

1. \[
\begin{bmatrix}
-(1-a_{11}) & -a_{12} & \ldots & -a_{1n} \\
-a_{21} & (1-a_{22}) & \ldots & -a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
-a_{n1} & -a_{n2} & \ldots & (1-a_{nn})
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix}
= 
\begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
d_n
\end{bmatrix}
\]

2. \[
(I - A)x = d
\]
Equation (8) shows that the output for each sector equals the final demand for that
sector multiplied by the inverse of the coefficient matrix \((I-A)^{-1}\). Output by sector in eq. 8 \((x^*)\),
we have defined as gross output. A sector’s gross output equals intermediate demand for itself
and by other sectors, local final demand—typically by households—domestic and foreign final
demand by governments, investors and exports.

Alternatively, an avenue of insight into the export base of an economy comes from
diagonalizing the vector of exogenous final demand \((d)\). To diagonalize means to place these
values of final demand along the major diagonal of a matrix with zeroes in the remaining off-
diagonal elements (Waters et al., 1999).

Eq. 8 is modified to include the change in the exogenous final demand vector to a
diagonalized matrix.

\[
x^* = \left( I - A \right)^{-1} d
\]

\[
X^* = \left( I - A \right)^{-1} \text{diag}(d)
\]

The matrix multiplication of the augmented Leontief inverse and the diagonalized matrix
of exogenous final demand reveals the demand for own and other sectors’ inputs needed to
produce a given sector’s exports. The sum of these elements across the row equals the total
industrial output for the sector, what we have defined as gross output. In addition, the sum of
the elements down the column for the sectors equals the export base of that sector. The export
base of a sector is the sum of the output across all sectors needed to produce the exports of a
given sector. These column sums measures of output, we define as export base output or
simply base output. The sum of base and gross output across sectors are equal, but not equal by sector.

The exports by sector are on the major diagonal—households and governments have transfers rather than exports—plus the indirect and induced effects for that sector (also included on the major diagonal) and other sectors’ output (off diagonal) also needed to produce the exports of the sector.

**Figure 1. Mecklenburg County, NC Base and Gross Output for the Top Ten Sectors in: 2012 ($1,000)**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Direct Base output</th>
<th>Indirect Base output</th>
<th>Total Gross output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary authorities and depository credit intermediation activities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Management of companies and enterprises</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wholesale trade businesses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Securities, commodity contracts, investments, and related activities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* Employment and payroll only (state &amp; local govt, education)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real estate establishments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Households (low tier)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Households (middle tier)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construction of other new nonresidential structures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Insurance carriers</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The data for this study includes the gross and base output data at the 20-sector level of aggregation for each of the one hundred counties in North Carolina, USA in 2012. For example, Fig. 1 shows the difference between gross and base output for the top ten sectors in Mecklenburg County, North Carolina. These are the data to which we derive the Shannon diversity measure to determine base and gross sector diversity by county.

Functional Form of Gross and Base Economic Diversity

Assume the demand for exports creates a derived demand for locally produced inputs, through import substitution. In particular, assume that the normalized diversity of exports \( D(H_B) \) drives the diversity of import substitution \( D(H_G) \) at rate \( r \) over time \( t \).

\[
S_G = S_B (1 + r)^t
\]  

(10)

Let \( t = 1 \) and solve eq. 10 for \( r \),

\[
\frac{S_G - S_B}{S_B} = r
\]  

(11)

Let the difference in normalized gross and base diversity in a region be a function of normalized base diversity, holding the per capita income constant.

\[
S_G - S_B = f(S_B, Y)
\]  

(12)

Interpretation

To understand these results, let’s examine the elements of gross and base output. Gross output for any sector equals the sum of intermediate demand plus endogenous and exogenous final demand.
Exogenous endogenous final demand represents a given sector’s supply use to support local production and consumption by other sectors and institutions. Without this local supply or locally-provided close substitutes, local producers and consumers are forced to meet their demand by substituting foreign or domestic imports. For this reason, we consider the intermediate and endogenous final demand portions of output of a sector to be a measure of import substitution.

Base output of a sector equals the sum of its direct, indirect and induces output:

\[ Q_B = Q_{Direct} + Q_{Indirect} + Q_{Induced} \]

The indirect and induced output represents the additional sales generated across the economy from the exogenous sales from a sector. Because exogenous final demand from domestic and foreign exports creates this additional economic activity, we consider the indirect and induced effects measures of output due to export expansion.

The difference between gross and base output equals the difference between their respective elements:

\[ Q_G - Q_B = Q_{Intermed} + Q_{FD_{Endog}} + Q_{FD_{Exog}} - (Q_{Direct} + Q_{Indirect} + Q_{Induced}) \]  

(13)

By definition and from eq. 7, we know that the exogenous final demand in the gross measure of output equals the direct output in the base measure such that:

\[ Q_{FD_{Exog}} = Q_{Direct} \]

By substitution, eq. (13) reduces to:
\[ Q_G - Q_B = Q_{\text{Intermed}} + Q_{\text{FDIndeg}} - (Q_{\text{Indirect}} + Q_{\text{Induced}}) \]

We know from eq. (9) that:

\[ \sum_i(Q_{\text{Intermed},i} + Q_{\text{FDIndeg},i}) = \sum_i(Q_{\text{Indirect},i} + Q_{\text{Induced},i}). \]

Equal output does not imply equal diversity. There are sectors producing primarily either for export or for the local market, base or non-base, through export expansion or import substitution. A company town represents an extreme example of the difference between measures of output and diversity. As a base measure, a single sector in a company town generates all the output. However, as a gross measure, many sectors contribute to the indirect and induced effect of the single exporting sector. Gross and base output is equal, while gross and base diversity is unequal.

The work of Cooke and Watson suggest that the diversity of production for the local markets through import substitution may be preferred over diversity of production for exports such that:

\[ S_G(Q_{\text{Intermed}}, Q_{\text{FDIndeg}}) > S_B(Q_{\text{Indirect}}, Q_{\text{Induced}}). \] (14)

When eq. (14) holds, we can assume that the diversity of sectors involved in import substitution exceeds that of sectors focused primarily on export expansion.

When the difference between gross and base diversity is positive, it reveals the additional effective number of sectors more involved in import substitution than export expansion as their primary focus.
\[ S_G Q_{Intermed} Q_{FD_{Induced}} - S_B Q_{Indirect} Q_{Induced} > 0 \]

**Statistical Measures of Shannon Index of Economic Diversity—Base & Gross**

An econometric estimation of eq. 12, gives the following result:

\[
S_G - S_B = 0.72S_B - 1.23S_B^2 + 0.0000156S_B Y \\
\text{adj}R^2 = .53, \quad F = 38.2
\]  

(15)

We checked for spatial dependency using a spatial error model and a Moran’s I-test.

Eq. (15) shows that proportion of effective number of sectors more involved in import substitution than export expansion as their primary focus. The proportion of sectors in North Carolina counties focused on local production increases until the ratio of local production to export diversity is forty-two to thirty percent. This ratio decreases to one when about sixty five percent of the sectors are focused primarily on local production, exports or both. See Figures 2 and 3.

The difference between effective gross and base diversity increases (.25 to .75%) with an increase in Income. See eq. (15) and Fig. 4. Perhaps as income increases households put pressure on sectors to increase local production through endogenous final demand—production that is somewhat less likely to also be exported thereby increasing import substitution diversity.

Solving eq. (15) for income provides insight into the effect of diversity on income.
\[
Y = \frac{641(-720S_B + 1229S_B^2 + 1000(S_G - S_B))}{S_B}
\]  

(16)

Solving eq. (16) with the range of \(S_B\) and \(S_G - S_B\) that includes the current range of \(Y\) found across the counties in North Carolina. See Fig. 5. When diversity progressed from fifty to fifty-five percent of the sectors focused on exports and from fifty-seven to sixty percent of the sectors produced primarily for local production, with some overlap for production in both, then per capita income increased most quickly on average in the counties of North Carolina in 2012.

Counties to the left of the (50-54/.07-.074) window of base diversity to difference in diversity base from gross, tend to have too many sectors focused on import substitution relative to those focused on export expansions. See window in Figures 2 and 4. For counties to the right of the window, the opposite situation holds.
Stages of Development: a Visualizations

Figure 2. Gross and Base Diversity Function across 20 to 29 sectors for the 100 North Carolina Counties, US: 2012
FIGURE 3. THE FUNCTIONAL FORM RELATING GROSS AND BASE DIVERSITY WITH INCOME

FIGURE 4. THE EFFECT OF INCOME ON DIVERSITY
Summary

The resulting coefficients reveal a predominantly negative and non-linear relationship between the difference in gross and base diversity and increasing base diversity—base diversity converges toward gross diversity as base diversity increases such that, when both are highly diverse, the difference between them approaches zero. For North Carolina, the standard interaction between base diversity and the range of income per capita observed in counties across the state reveals that income increases when a normalized Shannon measures of export sector diversity ranges approximately from fifty to fifty-four percent of the sectors and import substitution sector diversity leads by about seven to seven and one-half percent. Beyond this range, ceteris paribus, the greater the export sector diversity the greater the income per capita.
An Implan application of the Shannon-Weaver Diversity Index  

Kay and Cooke

Our interpretation of these measures suggests that the optimal range of export expansion and import substitution lies between company town with high import substitution diversity and low export diversity, on the one hand, and transfer-dependent towns where the diversity in sectors producing for local and export consumption is nearly equal, on the other. These normalized Shannon diversity measures of gross and base sector diversity of output suggest that, if communities in North Carolina wish to increase per capita income, they are well advised to achieve or exceed an export sector diversity of about fifty percent or more with an import substitution sector diversity of around seven percent or greater than export diversity. This result is consistent with the common sense notion that company towns need to increase export diversity to add stability to their economy, while transfer dependent regions need to increase their production for local consumption through import substitution to increase income.

Now with parameters applied to common sense notion of economic development, not only can a community measure their progress but also knows when they have reached an optimal range of export and import substitution diversity to maximize stability and income. In addition, national and state level policy makers can better focus support to the specific needs of a region to help it achieve its goals. An understanding of these relations suggests that import substitution becomes a viable development strategy when tied to the price signal provided by export expansion and knowledge of the interrelationship of the two.
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USDC. BEA. (2014d). SA27 - Full-time and part-time wage and salary employment by industry -- United States (number of jobs): since 1958. from U.S. Dept. of Commerce, Economics and Statistics Administration, Bureau of Economic Analysis [http://www.bea.gov/iTable/iTable.cfm?reqid=70&step=1](http://www.bea.gov/iTable/iTable.cfm?reqid=70&step=1)