**A DYNAMIC INPUT-OUTPUT MODEL FOR SMALL REGIONS:**

**UPDATED FOR THE MEXICAN CASE**

I. Introduction

In Mexico, attention has been drawn recently to the empirical construction of input-output regional matrices. As a result, a wide range of regional inter-sectoral matrices has been estimated (Armenta, 2007; Chapa, 2009; Cross, 2008; Callicó et al, 2003; Davila, 2002; Fuentes, 2005; Rosales, 2010; Albornoz et al, 2012; Fuentes et al, 2013). In all cases, the multi-sectoral model became a tool providing a basis for programming and for economic projection. However, it is yet to be used to analyze the temporal trajectories of regional variables and to explain how the system changes in time.

The purpose of this paper is to review the basics of the dynamic input-output (I-O) model adapted by Johnson (1986), and used in Bryden (2009) and Alva *et al.* (2011), for a small region, to analyze its dynamic behavior and establish its potential for regional analysis. The multi-sectoral and intertemporal regional model is applied as a study case of Baja California, Mexico (BC). The simulation software used was Stella / IThink (9.1.4).

The formulation of the regional inter-sectoral and intertemporal model arises from two different economic assumptions. First, we consider a balance condition in which the excess demand of each sector always tends to induce adjustments of the product that equals the excess demand. Secondly, we include the capital formation process involving the formation of inventory. Considering both, the model aims to solve issues related to: the degree of utilization of installed capacity, which changes considerably from one sector to another and from one period to another; the structure of delays in the actual process of investment; and the lags in inter-sectoral relations.

It is pertinent to mention that the simulation of the regional model, using the transaction table of Baja California (BC) aggregated to 12 sectors, produces a behavior that is considered satisfactory from the proposed hypotheses’ perspectives.

The study was organized into six sections. The second section explains the basic concepts of the input-output regional model in its static version and points out come of its limitations. The third section provides an analytical solution of the dynamic multi-sectoral regional model considering several limitations. The fourth section presents a variant of the dynamic model adapted by Johnson (1986). In the fifth section, the relations between the different elements of the model are displayed using diagrams. The sixth section discusses the dynamic model for BC, establishing its potential for regional analysis. An additional section reviews conclusions. Reflections are presented in relation to the use and benefits of the regional cross-sectoral dynamic model.

II. Regional input-output static model.

The I-O model offers the possibility of integrating location theory to the analysis of production. In that sense, the I-O regional models assume two possible sources of supply for each sector: local production and imports. The multiregional model, in its static version, can be written by equations (1) and (2).

 (1)

 (2)

(3)

In the balance equation (1) endogenous variables (X) represent sectoral levels of regional production of the *n* sectors of the economy that are expressed by a column vector *nx1*; exogenous variables (Y) are the final demands of the production sectors which are expressed by a vector of order *nx1*; (Xm) is a column vector of *nx1* of competitive regional imports; and (AX) is the intermediate demand, where matrix A, of order *nxn,* is the matrix of technical coefficients or outcomes.[[1]](#footnote-1) Equation (2) assumes that competitive regional imports (Xm) are proportional to the level of activity of the sector , where is a diagonal matrix of marginal import coefficients.[[2]](#footnote-2) Meanwhile, in the analytical solution (3), (X) is the sectoral output, (Y- Ym) is the autonomous vector that results from subtracting the vector of final demand (Y) and the imports of final demand (Ym), and is the inverse matrix of Leontief or matrix of multipliers[[3]](#footnote-3).

The regional inter-sectoral model in its static version has three important limitations. First, the analytical solution (3) includes the prediction of sectoral production based on structural changes in the components of the autonomous final demand, failing to predict changes in these components (such as investment, consumption, government spending and exports). Second, the analytical solution (3) is atemporal. Multipliers condense temporary reactions of sectoral production induced by a change in the components of final demand into a sum of atemporal effects. And third, the multi-sectoral regional model starts from a static equilibrium condition (1), this is, the model is incapable of describing the "movement" of the system against that balance.

III. Regional dynamic model based on the delayed accelerator.

Solving the above limitations means to transform the regional multi-sectoral model on an intertemporal one through the modeling of the evolution of the technical coefficients or the final demand (Leontief, 1953). The objective is to evolve from a static to a dynamic model, where the final demand and the technical coefficients are determined from former (or current) values of the system. With respect to final demand, the central element is the incorporation of the theory of investment based on a variant at the beginning of the accelerator; thus, the current investment demand depends on expected (or current) changes in the production. The regional dynamic model can be written using equations (4) and (5).



 (4)

 (5)

 (6)

(7)

where,

In the balance equation (4) we can separate the demand for investment, from the final demand component and the rest of the exogenous final demand (Yt).[[4]](#footnote-4) Equation (5) assumes that imports are used in the region’s final demand (), intermediate demand ( and investment (). Equation (6) represents the regional sectoral demand in terms of the local investment demand (), the domestic coefficients, local sectoral production (Xt) and final local demand (Yt). Meanwhile, the analytic solution (7) allows to predict the regional sectoral production when the growth rate of the exogenous final demand and the initial conditions are given.[[5]](#footnote-5)

The dynamic regional multi-sectoral model described above has three problems. First, the endogenization of the investment demand doesn't change the fact of starting from a static equilibrium condition. Second, the mathematical model does not have a plausible lag structure in the investment process,[[6]](#footnote-6) does not consider that the degree of utilization of installed productive capacity varies between sectors and in time,[[7]](#footnote-7) and does not consider that there are shortcomings in the intersectoral linkages - it is assumed that production and investment vary instantly with changes in final demand.[[8]](#footnote-8) Finally, the analytical solution of the cross-sectoral model implies that the growth occurring in the system is not a reflection of any regional growth —endogenous or another type— represented, but is the result of the intrinsic instability of the mathematical structure employed, so its empirical application can’t have useful results.

IV. Transformed regional dynamic model.

We can reformulate the regional cross-sectoral dynamic model by changing the balance equation through sectoral production adjustments caused by short-termed regional excess demand and the capital formation process.[[9]](#footnote-9) The cross-sectoral regional model can be rewritten using a new disequilibrium condition, as follows:

 (8)

 (9)

 (10)

 (11)

 (12)

 (13)

 (14)

 (15)

(16)

The new balance equation (8) includes a vector that can be interpreted as the demand for gross investment (It) and another vector of the excess demand of each sector per time unit (Et). This equation says that the production plus imports, has to satisfy four uses: current consumption, intermediate intersectoral demand, excess demand and the increase in the production capabilities for next period. Equation (9) determines regional imports. The excess regional demand is given by equation (10), which is similar to (8).[[10]](#footnote-10) Equation (11) is the dynamic adjustment of the sectoral production per time unit. This adjustment will eliminate the excess regional demand (in a new equilibrium) in a time unit, but only if the excess of regional demand and the production adjustments remains unchanged to the current rates. In practice, the excess regional demand will change in the short run, as the production adjustments will, resulting in a cat (production adjustments) chasing mouse (excess demand) game. If the process is convergent, the cat will catch the mouse in the long run.[[11]](#footnote-11) In this equation, is a diagonal matrix representing the accelerator coefficient (the marginal relationship between capital and product).[[12]](#footnote-12) In equation (12), the gross investment demand can be disaggregated between investment on capital replacement (depreciation), and (induced) demand for new investment, where is the actual production capacity and the investment demand distribution matrix.[[13]](#footnote-13) Equation (13) tell us that the actual production capacity is a function of desired production capacity , where is a diagonal matrix of marginal capital-product relationships (accelerator). Equation (14) proposes that the desired rate of production capacity , at any point in time is a linear function of the production rate, with α intercept that represents a vector representing the desired level of installed excess demand, and as a diagonal matrix representing the desired proportion of installed capacity. Equation (15) assumes that some sectors are restricted in the level of use of the installed product capacity. Finally, equation (16) assumes that to reproduce physically production, net investment has to be more or equal than depreciation.[[14]](#footnote-14)

The regional multi-sectoral and intertemporal model is now more robust from a conceptual perspective, however a huge effort is needed to solve it analytically, as it requires a new solution whenever a sector in the model is binding a restriction (i. e., at each stage of change). An alternative way of dealing with the solution of the model is to use a systemic approach, simulating the system from a general perspective, analyzing the evolution in time of the endogenous variables included for a predefined period, and periodically verifying the changes of phase or discontinuities.

V. Dynamic regional model simulation.

The regional intersectoral and intertemporal model in section IV will be simulated, in the sense applied numerical methods to find your solution, due to the difficulty to obtain an analytical solution.

Stella/IThink (version 9.1.4) software is a tool for modeling and simulation that allows to represent systems and simulate their behavior. It has the characteristic of being written in a friendly, flexible, simple and elegant language. This software allows to establish a relationship between the causal diagrams and equations written in text using the Forrester diagrams, which is very attractive for didactic purposes.[[15]](#footnote-15)

The Stella/IThink Software employs only four elements of the Forrester symbolism:

1. A "rectangle" representing stock or level variables, which are variables that accumulate, stock variables or background variables.

2. A "valve" that represents flow variables, which are variables that affect the behavior of the stock variables. Flows affect levels increasing or decreasing them. In fact, the only procedure to alter the value of a stock variable is through the actions of the flow variables.

3. A "circle" representing auxiliary variables, which affect the value of the flows. Therefore, the auxiliary variables are variables that help to explain the values of flows.

4.  An “Arrow” represents a material channel or an information channel. The first represents the action of a flow over a channel. The second represents the interrelationship between variables and variables and rates.

Thus, the regional intersectoral and intertemporal model can be reformulated for purposes of simulation. To do this, it can be decomposed into two modules. The first so-called "regional economy", that generates the temporary paths of sectoral production levels and allows project these levels towards the future if we know the rate of growth of the autonomous final demands and baseline levels of sectoral productions. The second call "regional capital formation", which introduces endogenously sectoral capital accumulation process in the system.

Equations (17-20) and (21) condition allow us to formulate the regional economic group.

 (Level of production) (17)

 (Unplanned inventories) (18)

 (Consumption level) (19)

 (Intermediate demand) (20)

(Capacity restriction) (21)

Equation (17) says that, in equilibrium, supply (PDNt) equals sectoral demand (CONSt) plus the change in sectoral unplanned inventories (INVt) and sectoral investment (It). Equation (18) is for sectoral unplanned inventories (INVt), which are a function of sectoral regional excess demand (PDNt - CONSt) and the formation of capital (It). In equation (19), sectoral product demand is the sum of the intermediate demand (DIt) and net final demand minus endogenous investment (DFt). Equation (20) defines the intermediate demand as the matrix of technical coefficients net of imports , multiplied by the level of regional sectoral production (PDNt). The equation (21) is a sectoral capacity restriction (CAPt) included simply as the minimum requirements of production and the productive capacity of each sector.

Equations (22) to (25) and condition (26) are sufficient to describe the formation of regional capital block. The equations can be rewritten as:

 (Gross investment demand) (22)

 (New investment demand) (23)

 (Capital depreciation) (24)

 (Desired capacity) (25)

(Physical reproduction) (26)

For the purposes of the model, sectoral investment demand should be disaggregated according to the origin and related to demand according to the destination. Equation (22) tells us that the vector of demands of sectoral investment according to the origin (It), has a vector of formation of new capital according to destination (NEWt), and a vector of depreciation according to destination (DEPt), multiplied by a matrix of distribution of investment (B) demands. Equation (23) shows the new investment demand, where  is a diagonal matrix with sectoral relations of capital-output (accelerator). The equation (24) shows the requirements for the investment of replacement that is assumed to be proportional to the capacity of existing production (CAPt), where  is a diagonal matrix of sectoral coefficients of depreciation. The equation (25) points out that the desired production capacity (CAPDESt), is proportional to the current production capacity. While the equation (26) tells us that the physical reproduction of the sectoral production implies that the gross investment must be greater or equal to the depreciation.



On the other hand, to find out if the projections of regional inter-sectoral and intertemporal model have some sense, we intend to compare them with any 'reasonable' pattern of temporary growth of regional sectoral productions. In particular, it was assumed that a reasonable behavior might be a uniform growth of initial values of sectoral productions according to the same rate of growth that the autonomous final demands.

Then, the proportional growth of production equation can be written as:

 (27)

Where (Xt TREND) is the temporal trajectory uniform of sectoral production; is the value of the initial production, g is the fixed sectoral growth rate and *t* is time. Thus, we examine the dynamics of the intersectoral regional model for each sector with proportional growth pattern in time, trying to compare the results of both projections.

Figure 1 presents the regional intersectoral and intertemporal model causal diagram. Large boxes are different blocks of the model, and each one contains variables that form them. Each circle in the causal diagram represents a variable, and it has a number if it is an initial value, or an algebraic expression if it is obtained from other variables. Shaded circles represent matrix variables; the rectangles show the stock variables, and thicker arrows that always reach them represent growth flows. The object code of the programme is translated into a code source (a set of lines of text that are instructions that must be follow by the program) that defines the dynamic multi-sectoral model. Therefore, the code source is described by full operation of the multisectoral dynamic model object code.

In Figure l, the causal diagram displays the circular flow between production and investment. At the top of the diagram, initial production and current production, through their respective technical coefficients, determines the levels of domestic demand. Domestic demand and final demand, which are functions of time, become the demand of the economy (consumption) which is located in different sectors of activity. The unplanned inventory accumulation, with lag in time, causes the growth of domestic industry production, based on the excess of supply over demand, thus closing the cycle. In the lower part of the diagram. The process of capital formation plays an important role, as it allows to incorporate the restriction of capital in the growth of the production process, and is calculated in the lower part of the diagram. That is, any increase or decrease of the internal sectoral production is channeled through temporary settings between the desired and the required sectoral capacity - in other words, the degree of utilization of installed capacity varies considerably in a sector to another and from one period to another — that affects the process of capital formation when there is a growth in the production sector.

Figure 1. Basic structure of the regional cross-sectoral dynamic model.



Source: Authors.

VI. Evidence for Baja California case.

With the model programmed in Stella, you can analyze the temporal behavior incorporating the restrictions of: (1) utilization of installed capacity; (2) the inertial effects of the investment process; and (3) the lags in the productive sector relations, considering the interrelationships and feedbacks. To do this, we use the transactions matrix of Baja California in 2003, which is divided into 72 sectors (IOTBC, 2003). However, for the purposes of showing the dynamic behavior of the model, we aggregated the 72 sectors to a total of 12 sectors, reclassified below:

1.- Non industrial agriculture

2.- Industrial agriculture

3.- Petroleum, gas and mining

4.- Modern manufacturing

5.- Traditional manufacturing

6.- Petroleum processing

7.- Construction

8.- Commerce

9.- Transport

10.-Infrastructure and services

11.- Public Administration

12.- Finances

The exogenous parameters of the inter-sectoral and intertemporal model are set as follows. First, sectoral aggregation performed in the transactions matrix (see Annex 1) comes from the fact that the degree of utilization of production capacity and the lags of investment in each of the sectors typically have very different effects on production. Second, the regional foreign trade was the incorporation of competitive imports on the diagonal of the technical coefficients matrix of the transactions matrix. This matrix is different from conventional as the interaction coefficient includes not only regional industries, but also competitive imports[[16]](#footnote-16). Third, in order to impose the investment requirements (depreciation), we opted for a procedure "ad hoc" that consists on the sectoral differentiation of the depreciation rates of sectoral investment, considering that they are a constant proportion of the capital[[17]](#footnote-17). And fourth, in terms of the components of final or autonomous demand, the procedure consisted in assuming that the growth rates were differentiated by sector.

The specification of the parameters above, allowed to project sectoral production levels by considering not only the growth of autonomous final demands, the restriction of sectoral production capacity and initial values of sectoral production, but also the formation of sectoral endogenous capital, necessary for such sectoral growth.

Figures 1 through 8 illustrate: the trajectories of regional sectoral output levels (PDNt), the formation of endogenous regional sector capital (It), the limitations of sectoral installed capacity (CAPt), the level of trend output (Xt TREND) -when all sectors grow at a proportional constant rate (0.05) –and the initial levels of the sectoral output and the growth rate of autonomous demands for major sectors of the aggregated transaction matrix are known.

Comparing the evolution of the regional multi-sectoral and intertemporal model with the proportional growth pattern, or trend production of the primary sectors, the following may be verified:

1. The projections of the dynamic model for the short, medium and long run are reasonable.
2. The restriction of the installed capacity in the primary sector determines the maximum production of this sector in the short run.
3. The required value of output at start is equal to the installed capacity, however, due to the growth of final demand, the production value is greater than the installed capacity in the short run. That is, there is an excess of regional demand.
4. The expansion of investment (driven by the increase in sectoral production) increases the production capacity in the medium run, which is subject to a greater number of lags than the sectoral production.
5. Subsequently, installed capacity continues to increase toward its target, despite the decline in investment.
6. In the area there are significant differences in the short term dynamics trajectories between model and pattern of trend growth. But in the medium and long term, the differences disappear.
7. In the short run, there are significant differences within the sector between the dynamic trajectories of the model and the pattern of trend growth. But in the medium and long run, these differences disappear.
8. The solution of the model appears more "realistic" than the solution of constant proportional growth because it considers the capital formation induced by the final demand growth.

Also, the movement of the variables in the secondary and tertiary sectors highlights that:

1. The behavior of the regional multi-sectoral dynamic model is quite reasonable in the period.
2. The secondary and tertiary sectors face no restrictions of installed capacity in the short run given the trajectory of production levels associated with the exogenous demand, as long as the final autonomous demand grow at a steady pace and there are no exogenous sectoral impacts.[[18]](#footnote-18)
3. In the tertiary sector, the production value required to start is less than the installed capacity; then a bottleneck is not generated in the production in the short run.
4. In the secondary sector, the required value of output at the beginning is equal to the installed capacity. However, the sectoral differentiation of the depreciation rate will have a slight impact on the dynamic trajectories of the role of investment in the first period. At longer runs the effect does not exist.
5. This result differs for the tertiary sector, since the sectoral differentiation of the depreciation rate generates an oscillatory behavior of short-run investment. However, in the long run the effect will be considerable.
6. This means that the sectoral differentiation of the depreciation rates have impacts on the trajectory of total investment.
7. In the secondary and tertiary sectors, projections based on the pattern of steady growth of the production level coincide with the projection of sectoral output levels in the medium run.
8. Finally, regarding the secondary and tertiary sectors, consistency of movement of the variables in the primary sector is verified as it is facing major limitations in installed capacity given an exogenous change in final demand.

As a result, we note that the simulation of the multi-sectoral and regional intertemporal model produces a behavior that can be considered satisfactory according to the hypotheses formulated: restriction of installed capacity per sector and per period, lags in inter-sectoral relations, and inertial effects in the investment process[[19]](#footnote-19).

Thus, the dynamic inter-sectoral model allows treating capital as primary factors that provide productivity rather than as components of final demand. Additionally, systemic approach allows to solve successfully some problems related to the model: (1) the degree of utilization of installed capacity; (2) the inertial effects of the investment process; and (3) delays in inter-sectoral relations.

Given the above, one can establish the potential of this specification and the utility of the inter-sectoral and intertemporal model for the analysis of a region.

Graph 1. Non industrial agricultura



Graph 4. Modern Manufacturing



Graph 7. Petrleum Proccesing



Graph 2. Industrial agriculture



Graph 5. Tarditional Manufacturing



Graph 8. Comerce



Graph 3. Petroleum, gas and mining



Graph 6. Construction



V. Conclusions.

In the last years, economics has shown great interest in understanding the temporal dimension from practically all phenomena. This is no exception for the input-output model analysts. When Leontief included the process of capital formation in the inter-sectoral model, then it was possible to transform it into an intertemporal model.

However, the problem of capital formation or investment is extremely complex and Leontief's initial proposal was quite simple. For this reason, some problems of the dynamic multi-sectoral model have waited for a satisfactory solution, for example regarding to: (1) the degree of utilization of installed capacity varies considerably from one sector to another and from one period to another; (2) that there are significant inertial effects of the investment process; and (3) that there are lags in inter-sectoral relations.

Additionally, the analytical approach used by the author to add time has the disadvantage that the detailed knowledge of the parts of intertemporal multi-sectoral model can lead to a solution that is obtained only with great effort.

Dynamic simulation is a technique recently used to model and study the behavior of any kind of system that has a cyclic behavior due to inertial effects or lags, and to retroactively effects or feedback loops. These characteristics of the systemic approach make it ideal to considerate the complexity of the investment process, restrictions of installed capacity sector, and especially the structure of lags involved in inter-sectoral relations. Moreover, the approach is useful when the analytical model is unsolvable.

Finally, an interest in building regional multi-sectoral matrices has emerged recently in the country. However, in all cases, the regional inter-sectoral matrix has been seen as a tool that provides only a basis for economic planning and projection, but in no case has been used in the development of dynamic simulation models. Thus, the regional multi-sectoral model presented will allow analysts and public and private institutions to have a new instrument that will facilitate knowledge and comprehension of the dynamic sectoral relations in a complex and heterogeneous regional reality in Mexico.

VI. References.

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Anexo 1

Matriz Input –Output Baja California



Fuente: Ana Cárdenas (2014).

1. The matrix A satisfies the property to be indecomposable and the Hawking-Simon condition (Park, 1975). [↑](#footnote-ref-1)
2. A more realistic equation would be Xm = AmX+Ym, where Am is a square matrix of imports. However, in practice it is difficult to obtain the Am matrix at regional levels. [↑](#footnote-ref-2)
3. The method for obtaining the multiplier matrix is known as power expansion method, (I – Ã)-1 = (1 + Ã + Ã2 +…+ Ãn)Y.Where, Ã = A-M, Y + Ã Y is the sum of the initial effect (Y) and the direct effect (Ã Y). The rest, (Ã2 + Ã3 +…+ Ãn)Y is the sum of the indirect effects. [↑](#footnote-ref-3)
4. When the technical coefficients (At) and the Capital-Product matrix (Bt+1) are variable in time the model is more complex. The balance Equation (4) would be: Xt = At Xt + Bt+1Xt + Yt + Xtm (Blanc y Ramos, 2002). [↑](#footnote-ref-4)
5. See Leontief (1953) for the periodic solution of the static I/O model. [↑](#footnote-ref-5)
6. The lag structure of investments is complicated, since it depends on technological, psychological, social, political and institutional factors, among others (Kozikowski, 1988). [↑](#footnote-ref-6)
7. The assumption of no restrictions in installed capacity or excess of production capacity in sectors or regions is not real (Kozikowski, 1988). [↑](#footnote-ref-7)
8. The instant adjustment of production and investment, when there are changes in the autonomous demand, generates reactions in the dynamic model that concludes in unacceptable predictions (Ryaboshlyk, 2005). [↑](#footnote-ref-8)
9. Johnson (1985) made the original adaption, we propose a variant by endogenizing imports. [↑](#footnote-ref-9)
10. Any change in Et is a change in production and, in turn, changes unplanned inventories. It is not equal to real inventories that are included in the capital stock, Kt, which have a positive relationship with the level of equilibrium output. So, Et has a zero value in equilibrium and the actual inventory affects only when the system moves toward equilibrium (Byden, 2009). [↑](#footnote-ref-10)
11. For the equations of dynamic and static balance represent a balance must specify when . Therefore, for the relationship to be stable we need that (Johnson, 1986). [↑](#footnote-ref-11)
12. This equation has temporal Lundberg lags (Burda y Wyolosz, 2005). [↑](#footnote-ref-12)
13. This equation introduces discontinuities in the model. the change will finite changes in which nothing appears, or certain amounts arise suddenly. Can use discontinuity theory. [↑](#footnote-ref-13)
14. This is equivalent to gross investment being equal or greater than zero. [↑](#footnote-ref-14)
15. Stella software uses a 4th order Runge-Kutta algorithm. The algorithm is appropriate for stiff systems, not sensitive to the initial conditions, nor to calculation accuracy. There are other similar programs such as Mathematica, GAMS or VENSIM. [↑](#footnote-ref-15)
16. Competitive imports are composed of goods that are imported, but may also be produced in the region. [↑](#footnote-ref-16)
17. Another method often used consists in relating the requirements of capital replacement with sectoral output levels. This is possible if sectoral capital-output ratios, *k*i, are known, and stable. This procedure has the advantage that allows to incorporate the coefficients in the matrix of technical coefficients original (A). [↑](#footnote-ref-17)
18. In reality, the different components of the final demand vector do not grow at the same rate, and consequently, the trajectories of the value of production are more complicated than those presented. [↑](#footnote-ref-18)
19. Even if the performance is not satisfactory from the standpoint of some historical patterns (or by subjective evaluation) is possible to modify the values of the parameters and the shape of the functional relationships. This procedure can be repeated until a "best fit" model is achieved. [↑](#footnote-ref-19)