JEL C67, E24, J24, O15

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The Dynamic Input-Output Model of the Russian Economy with a Human Capital Block

The article provides a mathematical description of the extended dynamic input-output model (DIOM) with a human capital block. The extended DIOM is based on the Input-Output Model from the KAMIN system (the System of Integrated Analyses of Interindustrial Information) developed at the Institute of Economics and Industrial Engineering of the Siberian Branch of the Academy of Sciences of the Russian Federation and at the Novosibirsk State University. The inclusion of human capital in the IO model permits to take into consideration the great influence of the factor on economic growth. It also allows us not only to distinguish labor force by the levels of education (as some of macroeconomic models do), but also to define the influence of human capital investment on productivity level of labor force as well as on gross product reproduction. It can help to build a more reliable model to analyze and forecast development of the Russian economy.

Keywords: input-output analysis, dynamic input-output model, human capital, national wealth, Russian economy

The description of the basic DIOM, which is extended in this paper, is presented in the article [1]. The basic model has been developed in several directions including a version of the model with an environmental protection block and the model with fuzzy parameters. Some of the modified versions can be found at [2]. The article presents a mathematical description of the DIOM with a human capital block.

The extended model uses the following parameters.

The model includes n sectors. Among them $1 \leq j \leq k$ can be defined as asset-building sectors, $k < j \leq \tilde{l}$ as sectors which produce human capital, $\tilde{l} < j \leq m$ as non-asset-building sectors in the first subdivision (which produce means of production and services included into intermediate consumption) and $m < j \leq n$ as non-asset-building sectors in the second subdivision (which produce commodities and services included into final consumption).

The following notations of parameters and variables are used in the model:

n – the number of sectors in the economy;

m – the number of the first subdivision sectors (m < n);

k – the number of asset–building sectors;

l – the number of labor force types;

$\tilde{l}$ – the number of human capital investment types;

T – years of the forecast period.

$a_{ij}(t)$ – direct consumption coefficients of a sector i for production in a sector j in the t time period;

$c_{ij}(t)$ – labor intensiveness ratios of a sector j for the type i of labor resources in the t time period;

$b_{ij}(t)$ – capital intensity ratios of a sector j for the type i of fixed assets in the t time period;

$h_{ij}(t)$ – human capital–output ratio, with human capital of the type i (in accordance with the type of human capital investment) and total output in a sector j;

θ_{ij} – construction lag in a sector j for fixed capital of the type i;

$\tilde{\theta}_{ij}$ – human capital formation lag (of the type *i*) in a sector *j*;

u – age of the fixed assets;

τ_{ij} – a year within the education or medical treatment process of human capital in a sector *j*, so as $0 \leq \tau \leq \tilde{\theta}_{ij}$;

$\kappa_{ij}(t, u)$ – replacement rate of fixed assets of the type *i* in a sector *j* aged *u* at the time *t*;

$\tilde{\kappa}_{ij}(t)$ – replacement rate of human capital of the type *i* in a sector *j* at the time *t*;

$B_{ij}(t)$ – fixed assets of the type *i* put in service in a sector *j* in the *t* time period;

$BH_{ij}(t)$ – output of the education sector, i.e. students of *i* level of education who get a job at the time *t*. They are included into new human capital of the type *i* in a sector *j*;

$K_{ij}(t, t + u)$ – fixed capital investment of the type *i* in a sector *j* at the time *t* into the facilities put into operation in the $(t + u)$ time period;

$H_{ij}(t, t + \tau)$ – human capital investment (of the type *i* in a sector *j* at the time *t*) into the output of students for the time $(t + \tau)$;

$K^{*}_{ij}(t)$ – total fixed capital investment of the type *i* in a sector *j* at the time *t*;

$H_{ij}(t)$ – human capital investment of the type *i* in a sector *j* at the time *t*;

$HC_{ij}(t)$ – the volume of human capital of the type *i* in a sector *j* by the end of a time period *t*;

$\mu_{ij}(t, t + u)$ – ratio showing the part of fixed assets input in a sector *j* at time $(t + u)$ that formed due to investment of the type *i* at the time *t* so as:

$$\mu_{ij}(t, t+u) – K_{ij}(t, t+u) \Big/ \left(\sum_{i-1}^{k} B_{ij}(t+u) \right)$$

$L_i(t)$ – size of the type *i* labor resources that potentially can be employed in the economy in the *t* time period;

$F_{ij}(t, t - u)$ – fixed assets of the type *i* in a sector *j* by the end of a time period *t* put into operation at the time $(t - u)$;

$F^*_{ij}(t)$ – total fixed assets of the type *i* in a sector *j* by the end of a time period *t*;

$N_{ij}(t)$ – construction–in–progress fixed assets of the type *i* in a sector *j* by the end of a time period *t*;

$NH_{ij}(t)$ – human capital (of the type *i* in a sector *j*) remaining in the education process (including “cultural education” and the process of receiving medical services) by the end of a time period *t*;

$x_j(t)$ – the produced gross output in a sector *j* at the time *t*;

$\bar{x}_j(t)$ – the used gross output in a sector *j* at the time *t*;

$S_j(t)$ – net export of a product *i* at the time *t*;

$\Delta z_j(t)$ – increase in inventory of a sector *j* at the time *t*:

$$\Delta z_j(t) – z_j(t) - z_j(t-1);$$

$\gamma_j(t)$ – losses of output in a sector *j* at time *t*.

The produced gross output in a sector *j* can be described as follows:

$$x_j(t) = \bar{x}_j(t) + S_j(t) + \Delta z_j(t) + \gamma_j(t).$$

The DIOM with lags describes the reproduction of the fixed assets as an exchange process of the used asset–building branches output at the time *t* with fixed assets put in service in the *t* time period mediated with changes of construction–in–progress.

Due to the lags, the output, export and import of asset–building branches at each particular moment of time can be coordinated with the same parameters at the previous and future moments of time.

Some part of the asset–building branches output is using in construction–in–progress process, while the rest can be exported. It defines the connection of fixed capital investment and its structure with previous investment and import in asset–building sectors.

Fixed capital put into service is formed due to the used output of the asset–building sectors, which are mechanical engineering and construction. The output of an asset–building sector *i* (in size $K^*_{ij}(t)$) is put into construction–in–progress in a sector *j* ($1 \le j \le n$) at the time *t*. Then it is being distributed by the layers of incomplete construction. Total investment of the type i in a sector j at the time t is defined as follows:

$$K_{ij}^{*}(t) = \sum_{u=0}^{\theta_{ij}-1} K_{ij}(t, t+u), \qquad i = 1, \dots, k;\ \ j = 1, \dots, n \tag{1}$$

Fixed assets of the type i put in service in a sector j in the t time period are formed from the used output of an asset–building sector i as follows:

$$B_{ij}(t) = \sum_{u=0}^{\theta_{ij}-1} K_{ij}(t-u, t), \quad i = 1, \dots, k;\ \ j = 1, \dots, n \tag{2}$$

Fixed capital investment into the facilities put into operation in the $(t + u)$ time period (K_{ij} $(t, t + u)$) can be defined with fixed assets put in service in the t time period:

$$K_{ij}(t, t+u) = \mu_{ij}(t, t+u) \sum_{i=1}^{k} B_{ij}(t+u), \quad i = 1, \dots, k;\ \ j = 1, \dots, n \tag{3}$$

Ratios $\mu_{ij}(t, t+u)$ characterize fixed assets put in service and depend on technology and intensity of construction process in a sector j. It should be noted that the construction technology consists of a finite number of stages. Fixed capital investment can be defined as follows:

$$K_{ij}(t, t+u) = \sum_{v} \left(\xi_j(t, t+u, v) \eta_{ij}(t+u, v) \sum_{i=1}^{k} B_{ij}(t+u) \right) \tag{4}$$

where $\eta_{ij}(t+u, v)$ is the part of fixed assets of the type i put in service in a sector j at the time $(t + u)$ that is formed at the construction stage v. $\xi_j(t, t+u, v)$ is a part of the stage v at the time t (u periods until the layer is put in service). The ratios $\eta_{ij}(t+u, v)$ should be determined by the methods of mathematical statistics using fixed capital investment, construction–in–progress, and forecast of fixed assets put into service data. Several construction stages can be finished within one period, as well as it can take several periods to finish one stage. The construction tempo depends on expecting fixed capital investment.

The main equations to define fixed capital investment are 2–4. Recurrent equations for re–computing construction–in–progress are:

$$N_{ij}(t) = N_{ij}(t-1) - \sum_{u=1}^{\theta_{ij}-1} K_{ij}(t-u,t) + \sum_{u=1}^{\theta_{ij}-1} K_{ij}(t,t+u) \quad (5).$$

The amount of fixed capital of the type *i* in a sector *j* aged *u* by the end of the time period *t* are described with the following recurrent equations:

$$F_{ij}(t, 0) = B_{ij}(t),$$

$$F_{ij}(t,u) = F_{ij}(t-1,u-1)\cdot\left(1-\kappa_{ij}(t,u)\right), \quad i = 1,\dots,k;\ j = 1,\dots,n \quad (6).$$

The model of fixed capital reproduction (1–6) is used to define the volume of investment and its technological structure taking into account a construction lag.

The basic model is extended with additional equations that allow to model human capital reproduction.

The output of students of *i* level of education ($BH_{ij}(t)$), which defines putting into operation of new human capital of type *i* in sector *j*, is determined from used human capital investment of a type *i* in the sector *j*:

$$BH_{ij}(t) = \sum_{\tau=0}^{\tilde{\theta}_{ij}-1} H_{ij}(t-\tau,\boldsymbol{t}) = \sum_{\tau=0}^{\tilde{\theta}_{ij}-1} \tilde{\eta}_{ij}(\tau)\cdot H_{ij}(t-\tau), \quad i = k+1,\dots,\tilde{l}; j = 1,\dots,n \quad (7),$$

where $H_{ij}(t-\tau,\boldsymbol{t})$ is a total amount of human capital investment of type *i* deposited in sector *j* in the ($t-\tau$) time period and provided for type *i* human capital at the time *t* in sector *j*;

$\tilde{\eta}_{ij}$ is a part of previous investment ($t-\tau$) of type *i* in sector *j* providing with putting into operation of human capital of the same type in sector *j* in the *t* time period with following conditions:

$\tilde{\eta}_{ij} \in [0,1]$ for any τ;

$$\sum_{\tau=0}^{\tilde{\theta}_{ij}-1} \tilde{\eta}_{ij}(\tau) = 1.$$

$H_{ij}(t-\tau)$ is human capital investment of type *i* deposited in sector *j* in the ($t-\tau$) time period. In addition, $\tau \geq 0$ as it allows to take into account some short educational programs (usually less than a year, eg. qualification courses) and shorter lag in medical sphere.

A necessary amount of human capital investment of type *i* in sector *j* for human capital output in the ($t+\tau$) time period is defined as follows:

$$H_{ij}(t,\boldsymbol{t}+\boldsymbol{\tau}) = \sum_{\tau=0}^{\tilde{\theta}_{ij}-1} \tilde{\mu}_{ij}(\tau)\cdot BH_{ij}(t+\tau), \qquad i = k+1,\dots,\tilde{l};\ \ j = 1,\dots,n \tag{8}$$

where *t* is a year of investment and *(t + τ)* is a year of students output, as well as "output" of people who underwent a course of medical treatment and can return to work, i.e. *(t + τ)* is a year of human capital output.

$\tilde{\mu}_{ij}(\tau)$ stands for ratio showing a share of human capital output in sector *j* at the time (t + τ) formed due to investment of type *i* in the *t* time period so as:

$$\tilde{\mu}_{ij}(\tau) \in [0,1] \text{ for any } \tau;$$

$$\sum_{\tau=0}^{\widetilde{\theta_{ij}}-1} \tilde{\mu}_{ij}(\tau) = 1.$$

Recurrent equations for re–computing construction–in–progress human capital of type *i* in sector *j* (i.e. people remaining in the education or medical treatment process) $NH_{ij}(t)$:

$$NH_{ij}(t) = NH_{ij}(t-1) - \sum_{\tau=1}^{\tilde{\theta}_{ij}-1} H_{ij}(t-\tau,\boldsymbol{t}) + \sum_{\tau=1}^{\tilde{\theta}_{ij}-1} H_{ij}(t,\boldsymbol{t}+\boldsymbol{\tau}) =$$

$$= NH_{ij}(t-1) - \sum_{\tau=1}^{\tilde{\theta}_{ij}-1} \tilde{\eta}_{ij}(\tau)\cdot H_{ij}(t-\tau) +$$

$$+ \sum_{\tau=1}^{\tilde{\theta}_{ij}-1} \tilde{\mu}_{ij}(\tau)\cdot BH_{ij}(t+\tau), \quad i = k+1,\dots,\tilde{l};\ j = 1,\dots,n \tag{9}$$

Total amount of human capital of type *i* in a sector *j* by the end of the *t* time period is determined as follows:

$$HC_{ij}(t) = BH_{ij}(t) + HC_i(t-1)\cdot\left(1-\tilde{k}_{ij}(t)\right),\ \ i = k+1,\dots,\tilde{l};\ \ j = 1,\dots,n \tag{10}$$

The produced gross output $x_i(t)$ of the asset–building sector *i* at the time *t* can be defined as follows:

$$x_i(t) = \sum_{j=1}^{n} K_{ij}^{*}(t) + S_i(t) + \gamma_i(t), \quad 1 < i \le k \tag{11}$$

The produced gross output $x_i(t)$ of a sector *i* that produces human capital at the time *t* can be described as:

$$x_i(t) = \sum_{j=1}^{n} \mathrm{H}_{ij}(t) + S_i(t) + \gamma_i(t), \quad k < i \le \tilde{l} \tag{12}$$

The balance between production and use of the output of the first subdivision of non–asset–building sectors is defined as follows:

$$x_i(t) = \sum_{j=1}^{n} a_{ij}(t)x_j(t) + S_i(t) + \gamma_i(t), \quad \tilde{l} < i \leq m \qquad (13).$$

The output of the second subdivision is defined as:

$$x_i(t) = Q_i(x_i(t-1), S_i(t-1), \lambda, t) + S_i(t), \qquad m < i \leq n \qquad (14).$$

where Q_i are maps synthesizing the structure and dynamics of consumption (usually these maps are monotonously growing functions of the parameter λ).

Labor resources limits are defined by the system of inequalities:

$$\sum_{j=1}^{n} c_{kj}(t) \cdot x_j(t) \leq L_k(t), \qquad k = 1, \ldots, l; \; j = 1, \ldots, n \qquad (15).$$

Fixed assets constraints are described as follows:

$$b_{ij}(t) \cdot x_j(t) \leq F_{ij}(t), \qquad i = 1, \ldots, k; \; j = 1, \ldots, n \qquad (16).$$

Along with the basic constraints and equations described above, an additional constraint for human capital should be added:

$$\sum_{j=1}^{n} h_{ij}(t) \cdot x_j(t) \leq HC_i(t), \qquad i = k+1, \ldots, \tilde{l}; \; j = 1, \ldots, n \qquad (17).$$

The same way as it is shown at the basic model, Ω defines a trajectory of the economic system development $x_i(t)$, which suits all basic constraints of the model as well as human capital restrictions 1–17 described above. Defining the trajectory Ω with given parameters (eg. amount of human capital $HC_{ij}(t)$, fixed assets put in service $B_{ij}(t)$, size of labor resources $L_i(t)$ etc.) for each moment from the [0;T] time period allows to get the system of economic development parameters. Among them are output $x_i(t)$, total fixed capital investment $K_{ij}^*(t)$, human capital investment $H_{ij}(t)$, human capital output $BH_{ij}(t)$, human capital $HC_i(t)$ and fixed capital by the end of the time period:

$$F_{ij}^*(t) = \sum_{u \geq 0} F_{ij}(t, u).$$

The optimization problem can be described as follows:

$$\sum_{t=1}^{T} \sum_{j=1}^{n} f_j(t)\, x_j(t) \Rightarrow max, x \in \Omega$$

with constraints described above and $f_j(t)$ that stand for weight coefficients of production in a sector j in the target function of the economic system.

Conclusion

The extended dynamic input–output model is using two important parameters, which are the amount of labor force and value parameter differentiating by the types (investment in human capital).

Two important problems was observed while constructing the DIOM. First, it is not so easy to distinguish asset–building and non–asset–building sectors producing human capital, as these sectors produce intermediate consumption at the same time with producing human capital (ex. in education). It is even more difficult to distinguish asset– and non–asset building healthcare sector.

The second problem is lack of information. The problem is especially topical for healthcare and culture sectors. Moreover, it is quite difficult to define a lag structure for these sectors (while at the education sector it can be described via years of education).

In addition, the economy development should be modeled in the long run that requires the developing of the information base for the DIOM.

All the problems the authors have faced remain the object of future researches.

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