Meeting Eigenvectors during a Square Dance on a Hilbert Cube with Interindustry Transactions: A Complex Story

Topic: Mathematical Treatments of Input-Output Relationships
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In recent years we have witnessed many papers using social network analysis, relying on methods of graph theory, in order to explore and evaluate input-output matrices. This has enabled the introduction of new tools for the input-output analysis such as measures of centrality of the nodes (sectors) of a graph; identification and evaluation of main paths who spread economic impulses within the industrial network; cohesion of economic networks, and other measures that help to know the characteristics of individual branches, groups of branches, and finally the topology of an inter-industrial network as a whole. However, some of these measures (the principal ones) face problems of information loss due to the use of algorithms from graph theory that usually cannot cope with asymmetric information inherent in the inter industry tables, so the analysts tend to filter, dichotomize or symmetrize the input-output matrices. To help overcome these limitations, in this article it is shown how to encode the input-output matrices in a complex numbers space. Each entry of the data table is recorded as a complex number, i.e. with two dimensions; one axis (real) for inputs and the other (imaginary) for outputs. Stated in graph theory terms, any node (sector) is linked to the others by two arcs, one for purchases and other for sales.

As a next step, the complex numbers matrix is rotated in order to get its conjugate, so it has been constructed a Hermitian matrix inserted in a Hilbert Space. Every Hermitian matrix is a normal matrix. The finite-dimensional spectral theorem says that any Hermitian matrix can be diagonalized by a unitary matrix, and that the resulting diagonal matrix has only real entries. This implies that all eigenvalues of a Hermitian matrix A are real, and that A has n linearly independent eigenvectors. Moreover, it is possible to find an orthonormal basis of consisting of n eigenvectors of A. Also, the matrix A can be written as a linear combination of orthogonal paired projectors, i.e. spectrally decomposed. The level or intensity of purchases and sales is deployed in the spectrum (set of characteristic values) and its associated eigenvectors show subgroups of related economic sectors through its commercial ties with the most influential sector of each group. This allows interpretation of the system beyond the main eigenpair. In fact, an interpretation for the totality of eigenvalues and associated eigenvectors is achieved.

Due to the novelty of the approach taken, it is necessary to show the main mathematical proofs needed to provide a clear understanding of the meaning of the new measures. Also, it is necessary to inform the reader on the fact that the data contained in the interindustrial flow matrices worked are not pre-processed, i.e. the data are not normalized, and is expressed solely in monetary terms without resorting to any sort of technical or distribution coefficients. Finally, two applications are presented: a) an analysis for a input-output flow table of Mexican economy, and b) an analysis of a bilateral trade matrix, taken from the OECD database.

Keywords: Input-Output Matrix, Hilbert Space, Complex Numbers, Asymmetry, Influential Sectors, Bilateral Trade.