Key sectors in economic development: a perspective from input-output linkages and cross-sector misallocation∗†‡

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Abstract

For a typical developing country, this paper shows that once inter-sectoral linkages are taken into account, closing the productivity gap in a number of services gives bigger gains in aggregate productivity than closing it in agriculture or in manufacturing, despite their larger gaps. This is performed in the context of an input-output economy and general equilibrium. Also, the importance of sector-specific distortions that produce cross-sector misallocation is addressed. I compute the effect of the removal of these distortions on aggregate productivity using the input-output model and find that this could increase productivity up to 68%, depending on whether the rents from distortions stay in the economy or not.

Keywords: key sectors, economic development, input-output linkages, cross-sector misallocation

JEL CODES: O1, O41, C67

∗This is Paper 2 in a set of two papers on input-output economies and aggregate outcomes. The companion paper of this set, Paper 1, is entitled “Equivalence between input-output and value-added economies.”
†Previous versions of this paper where entitled “Which sectors make poor countries so unproductive? A perspective from inter-sectoral linkages.” The first results of this paper were presented in LACEA-LAMES, 2013 in the session “Sources of cross-country income differences” under the title: “Productivity in Mexico: a Perspective on Resource Misallocation, Linkages, and Complementarity.”
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1 Introduction

Which sectors make poor countries so unproductive? One common idea is that there exist large distortions in a few key sectors that explain the bulk of the gap in aggregate productivity between rich and poor countries. The development literature have traditionally emphasized problems in agriculture or manufacturing.\(^1\) In contrast, a recent branch of this literature emphasizes distortions prevalent in services, such as those associated with the presence of informality.\(^2\) Thus, which sectors are the most important ones for explaining the differences in aggregate productivity across countries, is still an open question.

A prominent recent theory to explain these large differences in productivity is the one of resource misallocation across plants.\(^3\) In the same spirit, *cross-sector misallocation* might occur if sector-specific distortions are in place. What the quantitative importance of this type of misallocation on aggregate productivity is, is also an open question.

I make two main arguments regarding the questions at hand. First, I argue that to determine which sectors make poor countries so unproductive, it matters not only which sectors have the largest productivity gap with respect to the leader, but also the “degree of influence”\(^4\) of each sector. This degree of influence is determined by the way each sector is linked to the rest of the economy through input-output relationships. Some sectors play a central role in the input-output network because they are important suppliers of intermediate inputs in the economy, and thus, they have a high degree of influence.

The second argument in the paper is that there exist sector-specific distortions in developing countries that are not directly linked to low productivity at the *industry level*, but that could be a source of cross-sector misallocation, and thus, have an impact on aggregate productivity. The second goal is to measure the quantitative importance of these distortions on aggregate productivity, and to understand the economic channels through which this occurs.

To achieve these goals, I use a multi-sector model with inter-sectoral linkages based on Long Jr and Plosser (1987), Acemoglu et al. (2012), and Jones (2011b). In the model, there are \(N\)

\(^1\)Restuccia et al. (2008) blames the barriers to the use of intermediate inputs in Agriculture; Herrendorf and Trixeira (2005) emphasize barriers to international trade that directly affect industries that produce tradables; and Buera et al. (2009) argue that the problem is financial frictions that affect manufactures more than services.

\(^2\)For example, Prado (2011), and D’Erasmo and Moscoso Boedo (2012) argue that informality is associated with resource misallocation and other distortions.

\(^3\)For example, Restuccia and Rogerson (2008) and Hsieh and Klenow (2007).

\(^4\)See Acemoglu et al. (2012).
sectors (or industries) that produce different goods. The output of each sector can be used either as consumption or as an intermediate input in the production of the other sectors. This introduces the link between the performance of an individual sector and the performance of the rest. I calibrate this model to Mexico, an important developing country, and perform counterfactuals.

To study sector-level distortions in the model, I include wedges in the firm’s first order conditions (FOC). The introduction of these wedges is motivated by the observation of two facts that emerge when comparing data for Mexico and the U.S. economies: 1) the use of intermediate inputs is depressed in Mexico, relative to the US, for the majority of the sectors; and 2) the labor income share is lower in Mexico, for the majority of the sectors.\footnote{Gollin (2002) argued that the labor income share in developing countries is low due to measurement problems. In section 2, I show that even after performing Gollin’s measurement correction, the labor income share is still low for the majority of the Mexican sectors.} The first wedge enters in the firm’s problem as an output tax, and can also be interpreted as a markup that rises price over marginal cost. I call this distortion “the markup wedge.” The second distortion, is isomorphic to a payroll tax, and it captures policies that shift resources away from workers while increasing labor costs to firms. I call this distortion “the labor wedge.”

I use a calibration strategy that avoids the computation of productivity levels and instead focuses on productivity gaps. Assessing the extent of productivity gaps is enough to perform the counterfactual exercises of interest. To obtain the value of the distortions, I face an identification problem: I observe factor shares that are a function of both: technology parameters and distortions. The problem is how to separate the two. To address this problem, I make a couple of identifying assumptions. First, I take the U.S. as a relatively undistorted economy and use it as a reference point to measure the distortions in Mexico. Second, I assume that certain technology parameters are common in Mexico and the U.S. Thus, by comparing factor shares in the two countries, I obtain the values of parameters and distortions in Mexico. Although not without controversy, the procedure is clean and simple. Additionally, note that by focusing on a single country I can better determine the reasonableness of these assumptions and of the calibrated values of the resulting distortions using previously known information about the structure of the Mexican Economy and its policies (see section 5).

I use the calibrated model to provide a quantitative assessment of the effects on aggregate GDP per worker of two counter-factual exercises: 1) closing sectoral productivity gaps; and 2) eliminating sectoral wedges.

The results for the first exercise are as follows. First, in line with previous literature, I show that Mexico’s productivity gap is larger in manufactures. Because of this, the role of services
in economic development was underestimated by this literature. However, I show that once interconnections are taken into account, closing the productivity gap in a number of services, would give among the biggest gains in GDP per worker. To illustrate the mechanics behind this result, take two typical industries in manufacturing and services: Textile and Textile Products (sector 4), and Wholesale Trade (sector 20), respectively. The industry of Textiles in the US is 8 times more productive than the corresponding one in Mexico, while Wholesale Trade is only 3 times more productive. However, Trade is not only a much bigger sector than Textiles, it is also one of the most interconnected sectors in the economy: the degree of influence of Trade is 5 times bigger than the degree of influence of Textiles. Therefore, closing the productivity gap in Trade gives much bigger gains in GDP per worker than closing it in Textiles (15% vs. 4% gains), despite the fact that the productivity gap is higher in Textiles.

Why is it crucial to look at the degree of influence of a sector instead of at its value added share? In fact, there is a close connection between the two in this model. To understand this, one has to remind a basic property of input-output economies: aggregate final consumption is equal to aggregate value added. However, the value generated by a given sector $j$ can be used to support final consumption in other sectors different than $j$. And the way this value reaches other sectors is through the input-output network, i.e. through the use of sector $j$'s output as an intermediate input. Thus, sectors with a large degree of influence will also support a large share of aggregate final consumption, and hence, will have a large value added share. In fact, under certain assumptions, an input-output economy is fully equivalent to a value-added economy. I show this in the companion paper of Leal (2015).

A key assumption in this paper that prevents such equivalence is that technological change affects the efficiency of both, primary and intermediate inputs. Thus, a 1% increase in productivity maps into a more than 1% increase in the productivity of primary inputs. Put it differently, if productivity increases in all sectors by 1%, GDP increases by more than 1%. The counterpart effect on aggregate final consumption works through the existence of a gross output multiplier: start with an exogenous change that originally increases gross output by 1%, then you will end up with a final change in gross output of more than 1%.6 This original change can be given by a 1% increase in productivity. Since consumption is a constant fraction of gross output, this also means that a 1% increase in the productivity of all sectors, increases aggregate final consumption in more than 1%.

Regarding the second exercise, I first analyze the markup wedge, and then the labor wedge.

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6This is the same logic found in the Leontief multiplier: in order to increase final consumption by $1 dollar, you need to increase sales by more than $1 dollar. The reason is that in order to increase consumption you need to increase the purchases of intermediate inputs which are a constant fraction of gross output.
When eliminating these industrial wedges, it becomes relevant to distinguish between two cases regarding the distribution of rents from distortions: when the rents are given back to the household as lump-sum transfers (case 1); as opposed to when the rents are lost and taken out of the economy (case 2).

In the first case, three margins in the economy are affected. 1) the supply of the good; 2) the allocation of labor; and 3) the allocation of output between final and intermediate uses. The first one is intuitive as the markup enters in the profit maximization problem of the firm like an output tax, which, when reduced, it increases marginal revenue. Regarding the second one, the presence of wedges creates resource misallocation of labor but eliminating it does not necessarily improves it. This depends on whether wedge dispersion is reduced, or not. Finally, the third effect is present due to the negative income effect that occurs when reducing the rents associated with the distortion, this reduces aggregate demand and final consumption. As a result of these positive and negative forces, eliminating a markup does not necessarily increases GDP per worker. Moreover, the total effect of eliminating simultaneously all markups, is small.

In case 2, when the transfers are not given back to the household, the markup wedges are isomorphic to productivity. The wedges do not create resource misallocation in this case, but these can have a sizable effect on aggregate output. The reason for this is that, just like a decrease in productivity, a markup wedge reduces the amount of output per unit of input, affecting GDP and aggregate productivity. As a consequence, the effect of eliminating markups is much bigger than in case 1: when all markup wedges are eliminated simultaneously, aggregate productivity increases 68%. This large effect is also explained by the fact the multiplier also applies to the markup: a 1% decrease in the markup, increases aggregate output in more than 1%.

The contrasting effects on aggregate output between cases 1 and 2 is informative about the economic channel through which labor misallocation operates in the model. In particular, notice that the misallocation of labor is present in case 1 due to the extra income effect that transfers entail.

Finally, for the case of the labor wedge, I find that an important fraction of the difference in the labor income share between Mexico and the US is explained by the presence of the

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7One important feature of the model is that the equilibrium labor allocation across sectors is invariant to changes in productivity. This is a feature that makes cross-plant misallocation different than cross-sector misallocation. While in standard models of heterogeneous firms the allocation of resources is largely determined by relative productivity across firms; in standard input-output models, such as the one used in this paper, the allocation of resources across sectors is largely determined by the vector of influence, which, in turn, is affected by the specification of demand and by the nature of the inter-sectoral network.
markup wedge, the rest of course is explained by the labor wedge. This implies that policies that tend to decrease competition affect the labor income share, as well as policies that divert resources from workers and increase the cost of labor to the firms.

**Related literature.** A long tradition of studies argues that the productivity gap in poor countries manufacturing is higher than in services (e.g., Balassa (1964); Samuelson (1964); and more recently, Buera et al. (2009), and Herrendorf and Valentin (2012)). This is also true in the recent data from Inklaar and Timmer (2013), and in Herrendorf and Valentin (2012). This literature did not take into account the role of inter-sectoral linkages and cross-sector misallocation to assess which sectors are key for development.

A large literature intends to explain the sources of cross-country income differences. My paper is related to that literature and specially to a small subset studying the role of intermediate inputs in productivity. In particular: Moro (2011) and Jones (2011a, a). My paper is also related to the literature on resource misallocation across plants (e.g., Restuccia and Rogerson (2008) and Hsieh and Klenow (2007)). There is growing interest on extending the study of resource misallocation beyond the dimension of plants. Jones (2011b, b) argues that misallocation might be enhanced in input-output economies, this is perhaps the closest paper to mine. My paper distinguishes from Jone’s one in several dimensions. First, I make a quantitative assessment of the importance of the heterogeneity in inter-connections to determine the key sectors for development. Second, I analyze the economic channels through which distortions affect aggregate outcomes. Misallocation in input-output models is different than misallocation in heterogeneous firms models. In particular, while the labor allocation is invariant to productivity changes in input-output models; in heterogeneous firm models the allocation of labor is highly determined by relative productivity. In input-output models, the allocation obeys the structure of the demand side and the specification of the sectoral network. Finally, I show that, what is crucial for aggregate productivity is whether the rents associated from distortions stay in the economy or are taken away.

The paper is also related to the literature of economic networks such as in Acemoglu et al. (2012) and Acemoglu et al. (2015). This literature has focused on the role of networks in business cycles. This paper is an application of the concept of “degree of influence” coined by Acemoglu et al. (2012) to the literature of economic development.

The literature that studies the low labor income share in developing countries is also related. For example, Ayala and Chapa (2014) argue that this share is low in Mexico even after correcting for the measurement issues addressed by Gollin (2002). It is also related to the
literature studying a generalized recent decline of the labor share across countries, such as in Karabarbounis and Neiman (2014). In this paper, I argue that the low labor income share in Mexico is explained by the presence of the markup wedge, which in turn, might be related to lack of competition in product markets.

The organization of the paper is as follows. Section 2 presents relevant facts, section 3 presents the model and discusses the effect of distortions, section 4 presents the calibration strategy, section 5 the results, and section 6 concludes.

2 Facts

In this section, I present several facts that are relevant for the question at hand. First, I show that the productivity gap in developing countries is larger in manufactures. Second, I show that there is substantial heterogeneity in the degree of interconnections across sectors as well as in their final consumption shares. These two features are important to determine the “degree of influence” of each sector.\footnote{The concept of “degree of influence” is taken from Acemoglu et al. (2012) and it captures the idea that the stronger the inter-connections of a given sector are, and the higher its final consumption share is, the more “influential” a sector will be in the aggregate economy. Acemoglu et al. (2012) did not consider variation in consumption shares across sectors. However, once this variation is allowed in the model it turns out to be important for the vector of influence.} The variability on this degree of influence across sectors motivates one of the main arguments of the paper: mainly, that in order to determine how important the performance of a sector is for aggregate productivity, it is not sufficient to only look at its productivity gap; instead, one has to look at the degree of influence, too. Finally, I present two facts that support the existence of sector-specific distortions, which could potentially lead to low aggregate productivity as well. These facts motivate the introduction of “wedges” in section 3.

2.1 Productivity gaps in manufacturing and services

Here, I document that the productivity gap is larger in manufacturing relative to that in services. I use data from Inklaar and Timmer (2013) who compute cross-country relative prices at the industry level using data on prices of final goods. I plot their estimates of the relative productivity of services vs. GDP per hour worked in Figure 1. As the figure shows, the poorer the country, the larger the relative productivity of services with respect to manufactures. This implies that the productivity gap in poor countries is larger in manufactures.
Figure 1: The productivity gap is larger in manufactures


Table 1: Relative TFP US vs Latin America for the aggregate, services and goods.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate</td>
<td>$TFP_{US}/TFP_{LA}$</td>
<td>2.30</td>
</tr>
<tr>
<td>Services</td>
<td>$TFP_{s_{US}}/TFP_{s_{LA}}$</td>
<td>1.86</td>
</tr>
<tr>
<td>Goods</td>
<td>$TFP_{g_{US}}/TFP_{g_{LA}}$</td>
<td>3.58</td>
</tr>
</tbody>
</table>


A second piece of evidence is the one found in Herrendorf and Valentinyi (2012), where the authors compute Total Factor Productivity (TFP) for three main aggregates: GDP, services and goods. They report the ratio of TFP in the US to TFP in Latin America (LA). The data from Herrendorf and Valentinyi (2012) is presented in Table 1. The table tells a similar story as the one in Figure 1. Mainly, that the productivity gap is bigger in manufactures.

A third and final piece of evidence is the data on gross output labor productivity that can be constructed using Inklaar and Timmer’s PPP estimates at the industrial level and data on gross output and hours from the World input-output database (WIOD). I present such measures in Figure 2 for the Mexican sectors. The main message is similar to the one implied in the previous figure and table: the gaps tend to be larger in manufacture sectors.\(^9\)

\(^9\)The average relative sectoral productivity is 0.30, which implies that the gap is 2.33 ($= (1 - 0.3)/0.3$). Note that the figure includes a label on top of the bars that indicate whether the sector belongs to services (label=1), or otherwise (label=0). While 58% of the sectors that have a lower than average gap are services, only 38% of the sectors with a larger than average gap are services. Put it differently, the majority of the sectors with large productivity gaps are manufactures.
2.2 Sectoral interconnections

Mexican sectors exhibit considerable heterogeneity in the interconnections as measured using the information in the input-output tables. Figure 3 shows a network map for the Mexican economy. Each circle is a sector. The area of the circle is determined by the final consumption share of the sector (i.e., a measure of the size of the sector). A string between two circles indicates that the economic transactions between them are significant (i.e., above some threshold\(^{10}\)). The more concentric is a sector, the more interconnections the sector has. The figure shows that there is great heterogeneity across sectors in terms of, not only relative size, but also the number of inter-connections. Intuitively, the larger the consumption share, and the stronger its inter-connections, the more important role the sector will play in the economy.\(^{11}\)

\(^{10}\)Specifically, I define the threshold in terms of the share of intermediate inputs from sector \(j\) in the gross output of sector \(i\). If this ratio is higher than 0.025, a line is drawn between the two sectors. See Figure 3.

\(^{11}\)These two features of a sector will be important determinants of the “degree of influence” introduced in section 3.
Figure 3: Network map of Mexican Sectors.

Source: Author’s calculation.
2.3 Input shares as wedges

Previous literature has used variation in input shares either in gross output or in value-added across time and sectors to identify distortions on the optimal behavior of firms. When the production function is Cobb-Douglas and the firm operates under perfect competition, the equalization of the marginal product to the marginal cost of the inputs implies that the input shares are constant and equal to the coefficients in the production function. As a result, a discrepancy between input shares and the value of these coefficients, might be indicative of the presence of distortions. As explained by Cole and Ohanian (2013), deviations from perfect competition in product markets break the equality between the marginal product of inputs and the price of those inputs (the marginal cost). The reason is that under imperfect competition firms equate the marginal revenue product to the marginal cost, and not the marginal product, and thus, it depresses the quantity of inputs hired by the firm. One early contribution using the same basic idea is Hall (1988) who uses the ratio of labor compensation to total revenue to study the relation between price and marginal cost in US industries. More recently, there is the article of De Loecker (2011) who uses a similar property of firm’s maximization to identify markups in specific exporting industries.

In general, variation in input shares can occur for several reasons, a simple one being the existence of taxes. Taxes distort the equalization of marginal products and marginal costs because part of the marginal product has to be put aside by the firm in order to comply with tax laws. In general, any regulation, pecuniary or not, that rises the cost of inputs to the firms will create variation in input shares. Similarly, any regulation that affects marginal revenue, will also create variation in input shares.\(^\text{12}\)

2.3.1 Intermediate inputs share

Next, I show that the shares of value-added in gross output -the complements of the intermediate inputs shares- across Mexican industries have a strong correlation with the corresponding shares in the US. Figure 4 is a scatter plot of the US shares vs. the Mexican shares. The figure shows that if a share is relatively high for a specific industry in the US, then, the corresponding Mexican industry will also have a relatively high share.

\(^{12}\)I concentrate on two kinds of shares: the value-added share in gross output, and the labor share in value added. The value-added share in gross output is the complement of the intermediate inputs share in gross output. The focus in these two shares is because their measurement is relatively more accurate than other inputs in production, such as capital.
Figure 4: Share of value added in gross output

Source: Author’s calculation using data from WIOD.

Figure 5: Share of value added in gross output and a 45 degree line

Source: Author’s calculation using data from WIOD.

Figure 5 shows the same plot but adding a 45 degree line. This figure indicates that despite the close correlation between Mexican and US shares, the Mexican industries tend to have a larger value added-share on gross output relative to the US industries. Alternatively, the data shows that the intermediate inputs shares in Mexico are depressed relative to the US ones.

2.3.2 Labor share

The labor income share is low in México, as in many developing countries. It is commonly believed that this is due to the measurement arguments emphasized by Gollin (2002). The main argument made by Gollin is that in developing countries, there is a substantial fraction of labor income that is mistakenly recorded as non-labor income in national accounts. The main reason for this is the large presence of self-employment and unpaid family workers in developing countries.
Figure 6: Labor income share in value-added

![Figure 6: Labor income share in value-added](image)

Source: Author’s calculation using data from WIOD.

Table 2: Aggregate labor share: Naive vs. corrected calculation

<table>
<thead>
<tr>
<th>Labor share (2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico “naive”</td>
</tr>
<tr>
<td>Mexico “corrected”</td>
</tr>
</tbody>
</table>

Source: Author’s own calculation. The “naive” calculation refers to the exercise of taking the ratio of labor compensations to GDP straight from National Accounts. The “corrected” calculation refers to the exercise described in Conesa et al., 2007.

Figure 6 presents a scatter plot of the sectoral labor shares for Mexico and the United States calculated using WIOD data. The WIOD makes a correction of labor compensations in developing countries to take into account the large presence of self-employment (see Timmer et al., 2012 for details), however it does not take into account the presence of unpaid family workers. The Figure shows that labor shares are positively correlated between Mexico and the United States, however, Mexico consistently exhibits lower labor shares.

In Table 2, I present an exercise to correct for the measurement problems emphasized by Gollin following the methodology proposed by Conesa et al., 2007. Due to the lack of information by sector, this exercise is performed using aggregate data. The table shows that even performing this correction the Mexican labor income share remains well below to US share, which is roughly around 2/3.

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The methodology departs from the observation that the concept of “labor compensations” in National Accounts unambiguously corresponds to labor income. Thus, the idea is to identify the fraction of GDP that includes this concept and its corresponding capital income. Since ambiguous income is recorded as Net Mixed Income from the household sector, this is subtracted from GDP (together with Net indirect taxes), and then the ratio of “labor compensations” to this “adjusted” GDP is obtained.
3 Model

The model here is a version of the one found in Long Jr and Plosser (1987), which was also recently used by Jones (2011b) and Acemoglu et al. (2012). Consider an economy with $N$ sectors. The supply of labor ($H$) is exogenous and each sector uses labor and commodities from all other sectors (including its own) to produce. We assume that the production function of a representative firm in sector $i$ is represented by the following Cobb-Douglas technology:

$$Q_i = A_i(H_i)^{\alpha_i} (1 - \sigma_i \lambda_i) \prod_{j=1}^{N} x_{ij}^{\sigma_{ij}} M_i^{\lambda_i},$$  \hspace{1cm} (1)

where $x_{ij}$ represents the intermediate demand that industry $i$ makes from industry $j$ and $M_i$ is the quantity of a foreign intermediate good imported by sector $i$. $A_i$ and $H_i$ represent an exogenous productivity term, and labor used in sector $i$, respectively. Also, we define $\sigma_i = \sum_{j=1}^{N} \sigma_{ij}$. Notice that this production function exhibits Decreasing Returns to Scale (DRS), an assumption that is taken without loss of generality.\textsuperscript{14}

The output from each sector $Q_j$, can be used either as a consumption good ($c_j$), or as an intermediate input in the production of the other sectors. Thus, the resource constraint of each sector $j$ is given by:

$$Q_j = c_j + \sum_{i=1}^{N} x_{ij}, \forall i = 1, ..., N.$$  \hspace{1cm} (2)

Consumption ($c_1, ..., c_N$) is combined to produce a single final good, according to the following function:\textsuperscript{15}

$$Y(c_1, ..., c_N) = c_1^{\beta_1} c_2^{\beta_2} ... c_N^{\beta_N}.$$  \hspace{1cm} (3)

At this point, it is useful to note the Cobb-Douglas form of $Y(c_1, ..., c_N)$. This assumption will turn out to be important for the way labor resources are allocated across sectors in equilibrium (see section 3.1).

\textsuperscript{14}DRS has the advantage that it allows for a clear interpretation of the wedges, in particular, using this specification, makes it straightforward to relate the industrial labor share with the coefficient $\alpha_i$ (see also section 4).

\textsuperscript{15}Alternatively, I could have used a utility function to generate the demand side of the economy.
Problem of the representative household  This problem is quite trivial, but it is useful to write it down for future reference.

\[
\max_{\{c\}} \{u(C)\}
\]

\[s.t. \ C = wH + \Pi + T\]  

(2)

where \( C \) is aggregate consumption, \( w \) is the price of labor, \( \Pi \) are aggregate profits, and \( T \) are transfers. These transfers are financed with the rents associated with the distortions that affect optimal decisions of firms (see below). Provided \( u \) is increasing, the solution for this problem is trivial: the household will consume all the available income.

Problem of the final good producer  The problem of the final good producer consists on choosing \( \{c_i\} \), taking \( \{p_i\} \) as given, to solve:

\[
\max_{\{c_i\}} \left\{ c_1^{\beta_1} c_2^{\beta_2} \cdots c_N^{\beta_N} - \sum_{i=1}^{N} p_i c_i \right\}.
\]

The first order conditions are given by:

\[
\beta_i(Y/c_i) - p_i = 0 \iff \beta_i = \frac{p_i c_i}{Y}, \ \forall i.
\]  

(3)

Just like in the textbook Cobb-Douglas utility maximization problem subject to a budget constraint, the first order conditions of the problem above imply that the consumption shares are constant and equal to the coefficient of each consumption good in the production (or utility) function.

Problem of the representative firm in sector \( i \)  There exists a representative firm in each sector. Each firm faces distortions that are specific to the industry. I assume three distortions: \( \tau_i \), \( \psi_i \), and \( \phi_i \). The first distortion \( (\tau_i) \) represents output taxes that we will be able to pin-down using data on tax revenues at the industry level. The second distortion \( (\psi_i) \) enters in the firm’s problem in a way that resembles an output tax, but it is designed
to capture other distortions that are not captured by the tax revenue data. In general, this distortion introduces a wedge between marginal revenue and marginal cost. One possible interpretation is that a markup is in place due to the existence of imperfect competition. However, other forces might act through the same channel, and be therefore captured by $\psi_i$. For simplicity, I will refer to this wedge as the “markup” and will be defined in such a way that if $\psi_i > 1$, then it means that marginal revenue is above marginal cost, and vice-verse.

The last distortion, $\phi_i$, introduces a wedge between the marginal revenue product of labor and its marginal cost, and it enters in the firm’s problem as a labor tax. We define $\phi_i$ similarly to $\psi_i$, so that if $\phi_i > 1$, labor productivity is higher than the wage. For simplicity we will refer to this wedge as the “labor wedge”. Two alternative interpretations for this wedge are in place. The first one is that the marginal cost of labor faced by the firm is higher than the wage received by the workers due to policies and institutional constraints that make labor costs higher to firms. A second interpretation is that the value of the marginal productivity of labor is higher than the wage because of a low bargaining power of workers. The two interpretations differ in terms of who keeps the rents associated with the wedge. In the first interpretation, the rents are kept by agents involved in rent-seeking activities (not modeled), while in the second one are kept by the firms. In the model it is assumed that the household is the owner of the labor resources, of the firms, and of any rents associated with wedges. As long as all rents are given back to the household as lump sum transfers, the results are independent of the above alternative interpretations.

The problem of the representative firm in industry $i$ is given by:

$$\max_{H_i, \{x_{ij}\}, M_i} \left\{ \frac{(1 - \tau_i)}{\psi_i} p_i A_i(H_i)^{\alpha_i(1 - \sigma_i - \lambda_i)} \prod_{j=1}^{N} x_{ij}^{\sigma_{ij}} M_i^{\lambda_i} - \phi_i w H_i - \sum_{j=1}^{N} p_j x_{ij} - p_{M,i} M_i \right\}$$

and the first order conditions (FOCs) are as follow:

$$\frac{(1 - \tau_i)}{\psi_i} \alpha_i(1 - \sigma_i - \lambda_i) \frac{p_i Q_i}{H_i} = \phi_i w, \forall i \quad (4)$$

$$\frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \frac{p_i Q_i}{x_{ij}} = p_j, \forall i, j \quad (5)$$

$$\frac{(1 - \tau_i)}{\psi_i} \lambda_i \frac{p_i Q_i}{M_i} = p_{M,i}, \forall i \quad (6)$$

16
The interpretation of the above conditions is straightforward, the household chooses labor, and intermediate inputs to equalize the (distorted) marginal revenue to the (distorted) marginal cost in each case. Note that the markup $\psi_i$ affects the three conditions above identically: it increases marginal revenue above marginal cost for each input; while the labor wedge $\phi_i$ affects only the first order condition associated with the choice of hours (4). This feature will be useful in the Calibration part in order to identify the value of these wedges.

**Equilibrium** With this, I can provide a definition of competitive equilibrium. Given import prices, taxes, and wedges $\{p_{M,i}, \tau_i, \phi_i, and \psi_i\}$, a competitive equilibrium consists in quantities $\{H_i, x_{ij}, M_i, c_i\}$; and prices $\{p_j\}$ and $w$, $\forall i, j = 1, ..., N$; such that:

1. $\{c_i\}$ solves the representative final good producer problem at the equilibrium prices.
2. $H_i, \{x_{ij}\}$ and $M_i$ solve sector’s $i$ producer problem at the equilibrium prices.
3. Markets for labor, and goods $j = 1, ..., N$ clear.

An operative definition of equilibrium is obtained by writing the production function as $Q_i = A_i f(H_i, \{x_{ij}\}_j, M_i)$, where $f(H_i, \{x_{ij}\}_j, M_i) = (H_i)^{\alpha_i (1 - \sigma_i - \lambda_i)} \prod_{j=1}^{N} x_{ij}^{\sigma_{ij}} M_i^{\lambda_i}$. Using this expression, an operative definition of equilibrium consists of quantities $\{c_i, \{x_{ij}\}, H_i, M_i\}$, and prices $\{p_i\}, w$, $\forall i, j$; such that:

\[
\frac{(1 - \tau_i)}{\psi_i} \alpha_i (1 - \sigma_i - \lambda_i)p_i A_i f(H_i, \{x_{ij}\}_j, M_i) = \phi_i w H_i, \ \forall i \tag{7}
\]

\[
\frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} p_i A_i f(H_i, \{x_{ij}\}_j, M_i) = p_j x_{ij}, \ \forall i, j \tag{8}
\]

\[
\frac{(1 - \tau_i)}{\psi_i} \lambda_i p_i A_i f(H_i, \{x_{ij}\}_j, M_i) = p_{Mi} M_i, \ \forall i \tag{9}
\]

\[
\beta_i = \frac{p_i c_i}{\sum_{i=1}^{N} p_i c_i}, \ \forall i \tag{10}
\]

\[
A_i f(H_i, \{x_{ij}\}_j, M_i) = c_j + \sum_{i=1}^{N} x_{ij}, \ \forall i \tag{11}
\]
\[
\sum_{i=1}^{N} H_i = H \quad (12)
\]

This constitutes a system of \(N \times N + 4N + 1\) equations with the same number of unknowns, which has an analytic solution (see Jones, 2011, and the Appendix to this paper).

Note that the form of the resource constraint is related to the assumption on whether the rents from the distortions (\(\tau_i, \phi_i, \text{and} \psi_i\)) are given back to the household or not. For the baseline case, I assume that all rents from wedges and taxes are given back to the household, and therefore, \(T\) in the budget constraint 2 has three elements \(T = T^\tau + T^\phi + T^\psi\), which correspond to the aggregate rents associated with each distortion. As a result, these resources are available for consumption, and the resource constraints take the form in 11.

### 3.1 Analysis of equilibrium

In this section I describe the features of the equilibrium that are important for the results in the paper.

**Aggregate output and the vector of influence.** The first feature is related to the way each sector is connected with the rest of the economy, and how this determines the effect that changes in productivity of a given sector have on aggregate outcomes. To start, note that it can be shown (see Jones, 2011b, and the appendix here) that equilibrium aggregate output is given by:

\[
Y = AH^{\hat{a}} \quad (13)
\]

where \(H\) is aggregate labor, \(\hat{a}\) and \(A\) are constants that depend on parameters (see the Appendix). Furthermore, it can be shown that \(\ln A = m'a + \text{const}\), where:

\[
m'a = [m_1 \ m_2 \ m_3 \ ... \ m_N] \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}, \quad (14)
\]
\[ a_i = \ln A_i, \ \forall i, \] and the vector \( m \) is known as the “vector of influence” (Acemoglu et al., 2012) or the “vector of multipliers” (Jones, 2011b). Taking logs in both sides and deriving with respect to \( a_i \), we have:

\[ d\ln(Y) = m_i da_i \]  

(15)

Which states that the log change in aggregate output is a linear function of the log change in productivity \( A_i \), with the slope of this linear function given by the multiplier of sector \( i \): \( m_i \).

The vector of multipliers is defined by

\[ m' = \frac{\beta' (I - B)^{-1}}{1 - \beta' (I - B)^{-1} \lambda}, \]

where \( \beta \) is the vector of consumption shares, \( B \) is the input-output matrix of technical coefficients with typical element \( \sigma_{ij} \), and \( \lambda \) is a vector with typical element \( \lambda_i \). The interpretation of an element \( m_i \) is that a 1\% increase in productivity \( A_i \), rises aggregate GDP in \( m_i \)\%. 16 To gain more intuition consider the case of a closed economy. In this case the vector of influence boils down to:

\[ m' = \beta' (I - B)^{-1} \]  

(16)

Thus, the elements of this vector depend on two terms \( \beta \) and \( (I - B)^{-1} \). The first term collects the final consumption shares, while the second one is a NxN matrix known as the Leontief inverse. The traditional interpretation of this matrix is that a typical element \( l_{ij} \) of it gives the change in sales of sector \( j \) needed to achieve an increase in final consumption expenditures in sector \( i \) of $1 dollar. This includes all the direct and indirect effect that occur through the input-output network.

One key observation in Leontief (1986) is about the existence of a multiplier: an original increase of $1 dollar in final consumption expenditures of sector \( i \) leads to an increase of sales of more than one dollar \( (l_{ii} > 1) \). 17 In a companion paper (Leal, 2015), I show that this multiplier also applies to gross output: start with an exogenous change that produces an original increase in gross output of sector \( i \) of 1\%. Take, for example, a 1\% increase in productivity of sector \( i \). Thus, \( (l_{ij})(1\%) \) gives you the percentage increase in gross output of sector \( j \) due to that original change. Moreover, since final consumption is a constant

---

16In fact, this interpretation depends on the accuracy of the logarithmic approximation which it is only valid for small changes in \( A_i \). In general, and specially for the exercise of interest in this paper where closing productivity gaps requires large changes in \( A_i \), this interpretation will not be accurate.

17The reason for this is that in order to increase consumption it is necessary to also increase the purchases (and the sales) of intermediate inputs from other sectors, which also use sector \( i \) as an intermediate input.
fraction of gross output, \((l_{ij})(1\%)\) also gives you the percentage change in final consumption of sector \(j\) as a response to the exogenous change that occurred in sector \(i\). Thus, to obtain the total change in aggregate consumption (which equals GDP), one only needs to add-up these changes across sectors using the appropriate weights, which in this case correspond to the \(\beta_i\) coefficients. Therefore, the aggregate change in GDP due to an increase of 1% in the productivity of sector \(i\) is given by: \(\sum_{j=1}^{N} \beta_j l_{ij}\). Which in vectorial notation corresponds to equation 16.

Since aggregate final consumption expenditures is equal to aggregate value added, there is a close connection between the degree of influence of a sector and its value added share. The value generated in one sector can potentially be used to support final consumption in all other sectors. The way this value reaches other sectors is through the input-output network because the output of one sector can be used as an intermediate input and indirectly produce the final consumptions of many other sectors. In fact, when productivity affects only the efficiency of primary inputs (for example, when \(Q_i = (Z_i H_i^{\alpha_i})^{(1-\sigma_i)} \prod_{j=1}^{N} x_{ij}^{\sigma_{ij}}\)), such as in Acemoglu et al. (2012), the vector of influence is equal to the vector of value added shares, and the input-output economy is equivalent to a value-added economy. Note that in this case, a 1% increase in the productivity of all sectors leads to a 1% increase in GDP, as one would expect in a value-added economy (see Leal, 2015 for more details).

A key assumption in this model is that productivity changes preserve a balance between primary and intermediate inputs, affecting the efficiency with which all inputs are used in production (see equation 1). As a result, a 1% increase in productivity increases the productivity of primary inputs by more than 1% (note that \(Z_i = A_i^{1-\sigma_i}\), and \(\frac{1}{1-\sigma_i} > 1\)). In this case, the vector influence is equal to sales divided by GDP - the Domar (1961) weights:

\[
m_i = \frac{p_i Q_i}{Y},
\]

As a result, if the productivity of all sectors increases by 1%, aggregate output increases by more than 1%. The total effect would be given by \(\sum_{i=1}^{N} m_i = \frac{\sum_{i=1}^{N} p_i Q_i}{Y} = \frac{\sum_{i=1}^{N} p_i Q_i}{\sum_{i=1}^{N} (1-\sigma_i) p_i Q_i}\), which is larger than 1.\(^{18}\)

\(^{18}\)Note also how closely related are the value added shares to the Domar weights: \(v_i = \frac{(1-\sigma_i) p_i Q_i}{Y} = (1-\sigma_i) m_i\).
3.1.1 The effect of distortions.

I divide the analysis on the effect of distortions in three parts. First, I analyze the effect of distortions on the allocation of labor across sectors; then, I move forward to analyze the effects of distortions on the allocation of output between final and intermediate uses; finally, I analyze the total effect of distortions on aggregate output. It will be convenient for didactic purposes to focus on the case of a closed economy facing wedges between marginal revenue and marginal cost ($\psi_i$).

**Effect on labor allocations.** In the undistorted economy, the equilibrium allocation of labor is determined by the equalization of marginal productivity (MP) of labor across sectors. What matters for this allocation is the way in which each unit of labor across the different sectors affects the supply of aggregate output $Y$. To gain intuition, consider a simple 2-sector model without inter-sectoral linkages; thus, $B = 0$, $\sigma_i = 0$, $\forall i$, and $Q_i = c_i$, $\forall i$. In this case, the trade-off is quite simple: the more labor is allocated to sector 1 and the more $c_1$ is produced; the less labor is allocated to sector 2, and the less $c_2$ is produced. The equilibrium allocation of labor is determined by the following efficiency condition:

$$
\left( \frac{\partial Y(c_1, c_2)}{\partial c_1} \right) \left( \frac{dc_1}{dh_1} \right) = \left( \frac{\partial Y(c_1, c_2)}{\partial c_2} \right) \left( \frac{dc_2}{dh_2} \right),
$$

(17)

where I have used a lower-case $h$ to denote labor in this simple 2-sector model and make a difference with the labor allocation in the richer model with inter-connections.\(^{19}\) The left hand side is the marginal productivity of the composite with respect to labor $h_1$, while the right hand side is the marginal productivity with respect to $h_2$. The efficiency condition above indicates that labor should be allocated to sector 1 until the marginal productivity of $h_1$ is equal to the marginal cost, which is precisely the lost production in sector 2 (due to the reduction in $h_2$). In this simple model the efficient allocation of labor is given by:

$$
\hat{h}_i = \frac{\alpha_i \beta_i}{H} = \frac{\alpha_i p_i Q_i}{\sum_{s=1}^{N} \alpha_s \beta_s} = \frac{\alpha_i p_i Q_i}{\sum_{s=1}^{N} \alpha_s p_s Q_s},
$$

(18)

which depends on the influence of each sector ($\beta_i$) and the labor income shares ($\alpha_i$). Note that since $B = 0$, the vector of influence is simply $m = \beta$. Also note that I have used a hat to indicate that this allocation corresponds to the undistorted economy. In addition,\(^{19}\)

\(^{19}\)Note that I have arrived to the above equation by combining the first order conditions of the firm’s problem in each sector with the first order conditions in the problem of the composite producer. Alternatively, it can be derived as the optimal condition of a social planner’s problem.
the second equality reveals that the fraction of labor allocated to sector $i$ ($\hat{h}_i/H$) equals the share of labor compensations of sector $i$ on aggregate labor compensations. Note that labor compensations in sector $i$ are given by the fraction $\alpha_i$ of the value added in sector $i$, which in this economy is simply $p_iQ_i$.

For the economy with inter-sectoral linkages a condition similar to 17 also holds, and it is easy to show that equilibrium labor is given by:

$$\frac{\hat{H}_i}{H} = \hat{\theta}_i = \frac{\alpha_i(1 - \sigma_i)m_i}{\sum_{s=1}^{N} \alpha_s(1 - \sigma_s)m_s} = \frac{\alpha_i(1 - \sigma_i)p_iQ_i}{\sum_{s=1}^{N} \alpha_s(1 - \sigma_s)p_sQ_s},$$

The expression above says that labor ought to be allocated taking into account the relative influence of each sector ($m_i$), the share of labor income in value added ($\alpha_i$), and the share of value added in gross output ($1 - \sigma_i$). Note that, by the second inequality, relative labor also equals relative labor compensations, just as in the previous case.\(^\text{20}\) Finally, note that when there are no linkages ($B = 0$, and $\sigma_i = 0, \forall i$), the expression above converges to equation 18.

The allocation of labor is independent of the productivity parameters $\{A_i\}_{i=1}^{N}$. The reason for this is that there is some degree of complementary between any two consumption goods $c_i$ and $c_j$ in the production of the composite. The social planner wants to allocate labor in such a way that the marginal productivity of labor in the composite production function is equalized across sectors (equation 17). However, when $A_i$ increases, this increases not only the marginal productivity of $H_i$, but also the marginal productivity of $H_j$ (because more $c_i$ is produced and, thus, $j$ has more $c_i$ to produce with). It turns out that due to the Cobb-Douglas form of the production functions, the marginal productivity of both, $i$ and $j$, shift up by the same magnitude, and the allocation of labor remains unaltered in response to a change in productivity.\(^\text{21}\)

An important point regarding the literature on resource misallocation is opportune at this point. While in standard models of heterogeneous firms the allocation of resources is largely

\(^{20}\)There is a slight difference, though, in the case of the economy with sectoral linkages, only a fraction ($1 - \sigma_i$) of gross output corresponds to value added, and labor compensations in sector $i$ are given by $\alpha_i(1 - \sigma_i)p_iQ_i$ (see equation 7).

\(^{21}\)From the perspective of the decentralized equilibrium, there are countervailing forces that affect the demand for labor in each sector in response to a change in productivity. For example, the increase in $A_i$ increases demand for labor in sector $i$ (a quantity effect), but the price of $i$ decreases (due to increased supply) which tends to reduce the demand for labor (a price effect). Similarly, the demand for labor of the other sectors is also affected by opposite forces. At the end, wages and prices change in such a way that labor demands remain unaltered by the original change in productivity. Key in this mechanism is the fact that the cross-price elasticity of demand is zero when the production function of the composite is Cobb-Douglas.
determined by relative productivity across firms; in standard multi-sector models, the allocation of resources across sectors is invariant to changes in productivity, and is largely determined by the vector of influence, which, in turn, is affected by the specification of demand (either through preferences or via the production function of the composite) and by the nature of the inter-sectoral network. As a result, misallocation across sectors will result different in nature than misallocation across plants.

Consider now the equilibrium allocation in the distorted economy, which is denoted without a hat to separate it from the allocation in the undistorted economy. It becomes relevant to distinguish between two cases regarding the distribution of rents from distortions:

Case 1.- The rents are given back to the household as lump sum transfers.

Case 2.- The rents are lost in the sea.

Take Case 1 and consider how the labor allocation looks like for the simple 2-sector model without inter-sectoral linkages and when only the markup wedge is present. In this case, the equalization of marginal productivity of labor across sectors is broken and the labor allocation is given by:

$$\frac{h_i}{H} = \frac{\alpha_i \left(\frac{1}{\psi_i}\right) \beta_i}{\sum_{s=1}^{N} \alpha_s \left(\frac{1}{\psi_s}\right) \beta_s}, \quad i = 1, 2$$

Thus, the presence of wedges in the simple economy creates misallocation of labor across sectors by shifting resources away from those sectors where wedges are large, and into those sectors where wedges are low. Note also that, there is no misallocation when wedges are the same across sectors ($\psi_i = \psi, \forall i$). The reason for this is that in such case, marginal productivity is affected proportionally across sectors. Furthermore, in the simple model, the level of distortions does not affects aggregate output as long as wedges are homogeneous across sectors. This occurs in equilibrium because the only primary factor of production is inelastically supplied, and the wage rate absorbs all the burden imposed by distortions.

In the economy with inter-sectoral linkages, the distorted equilibrium allocation of labor is similar:

$$\frac{H_i}{H} = \theta_i = \frac{\alpha_i (1 - \sigma_i) \left(\frac{1}{\psi_i}\right) \bar{m}_i}{\sum_{s=1}^{N} \alpha_s (1 - \sigma_s) \left(\frac{1}{\psi_s}\right) \bar{m}_s} = \frac{\alpha_i (1 - \sigma_i) \left(\frac{1}{\psi_i}\right) p_i Q_i}{\sum_{s=1}^{N} \alpha_s (1 - \sigma_s) \left(\frac{1}{\psi_s}\right) p_s Q_s}$$

23
where $\tilde{m} = \beta'(I - \tilde{B})^{-1}$, is a vector similar to the vector of influence but computed using the matrix $\tilde{B}$ instead, for which the typical element is $\sigma_{ij}/\psi_i$. Thus, labor is misallocated away from those sectors with high markup wedges, and into sectors with low wedges.\footnote{Note also that in this distorted equilibrium it is still true that the fraction of labor in sector $i$ equals the share of labor compensations in that sector (second equality). The only difference now is that those compensations are affected by distortions.}

Next, consider Case 2 which assumes that the rents from distortions are taken out of the economy. In this case, the markup wedge $1/\psi_i$ is isomorphic to sectoral productivity $A_i$. To see this, note first that, since $T = 0$, the budget constraint in the household problem, 2, becomes $C = \Pi + wH$, and markup rents are not available for consumption. This implies that the resource constraint has to be replaced with:

$$
\frac{1}{\psi_i} A_i f(H_i, \{x_{ij}\}, M_i) = c_j + \sum_{i=1}^{N} x_{ij}, \forall i
$$

(19)

Note that if we replace equation 11 with equation 19, then both parameters $(1/\psi_i$ and $A_i$) affect equilibrium conditions (7 to 10 plus 19) in exactly the same way. Thus, changes in distortions in this case, can have large impacts on aggregate output through a feasibility channel.\footnote{Note also that since changes in productivity do not affect the allocation of labor in equilibrium, this means that distortions in Case 2 do not misallocate labor across sectors neither.}

To further illustrate this point, take again the simple 2-sector model without inter-sectoral linkages, and make it even simpler by assuming that $\alpha_i = 1, \forall i = 1, 2$. In the absence of distortions, labor is given by $\hat{h}_i/H = \beta_i$, and aggregate output is $\hat{Y} = (A_1\beta_1)^{\beta_1} (A_2\beta_2)^{\beta_2} H$. When distortions are introduced, and the rents from distortions are not given back to the household (Case 2), aggregate output is given by $Y = (\tilde{A}_1\beta_1)^{\beta_1} (\tilde{A}_2\beta_2)^{\beta_2} H$, where $\tilde{A}_i = \frac{A_i}{\psi_i}$.

The contrasting effects on aggregate output between Cases 1 and 2 is informative about the economic channel through which labor misallocation operates in the model. In particular, notice that the misallocation of labor is present in Case 1 due to the extra income effect that transfers entail. When we reduce the distortion of sector $i$ ($\psi_i$) the rents associated with that distortion are also reduced, and, as a result, there is less income to consume, overall. This, in turn, translates into less labor being allocated to every sector. However, the reduction of $\psi_i$ increases the marginal revenue product of labor in sector $i$, which mitigates the negative income effect on that sector. These forces, reallocate labor into sector $i$ and away from every other sector. \footnote{Note also that, in the simple 2-sector model with distortions, when transfers are given back to the}
Effect on the allocation of output between consumption and intermediates. In the basic model without inter-sectoral linkages, distortions can’t have an impact on the supply of labor, because this factor is supplied inelastically. In the richer model with inter-sectoral linkages, there are N-inputs in addition to labor, which are provided using output from the sectors. In contrast to the supply of labor, the supply of these N-inputs is not inelastic and can be affected by the level of wedges. In fact, one important margin that is affected by the presence of the markup wedge is the allocation of gross output between consumption and intermediate uses.

To see this, note that the ratio of intermediate inputs to gross output is a function of distortions:

$$\frac{X_i}{Q_i} = 1 - \frac{\beta_i}{\tilde{m}_i(\psi_1, ..., \psi_N)}.$$  

where $\tilde{m}_i$ is a typical element of $\tilde{m} = \beta'(I - \tilde{B})^{-1}$. This equation says that a reduction in one distortion, leads to an increase of the use of intermediate inputs, and hence, to a reduction in the fraction of gross output that goes to final consumption. To see it, note that the elements $\tilde{m}_i$ depend negatively on the level of distortions (because a typical element of $\tilde{B}$ is $\frac{\sigma_{ij}}{\psi_i}$) and that a single distortion $\psi_i$, affects all elements of $\tilde{m}_i$, simultaneously. Thus, when we reduce a distortion, all $\tilde{m}_i$’s increase and the fraction of gross output that is used as intermediate inputs increases in every sector.

This result is intuitive, a reduction in $\psi_i$ increases the ratio of expenditures in intermediate inputs over gross output in sector $i$ because $\left(\sum_{j=1}^{N} p_j x_{ij}\right)/p_i Q_i = \sigma_i/\psi_i$ (see equation 5). As a result, intermediate demand increases for all sectors. Additionally, a reduction in $\psi_i$, reduces the transfers associated with the rents, which affects the household’s demand for consumption. These effects combined lead to an increase in the ratios $\frac{X_j}{Q_j}, \forall j$.

The total effect of eliminating the markup wedge. The total effect of changing individual distortions on aggregate output can be first illustrated in the context of the 2-sector model. Denote values of the variables before the change in $\psi_i$ with a 0 superscript, and those after the change with a 1 superscript. The effect of changes in $\psi_i$ on (log) aggregate output is given by:

household (Case 1) the allocation of labor is given by $\frac{h_i}{H} = \theta_i = \frac{\beta_i/\psi_i}{\sum_{i=1}^{N} \beta_i/\psi_i}$, and aggregate output is $Y_1 = (A_1 \theta_1)^{\beta_1} (A_2 \theta_2)^{\beta_2} H$.  

25
\[ \ln \left( \frac{Y_1}{Y_0} \right) = \sum_{s=1}^{2} \beta_s \ln \left( \frac{\theta^1_s}{\theta^0_s} \right). \] (20)

The above equation shows that, a reduction in a single wedge \( \psi_i \) will not necessarily increase aggregate output. To see this mathematically, assume that we reduce the distortion of sector 1 (\( \psi_1^1 < \psi_0^1 \)), then more labor will be allocated to sector 1 (\( \theta_1^1 > \theta_0^1 \)), and less labor to sector 2 (\( \theta_2^1 < \theta_2^0 \)). Since the total effect on aggregate output is equal to the sum of these two effects, it is not clear which one will dominate. In general, output will increase if the movement in \( \psi_i \) is in the direction that brings the two distortions (\( \psi_1 \) and \( \psi_2 \)) closer to each other, because, this way, misallocation is reduced; otherwise, output will decrease. Also important is each sector’s “degree of influence” which in this case is simply the weight \( \beta_i \), \( i = 1, 2 \). For example, when \( \beta_1 = 0 \), then, it does not matter for aggregate output what the value of \( \psi_1 \) is.

It can be shown that the effect on aggregate output of changing the markup wedge in sector \( i \), \( \psi_i \), in the input-output model, is given by the following equation:

\[ \ln \left( \frac{Y_1}{Y_0} \right) = \sum_{j=1}^{N} m_j \alpha_i (1 - \sigma_i) \ln \left( \frac{\theta^1_j}{\theta^0_j} \right) + m_i \sigma_i \ln \left( \frac{\psi^0_i}{\psi^1_i} \right) + \sum_{j=1}^{N} m_j (1 - \sigma_j) \ln \left( \frac{\tilde{m}_0^j}{\tilde{m}_1^j} \right) \] (21)

The equation above is analogous to equation 20. The first term is again the effect of misallocation of labor across sectors. This can be improved or worsened depending on whether the movement in \( \psi_i \) goes in the direction of equalizing wedges. Note that there is a slight difference between this term and the right hand side of equation 20: the weights of the relative \( \theta's \) are now given by the degree of influence (\( m_i \)) adjusted by the coefficient of labor in the production function \( \alpha_i(1 - \sigma_i) \). Finally, note the importance of the degree of influence to determine the final sign of the misallocation effect: if \( m_i \) is large, then the sum will give a larger weight to the positive effect (\( \theta_1^1 > \theta_0^0 \)) and less weight to the negative one.

The second term in the equation above is a direct effect of the change in markup \( \psi_i \) on aggregate output. If the markup is reduced, this term is positive. This effect is present because the markup is affecting the supply of an input. We did not have this effect before, in the simple model, because the only input in the sectorial production functions was labor, and it was not being produced. Thus, the second term captures the common idea that less taxes/distortions on a factor, induce a higher supply of this factor.
The third term captures the effect of $\psi_i$ on the degree of misallocation of gross output between final and intermediate uses. Note that the weights are the degrees of influence ($m_i$), adjusted by the shares of value added in gross output ($1 - \sigma_i$). As explained above, $\tilde{m}_i$ controls the way in which gross output is divided between the two uses: consumption $c_j$, vs., intermediate inputs $X_j = \sum_{s=1}^{N} x_{sj}$. Thus, when $\psi_i$ is reduced, $\tilde{m}_j$ increases $\forall j$, and the whole term is negative. The result is intuitive, since a reduction in the wedge translates into a lower ratio $c_i/Q_i$ through this channel.

4 Calibration

There is a fundamental identification problem in the calibration that consists on the following. I observe factor shares for the intermediate goods and for labor, which are a function of technology parameters (the Cobb-Douglas exponents) and distortions. The identification problem is how to separate these two. I address this problem by focusing on the differences between México and the U.S. The advantage of this approach is that, by focusing on a single country, I can assess the reasonableness of the assumptions needed for identification.

I proceed by making two assumptions:

1. The U.S. has no distortions.

2. The labor exponent ($\alpha_i$) and the total intermediate goods exponents ($\lambda_i + \sigma_i$) are the same in the U.S. and Mexico.

Assumptions 1 and 2 imply that we can read technological parameters for both economies from the U.S. factor shares data.\(^{25}\) To see this, note that from equations 5 and 6, we obtain (see also the appendix):

$$\sigma_i + \lambda_i = \sum_{j=1}^{N} \sigma_{ij} + \lambda_i = \left( \frac{\sum_{j=1}^{N} p_{ij} x_{ij}}{p_i Q_i} \right)_{US} + \left( \frac{p_{M_i} M_i}{p_i Q_i} \right)_{US},$$

similarly, we can use equation 4 and data of the labor share in the U.S. to obtain:

\(^{25}\)The data used for calibration is available in the input-output tables of Mexico and the U.S. published by the World Input-Output Database (WIOD).
\[ \alpha_i = \left( \frac{wH_i}{(1 - \sigma_i - \lambda_i)p_iQ_i} \right)_{US} . \]

Given these technological parameters, we can use the factor shares in Mexico to obtain the value of the distortions. Using again equations 5 and 6, and data for Mexico, we obtain:

\[ \psi_i = \frac{(\sigma_i + \lambda_i)}{\left( \sum_{j=1}^{N} \frac{p_jx_{ij}}{p_iQ_i} \right)_{MX} + \left( \frac{p_{M,i}M_i}{p_iQ_i} \right)_{MX}} , \]

and given the value of the parameters and the markup wedge we can use the first order condition with respect to labor (eq. 4) to obtain \( \phi_i \):

\[ \phi_i = \psi_i \frac{wH_i}{(1 - \sigma_i - \lambda_i)p_iQ_i}_{MX}  \]

The procedure could be interpreted as using the deviations from the 45 degree line in Figure 5 as the size of the markup wedges in Mexico. Notice also that, the calibrated labor wedge depends on the value of the markup wedge. Put it differently, the observed labor income share in Mexico, at a given industry, is affected by both: the markup wedge and the labor wedge. Thus, part of the explanation of the low labor income shares in Mexico relies on the existence of the markup wedges.²⁶

In addition to assumptions 1 and 2 above, note that, since the set-up of the model I have also made an assumption regarding the nature of the distortions. In particular, note that, I don’t allow for differential distortions across different intermediate goods, which implies that differences in factor shares of individual intermediate inputs are picked-up by differences in technology parameters (i.e. \( \sigma_{ij}^{US} \neq \sigma_{ij}^{MX} \) and \( \lambda_i^{US} \neq \lambda_i^{MX} \)).²⁷

It is emphasized that thanks to this calibration strategy, all parameters of the technology (for a given industry) are country-specific, except for two: \( \alpha_i \) and \( \sigma_i + \lambda_i \). Thus, one advantage of this strategy is that a significant amount of heterogeneity in the technologies of the two

²⁶This is the intuition used by Hall (1988), when he uses the ratio of labor compensation to total revenue to study imperfect competition in US industries.

²⁷Given the value of \( \psi_i \), I can use equations 5, 6, and data of factor shares to obtain the calibrated values of \( \{ \sigma_{ij} \} \) and \( \lambda_i \).
Table 3: Calibrated parameters (averages)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
<th>$\psi$</th>
<th>$\phi$</th>
<th>$\tau$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.521</td>
<td>0.037</td>
<td>0.388</td>
<td>0.134</td>
<td>0.053</td>
<td>1.30</td>
<td>1.42</td>
<td>0.003</td>
<td>0.072</td>
</tr>
</tbody>
</table>

countries is still allowed. Table 3 shows the simple average across industries of the calibrated parameters.\(^{28}\)

The assumptions above are not uncontroversial ones. Certainly, the fact that Mexico and the U.S. are in different stages of development might indicate the presence of different technologies, which in turn would be reflected in factor shares. Similarly, distortions might be present in a variety of ways leading to differences in factor shares of individual intermediate inputs. Nonetheless, these assumptions provide a simple way to deal with the identification problem above. Moreover, I think these are reasonable, despite the controversy. Note that the assumption of equal coefficients of labor across countries is a standard one in the development literature. Similarly, the assumption of equal coefficients of total intermediate goods is consistent with the fact that the factor shares of total intermediate goods in the two countries are highly correlated (see Figure 5).\(^{29}\) Furthermore, the way distortions are modeled, allows me to focus on one possible interpretation: the presence of market power. Finally, since I have focused on Mexico, I can assess whether the calibrated values of these distortions are consistent with the facts about Mexico’s market structure (see section 5).

To calibrate $\beta_i$, I use equation 3 and data on final consumption by industry.\(^{30}\) Also, measures of productivity gaps at the industry level are needed. In principle, I could use the production function in equation 1 to pin-down the value of $A_i$. For this, I would need data on $Q_i$, $x_{ij}$, $M_i$ and $H_i$. Unfortunately, what is observed in the data is not $Q_i$, but $p_iQ_i$, and similarly for $p_jx_{ij}$ and $p_{M,i}M_i$. Thus, also needed are the relative prices of gross output and imports at the industry level to perform this operation. This is a challenge since there are just a few sources available on relative prices across countries. In addition, prices are regularly collected on final goods and services, not on the output of industries as required by the model. One database available containing prices is the one used by Inklaar and Timmer (2013), who computed gross output prices for 35 industries departing from data on the prices of final

\(^{28}\)Note that the output tax rates can be calibrated using data tax revenue data. It turned out that output taxes are quite small in both countries, and therefore irrelevant.

\(^{29}\)The assumption implies that, in the absence of distortions, the share of value-added in gross output for a given industry, should be the same in both countries. As showed in Figure 4 of Section 2, there is a strong correlation in the shares of value added in gross output between the two countries.

\(^{30}\)Consumption is defined as in the model: the difference between gross output of sector $i$ and the value of purchases of sector’s $i$ output made by the rest of the sectors.
goods and services.\textsuperscript{31} Using these prices, for Mexico and the U.S., and data on gross output and hours worked by sector from the WIOD, I compute gross output labor productivity in sector $i$ ($Q_i/H_i$) for the two countries.\textsuperscript{32} Given this, I use the model to obtain the change in $A_i$ that is necessary to close the observed labor productivity gap between Mexico and the US (see the appendix).\textsuperscript{33}

5 Results

5.1 Key sectors: vector of influence and productivity gaps

The first set of results to report correspond to the calibrated vector of influence and the distortions for Mexico. Figure 7 reports the calibrated vector of influence. The multipliers of industries in services are typically large. Focusing on those industries with “influence” larger than 0.10, we see that 5 out of 7 are in services. The two non-services industries in this group are Construction\textsuperscript{34} and Food products, beverages and tobacco.

One simple approach to identify key sectors is to rank them in two dimensions: their relative productivity with respect to the US and its “influence”. Figure 8 presents this in a scatter plot.\textsuperscript{35} The Figure additionally shows two straight lines drawn at the simple averages of the two variables. According to this ranking, the key sectors are the ones with labor productivity below average and multipliers above average, which correspond to those in the area located at the southeast of the intersection of the two straight lines. The sectors in this area are: trade, construction, transport, real estate activities, transport equipment, and agriculture.

\textsuperscript{31}They implemented a methodology that includes the use of input-output tables to go from final good prices to industry output prices. To my knowledge, this is the only publicly available data set on gross output prices at the industry level that includes a comprehensive set of developed and developing countries.

\textsuperscript{32}See Figure 2 of Section 2

\textsuperscript{33}Specifically, I use an equilibrium equation that relates changes in the productivity parameter, $A_i$, with changes in labor productivity, $Q_i/H_i$, at the industry level. See equations 37 and 38 in the appendix.

\textsuperscript{34}Construction, is the one industry with the highest multiplier and it is typically considered manufacturing. However, in contrast to most manufacturing industries, construction is a non-tradable, a characteristic that shares with most service industries.

\textsuperscript{35}Notice that the measure of productivity used is the gross output labor productivity in Mexico relative to the US for each sector.
Figure 7: Calibrated vector of influence

Figure 8: Relative productivity and multipliers
5.2 Distortions

Figure 9 reports the value of $\psi_i$ for all the 33 sectors included in the analysis. A number of values are close to 1, which imply no distortions. However, for the majority of the sectors, when the value of $\psi_i$ significantly differs from 1, it is in the direction that implies marginal revenue above marginal cost ($\psi_i > 1$). One advantage of focusing on Mexico is that we can relate the measures of distortions to specific policies that are known to be present in this country. The interpretation of the markup distortion as the presence of market power is consistent with the common idea that Mexico is a very concentrated economy. Some facts give support to this idea. For example, in contrast to the US, which had a strong and operating antitrust authority since the end of the 19th century, Mexico only reached such status in 2013. Through most of the entire twentieth century, Mexico did not have an antitrust authority at all (see, Van Fleet, 1995 and Van Fleet, 2015). For this reason, the pro-competitive culture nowadays is seriously limited.\footnote{The first antitrust commission was created in 1992. However, it was only until 2013 that the commission was granted by congress the power to punish anti-competition behavior with jail time and large fines (e.g., 10\% of sales).} An illustrative case is the one of a trade association of the transportation industry that used to enforce agreements among its members to simultaneously increase prices. The association would do so by posting the agreement in its website, using press releases describing the agreement, and providing training for workers of the member firms to “correctly” apply the increases (See COFECE, 2015). Moreover, Mexico ranks among the worst in the “Extent of market dominance” index from the World Economic Forum.\footnote{This index is constructed with the following question among business managers: “In your country, how do you characterize corporate activity? [1 = dominated by a few business groups; 7 = spread among many firms].” Mexico ranks 103 out of 140 alongside with Guinea, Tunisia and Jamaica. In contrast, the United States ranks 11 in this index close to Netherlands.} Thus, consistent with this view is the general picture described by the calibrated markups, which reflect a highly concentrated economy.\footnote{Nonetheless, some sectors do show markups less than one. To accurately interpret this case it is important to emphasize that, given the calibration strategy followed in the paper, distortions in Mexico are measured relative to the ones in the US. Thus, a markup less than 1 implies that the distortions faced in Mexico are smaller than the ones in the US.} The unconditional average of industrial markups is 1.3, while if we only take the industries with markups above 1, the average is 1.6. The Mexican Federal antitrust commission (COFECE by its Spanish acronym) has signaled interest in performing market investigations and inquiries in several industries. Currently the authority is performing research regarding possible monopoly practice in high-impact sectors, including: finance, agro-food industry, transportation, and health. But the commission has signaled interest in several other sectors such as: beverage production, medicines, education, personal care, and pension managers (see Urzúa, 2008, p. 22). Additionally, specific reg-
ulatory institutions have been created to deal with the telecommunication and the energy industries. This set of sectors, altogether intersects with the set of sectors where I find a calibrated markup greater than one. 39

Regarding the labor wedge, the estimates are presented in Figure 10. In this case, an overwhelming majority of the wedges are above 1. On average, the value of the marginal productivity of labor (net of the effect of markups) is 42% above the marginal cost of labor. Conditional on having a positive wedge, this number increases to 68%. 40

39Markups significantly above 1 are obtained in Water Transport, Wholesale Trade, Retail Trade, Food, Beverages and Tobacco, Other Non-Metalic Mineral, Wood Products, Business Services, Agriculture, Forestry & Fishing, Health and Social Work, Other Services, Inland Transport, Financial Intermediation, Hotels and Restaurants, Mining and Quarrying, Education, and Real Estate Activities.

40Labor wedges are significantly above 1 for Other Services, Utilities, Transport Equipment, Retail Trade, Post and Telecon, Leather and Footwear, Wholesale Trade, Construction, Air Transport, Motor Vehicle and Fuel Trade, Basic and Fabricated Metal, Transport Services, and Electrical and Optical Eq.
5.3 Counterfactuals

5.3.1 Effect of closing gaps

With this at hand, it is now possible to perform counter-factual exercises. The first exercise I am interested in is closing productivity gaps of individual sectors to assess its effect on equilibrium GDP per hour worked. Since there is high variation on the size of the gap, and on the degree of influence, I expect to see large differences on the effects of each sector.\footnote{To perform this exercise I proceed in the following way. Given the value of relative productivity from the data, $\ln \left( \frac{Q^m x / H^m x}{Q^u x / H^u x} \right)$, I use equation 38 to compute the change in $A_i$ needed to close this gap. Then, I feed this estimated change in $A_i$ into the model to compute the associated change in aggregate GDP. The advantage of this procedure is that the computation of levels of the productivity parameter $A_i$ is avoided.}

The effect in GDP associated with the elimination of each sectoral gap is presented in Figure 11. Closing the productivity gap in construction would increase aggregate output and aggregate labor productivity by around 20%! This is the sector that would give the biggest gain, but it is closely followed by: Wholesale Trade, Retail Trade, Food, Beverages and Tobacco, Agriculture, Forestry & Fishing, Business Services, and Real Estate Activities. Thus, once the model is used to assess the importance of each industry, the conclusion is that key sectors...
Figure 11: Effect in Y of closing the productivity gap

that give the biggest gains in aggregate output, are mostly in services.\textsuperscript{42}

To illustrate the mechanics behind this result take two typical industries in manufacturing and services: Textile and Textile products (sector 4), and Wholesale Trade (sector 20), respectively. The industry of Textiles in the US is 8 times more productive than the corresponding one in Mexico, while Wholesale Trade is only 3 times more productive. However, Trade is not only a much bigger sector than Textiles in terms of its consumption share, it is also one of the most interconnected sector in the economy: the multiplier of Trade is 5 times bigger than the multiplier in Textiles. Therefore, closing the productivity gap in Trade gives much bigger gains in GDP per worker than closing it in Textiles (15\% vs. 4\% gains), despite the fact that the productivity gap is higher in Textiles.

One way to fully appreciate the importance of the variation in “influence” across sectors is by comparing the effect of closing productivity gaps when this feature is not present in the model. This exercise is presented in Figure 12 which plots the productivity gaps (in logs) in the x-axis, and the effect of closing the gaps in the y-axis (in logs). Remember from equation 15 that the change in log aggregate output is a linear function of the change in log individual productivity:

\textsuperscript{42}Some non-services industries that also give large gains in aggregate output are: Food, Beverages and Tobacco, and Coke and Refined Petroleum, as well as Agriculture and Construction.
\[ d\ln(Y) = m_i d\alpha_i \]

The degree of influence is given by \( m_i \) which, in turn, depends on the consumption shares \( \beta_i \), and the interconnections captured by the Leontief matrix \((I - B)^{-1}\) (see equation 16). Consider three different cases. In the first one, sectors do not differ in their consumption shares and there are no interconnections, that is: \( \beta_i = 1/N \), \( \forall i \), and \((I - B)^{-1} = I\). In this case \( m_i = 1/N \) and thus, the function above becomes identical for all sectors. In this case, as the markers in circles in Figure 12 show, the larger is the gap, the larger will be the associated change in log aggregate output when this gap is closed. In the second one, I allow for variation in the consumption shares and set the \( \beta_i \)'s equal to their calibrated values. Thus, the linear relationship between the size of the gap and the effect in \( \log Y \), is now broken. This is represented with the plus (+) markers in the Figure. Finally, in the third case, I allow for both: differences in \( \beta_i \) and inter-connections \((B \neq 0)\). In this case, the correlation between log gap and the change in log \( Y \) is even worse: the red stars (*) markers in the Figure represent this last case, these markers are all over the place, forming a cloud.

Figure 12: Decomposing the effect in \( Y \) of closing the productivity gap

5.3.2 Effects of eliminating markups and labor wedges

The next counterfactual exercise of interest consists on reducing the distortions in the model: the markups and the labor wedges. It is convenient to split the analysis into the two previous cases: Case 1, when the rents from distortions are given back to the household; and Case 2, when the rents are lost. In both cases, I set distortions equal to 1, industry by industry, and compute its effects on aggregate output. This exercise is presented in Figure 13 for the case of the markup wedges.
In general, the effect of eliminating markups in Case 1 is smaller than the effect of eliminating productivity gaps, and, for the majority of the sectors, this effect is negligible. This is due to the fact that there exists countervailing effects on aggregate output in response to changes in markup wedges.

Note that, the higher is the influence of the sector, the more important will be the direct positive effect of reducing the markup, and the less important will be the indirect countervailing effects associated with the reduction of the rents from distortions. For example, even when the markup in Wholesale Trade is smaller than the markup in “Other Services”, eliminating the markup in the former gives bigger gains than eliminating it in the later. The reason for this is that the degree of influence of Trade is 5 times the degree of influence of Other Services. Consider one more example. Figure 9 shows that the two sectors with the largest markups are Education and Real Estate. Note, however, from Figure 7, that Real Estate has one of the largest multipliers, while Education does not have a large one. Thus, the first term in equation 21 will be big for the case of Real Estate, while it won’t be as big (or even negative) for the case of Education. Consistent with these observations, Figure 13 shows that the effect of eliminating the markup wedge in education is negative, while the effect is positive for the case of Real Estate.

We also computed the effect in $Y$ of closing the labor wedge, under the assumption of Case 1. The results are presented in Figure 14. Similar to the the case of the markup wedge, there are also two opposing effects when eliminating the labor wedge. In general, the main message from Figure 14 is that the net effect of eliminating the labor wedge is small.

Finally, I study the effect of eliminating the markup wedge in Case 2, when the rents from distortions are not given back to the household. This exercise is presented in Figure 15. As discussed in section 3.1.1, this assumption shuts down the extra income effect of Case 1, and makes the markup isomorphic to productivity $A_i$. As a consequence, the results of eliminating markups are much bigger than in Case 1. When all markup wedges are eliminated simultaneously, aggregate productivity increases 67.7%.

---

43If the markup of sector $i$ was above 1, then the effect of eliminating this markup is to increase the marginal revenue product in sector $i$, but to reduce the income available for consumption in all sectors. Thus, aggregate output could increase or decrease when a markup is eliminated. See also section 3.1.1

44Note, however, that there are important gains in reducing the labor wedge in trade, construction and the production of electrical and optical equipment.

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37
Figure 13: Effect in Y of reducing markups under Case 1: when rents are given back to the household.

Figure 14: Effect in Y of reducing the labor wedges.
6 Conclusion

For a typical developing country, I have shown that once inter-sectoral linkages are taken into account, closing the productivity gap in an important number of services gives bigger gains in aggregate productivity than closing it in agriculture or in manufacturing, despite their larger gaps. This was done in the context of a general equilibrium framework with inter-sectoral linkages calibrated to Mexico and the US using input-output tables. Also, sector-specific distortions were computed: one similar to a markup which introduces a wedge between marginal revenue and marginal cost, and another one similar to a labor wedge that introduces a discrepancy between the marginal productivity of labor and the marginal cost of labor. I provided a quantitative assessment of the importance of these distortions for aggregate productivity.

The results suggest that analyzing distortions that lead to low productivity in services is a promising area of research in the development literature. The results also suggest that policies that tend to reduce the wedge between marginal revenue and marginal cost, in general, and for the labor market, can also increase measured productivity in a significant way. These policies include anti-trust reforms that aim to increase competition in product markets.
References


Appendix

Derivation of Equilibrium

In this appendix we follow closely Jones (2011b, b) and Acemoglu et al. (2012) in order to solve for equilibrium $Y$. We also show that changes in gross output and in aggregate output are proportional to changes in the exogenous productivity term $A_i$. In addition, we show that changes in aggregate output depend on the distortions in a non-linear way.

Consider the profit maximization for the composite:

$$\max \{\alpha\} \left\{ \prod_{i=1}^{N} c_i^{\beta_i} - \sum_{i=1}^{N} p_i c_i \right\}.$$

FOC:

$$\beta_i = \frac{p_i c_i}{Y},$$

(22)

where $Y = \sum_{i=1}^{N} p_i c_i = \prod_{i=1}^{N} c_i^{\beta_i}$.

Next, consider the maximization problem for the representative firm in sector $i$: 
\[ \max_{H, (x_{ij}), M_i} \left\{ \frac{(1 - \tau_i)}{\psi_i} p_i A_i (H_i)^{\alpha_i (1 - \sigma_i - \lambda_i)} \prod_{j=1}^{N} x_{ij}^{\alpha_{ij}} M_i^{\beta_i} - \phi_i w H_i - \sum_{j=1}^{N} p_j x_{ij} - p_{M,i} M_i \right\} \]

With first order conditions

\[ \frac{(1 - \tau_i)}{\psi_i} \alpha_i (1 - \sigma_i - \lambda_i) \frac{p_i Q_i}{H_i} = \phi_i w, \forall i \quad (23) \]

\[ \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \frac{p_i Q_i}{x_{ij}} = p_j, \forall i \quad (24) \]

\[ \frac{(1 - \tau_i)}{\psi_i} \lambda_i \frac{p_i Q_i}{M_i} = p_{M,i}, \forall i \quad (25) \]

Now, using the first order condition for \( x_{ij} \) (equation 24) in the resource constraint for sector \( j \), and multiplying both sides by \( p_j \), we have:

\[ p_j Q_j = p_j c_j + \sum_{i=1}^{N} \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} p_i Q_i \]

Now, define \( \gamma_i = \frac{p_i Q_i}{Y} \), and use equation 22 to obtain:

\[ \gamma_j = \beta_j + \sum_{i=1}^{N} \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \gamma_i \]

and using matrix notation we have:

\[ \gamma = \beta + \tilde{B} \gamma \]

\[ \Rightarrow \gamma = \beta' (I - \tilde{B})^{-1}. \]

where \( \tilde{B} \) is an NxN matrix with typical element \( \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \).
Next, using equations 23, 24, and 25, we write expression for $x_{ij}$, $M_i$ and $H_i$ in terms of $\gamma$.

We will use these expressions later on, when we solve for $Q_i$ and $Y$.

$$x_{ij} = \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \frac{p_i Q_i}{p_j} = \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \gamma_i Q_j$$  \hspace{1cm} (26)$$

and

$$M_i = \frac{(1 - \tau_i)}{\psi_i} \lambda_i \frac{p_i Q_i}{p_{M,i}} = \frac{(1 - \tau_i)}{\psi_i} \lambda_i \gamma_i \frac{Y}{p_{M,i}}$$  \hspace{1cm} (27)$$

$$H_i = \frac{(1 - \tau_i)}{\psi_i} \alpha_i (1 - \sigma_i - \lambda_i) \frac{p_i Q_i}{\phi_i w} = \frac{Y (1 - \tau_i) \alpha_i (1 - \sigma_i - \lambda_i) \gamma_i}{\psi_i \phi_i w}$$  \hspace{1cm} (28)$$

If we define $H = \sum_{i=1}^{N} H_i$, we have:

$$\frac{H_i}{H} = \frac{(1 - \tau_i) \alpha_i (1 - \sigma_i - \lambda_i) \gamma_i}{\psi_i \phi_i \sum_{j=1}^{N} (1 - \tau_j) \alpha_j (1 - \sigma_j - \lambda_j) \gamma_j} \equiv \tilde{\theta}_i$$  \hspace{1cm} (29)$$

and, $\theta_i \equiv \tilde{\theta}_i \frac{\psi_i}{(1 - \tau_i)}$. With this, we can use the above expressions for $x_{ij}$, $M_i$ and $H_i$ into the production function $Q_i$:

$$Q_i = A_i H_i^{\alpha_i (1 - \sigma_i - \lambda_i)} \prod_{j=1}^{N} x_{ij}^{\sigma_{ij}} M_i^{\lambda_i}$$

$$\Rightarrow Q_i = A_i (\theta_i H)^{\alpha_i (1 - \sigma_i - \lambda_i)} \prod_{j=1}^{N} \left( \frac{(1 - \tau_i)}{\psi_i} \sigma_{ij} \gamma_i Q_j \right)^{\sigma_{ij}} \left( \frac{(1 - \tau_i)}{\psi_i} \lambda_i \gamma_i Y \right)^{\lambda_i}$$

$$\Rightarrow Q_i = A_i \left( \frac{(1 - \tau_i)}{\psi_i} \right)^{\alpha_i (1 - \sigma_i - \lambda_i) + \sigma_i + \lambda_i} (\theta_i H)^{\alpha_i (1 - \sigma_i - \lambda_i)} \prod_{j=1}^{N} \left( \sigma_{ij} \frac{\gamma_i}{\gamma_j} Q_j \right)^{\sigma_{ij}} \left( \frac{\lambda_i \gamma_i Y}{p_{M,i}} \right)^{\lambda_i}$$  \hspace{1cm} (30)$$

Taking logs of equation 30 above, gives us:

$$q_i = lnQ_i = lnA_i + (\alpha_i (1 - \sigma_i - \lambda_i) + \sigma_i + \lambda_i) ln\left( \frac{(1 - \tau_i)}{\psi_i} \right) + \alpha_i (1 - \sigma_i - \lambda_i) ln\theta_i +$$
α_i(1 − σ_i − λ_i)lnH + \sum_{j=1}^{N}{σ_{ij}ln(\sigma_{ij}\frac{γ_i}{γ_j})} + \lambda_i ln(\lambda_i\frac{Y}{p_{M,i}}) + \sum_{j=1}^{N}{σ_{ij}q_j}

Now, define \( a_i \equiv lnA_i \), \( δ_i \equiv α_i(1 - σ_i - λ_i) \) and \( const_{q_i} \equiv (δ_i + σ_i + λ_i)ln(\frac{1 - τ_i}{ψ_i}) + δ_i lnθ_i + \sum_{j=1}^{N}{σ_{ij}ln(\sigma_{ij}\frac{γ_i}{γ_j})} + λ_i ln(\frac{λ_iγ_iY}{p_{M,i}}) \) and write the above expression in vector notation:

\[
q = a + const_q + δlnH + Bq + λlnY
\] (31)

This equation can be solved for \( q \) to yield:

\[
q = (I - B)^{-1}\{a + const_q + δlnH + λlnY\}
\] (32)

Finally, using the composite production function and the fact that \( γ_i = p_iQ_i/Y = β_iQ_i/c_i \), we have:

\[
lnY = \sum_{i=1}^{N}{β_i ln(c_i)} = \sum_{i=1}^{N}{β_i ln(\frac{β_iQ_i}{γ_i})} = \sum_{i=1}^{N}{β_i(ln(\frac{β_i}{γ_i}) + q)} = \sum_{i=1}^{N}{β_i(const_{c_i} + q)}
\]

where \( const_{c_i} \equiv ln(\frac{β_i}{γ_i}) \). Now stacking this last equation into a vector we have:

\[
lnY = β'(const_c + q)
\] (33)

Using equations 32 and 33 we can find a solution for \( lnY \):

\[
lnY = \frac{β'const_c + β'(I - B)^{-1}\{a + const_q + δlnH\}}{1 - β'(I - B)^{-1}λ}
\] (34)

Which is precisely the desired equilibrium aggregate output in equation (8).

Next, I show that changes in gross output and in aggregate output are proportional to changes in the exogenous productivity. Consider a change in productivity of sector \( i \) from \( A_i^0 \) to \( A_i^1 \). Let \( Q_i^1 \) be the value of gross output of sector \( i \) after the change in \( A_i \) and \( Q_i^0 \) the value before the change, we will show that:
\[ \ln \left( \frac{Y^1}{Y^0} \right) \propto \ln \left( \frac{A^1_i}{A^0_i} \right) \tag{35} \]

and

\[ \ln \left( \frac{Q^1_i}{Q^0_i} \right) \propto \ln \left( \frac{A^1_i}{A^0_i} \right) \tag{36} \]

It is easy to show from equation 33 that

\[ \ln \left( \frac{Y^1}{Y^0} \right) = m_i(a^1_i - a^0_i) = m_i \ln \left( \frac{A^1_i}{A^0_i} \right) \propto \ln \left( \frac{A^1_i}{A^0_i} \right) \]

Taking the difference of equation 32 evaluated at \( A^1_i \) and \( A^0_i \), we have

\[ q^1_i - q^0_i = b_{ii}(a^1_i - a^0_i) + \sum_{j=1}^{N} b_{ij}\lambda_j \ln \left( \frac{Y^1}{Y^0} \right) \]

where \( b_{ij} \) is a typical element of \((I - B)^{-1}\) and \( Y^i \) denotes that \( Y \) is evaluated at \( A^i \). Therefore,

\[ \ln \left( \frac{Q^1_i}{Q^0_i} \right) = q^1_i - q^0_i = (b_{ii} + \sum_{j=1}^{N} b_{ij}\lambda_j m_i)(a^1_i - a^0_i) = (b_{ii} + \sum_{j=1}^{N} b_{ij}\lambda_j m_i) \ln \left( \frac{A^1_i}{A^0_i} \right) \tag{37} \]

\[ \Rightarrow \ln \left( \frac{Q^1_i}{Q^0_i} \right) \propto \ln \left( \frac{A^1_i}{A^0_i} \right) \]

The effect of changes in wedges on aggregate output and productivity can be derived similarly.

**Useful equilibrium relationships.** Here I discuss equilibrium relationships that are expressed in ratios instead of levels. This feature of equilibrium is useful in the calibration and results sections. Notice that using equation 13, I can obtain expressions for the changes in each industry’s equilibrium gross output, and equilibrium aggregate GDP that result from changes in sectoral productivity \( A^i \), and in distortions \( \psi^i \) and \( \phi^i \). Since the amount of labor in the whole economy is fixed the change in aggregate GDP will be equivalent to the change in GDP per worker. In particular, suppose that we change productivity of sector \( i \) from \( A^0_i \) to \( A^1_i \), such that \( A^1_i > A^0_i \), and we keep the productivity of all other sectors constant. Call
$Q_1^i$ to the value of gross output of sector $i$ after the change in $A_i$ and call $Q_0^i$ to the value before the change. Similarly, let $Y^1$ be the value of aggregate GDP associated with $A_1^i$, and let $Y^0$ be the value for $A_0^i$. We show in the appendix that in equilibrium:

$$\ln \left( \frac{Q_1^i/H_1^i}{Q_0^i/H_0^i} \right) \propto \ln \left( \frac{A_1^i}{A_0^i} \right),$$

That is, the change in labor productivity of sector $i$ is proportional to the change in productivity $A_i$. For the counter-factual exercises performed in section 5, I take advantage of this relationship to avoid the computation of equilibrium levels. Thus, only changes in the equilibrium levels are computed.

Now consider the distorted economy in equations 1 through 6. In this case, it can be shown that equation 38 also holds for this economy, and in addition:

$$\ln (Y^1/Y^0) = f_{\psi}(\psi_i^0, \psi_i^1),$$

$$\ln (Y^1/Y^0) = f_{\phi}(\phi_i^0, \phi_i^1).$$

Which implies that we can compute the change in aggregate output associated with given changes in distortions. Notice that in contrast with the case of changes in the productivity parameter $A_i$, we do need to have both, the initial and the final levels for $\phi_i$ and $\psi_i$ in order to perform the above computations. Regarding the initial levels, in section 4, I describe the way in which these are calibrated. Then, in the counterfactual exercises of section 5, I will change the levels of these wedges to eliminate distortions in particular industries, and will make use of equations 39 and 40 to compute the effect of these changes in aggregate output.

**Expenditure shares in equilibrium.** An important feature of the equilibrium is related to how the coefficients of the production function can be related to expenditure shares of firms. Consider the equilibrium allocation for an economy with no distortions, that is $\tau_i = 0$ and $\psi_i = \phi_i = 1$. Since the production function is Cobb-Douglas, we can relate expenditure shares to the coefficients. Equation 5 implies that $\sigma_{ij} = \frac{p_j x_{ij}}{p_i Q_i}$, and, as a result $\sigma_i = \sum_j \sigma_{ij}$ is the fraction of domestic intermediate inputs on gross output of industry $i$:
\[
\sigma_i = \sum_{j=1}^{N} \sigma_{ij} = \sum_{j=1}^{N} \left( \frac{p_j x_{ij}}{p_i Q_i} \right) = \left( \frac{\sum_{j=1}^{N} p_j x_{ij}}{p_i Q_i} \right).
\] (41)

Similarly, equations 5 and 6, imply that:

\[
\sigma_i + \lambda_i = \left( \frac{\sum_{j=1}^{N} p_j x_{ij}}{p_i Q_i} \right) + \frac{p_{M,i} M_i}{p_i Q_i}
\] (42)

In this undistorted economy, \(\sigma_i + \lambda_i\) is the share of intermediate inputs (domestic and imported) in gross output. This also implies that \(1 - \sigma_i - \lambda_i\) is the share of value added in gross output. The reader is referred back to figures 4 and 5, where it was shown that there is a strong correlation between the share of value added in gross output of Mexico and the US, and that Mexico tends to have higher shares of value-added in gross output for the majority of the sectors with respect to the US. Taking the US as a relatively undistorted economy, it is possible to use equations 5, 6 and 42 to obtain estimates of the Mexican markups \(\psi_i\), \(\forall i\). More details of this strategy are provided in section 4.
Table 4: Industry codes

| 1 | Agriculture, Hunting, Forestry and Fishing |
| 2 | Mining and Quarrying                      |
| 3 | Food, Beverages and Tobacco               |
| 4 | Textiles and Textile Products             |
| 5 | Leather, Leather and Footwear             |
| 6 | Wood and Products of Wood and Cork        |
| 7 | Pulp, Paper, Paper, Printing and Publishing |
| 8 | Coke, Refined Petroleum and Nuclear Fuel   |
| 9 | Chemicals and Chemical Products           |
| 10| Rubber and Plastics                       |
| 11| Other Non-Metallic Mineral                |
| 12| Basic Metals and Fabricated Metal         |
| 13| Machinery, Nec                            |
| 14| Electrical and Optical Equipment          |
| 15| Transport Equipment                       |
| 16| Manufacturing, Nec; Recycling             |
| 17| Electricity, Gas and Water Supply         |
| 18| Construction                              |
| 19| Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel |
| 20| Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles |
| 21| Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods |
| 22| Hotels and Restaurants                    |
| 23| Inland Transport                          |
| 24| Water Transport                           |
| 25| Air Transport                             |
| 26| Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies |
| 27| Post and Telecommunications               |
| 28| Financial Intermediation                  |
| 29| Real Estate Activities                    |
| 30| Renting of M&Eq and Other Business Activities |
| 31| Education                                |
| 32| Health and Social Work                    |
| 33| Other Community, Social and Personal Services |