The Linear Matrix-Valued Cost and Production Functions in the Rectangular and Square Inputâ€“Output Models

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A general linear problem of inputâ€“output analysis (GLP IOA) is considered in this study as a system of equations written in terms of free variables for any rectangular supply and use table given. This system spans the regular linear equations for material and financial balances, a batch of predetermined values for exogenous variables chosen in advance and an additional set of linkage equations that provides the exact identifiability for all unknown variables.

The study is concerned with some operational opportunities for constructing a set of the identifying linear equations in the cases of evaluating the response of the economy to exogenous changes in final demand and value added vectors. To this end, it is expedient to involve into consideration a pair of the matrix-valued linear cost functions with product and industry outputs as their arguments respectively as well as the product-mix and market shares contours of supply (production) matrix. These four linear equations generate four different specifications of GLP IOA. On the other hand, one can consider a pair of the matrix-valued linear production functions with product and industry intermediate inputs as their arguments respectively as well as the product-mix and industry shares contours of intermediate consumption (use) matrix. As a result, the other four different specifications of GLP IOA arise.

It is shown that there are three main types of economy's response to exogenous changes in final demand and value added vectors while all of eight specifications are using for its evaluating, namely, in terms of volume changing exclusively, in terms of price changing only, and in terms of combined price and volume changes. The latter type of economy's response together with four associated specifications of GLP IOA seem to be implausible artefacts that are out of economic sense. In particular, there are some certain doubts about plausibility of underlying background for an industry technology assumption and a fixed product sales structure assumption, which are used in the transformation of supply and use tables to symmetric input-output tables.

The other four specifications of GLP IOA appear to be pairwise equivalent. First, the specification with linear dependency of intermediate consumption matrix from industry output vector (based on Leontief technical coefficients to be fixed) and invariable â€œverticalâ€• (product-mix) structure of production matrix is equivalent to the specification with linear dependency of production matrix from vector of industry expenditures for intermediate consumption and invariable â€œverticalâ€• structure of intermediate use matrix. Secondly, the specification with linear dependency of intermediate consumption matrix from product output vector (based on Ghosh allocation coefficients to be fixed) and invariable â€œhorizontalâ€• (industry shares) structure of production matrix is equivalent to the specification with linear dependency of production matrix from vector of product amounts in intermediate use and invariable â€œhorizontalâ€• structure of intermediate consumption matrix. Thus, technical and allocation coefficients should be regarded as helpful ways of economic interpretation rather than as operational tools for modeling.

It is shown that in a square case (when all matrices are square) the first pair of specifications forms an underlying algebraic framework of Leontief demand-driven model, whereas the second pair provides an algebraic foundation of Ghosh supply-driven model. For a symmetric (when production matrix is diagonal) square case, the equivalence of Ghosh supply-driven model and Leontief price model as well as the equivalence of Leontief demand-driven model and Ghosh quantity model are proved.