Eigenvalue distribution, matrix size and the linearity of wage-profit curves

Topic: Mathematical Treatments of Input-Output Relationships
Author: Luiza Nassif Pires
Co-Authors: Anwar Shaikh

While experimenting with equilibrium computations using randomly generated matrices, Brody (1997) noticed that the speed of convergence towards equilibrium increased with matrix size. The relative size of the second eigenvalue with respect to the first determines the convergence speed, so Brody conjectured that this relative size tended to fall as a random matrix got larger. While this does not appear to hold for observed direct requirements input-output matrices (Mariolis and Tsoulfidis, 2012, Table 1, p. 6), Bidard and Schatteman (2001) proved that in a random matrix with independently and identically distributed entries the speed of convergence increases with the size of the matrix because the relative size of all subdominant eigenvalues tends to zero as the matrix size approaches infinity. Schefold (2010) then showed that zero subdominant eigenvalues imply linear wage-profit curves for any given numeraire. The linearity of wage-profit curves are important to answer questions such as the existence of re-switching of techniques and reverse capital deepening. More complex forms of the wage curve are attributed by Schefold (2010) to non-vanishing subdominant eigenvalues and even though the existence of those complex curves are accepted in theory they have not ben proven to exist in real economy. Our concern is with actual input-output matrices. We successively aggregate the BEA US 2002 make and use tables using the NAICS codes, in order to build 176 squared industry by industry direct requirements tables ranging in size from 403 to 15 industries. This allows us to assess the effects of the size of the matrix on the distribution of the moduli of eigenvalues and on their arithmetic and geometric means. We find that the distribution of the moduli does indeed shift downward as the size of the matrix increases, so that the average size of the moduli of subdominant eigenvalues falls by either measure. In this particular year (2002), as the matrix size increases the arithmetic mean seems to stabilize around 0.05 and the geometric mean around 0.02. This stabilization seems to contradict the random matrix hypothesis, but its implications are unclear: on the one hand, input-output matrices are not random; on the other hand, the random matrix hypothesis only applies in the limit as matrix size approaches infinity. In any case, the fact that the subdominant moduli do fall to some small level has major implications for the analysis of relative prices and choice of technique.

References:
Bidard, Christian and Tom Schatteman. 2001. "The Spectrum of Random Matrices."Economic Systems Research, 13(3), 289-298.
Brody, Andras. 1997. "The Second Eigenvalue of the Leontief Matrix." Economic Systems Research, 9(3), 253-258.
Mariolis, Theodore and Lefteris Tsoulfidis. 2012. "On Brody's Conjecture: Facts and Figures from the US Economy." unpublished paper, University of Macedonia.
Schefold, Bertram. 2010. "Approximate surrogate production functions." unpublished paper, Institut fÃ¼r Volkswirtschaftslehre, Johann Wolfgang Goethe-UniversitÃ¤t.