The Hadamard-multiplicative GLS-based Model for Matrix Updating with a Solution Space of Reducible Dimension

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Author: Vladimir MOTORIN

The purpose of this study is to construct an operational Hadamard-multiplicative model for economic matrix updating within RAS method's multiplicative pattern of target matrix that can be represented as Hadamardâ€™s product of the initial matrix and a factor matrix of the same dimension formed by matrix multiplication of column and row vectors of unknown parameters. A natural way to generalize RAS pattern is to abandon its biproportional framework in favor of multiparametrical approach with the most common factor matrix. To satisfy row and column total constraints, one can disturb the elements of a factor matrix in some minimalistic manner that requires to be defined well.
According to RAS multiplicative pattern, all matrices from homothetic family of initial matrix have an excellent structural similarity. This feature allows to introduce the notion of a homothetic ray for vectorized factor matrix (directed by summation vector with unit elements) in multidimensional Euclidean space, whose dimension can be meaningly reduced by an elimination of those components that are corresponding to zero entries of initial matrix. Homothetic measure for matrix similarity is defined as a shortest path from unknown parametrical point (determined by vectorized factor matrix) to homothetic ray in GLS terms. Further, angular measure for matrix similarity is then defined as an angle between vectorized factor matrix and homothetic ray to be minimized. Note that both measures are geometrically conjugate and invariant with respect to a certain way of matrix vectorization.
It is shown that a minimization of angular measure can be implemented via the minimization of homothetic measure by solving the uniparametrical GLS-based mathematical programming problem with the quadratic objective function, two sets of the mutually dependent linear row and column total constraints, and scalar linear condition of orthogonal projecting onto a homothetic ray. This optimization model is fully represented in matrix notation, and its solution is obtained in analytical form that is further used in formulation and analytical solving the univariate unconstrained minimization problem for angular measure. Special attention is paid to sensitivity of optimal solution for homothetic measure to small changes in row and column totals of the target matrix (when the row and column total constraints are assumed non-binding).
Main advantage of the proposed model is its immanent flexibility because of great number of adjusting variables. The model is quite applicable for updating the economic matrices and tables with some negative entries. Several illustrative numerical examples are given.