A proposal for the construction of a regional input-output model using a bottom-up approach with hybrid methods[1].

Regional output modeling

Working Paper

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Introduction

The main objective of this essay is to develop and apply a methodology for the construction of a regional input-output model, which, based on a hybrid "bottom-up "approach, allows us to pick up the essential aspects of the particularities of a regional economy, as well as to make it compatible it with national data. This is done through the construction of a basic regional statistical system that leads us both to orient the process of regionalization of the national accounts, and the construction of the key variables for the construction of the regional supply and use input-output tables. In our view, this fulfills the regional data requirements and the use of superior data at national level, making possible the construction of regional bottom-up I-O matrices.

It is well known that the methodology for the construction of a regional I-O model depends on the availability of regional data. This information plays a key role in the construction of a national I-O matrix. As a matter of fact, its construction is supported by a statistical system, which relies on the elaboration of national accounts. This is one of the main international achievements that lead to the development of national I-O matrices and their consequent use as basic tools for measuring national economic performance

According to Miller and Blair (2009), among the most formidable challenges in using input–output analysis in practice, is assembling the detailed basic data of the economic area of interest – national, regional, or perhaps multiple-regions for the construction of input–output tables.

This challenge is traditionally combined with the argument of the lack of regional data, leading regional matrices to depend on the national ones. Although, in our view, the problem relies on the lack of a regional statistical system, which motivated us to elaborate a regional database with only those needed variables for the creation of regional I-O matrices.

Furthermore, depending on the national statistical system, especially for the development of national accounts and the availability of regional data it is possible to establish a database in order to estimate, through hybrid methods the key regional variables that are crucial for the construction of a regional input-out table.

In the case of Mexico, we have a solid system of national accounts, as well as consolidated accounts at a national level of non-financial corporations, government, institutions and no profit-making sectors. Besides, there are estimates of value generated by the states, considering the sectorial classification of the North American Industrial Classification System (NAICS). We conclude then, that we have enough elements for the development of our research.

In our opinion, the results from this research can contribute to the development and application of a "bottom-up" methodology.

Furthermore, in the literature, there are few but important recommendations concerning regionalization procedures for the construction of regional input-output matrices through the construction of regional accounts, considering as a key feature the regionalization of national tables (Jackson, 1998; Lahr, 2001). In México there is not such type of analysis.

Our main interest is to develop a methodology based on the creation of a regional data base that lead to build a supply and use regional matrix and regionalize the national table and the creation of the regional make and use IO tables, taking into account the regional data and its combination with the national one based on hybrid methods.

This article is exploratory in nature and it attempts to develop and deepen in the methodology we have been used for the construction of bottom-up regional matrices, using as starting point previous essays in our research agenda. Hence, this essay was written using the theoretical elements developed in the conception of the economic spatial concentration. Its analytical instrumentation is realized through the construction of a IO regional matrix through the bottom-up approach, using a case study, the state of Sonora, Mexico.

The methodology consists of the following steps: 1) Review of the main theoretical concepts and regional input-output models; 2) The methodological proposal and 3) Preliminary empirical results.

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1. Review of the main theoretical concepts and of regional input-output models.

According to Miller and Blair (2009), the main problems in the construction of a regional input output matrix are related to two specific characteristics that involve a regional dimension. It becomes evident and necessary the distinction between national and regional input-output models. First, the productive structure and the technological levels of each region are specific and probably very different from the nation, For example, the age of regional industries, the characteristics of input markets or the education level of the labor force are important factors that may cause the regional technology of production to deviate from the national one; second, the smaller the focus on the economy, the more it depends on the exterior world, including sub national regions and other countries. This gives exports and imports more importance in determining the region's demand and supply. Therefore, in order to construct a proper I-O regional matrix, it is absolutely necessary to count on the regional input-output technical coefficients, as well as the regional trade coefficients Nevertheless, in the literature, this lack of information is known, as well as the need to elaborate solid and accurate estimates of both types of coefficients based on hybrid methods.[2].

The main methodological attempts for the construction of I-O regional models have been based on simplified approaches, such as the estimation of location quotients or mathematical responses to the lack of regional information mainly through the application of the RAS method. [3]

Hence, as we stated before, we stress that, from our point of view, what is really needed is a spatial, theoretical and methodological approach from "below"; in order to address the issue of regional analysis and to create a database, from which a regional input-output matrix could be constructed. We assume, then, that its construction, using a top-down approach, is inadequate for the full comprehension of economic behavior, structural attributes and spatial characteristics of an economic region, and consequently, it is unsuitable for decision-making in terms of

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[2] See Miller and Blair, 2009, Organization of Basic data for Input-output models, Chapter 4, p.119

[3] See Asuad and Sanchez 2016, A methodological proposal for the construction of a regional input-output matrix using a bottom-up approach and its statistical assessment, Investigación Económica, vol. LXXV, núm. 298, octubre-diciembre de 2016, pp. 3-56

regional and territorial economic policy, due to its inability to grasp the spatial heterogeneity of the regional economic structure and its economic interactions and spatial interdependencies.

Furthermore, it distorts the estimation of the technical coefficients and economic linkages within a region. This is due, mainly to the lack of the spatial location of sales and purchases between places of origin and destination within a region and between sub-regions. This arises from a sectorial bias, which in turn, it is inherent to regional input-output matrices constructed according to a top-down approach.

We also consider that, to some extent we have provided empirical evidence that shows the importance of the bottom-up approach when compared to the top- down. It mainly consists of the grasping of the economic particularities of a region, while the latter only shows the economic similarities between the region and the nation.

However, despite the latter, since Walter Isard´s proposals (1951 and 1956), it is well known that the predominant agenda of the research in regional science should focus more on the economic space, in order to complement the traditional economic analysis based on a wonderland with no dimensions. Regional I-O models have been developed without a spatial dimension, which in other words, means that in their analysis, the location of economic activity is omitted.

The most important regional I-O models developed since the 1950´s, take into account regions as a whole, only differentiating their analysis by different types of spatial aggregation levels: An isolated region, two regions and multiple regions. Despite this, they do not take into account the spatial dimension of a regional economy, given that the starting point in their analysis is the region as a whole, without explaining how the economic regions are spatially integrated and what the implications are for their economic behavior.

Furthermore, traditionally, the analysis has been based on political-administrative units, being states, municipalities, metropolitan areas and communities, assuming they behave as economic regions, which leads to misinterpretations.

Therefore, it is clear that an adequate level of spatial disaggregation of an economic region and also a proper specification of spatial unities, and its economic nature are central issues for the construction of an accurate regional I-O matrix.

However, their appropriate specification depends on the theoretical bases from which this interpretation is created and on the required data base for its practical application. Thus, we proceed to establish our theoretical framework based on the spatial dimension of the economy.

1.1 The interpretation of economic space

The analytical orientation of the construction of regional input-output matrices is conceived considering the economic sub regions with which a region is defined. This approach leads us to rely on a multi sub regional input-output analysis, characterized by having a more detailed spatial analysis at intra-regional level. As a starting point, we identify functional economic sub regions and their patterns of economic interactions as well as their market areas: local sub regional, intra sub regional, regional and those national and international regions.

In this approach we use, on the one hand the theoretical and methodological approach of the economic concentration, which is part of a broader perspective of the spatial dimension of the economy. (Asuad, 2016, 2014 and 2001). This has already been developed for and applied to the construction of regional I-O matrices (Asuad & Sánchez, 2016).

The foundations of the spatial dimension of the economy have their roots in the works of Walter Isard (1951). Niles Hansen (1975), establishes that Isard´s view on the spatial dimension is closely related to regional matters:

"In brief, regional science as a discipline concerns the careful and patient study of social problems with regional or spatial dimensions, employing diverse combinations of analytical and empirical research." (p. 2). "In other words, given problems that have a spatial dimension, regional science is what regional scientists do about them".

Isard stresses the importance of the spatial dimension in economic analysis but without a precise definition. A similar view is shared by Harry Richardson (1988), who focuses on the importance of the economic spatial dimension but neither defined their main concepts nor mention how can it be practically applied to economic analysis. In the literature, there are plenty of concepts related to space without a clear definition. (See Asuad, 2014)

The interpretation of the economic concentration under the approach of the economic spatial dimension is based on the concept of economic space which is the main category while economic territory and region are of secondary nature.

Therefore, we assume that economic development and growth tend to be spatially unbalanced, due to the heterogeneity of both natural and economic spaces, which by the way, are not politically bounded to states or municipalities. Also, the fact that the spatial distribution of economic activity is highly concentrated in very few areas, means that economic and population nodes emerge. These are characterized by their economic interactions through production, exchange, consumption, capital accumulation, trade and government involvement in economic affairs as well as the regulation it imposes.

Thus, a node or hub is defined as a site or place, whose economy is characterized by its economic dominance over and connection with a set of minor economic sites that interact and compete, whereas a traditional economic site is defined as a place on the economic space, where economic activities are highly concentrated and from which a set of economic impulses are exerted through economic exchanges; this guides the spatial economic behavior as a whole.

Economic nodes are spatial economic sub-units distributed in a given geographical or political space, with extremely dense economic activity and demographic concentration. Indeed, they behave as the centers of a given market area where most of the spatial concentration of production and consumption are located. Furthermore, they are connected by the economic flows established among them, which in sum, integrate the economic space.

The economic importance of nodes depends on their economic interactions, which are an outcome of the type of connection and market relationships they establish. These can be thought of as economic complementarities or competition between themselves, or just a mixture of both economic interactions. If these interactions were relevant, they would lead to the creation of sub-economic spaces. Therefore, economic space, to exist, requires at least the existence of a pair of economic sites or nodes, interacting with each other.

Of course, they do not necessary coincide with any geographical or political unit, despite their influence on economic decision-making processes. Only those economies based on market behavior and territorial development define how the economy is structured in space.

These can be measured through their economic interactions, mainly purchases and sales carried out by companies and consumers. This sectorial-spatial economy and its synergy with the natural and territorial space in a given area, leads to the development of a region or sub-regions, integrated by a system of cities and networks of transportation routes, that link them.

In a generic way, the development of regions as spatial economic units is defined as spatial economic functional units, SEFU[4] , which are an outcome of economic growth and development on space, that is to say, the economic space as a whole. Thus, the development of this spatial unit allows us to know how economic activities have been spatially distributed, defining the spatial structure and behavior of their economy.

The main components of functional economic regions are the economic sub regions which are characterized by being an economic system that, just as Leontief (1963, p.119) pointed out, not only behave as interdependent industries but also as several interrelated regions- sub regions-. The output of each sub region is defined as a combination of outputs of economic activities carried on within its geographic boundaries -spatial boundaries; their inputs, on the other hand comprise the direct inputs of these industries and the goods and services absorbed directly by the final demand sectors of that sub region.

The economic interdependence between the sub regions comes from industries and final consumers located within their own spatial boundaries. The movement of commodities or services from one sub-region to other reflects the existence of

[4] In the regional and urban literature, there is a consensus on the concept of functional regions, which are defined as spatial units that result from the organization of economic and social relationships in space. Furthermore, theoretically, this concept has been treated through different perspectives: theory of location, theory of market areas, theories of poles of economic growth along with their respective debates in explaining urban territories, especially from Christaller (1933) and Losch (1944) as well as in current urban policy issues (e.g., oecd, 2002 and 2013). According to our conception, the essential aspect of functional economic regions is the identification of economic activities in space through its location and economic sectorial characteristics, as well as the role and interactions they establish, which give rise to a economic structure on space, and leads to the creation and development of an economic spatial unity (See Asuad Sanén, 2014, pp. 339-356). For the specific economic functions in a city, see McDonald (1997). However, there are different techniques for the identification and measurement of the economic spatial functional region: gravity models, labor market models and commercial interactions areas. Nevertheless, we have developed a methodology for the identification and analysis of these units and their economic interactions through the identification of the sub-regional productive chains.

direct input-output relationships through industries and final consumers located in their own sub regions. This leads to multi sub-regional economic interactions.

Therefore, there are multiple economic sub-regional interactions in the economic region, characterized by their intra-regional, sub-regional, and local economic interactions, making an inner- regional economy. It is also characterized by their multiple inter-regional economic interactions with other national and international economic regions.

1.2 Regional input-output models

The first applications of the input-output model were done at a nation-wide level. However, the interest in extending the application of the same framework to different spatial units (usually, sub-national regions) led to some modifications in the national model, originating a set of regional input-output models. (Sargento, 2009, p. 7).

Accordingly, the differences between regional input-output models lie on the following criteria: (1) the number of regions taken into account; (2) the recognition (or not) of interregional linkages; (3) the degree of detail implicit in interregional trade flows (which is related to the degree of detail demanded for the input-output data) and (4) the kind of hypotheses assumed to estimate trade coefficients.

The first criterion is used to distinguish the single-region model from the several types of models designed for systems with more than one region. This single-region model, seeks to capture intra-regional effects alone. So, its crucial limitation consists of the fact that it ignores the effects caused by the linkages between this region and the others. Thus, the second I-O regional model captures the effects caused by the interconnections between al least two regions, which is known as interregional model (ICOR) or Isard´s model. Finally, the third model is applied to more than two regions and it is known as the multiregional model (MICOR). This model is associated to the original proposal of the Chenery-Moses model. The main differences are in the number of regions taken into consideration for their analysis, but also in the implicit details of the flows of interregional data and the hypothesis for the estimation of the trade coefficients.

1.2.1 Single - region model [5]

The starting point for a single-region model is, obviously, a single-region input-output table, very similar to the national one. It can be presented in two different versions: A total-use table or an intra-regional flows table.

The development of this model followed closely the development of the national input-output model.. Consequently, as an integrated spatial unity, it includes a set of economic activities that constitute the sectors of productive activity ,assuming that each sector produces a single product. So, Input-Output accounts, would capture the interconnections or transactions within a regional economy through its computation in a period of time, say a year. These transactions, correspond to sales and purchases within that economy. In addition, the regional input-output analysis, according to the Leontief framework, is governed by demand and its effects on productive sectors located in the region. As a result, having available information on the total of purchases and sales flows between the n sectors of the region and the total gross production of sector j in region r, we denote the technical coefficients of the region as:

[5] We develop this section following the document of Sargento 2009, Introducing Input-output analysis at the regional level: Basic notions and specific Issues and the Chapters on the subject in Miller and Blair 2009, that is Chapter 3, Input-Output Models at the regional level and the chapter 8, Non survey and Partial – Survey Methods: Extensions, contained in the book of the mentioned authors which is titled: Input-Output Analysis, Foundations and Extensions.

$$a_{ij}^{rr} = \frac{z_{ij}^{rr}}{x_j^r}$$

$$Where:$$

$$z^{rr} = \left[z_{ij}^{rr}\right]$$

$$x^r = \left[x_j^r\right]$$

$$\therefore A^{rr} = z^{rr}(\hat{x}^r)^{-1} \ldots\ldots(1)$$

Hence, when considering these coefficients, the regional production equation can be stated as:

$$x_i^r = z_{i1}^r + z_{i2}^r + \ldots + z_{ii}^r + \ldots . z_{in}^r + y_i^r$$

$$So:$$

$$x_i^r = a_{i1}^r x_i^r + a_{i2}^r x_2^r + \ldots + a_{ii}^r x_i^r + \ldots a_{in}^r x_n^r + y_i^r$$

Thus, the compact matrix representation is:

$$\left(I - A^r\right).\, x^r = \mathrm{y}^r \quad \Leftrightarrow \quad x^r = \left(I - A^r\right)^{-1} \mathrm{y}^r$$

Then, if we want to quantify the impact on the total output available at region r caused by a change in regional final demand, we use the following equation:

$$\Delta x^{r} = \left(I - A^{r}\right)^{-1} \Delta y^{r}$$

However, if we want to consider the impacts outside the region through imported production, it is possible to estimate these impacts on regional production if we pre multiply both sides of the previous equation by $(I - \hat{C})$, in which $\hat{C}$ is a diagonal matrix of the import regional propensities.

According to Sargento (2009), in an intra-regional model the I-O table relies on an input supply coefficient, instead of a regional technical coefficient. It is denoted as follows:

$$a_{ij}^{rr} = \frac{z_{ij}^{rr}}{e_{j}^{r}}$$

A_{ij}^{rr} = *Matrix composed* by intra-regional coefficients a_{ij}^{rr}

e_{j}^{r} = *Vector* of output produced in the region r

The final equation for the simple region model considering intra-regional transactions, is the following:

$$e^{r} = \left(I - A^{rr}\right)^{-1} f^{r}$$

$$e^{r} = B^{rr} f^{r}$$

$$\Delta e^{r} = B^{rr} \Delta f^{r}$$

B^{rr} = Impact of changes in final demand of regional produce output

f^{r} = Regional final demand including national and international exports

The practical application of the model is difficult due to the lack of reliable regional information, so it is required to apply a set of hypothesis in order to estimate A^{rr}.

What is more, this model ignores the effects caused by the linkages between regions, thus, it is needed to add the interregional feedback effects.

1.2.2 The balanced regional model

Also known as Leontief's intra-national model, it is the first spatial input-output model, developed by that author in 1953. It is made of a combination of traditional input-output analysis with spatial elements). Leontief (,1953, p-93) pointed out that "…*some commodities are produced not far from where they are consumed, while the others can and do travel long distances between the place of their origin and that of their actual utilization*".

Thus, it explains spatial interactions by distinguishing two classes of commodities: regional and national. Regional commodities are supposed to be regionally balanced, which means that all the regional production is consumed in the same region, while national commodities are those which are "...easily transportable..." (Leontief, 1953, p. 94) and whose production-consumption balance occurs only at the national or even international levels.

This implies that one region may have an excess of production in some "national" product, generating exports to the rest of the country or, instead, it may have a deficit, which leads to imports from the rest of the country.

The model only computes net trade flows and it does not determine the region of origin of imports or the destination of exports. This is the reason why the author prefers to label this model intra-national, instead of inter-regional.
This model takes into consideration *n* products, and it divides regions from I to *h* and nations from *h* to *n*. This classification has to be done previously.

The technical coefficients are known and the same technological matrix is used for the regions and for the nation as a whole, being the technical coefficients denoted as:

$$a_{ij} = \frac{z_{ij}}{x_j}$$

It also considers that national and regional demand are known *a priori*. Finally, the model is based on sectorial economic differences based on the type of products related to their respective markets: Subnational and national, rather than their respective spatial locations.

This intra-national model is based on the fact that a national economy is composed of products with different national and regional market areas, which tend to balance themselves at a national level However, there are products which can balance on a smaller geographic scale, as they serve regional or local markets.

The basic structure of the model is identical to that of the inter-regional models; the differences are in the interpretation of its components, which differentiate the market balance at national or regional level. It classifies national sectors in terms of their market orientation: National or regional.

If we suppose that all sectors can be assigned to either a national (N) or a regional (R) category, considering as criteria the percentage of inter-regional deliveries of a sector´s products, as opposed to intra-regional ones in a simple economy integrated by sectors 1, 2, . . . , n. which represent the regionally balanced sectors

Let sectors r + 1, . . ., n represent nationally balanced sectors. Then, we have the following table of national input coefficients:

$$A = \begin{bmatrix} A^{RR} & \mathrm{A}^{RN} \\ A^{NN} & \mathrm{A}^{NR} \end{bmatrix}$$

If we have x^R and f^R (r elements of column vectors) representing total output and final demand for the regional sectors, and x^N and f^N, which are (n – r)-element column vectors, represent output and final demand for the national sectors, we denote the following expressions:

$$X = \begin{bmatrix} x^R \\ x^N \end{bmatrix} \qquad \mathrm{f} = \begin{bmatrix} f^R \\ f^N \end{bmatrix}$$

$$\text{Then, if we have:}$$

$$(I - A)x = f$$

$$\left(I - A^{RR}\right)X^R - A^{RN}X^N = f^R$$

$$-A^{NR}X^N + \left(I - A^{NN}\right)X^N = f^N$$

Accordingly, in a partitioned matrix form we have:

$$\begin{bmatrix} \left(I - A^{RR}\right) & -\mathrm{A}^{RN} \\ -\mathrm{A}^{NR} & \left(I - A^{NN}\right) \end{bmatrix} \begin{bmatrix} x^R \\ x^N \end{bmatrix} = \begin{bmatrix} f^R \\ f^N \end{bmatrix}$$

$$\textit{and} \text{ so:}$$

$$\begin{bmatrix} x^R \\ x^N \end{bmatrix} = \begin{bmatrix} \left(I - A^{RR}\right) & -\mathrm{A}^{RN} \\ -\mathrm{A}^{NR} & \left(I - A^{NN}\right) \end{bmatrix}^{-1} \begin{bmatrix} f^R \\ f^N \end{bmatrix}$$

1.2.3 Interregional input-output model (IRIO)

In the literature, regional and interregional input-output models are known as IRIO (Interregional Input-Output model), which in fact, are related to the analysis of two regions: The one of interest or *base* and the complementary. This model was proposed initially by Walter Isard in 1950 and developed further in 1961.

In fact, this gave rise to the development of literature oriented to the consideration of input-output transactions between two regions. This was incorporated in the conception and in the methodology for the construction and analysis of input-output regional matrices.

Consequently, if we have two regions *r* and *s,* we then have X^r, X^s, f^r, f^s. In region r we have three sectors, and in region s two sectors. As a result, the following tables of intra-regional flows of inputs and inter-regional trade are given[6]:

$$Z = \begin{bmatrix} Z^{rr} & Z^{rs} \\ Z^{sr} & Z^{ss} \end{bmatrix}$$

However, it is necessary to clarify that inter-regional sales are between productive sectors, or in other words, sales of inputs. On the other hand, sales to final consumers are specified explicitly in the final demand of each region.

In consequence, we have two types of coefficients of regional inputs at an intra-regional level, and coefficients of inter-regional[7] inputs for each region, defined as:

[6] This matrices have to be squared in their main diagonal, that is to say, the number of selling sectors have to be equal to the number of buying sectors. In other words m x n, where m=n. If this is not the case, the matrices must be divided into sub matrices, which denoted by lines or dots. So, if there are matrices of a 4 by 4 size, they can be transformed into 4 matrices.

[7] In the literature, the pattern purchases and sales between regions is called inter regional trade, while the concepts regional exports and imports are reserved for external trade, in other words, the one that crossed international borders, see Miller and Blair (2009, pp.78).

$$a^{rr}_{ij} = \frac{z^{rr}_{ij}}{x^{r}_{j}}; \; a^{ss}_{ij} = \frac{z^{ss}_{ij}}{x^{s}_{j}}$$

Where :

$$Z^{rr} = \left[z^{rr}_{ij}\right] \quad Z^{ss} = \left[z^{ss}_{ij}\right]$$

$$X^{r} = \left[x^{r}_{j}\right] \quad X^{s} = \left[x^{s}_{j}\right]$$

$$\therefore \quad A^{rr} = Z^{rr}\left(\hat{x}^{r}\right)^{-1} \qquad A^{ss} = Z^{ss}\left(\hat{x}^{s}\right)^{-1}$$

$$a^{rs}_{ij} = \frac{z^{rs}_{ij}}{x^{s}_{j}}; \; a^{ss}_{ij} = \frac{z^{sr}_{ij}}{x^{s}_{j}}$$

Where :

$$Z^{rs} = \left[z^{rs}_{ij}\right] \quad Z^{sr} = \left[z^{ss}_{ij}\right]$$

$$X^{s} = \left[x^{s}_{j}\right] \quad X^{r} = \left[x^{r}_{j}\right]$$

$$\therefore \quad A^{rs} = Z^{rs}\left(\hat{x}^{s}\right)^{-1} \qquad A^{sr} = Z^{sr}\left(\hat{x}^{r}\right)^{-1}$$

As a result, the production of each of the regions r and s, and their final demand function, considering two sectors of economic activity,1 and 2, are integrated as follows:

$$(I - A^{rr})x^{r} - A^{rs}x^{s} = f^{r}$$

$$-A^{rs}x^{r} + (I - A^{ss})x^{s} = f^{s}$$

The solution to the system of equations for the two regions, in a matrix form, and according to the input-output contextual frame, is represented in the following expression[8]::

$$(I - A)x = f$$

$$\left\{ \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \left[\begin{array}{c:c} A^{rr} & A^{rs} \\ \hdashline A^{sr} & A^{ss} \end{array} \right] \right\} \begin{bmatrix} x^{r} \\ \cdots \\ x^{s} \end{bmatrix} = \begin{bmatrix} f^{r} \\ \cdots \\ f^{s} \end{bmatrix}$$

Multi-regional Input – Output model (MRIO)

The previous models do not recognize the links or interconnections between regions. It is assumed that a region exchange with just one region, which is its complement. This is the case of the inter-regional model (ICOR), which does not take into account the activities in the rest of the country ignoring their influence, despite being located in that space.

In the case of a country integrated by many regions, it is implied that there are economic transactions between regions, which in the literature is known as the multi-regional input-output model (MRIO); its conception and methodology is very similar to the inter-regional model. However, among its differences are the number and size of matrices. For example, if we have three regional matrices of 2 by 2 size, it is necessary to construct 6 additional inter-regional matrices, which gives us 9 matrices in total: 3 of technical coefficients of intra-regional inputs and 6 of coefficients of inter-regional trade. In addition, there are 3 identity matrices I, as well as the vectors of gross production and final demand. In other words, there would be 12 regional input-output matrices.

Therefore, if we have three regions *r, s* and *l*, with the same amount of sectors (2), the coefficients of inputs and trade of regions r and s, need to consider the coefficients of region l, in addition to the inclusion of transactions of production and the estimation of final demand for each region. This is expressed as follows:

[8] It should be clarified that for matrix multiplication to take place, the matrix (I-A) has to be partitioned, that is to say to separate the matrices of technical and trade coefficients in order to make them square and conformable. Dividing the matrices by points or lines denotes this operation.

$$(I - A^{rr})x^r - A^{rs}x^s - A^{rl}x^l = f^r$$

$$-A^{rs}x^r - A^{rl}x^l + (I - A^{ss})x^s = f^s$$

$$-A^{rs}x^r - A^{rl}x^l + (I - A^{ll})x^l = f^l$$

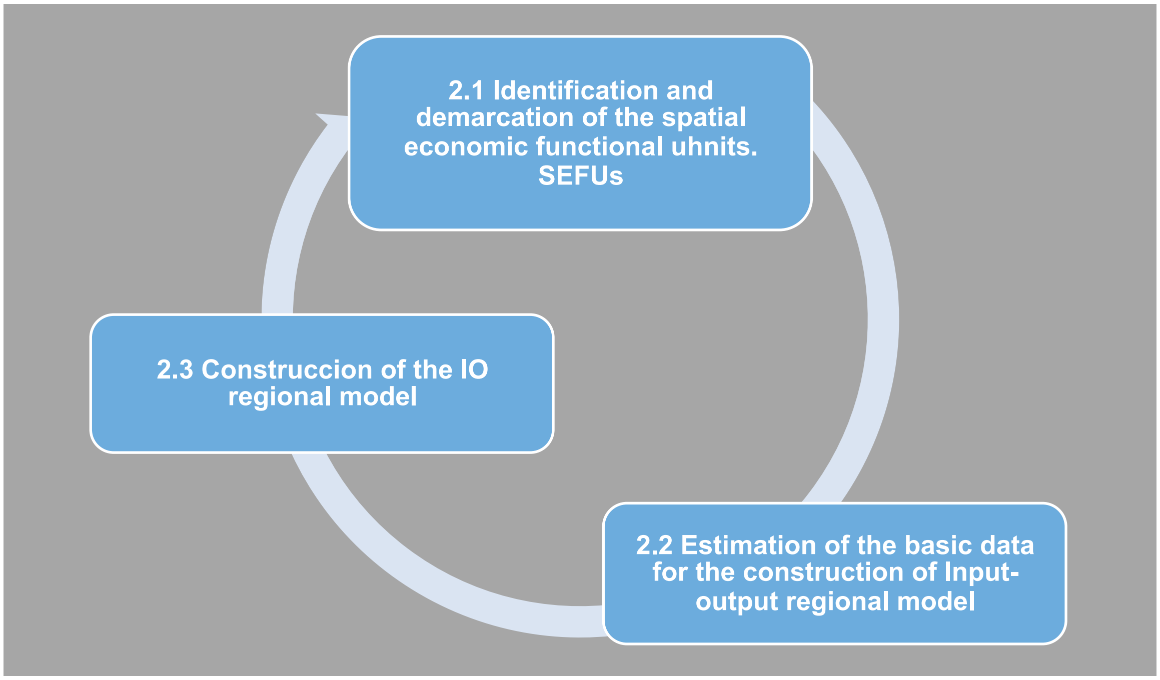
Then, the solution to the systems of equations in a matrix form is:

$$(I - A)x = .$$

$$\left\{ \begin{bmatrix} I & 0 & 0 \\ 0 & \mathrm{I} & 0 \\ 0 & 0 & \mathrm{I} \end{bmatrix} \right.$$

2. Methodological proposal

In accordance to this theoretical framework, we propose a methodology for the construction of a regional input-output matrix using a *bottom-up* approach, with the following steps:



2.1 Identification and demarcation of SEFUs

The identification and demarcation of the spatial economic functional units in the spatial economic system of a region, requires us to specify the importance and the economic specialization of that region, as well as its spatialization by pointing out the particularities of its location. We first identify the nodes using an index of concentration of economic activity and population, and then, assuming that a pair of nodes spatially close to each other are in competition, we establish their areas of influence using a Reilly index, which takes into account their sizes and distances with respect to each other. Actually, as already mentioned, this is an economic space. However, in the first step we first analyzed the economic structure of Sonora, and later the role and importance of its economic and population nodes, as well as their areas of influence. This led us to identify the SEFUs within the Sonora region. The concentration index is the ratio between the share of a output i of a sub-region r with respect to the total regional production j, and the same national ratio; this is denoted as:

$$Icee_i^r = \frac{\frac{q_i^r}{q_j^r}}{\frac{q_i^n}{q_j^n}}$$

The Reilly index (Asuad, 2016, pp. 362-364), which measures the border point of pair of nodes that compete with each other, is an inverse relationship between their size and distance, and is denoted as:

$$BP = \frac{Dab}{1 + \sqrt[2]{\frac{Pa}{Pb}}}$$

Where: *BP* is the border point; *Pa* is the population of site *a*; *Pb* is the population of site *b* and *Dab* is the distance between sites *a* and *b*.

2.2 Estimation of basic data for the construction of a regional Input-Output model

According to Miller and Blair (2009), among the most formidable challenges in using input–output analysis in practice, is assembling the detailed basic data of the economic area of interest – national, regional, or perhaps multiple-regions for the construction of input–output tables.

The methodology for the construction of a regional I-O model depends on the availability of regional data. This information plays a key role in the construction of a national I-O matrix. As a matter of fact, its construction is supported by a statistical system which relies on the elaboration of national accounts. This is one of the main international achievements that lead to the development of national I-O matrices and their consequent use as basic tools for measuring national economic performance.

Nevertheless, according to Eurostat 2013, regional accounts should ideally cover the same set of accounts as the national accounts, in the sense that they make visible traits such as regional economic structures, development and certain differences. However, due to the specific conceptual and measurement problems they are usually more limited in scope and detail. This is evident mostly in the case of imports from and exports to other regions, the financial transactions between enterprises in several different regions or in the regional allocation of collective consumption.

Therefore, because of this situation, the 2008 System of National Accounts (SNA 2008) concluded: *"these conceptual difficulties partly explain why no country establishes the complete SNA accounts for every region".*

This situation is traditionally combined with the argument of the lack of regional data, leading regional matrices to depend on the national ones. Although, in our view, the problem relies on the lack of a regional statistical system, which motivated us to elaborate a regional data base with only those needed variables for the creation of regional I-O matrices.

Furthermore, depending on the national statistical system, especially for the development of national accounts and the availability of regional data it is possible to establish a data base in order to estimate, through hybrid methods the key regional variables that are crucial for the construction of a regional input-out table.

In the case of Mexico, we have a solid system of national accounts, as well as consolidated accounts at a national level of non-financial corporations, government, institutions and no profit-making sectors. Besides, there are estimates of value generated by the states, considering the sectoral classification of the North American Industrial Classification System (NAICS). So we conclude, that we have enough elements for the development of our research.

On the other hand, there is information from population and economic censuses by state, and other surveys, mainly of employment, income and expenditure. Economic censuses provide a significant amount of information, for instance, around 60% of total national production is computed, which means that 40% is not registered. contrasting with intermediate consumption which is almost totally computed, although there is a lack of data related to agriculture, forestry, education services, medical services and public administration, as well as to imports and exports. However, economic censuses, not only are the most dissagregated (by state) source of information, but also the most complete, mainly in terms of the productive sector. On the other hand, there are other sources of information that supplement the lack of data in the economic censuses.

Consequently, one approach when putting together the regional data base, was to use information by state provided by the economic censuses, and supplementing it with national information.

This allowed for the application of the methodology for the construction of “bottom-up” regional matrices, a fundamental approach to grasp the essential and particular aspects of regions, such as the economic performance of its sub-regions.

It is worth stressing that this approach borrows the ideas established by Jackson (1998) and Lahr (2002), regarding the elaboration of regional accounts through the “regionalization” of national data. However, our analysis is supported by the creation of regional data bases.

Consequently, the estimation of critical regional data for the construction in regional input-output tables was based on the following procedures:

1. Out of the consolidated data of the 2008 National Accounts System, we decided to use as main variables gross production, intermediate consumption, imports, exports, valued added, investment, and both private and government consumption.

2. The available regional information was compiled considering states as regions and then comparing them to nationally aggregated information data.

3. We found missing information in variables such as government spending and investment, investment of financial societies, and investment of families.

4. Considering the consolidated data of the government national accounts, we regionalized data from financial societies, non-financial societies and non-profit societies. Different assignment measures were applied to each state, following the recommendations of EUROSTAT 2013 regarding regional accounts and those of Jackson 1998 and Lahr 2002. The indices we used are presented in the attachment.

5. We elaborated a data base of exports and imports by state and sector (NAICS).

6. . Private consumption of families was estimated based on the national consumption amounts recorded in the Income-Expenditure survey.

7. Our estimates by state were added up and compared to the national selected variables, resulting in around 60% of the national total data. Consequently, those variables were distributed by state according to their shares in the representative variables, so as to promote a regionalization that consider the essential regional information, which in this case, was represented by states. This, in order, to carry out a regionalization of the national variables based on regional information.

2.3 Construction of the I-O regional model

According to Osterhaven (1984), the accounting system used in the table construction phase, that further determines the set of hypotheses to assume in the modelling phase, depends on the amount and format of the available national and regional data.

In fact, the procedures used and hypotheses adopted in the construction and modelling of regional input-output tables are strongly connected to the type of information contained in the national table, which is used as a starting point to achieveboth a regional table and the format in which this information is presented.

The I-O national tables have been presented according to the traditional symmetric input-output format, which can be of product-by-product or industry-by-industry nature[9]. The first format consists of symmetric input-output tables presenting products as the dimension of both rows and columns, which show the amounts of each product used in production . The second format, on the other hand,has symmetric input-output tables with industries as the dimension of both rows and

[9] The term "product" is refers to all goods and services generated in the context of productive activity (EUROSTAT, 1996, paragraph 3.01) while the term "industry" involves more complexity. According to the 1993 System of National Accounts' definition, "an industry consists of a group of establishments engaged in the same or similar kinds of activity" (UN, 1993, paragraph 5.40). However, an industry has many activities, besides its most important, (the one responsible for the creation of most of value added) such as secondary and ancillary activities. (Sargento, 2009).

columns, showing the amounts of output of each industry used in the production of other industries. (UN, 1993).

However, their format consideration is very important when deriving I-O regional tables, either for the accuracy of the regional model or for the analysis of the economic interdependence between economic activities and for regional policy making, as well as at a national level.

Therefore, it is required that input-output models be created in a commodity-by-industry format which displays the input output table in two versions: Make and use input-output tables . Both of them[10] reveal how supplies of different products come from domestic industries and imports and how those products are used by the different intermediate or final users, including exports (UN, 1993).

This framework was set up in the 1960's, when the United Nations introduced the 1968 System of National Accounts and most countries started to compile and published their input-output tables in the commodity-by-industry format.

The supply and use accounts are presented in an integrated way in a basic table that relates the tables of supply and use of product-by-product and industry-by-industry, establishing a product-industry format; the U matrix of utilization; and a format of industry- products; the V make or supply matrix, as it is ilustrated in the following table:

[10] The use table describes the consumption of products j by the several industries i, and the make or supply table represents the distribution of the industries' output throughout the several products

Basic structure of a National Make and Use Table with Flows				
	Products	**Industries**		
Products		**U**	**Y**	**P**
Industries	**V**	**------**	**------**	**g**
	m	**w**		
	p	**g**		

However, the tables U and V are assymetric due to the higher number of products compared to the considered industries, which should be transformed, in order to have a symmetrical I-O matrix, according to the availability of data and assumptions concerning technology and sales structure. EUROSTAT 2008.

The elements of these matrices are represented by u_{ji}, which includes both imported and domestic inputs, while v_{ji}, describes the domestic production of product *j* by industry *i.*

The component of final demand is denoted by ***y*** which is the vector that adds all its categories up: Private consumption, government consumption, investment and exports, which also include imports and domestic production.

The integrated format of the supply and use matrix allows for the balancing of supply and demand for product and industry . So, if we add up columns in ***U*** and and add ***y***, we get vector ***p***, that accounts for total output of each product. Thus, the same transposed vector can be obtained summing all the entries of matrix ***V*** and adding the imported products comprised in ***m:***

$$P_j = \sum_j v_{ij} + m_j \quad = \sum_j v_{ji} + y_j$$

For industries, a similar balance can be done, so, if we define *w* as a valued added vectorby industry, we have:

$$g_i = \sum_j v_{ij} \quad = \sum_j v_{ji} + w_i$$

Hence, from these fundamental identities, an input-output model may be derived as a traditional symmetric input-output table, and an inverse matrix can be obtained. To develop such a model, at least two hypotheses have to be considered: 1) Fixed technical coefficients and (2) The assumption that relates industry's output with commodity's output.(Sargento 2009)

Furthermore, according to Eurostat (2013), the basic models for the transformation of supply and use tables into symmetric input-output tables are:

- Model A: Product-by-product input-output table based on the assumption of product techcnology

  Each product is produced in its own specific way, irrespective of the industry where it is produced.

- Model B: Product-by-product input-output table based on the assumption of industry technology

  Each industry has its own specific way of production, irrespective of its product mix.

- Model C: Industry-by-industry input-output table based on the assumption of fixed industry sales structure

  Each industry has its own specific sales structure, irrespective of its product mix.

- Model D: Industry-by-industry input-output table based on the assumption of fixed product sales structure.

Each product has its own specific sales structure, irrespective of the industry where it is produced.

 Model E: Product-by-product input-output tables based on the assumption of hybrid technology .

 Model F: Product-by-product input-output tables based on Almon procedure

The symmetrizaton of the input-output supply and make or use tables of is needed, in order toequal the number of products and industries. Nevertheless, it is also required to adequate this matrix to the product-industry format, to avoid insuficient and distorted information about the products produced by industry in the creation of the input -output matrix.

As jackson (1998) and Lahr (2002), had mentioned that it is needed the domestication of the national technology matrix, which implies the estimation of the technical coefficients product –by- industry. Therefore, this technical coefficient matrix of products- by –industry ***A,*** is a function of the premultiplication of the market shares matrix, that represents the contribution of each industry to the output of a product, ***D***, by the matrix of input requirements for products per unit of output of an industry ***B***, which is denoted as:

$$A = \overline{D}B$$

Where:

$$B = U\hat{g}^{-1}$$

$$\overline{D} = \begin{pmatrix} V - D\hat{x} \\ m' \end{pmatrix}(\hat{q} - \hat{x} + \hat{m})$$

$$D = V\hat{q}^{-1}$$

$$So:$$

$$A = \begin{pmatrix} V - D\hat{x} \\ m' \end{pmatrix}(\hat{q} - \hat{x} + \hat{m})^{-1}B$$

Furthermore, Lahr (2002), validated Jackson´s proposal including a trade adjusted parameter, p , which incorporated the relationships between domestic production and total supply, taking into account imports, denoted as follows:

$$\rho = \frac{q}{q+m}$$

Thus, his final proposal is the following :

$$A = D\left(\frac{\hat{q}}{q+m}\right)B$$

Subsequently, we proceeded tothe regionalization of the product by industry taking into account the regional data base in order to begin the construction of the regional matrix. The first stage is concerned with the estimation of the regional technical coefficients matrix, which was done taking the national estimated technical coefficients of the new format matrix through their adaptation to the regional economy by applying a regional productivity factor taking into account the criteria that Lahr (2002 p.171) had pointed out.

This factor tries to grasp regional and national differences in productivy under the assumption that these differences can be seen in in the use of labor and capital, measuring the distance among them, with a ratio between the share of regional valued added in production andthe same relationship at the national level.Thus, we have a factor of regionalization as a measurement tool for the adjustment of the national technical coefficients to the region, based on the assumption that technology between the region and the nation is spatially variable, which is denoted as follows:

$$A^{r} = \hat{\tau} A^{n}$$

Where:

$$\hat{\tau} = \frac{\frac{VA^{r}}{P^{r}}}{\frac{VA^{n}}{P^{n}}}$$

The second stage is related to the estimationof the supply and use input-output regional table, relying on the regionalization matrix and on the basic relationships of these accounting identities as can be seen in the following expressions::

$$U^{r} = i\hat{C}^{r} A^{r} + y^{r}$$

iC^{r} =*Intermediate consumption*

y^{r} =Components of final demand: Private consumption+ Goverment consumption+

investment+exports

$$V^{r} = \hat{g}^{r} A^{r} + m^{r}$$

It is worth mentioning that from the regional data base, we used the required data for the estimation of variables such as intermediate consumption and the components of final demand. However, in the case of private consumption we estimated it, based on the data provided by the 2008 household´s survey of income and expenditure. We also used the indicators we developed for the regionalization of the crucial variables in order to build the integrated regional framework of the input-output tables of Make and Use.

In the third stage, we applied the transformation model in accordance with the methodology recommended by EUROSTAT 2013, to get a regional symmetric matrix.

Finally we get a regional model based on the traditional Leontief´s formulation, as follows:

$$A^{rr} = \hat{\tau}^{r} A^{n}$$
$$X^{r} = \left(I - A^{rr}\right)^{-1} y^{r}$$
$$\Delta X^{r} = \left(I - A^{rr}\right)^{-1} \Delta y^{r}$$

However, a note of caution: The regional I-O model we presented in this essay, is preliminary , given that we needto aggregate the inter regional effects caused by the linkages between a single region and the others. Thus, in this paper we merely applied the regional model in order to capture intra-regional effects alone as a first step of the methodology for the construction of ”bottom-up” regional I-O model ,built as a single region, through the formulation of the crucial regional variables, making them compatible with the national accounts and regionalizing them using the states of Mexico as a first step of the spatial dissagretation process, in order to reach in the future, a sub regional level of analysis.

The following steps in the development and application of the model required the development and application of trade coefficients, that take into account the economic inter-linkages between regions. This will also be extended first to develop and apply a multi-subregional I-O model, given our intention to first construct a regional model at a sub regional level and second, a multi regional model. Both are stages in our research agenda, that we are going to develop in the near future.

3. Preliminary empirical results

1. The methodology for the construction of regional matrices using a “bottom-up” approached was developed, based on the use of crucial information from the

regional accounts of the 32 states compatible with the national accounts and their extension and application to sub regional matrixes.

2. The economic sub regions of Sonora were identified and delimited.

3. A 2008 database of crucial variables was constructed at a regional level, considering the 32 states of the country to construct regional supply and use tables, taking into account their compatibility with the national accounts. A series of indicators were developed for the regionalization of national accounts, in accordance to the regional data base

4. The key variables of Sonora were estimated using the 2008 the regional data base, considering the total participation of each state.

5. We estimated the 2008 Mexican national input-output matrix, under the format of product–by-industry, in accordance with the recommendations of Lahr (2002) and Jackson (1998).

6. The technical coefficients of the region of Sonora were estimated, through an adjustment of the national ones with regional productivity factor, under the assumption of spatial invariability in the technology of production.

7. Aggregated regional supply and utilization tables for the state of Sonora were constructed.

ATTACHMENT

Regional value added was estimated using census values and information related to valued added from the state governments in sectors 61,62 and 93. This information was added up by state and sector, and then multiplied by the national value added of the sectors in the System of National Accounts.

$$VA_s^E = \left(\frac{VA_{ce,s}^E + VAg_{sna,s}^E}{VA_{ce,s}^T + VAg_{sna,s}^T} \right) GDP_s^E$$

Where :

$VA_s^E =$ Value added by state E and sectors s

$VA_{ce,s}^E$ = Value added by state E , from the economic census ce, by sectors s

$VA_{ce,s}^T$ = Total Value added T, from the economic census ce, by sectors s

$GDP_{,s}^E$ = Gross Domestic Product by state* E and sectors s

$VAg_{sna,s}^E =$ Government Value Added by state E and sectors s, from the SNA

$VAg_{sna,s}^T$ = Total Government Value Added by sectors s, from the SNA

- In order to clarify, this measurement, according to a national estimate of National Institute of Geography and Statistics, is equal to value added.

Intermediate consumption was equally estimated as value added, using information from the 2008 economic census and from the SNA in the case of state governments, adding them up and then multiplying them by the national values of intermediate consumption reported in the SNA.

$$IC_s^E = \left(\frac{IC_{ce,s}^E + ICg_{sna,s}^E}{IC_{ce,s}^T + ICg_{sna,s}^T} \right) IC_{sna,s}^N$$

IC_s^E = Intermediate Consumption by state E and sector s

$IC_{ce,s}^E$ = Intermediate Consumption by state E from economic census ce by sector s

$IC_{ce,s}^T$ = Total Intermediate Consumption from economic census ce, by sectors s

$IC_{scn,s}^N$ = National Intermediate Consumption from the SNA by sectors s

$ICg_{sna,s}^E$ = Government Intermediate Consumption by state E and sector s from the SNA

$ICg_{sna,s}^T$ = National Government Intermediate Consumption by sector s from the SNA

In the case of gross total production by state, once determined the estimated state value added and intermediate consumption, they are added up for each state and sector of activity, in order to get the GTP by state and sector.

$$GTP_s^E = VA_s^E + IC_s^E$$

GTP_s^E = Gross Total Production by state E and sector s

VA_s^E = Value Added by state E and sector s

IC_s^E = Intermediate Consumption by state E and sector s

The estimates of the private consumption of households for the state of Sonora, is based on the income and expenditure survey of households 2008 by the following procedure : 1. The household consumption by NAICS sector was estimated at national level and applied to the number of households in Sonora, and 2. The national household consumption by sector was multiplied by the number of Sonora´s households, in order to obtain the total consumption of the households of that state.

To do so, we first estimate the share of total private consumption of Households by sector, $^{SN}{}_{HCs}$ by dividing the total consumption of households by sector between the total consumption of households, denoted as follows:

$$S^N_{HCs=}\frac{HCs}{Hct}$$

Subsequently, the number of consumption households by NAICS sector is obtained, for which the number of families is divided between the share of consumption by NAICS ´s consumption sectors :

$$n^N_{HCs} = \frac{n^N_{HC}}{S^N_{Hct}}$$

Next, the participation of the households of Sonora in the consumption by sectors NAICS was estimated, for which the number of households of consumers of Sonora was divided by the participation of the households in the consumption by sectors NAICS at national level:

$$S^R_{HCs} = \frac{n^R_{HCt}}{S^N_{Hct}}$$

After, the Sonora´s number of households per consumption of NAICS sector is estimated by the division of the number of households in Sonora by consumption of the NAICS sector among the total number of households in Sonora:

$$n^{R}_{HCs} = \frac{n^{R}_{HCs}}{n^{R}_{HCt}}$$

Finally, the Houselholds´s consumption by NAICS sector is estimated by multiplying the national consumption of households by the participation of the household´s consumption of Sonora by sector NAICS:

$$HC^{R}_{s} = HC^{N} * S^{R}_{HCs}$$

In the case of government consumption, and according to the accounting framework of the national accounts, the government final consumption equals the totallity of its production given that it is not market related. For this reason, we took total production from the state governments data bases, specifically from sectors 61, 62 and 93 in order to express them as shares and multiply them by national government final consumption and by sectors from the SNA.

$$GC_{s}^{E} = \frac{GTPg_{sna,s}^{E}}{GTPg_{sna,s}^{T}} GC_{sna,s}^{N}$$

GC_{s}^{E} = Government Consumption by state E , by sector s

$\mathrm{GTPg}_{\mathrm{sna.s}}^{\mathrm{E}}$ = Gross Total Production of state governments by state E, from SNA by sectors s

$\mathrm{GTPg}_{\mathrm{sna.s}}^{\mathrm{T}}$ = Total Gross Total Production of state governments, from SNA, by sectors s

$\mathrm{GC}_{\mathrm{scn.s}}^{\mathrm{N}}$ = Government Consumption, from SNA by sectors s

In the case of investment, we did regional estimates for the institutional sectors since the economic censuses have less information that the Mexican System of National Accounts. So, we used information of investment of societies from the economic census, we estimate household investment and government investmet (sector 23 *Construction)*. Each of these three calculations were added up to the reported information of total investment by the sectors in SNA.

Total investment is stated as:

$$I_s^E = HI_s^E + GI_s^E + SI_s^E$$

I_s^E = Total investment by state and by sector

HI_s^E = Household investment by state and by sector

GI_s^E = Government investing by state and by sector

SI_s^E = Investment of Societies by state and by sector

Household investment is obtained with population data by state obtained from the *National Council of Population and Housing*¸ and then expressed as shares to be used as indicated, in order to obtain household investments from sector 23, when multiplied by the total households investment resported by SNA.

$$Ih_s^E = \frac{POB^E}{POB^N} Ih_{scn,s}^N$$

Ih_s^E = Household Investment by state and by sector

POB^E = Population of the state

POB^N = National Population

$HI_{sna,s}^N$ = National Households Investments from SNA by sectors s

Investment of state governments is available in the data base used by those governments in variables such as value added and intermediate consumption. That database does not have a sectorial classification, so it was assigned to sector 23 *Construction*. We generated shares and multiplied them by state total investments from the SNA. To this estimate, we added the federal investment and state investment.

$$GI_s^E = \frac{GI_{sna}^E}{TSGI_{sna}^T} TNSI_{sna}^N + \frac{ICG_{shcp}^E}{TICG_{shcp}^T} ICGF_{shcp}^N$$

GI_{s}^{E} = State Government Investment by state E and sector s

GI_{scn}^{E} = State Government Investment by state E, from the SNA

$TSGI_{scn}^{T}$ = Total State Government Investment T, from the SNA

$TNSI_{scn}^{N}$ = Total National State investment from the SNA

$\mathrm{ICG}_{\mathrm{shcp}}^{\mathrm{E}}$ = Investment from the Central Government by state, from Secretary of Treasury

$\mathrm{TICG}_{\mathrm{shcp}}^{\mathrm{T}}$ = Total Investment from the Central Government in the states, from the Secretary of Treasury

$\mathrm{ICGF}_{\mathrm{shcp}}^{\mathrm{N}}$ = National Investment from the Central Government and Federal Funding, from SNA

When it comes to investment of societies, we used information from the economic census by state and by sector. It is multiplied by the value of investments of societies reported by the SNA at a national level and by sector.

$$Is_{s}^{E} = \frac{Ice_{s}^{E}}{Ice_{s}^{T}} Is_{scn,s}^{N}$$

Is_{s}^{E} = Investment of Societies by state and by sector

$\mathrm{Ice}_{\mathrm{s}}^{\mathrm{E}}$ = Investment of Societies by state and by sector from economic census

$\mathrm{Ice}_{\mathrm{s}}^{\mathrm{T}}$ = Total investment by state and by sector from economic census

$\mathrm{Is}_{\mathrm{scn,\,s}}^{\mathrm{N}}$ = Total investment by state and by sector from SNA

For exports, there is information by sectors (11,21, 31-33) and by state, from the data base *Atlas of Complexity.* This data is expressed as shares that are multiplied by national exports of the corresponding sectors (SNA). Fort he rest of the sectors, we did a calculation trough an index of export base, expressed as shares by sector for each state, also, to be multiplied by all exports registered in the SNA.

$$X_{s}^{E} = \frac{X_{a,s}^{E}}{X_{a,s}^{T}} X_{scn.s}^{N} + \frac{IBE_{,s}^{E}}{IBE_{,s}^{T}} X_{scn,s}^{N}$$

X_s^E = Exports by state and sector

$\frac{X_{a,s}^E}{X_{a,s}^T}$ = Share of exports of each state, by sector

$\frac{EBI_s^E}{EBI_s^T}$ = Share of EBI by state and by sector

$X_{sna,s}^N$ = National exports from SNA, by sector

$$EBI_s^E = VA_{ce,s}^E - \left(\frac{VA_{ce,s}^E}{IES_s^E} \right)$$

EBI_s^E = Export - base index by state and by sector

$VA_{ce,s}^E$ = Value Added from economic census ce, by state E and by sector s

IES_s^E = Index of Economic Specialization by state E and by sector s

$$IES_s^E = \frac{VA_s^E / VA^E}{VA_s^N / VA^N}$$

Finally, with imports we followed the same procedure as exports, given that information from sectors 11,21 and 31 to 33 is found in the *Atlas of Complexity*. We express them as shares, and for the rest of the sector we again used indexes of export base.

$$M_s^E = \frac{M_{a,s}^E}{M_{a,s}^T} M_{scn.s}^N + \frac{IEB_s^E}{IEB_s^T} M_{scn,s}^N$$

M_s^E = Imports by state and sector

$\frac{M_{a,s}^E}{M_{a,s}^T}$ = Share of imports by state and by sector

$M_{scn,s}^N$ = National imports from the SNA, by sector

$\frac{IBE_s^E}{IBE_s^T}$ = Share of each sector in the states from the export base index

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