Theoretical and Empirical Considerations towards the Integration of the Leontief and Sraffa Systems

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ABSTRACT

The production equations of the Sraffa system have an agrarian point-input point-output character. This paper presents a generalization of the Sraffa system to cover continuous-input continuous-output processes. The resulting system turns out to be identical with Leontief's dynamic price model. It is shown that the generalized model possesses all the important properties of the usual Sraffa system such as the impossibility of devising a physical measure of capital, possibility of reswitching of techniques, etc. A further generalization to cover fixed capital has been made to show how the problems of joint utilization, transferability of fixed capital between industries and changing efficiency of machines over their lifetimes can be tackled. Some empirical observations on the general relation between the rate of profit, the on-cost markup rate and the rate of capital turnover have also been presented. These form the basis for a new formulation of the dynamic price and output systems that can facilitate the empirical application of the dynamic systems. The new formulation automatically incorporates imperfectly competitive industries into the system of inter-dependent industries.

1. Introduction

The production equations of the Sraffa system [Sraffa 1960] have an 'agrarian' flavor - inputs in all industries are applied at one point of time and outputs of all commodities are obtained at the end of the 'season'. These point-input point-output production processes do not properly describe modern industrial production which is characterized by continuous-input continuous-output processes. Industrial enterprises belonging to sectors like engineering, chemicals, electronics, equipment manufacturing, power, etc., typically operate continuous production processes. Their purchases and sales are not concentrated respectively at the "start of the season" and "end of the season" as is typical of agriculture. Instead in these industries enterprises buy inputs every day, produce every day and sell every day. The "stocks consumed" during a year's production may be several times the "stock held", the proportion varying depending upon the rate of stock turnover. The process of production itself is rendered continuous by the carrying of stocks of raw materials, semi-finished goods and finished goods which enable the enterprises to eliminate the time gaps between the purchase of materials and their use in production, between production at various stages of finish and between the production of finished goods and their sale respectively.

The purpose of this paper is to present a generalization of the Sraffa system that incorporates continuous industrial production and to investigate its properties. The paper is divided into ten sections. Sections 3 and 4 show that all the essential properties of the usual Sraffa system carry over to the generalized Sraffa system except one viz. the on-cost profit markup rate and the rate of profit earned on invested capital differ depending on the capital-turnover rates in the different industries. Interestingly, the generalized Sraffa system, it turns out, is identical with the price system associated with Leontief's dynamic open system. In effect all the essential properties of the Sraffa system are shared by Leontief's dynamic system even though they have never been discussed in the latter context. Section 5 contains a doctrinal discussion of the Leontief and Sraffa systems. Section 6 introduces fixed capital into the primal and dual Leontief-Sraffa system and shows (a) how prices can be reduced to dated labour terms and (b) how the standard system can be constructed, under the usual general conditions. Section 7 goes further to treat problems such as joint utilization of machines, transferability of fixed capital between industries and treatment of changing efficiencies of machines over their lifetimes. Section 8 offers some brief but suggestive empirical evidence on the relationship of on-cost markup rates and the capital turnover rates from Indian industrial data. Section 9 proposes a new formulation of the Leontief-Sraffa dynamic systems which, although it is cast in the static form.

2. The generalized Sraffa System

It has already been remarked that continuous industrial production becomes possible only by continually replenishing stocks of raw materials, semi-finished goods and finished goods by continuous purchases, production at successive stages of finish and the production and sale of finished goods. The rate of stock turnover, that is to say, the ratios of stock consumed to stock held (as well as the ratios of stock sold to stock held of the finished goods) is different for different items of stock within each industry depending upon whether the item is of "fast" or "slow moving" variety and also across industries depending upon technology and the nature of demand. In the usual Sraffa system the ratio of the stock consumed during the year to the stock held at the start of the year is equal to 1 for all items of stock in all the industries. The generalization that is envisaged simply requires that the ratios be allowed to be different. Accordingly we write

$$
\begin{aligned}
&(S_{11}p_1 + S_{21}p_2 + \ldots + S_{n1}p_n)r + A_{11}p_1 + A_{21}p_2 + \ldots + A_{n1}p_1 + A_{n1}p_n + wL_n = X_1 p_1 \\
&\underline{(S_{12}p_1 + S_{22}p_2 + \ldots + S_{n2}p_n)r + A_{12}p_1 + A_{22}p_2 + \ldots + A_{n2}p_2 + A_{n2}p_n + wL_n = X_2 p_1} \\
&(S_{1n}p_1 + S_{2n}p_2 + \ldots + S_{nn}p_n)r + A_{1n}p_1 + A_{2n}p_2 + \ldots + A_{nn}p_n + A_{nn}p_n + wL_n = X_n p_n \qquad (1)
\end{aligned}
$$

where S_{ji} is the stock of commodity *j* held by the *i*[th] industry, A_{ji} (i.e. stock of *j* consumed in the production of *i*) is the flow input requirement, L_i is the labour used, X_i is the output produced and *w* and *r* are the wage rate and rate of profit respectively. The flow input A_{ji} arises from the following accounting formula,

Stock Consumed = Opening Stock + Purchases - Closing Stock

The throughputs from any one stage of value addition to the next are similarly defined. For a steadily growing economy the closing stock = (1+g) (opening stock) and the value of closing stock is the working capital employed in the next year. The value ratio $\sum A_{ji}p_j / \sum S_{ji}p_j = T_i$ is called the stock turnover rate. The value ratio $X_i\, p_i /(\sum A_{ji}p_j + wL_i) = 1 + m_i$ is the gross profit markup factor which, in the absence of fixed costs, is also the net profit markup factor. Being ratios of value magnitudes, the stock turnover ratios and the gross profit markup rates are not determined until the prices of commodities are determined. In system (1) technology will be described by three sets of coefficients, S_{ji} / X_i , A_{ji} / X_i , $l_i = L_i / X_i$ are the usual input, capital and labour coefficients and S_{ji} / X_i are the physical stock to output coefficients. The stock-turnover ratios for individual items of stock are $s_{ji} = A_{ji} / S_{ji}$. $\sum\sum S_{ji} p_j$ is best understood to be "permanent working capital" on which owners earn profit at the rate prevailing under free competition. The special case $A_{ji} / S_{ji} = 1 \ \forall i, j$ reverts us to the usual Sraffa system. Postponing the doctrinal discussion of system (1) and its generalization to include fixed capital to section 5 let us first proceed to study some of its properties in relation to the well-known properties of the regular Sraffa system.

3. Properties of System (1)

In this section, we show that the generalized system (1) possesses all the essential properties of the usual Sraffa system. To start with there are *n* equations in *n+1* unknowns, viz., *n-1* relative prices, the real wage rate and the rate of profit. Also the behavior of relative prices due to changes in the distribution of income between wages and profits depends only on the capital to labour ratios in the industries - it is independent of the A_{ji} / S_{ji} or A_{ji} / L_i ratios. To see this consider a two-goods economy and suppose $P_2 = 1$ to be the *numeraire,*

$$\begin{aligned} (S_{11} p + S_{21}) r + A_{11} p + A_{21} + w^* L_1 &= X_1 p \\ (S_{12} p + S_{22}) r + A_{12} p + A_{22} + w^* L_2 &= X_2 \end{aligned} \qquad (2)$$

where $p = p_1 / p_2$ and $w^* = w / p_2$. The solution for the relative price is

$$p = \frac{(X_2 - A_{22} - rS_{22})L_1 + (A_{21} + rS_{21})L_2}{(A_{12} + rS_{12})L_1 + (X_1 - A_{11} - rS_{11})L_2}$$

Differentiating with respect to *r* we find that

$$\frac{dp}{dr} \gtrless 0 \quad \text{as} \quad p \gtrless \frac{L_2 S_{21} - L_1 S_{22}}{L_1 S_{12} - L_2 S_{11}}$$

or

$$\frac{S_{11} p + S_{21}}{L_1} \gtrless \frac{S_{12} p + S_{22}}{L_2} \qquad (3)$$

i.e. the price of commodity 1 increases (decreases) with an increase (decrease) in the rate of profit if it is more (less) capital intensively produced.

Further, a unique standard system for system (1) can be constructed just like the usual Sraffa system: Expressing (1) in matrix notation for unit outputs,

$$S^T P r + A^T P + wL = P$$ (4)

$$(I - A^T)P = S^T P r + wL$$

$$P = (I - A^T)^{-1} \; S^T P r + (I - A^T)^{-1} wL$$

$$P = [I - (I - A^T)^{-1} S^T \; r]^{-1} (I - A^T)^{-1} wL$$ (5)

At *w=0* the price system is

$$P = (I - A^T)^{-1} S^T P r$$

With $\lambda = (1/r)$ it can be expressed as

$$[\lambda I - (I - A^T)^{-1} S^T]P = 0$$ (6)

If $(I - A^T)^{-1}$ is positive (Hawkins-Simon conditions), $(I - A^T)^{-1} S^T$ must be non-negative. By the Perron-Frobenius theorem it has a dominant eigenvalue λ_d with which is associated a non-negative eigenvector X_d. Thus for values of $r < R$ the matrix $[I - (I - A^T)^{-1} S^T r]$ has positive minors so the price solution of (6) is strictly positive.

Multiplying (4) by the eigenvector X_d we get,

$$X_d S^T P r + X_d A^T P + w X_d L = X_d P$$ (7)

At *w=0*

$$r = R = \frac{X_d (I - A^T) P}{X_d S^T P}$$ (8)

If $X_d (I - A^T) P$, the standard net product is set equal to 1 and further if $X_d L = 1$ then substituting (8) into (7) gives the linear wage-profit frontier

$$r = R(1 - \omega)$$ (9)

where ω is the share of wages in the standard net product.

Finally, we can expand (5) in a matrix power series which must be convergent for *r < R*,

$$\begin{aligned} P &= [I + rS^T (I - A^T)^{-1} + r^2 S^{T2} (I - A^T)^{-2} + \ldots](I - A^T)^{-1} wL \\ &= [(I - A^T)^{-1} + rS^T (I - A^T)^{-2} + r^2 S^{T2} (I - A^T)^{-3} \ldots] wL \\ &= wL_1 + wrL_2 + wr^2 L_3 \end{aligned}$$ (10)

where $L_t = (S^T)^{t-1}(I - A^T)^{-t} L_t (t = 1..\infty)$. In other words system (1) is amenable to a reduction to dated labour terms just like the usual Sraffa system.

It is easily possible to construct numerical examples using equation (10) in which it can be shown

(i) that relative prices of two commodities show a non-monotonic behavior, i.e. first a rise, then a fall, then a rise again, etc. with respect to a monotonic rise in the rate of profit, thus showing the impossibility of measuring capital independently of the distribution of income [Sraffa (1960), Chapter 6, Section 48].

(ii) that the choice between two alternative technologies of producing the same commodity may switch two or more times at different rates of profit [Sraffa (1960), Chapter 12].

4. Profit Markups and Profit Rates

The only respect in which system (1) differs from the Sraffa system is in the relationship of the on-cost profit markup rate and the rate of profit on invested capital. It is well-known that most industries follow the full cost pricing principle, i.e. they apply a gross profit markup on prime cost to arrive at the price. The prime cost includes the cost of stocks consumed, energy costs, wage costs and other variable costs. If unit prime cost is u_i the price charged is

$$P_i = u_i (1 + m_i) \tag{11}$$

The rate of profit on invested capital is

$$r = \frac{u_i m_i X_i - F_i}{K_i} = m_i T_i - F_i / K_i \tag{12}$$

where X_i is the output sold, F_i is the fixed cost, K_i is the equity capital and T_i the capital turnover ratio is the prime cost $u_i X_i$ to capital K_i ratio. Long run considerations mean that $F_i = 0$ and all costs are variable and the formula simply reduces to

$$r = m_i T_i \tag{13}$$

For the usual Sraffa system, the material input cost $\sum A_{ji} p_j$ is also the invested capital in each industry so the markup on material cost is charged to recover the wage cost and net profit and the relationship becomes

$$r = m_i - \frac{wL_i}{K_i} \tag{14}$$

Even if, in the Sraffa system, the wage is assumed to be paid pre-factum the invested capital equals the input plus wage costs and the relation becomes

$$r = m_i$$

In either case, $T_i = 1 \ \forall i$, i.e. the capital turnover ratio is implicitly assumed to be equal to 1 in all industries and for every item of stock in every industry.

However, for the generalized Sraffa system shown in equation (1), even without supposing that wages are advanced,

$$\begin{aligned} r &= \frac{p_i X_i - \sum A_{ji} p_j - wL_i}{\sum S_{ji} p_j} \\ &= \frac{u_i (1+m_i) X_i - u_i X_i}{K_i} \\ &= m_i T_i \end{aligned} \quad (15)$$

which is in accord with the usual full-cost pricing method. The markup rate varies inversely with the capital turnover ratio; industries having high turnover ratios (e.g. retail, FMCG) need to apply a lower markup as compared to industries with low turnover ratios (e.g. shipbuilding, aircraft manufacturing) to attain the uniform competitive rate of profit (1). These remarks also hold good for the Leontief static open price system which supposes $r = m_i = 0$ and $T_i = 1$.

5. Doctrinal Discussion of the Leontief and Sraffa Systems

Attention is now called to the fact that the system (1) which is expressed for unit outputs in matrix notation in equation (4) is exactly identical with the price system corresponding to Leontief's dynamic open model. (The notation B is generally used in place of S for the stock coefficients in the literature). First formulated by Georgescu-Roegen (1951), its properties have been extensively studied by Morishima (1958), Dorfman, Samuelson and Solow (1958), Solow (1959), Zaghini (1971) and Szyld (1985) among others. The differences between the original motivations underlying the formulations of the Sraffa system and the Leontief dynamic system (rehabilitation of classical economic methods and the study of periodic fluctuations in business activity in a multisector context in the course of the attainment of a steady state respectively) do not detract from the fact the two systems are formally identical. Accordingly, their dual is Leontief's dynamic open output system,

$$SXg + AX + C = X \quad (16)$$

where C and X are the *n x 1* vectors of final consumption and gross output vectors respectively and g is the rate of growth. At C = 0 the economy attains its maximum growth rate G = R. So for values of g < G it can be shown that (16) has a strictly positive solution provided the principal minors of I-A are positive. It may be observed that G is the reciprocal of the dominant eigenvalue of the matrix $(I-A)^{-1}S$. Symmetrically with (5) the solution of (16) may be written as,

$$X = [I - (I-A)^{-1} Sg]^{-1} (I-A)^{-1} C \quad (17)$$

The foregoing analysis has some bearing on the doctrinal aspects of a) the interpretation of the Sraffa system itself, (b) the relation between the Leontief and Sraffa systems and (c) the capital controversies of the sixties. Sraffa (1960) himself presented his system as being a purely static system in which no changes in inputs and outputs take place, and that approach has been adopted by the Sraffian literature. However, it appears that this view will need to be reconsidered in the light of the fact that the generalized Sraffa price system is identical with the Leontief dynamic price system so that their dual and its solution, Leontief's dynamic inverse, are identical too. And this remark holds whether or not S = A; S = A is a special case for which the dual is

$$(AX)(1+g)+C=X \tag{18}$$

That the system of equations (18) is a satisfactory expression of the dual of the Sraffa price system $A^TP(1+r)+wL=P$ has been shown conclusively by Kurz and Salvadori (1995).

Kurz and Salvadori (2006) also found that the works of Leontief and Sraffa had common sources of inspiration and striking similarities of approach. They pointed out however that Leontief's assumption of an exogenously given value-added vector was not theoretically sustainable. It amounts to treating capital as an exogenously given magnitude independently of the prices of capital goods; this treatment is at odds with the Sraffa system. Kurz and Salvadori's remark about the theoretical unsustainability of Leontief's assumption is even more true of the dynamic price system (4); assuming an exogenous value-added vector in (4) amounts to assuming away completely the term S^TP and reverting the dynamic system back to the static open system making it impossible to capture the independent influence of the stock coefficients on relative prices and defeating the very purpose of formulating the dynamic model. In other words, if the dynamic model is employed, whether for theoretical or empirical purposes, the problem of the determination of income distribution along with relative prices will have to be squarely faced, it cannot be bypassed. But of course that has proved to be most intractable. The presence of the rate of profit *r* in the production equations in the Sraffa system and Leontief's dynamic price system has led to a very curious situation methodologically speaking. The presence of the rate of profit greatly enhances the realism of the theoretical model. But ironically it almost entirely eliminates the empirical applicability of the model. Neither the Sraffa system nor Leontief's dynamic price system have been fruitfully employed in empirical work. [Perhaps Han and Schefold's (2006) investigation of reswitching and reverse capital deepening employing Leontief's dynamic model is the lone exception]. And it is only when *r* is treated in theoretically and empirically unsustainable ways, that is to say, by assuming it to be zero or as part of an exogenously given value added vector, that the avenues for empirical application are opened. Witness the various empirical applications of the Leontief static open price system. So we are landed in a frightful mess – increasing the realism of the model renders it practically useless for empirical application. On the other hand empirical application is precisely what we rely upon to understand reality. (One way out of this embarrassing dilemma has been proposed in Section 9 below).

It has been shown in section (3) that all the important properties of the usual Sraffa system viz., the impossibility of measuring capital independently of the distribution and prices, the possibility of reswitching of techniques, the existence of the standard commodity and the general inapplicability of marginal productivity theory are shared by Leontief's dynamic price system. It is strange that Samuelson and Solow who made a deep study of the Leontief dynamic model in the decade of the fifties never encountered any of them. Even more strange it is that when they were confronted with those properties [Sraffa (1960)], they were stunned into incredulity and entered into a long debate, the famous capital controversy of the sixties, which has caused an irreconcilable division among economic theorists ever since. I may be permitted to ask one counterfactual "what if" question at this point, which

I hope will not be considered entirely pointless!, "What if they themselves had independently discovered some of those properties?"

6. Fixed capital

The use of fixed capital is of obvious interest for any discussion of continuous industrial production. Of the several methods of incorporating fixed capital into value theory, Sraffa's treatment of fixed capital and depreciation has been the most satisfactory as compared to all other methods proposed so far because it is the only method that gives a uniform price of the product irrespective of the age of the machine by which it may be produced. Sraffa's treatment, based on a detailed joint-products approach, is equivalent to the simple annuity method only if machines work with constant efficiency over their lifetime. Section 7 below shows how the annuity method can be suitably modified to the case of machines that work with variable efficiencies over their lifetime.

In the case of fixed capital, say machines, the relationship between the book-values of the machines at successive ages is as given in equation (19).

$$
\begin{aligned}
p_0 &= p \\
p_1 &= [(1+r)-\psi]p \\
p_2 &= [(1+r)^2-\psi\{(1+r)+1\}]p \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
p_k &= [(1+r)^k-\psi\{(1+r)^{k-1}+\ldots+(1+r)+1\}]p = 0 \qquad (19)
\end{aligned}
$$

$$\text{where } \psi = \frac{r(1+r)^k}{(1+r)^k-1} \qquad (20)$$

Suppose M_0, M_1, …, M_{k-1} be the numbers of machines of ages 0…, k-1 used in production of commodity *i* (along with other inputs and stocks) during a year. They emerge one year older at the end of the year; the price equation is,

$$
\begin{aligned}
&(M_0 p_0 + M_1 p_1 + \ldots + M_{k-1} p_{k-1})(1+r) + (\sum S_{ji} p_j)r + \sum A_{ji} p_j + wL_i = \\
&p_i X_i + M_0 p_1 + M_1 p_2 + \ldots + M_{k-1} p_k
\end{aligned} \qquad (21)
$$

Substituting into these the book values above and rearranging the machine terms gives the value $Fp_0\psi$ where $F = M_0 + M_1 + \ldots + M_{k-1}$ because $p_t(1 + r) - p_{t+1} = p_0\Psi\ \forall$ t=0….k. This method of applying book prices to the old machines and adding them into a single term can be called as "reduction to new machines". This method can always be applied when (a) machines work with constant efficiency over their lives or (b) when they are made to work with constant efficiency by incurring repairs and maintenance expenses, which are included in the A^T and L. By reducing all machines to their new machine equivalents the technical coefficients of the fixed capital items can be simply defined as $f_{ji} = F_{ji} / X_i$ irrespective of the age of those items. The effect of reducing all old machines to their new machine equivalents whether by the use of book-values or by Sraffa's more general method is to eliminate all the processes that use or produce old machines to one single process with inputs on one side and the principal product of that industry on the other thus replicating the simplicity of single-product industries. Accordingly the dynamic open price system can be written as

$$F^T(r)Pr + S^T Pr + A^T P + wL = P \tag{22}$$

where $F^T(r)$ is an *n x n* matrix containing the elements $f_{ji}\psi'_{ji}$ for cells that correspond to the use of fixed capital items and 0's in those cells that do not represent durable capital goods, where

$$\psi'_{ji} = \frac{(1+r)^{k_{ji}}}{(1+r)^{k_{ji}} - 1} \tag{23}$$

where k_{ji} is the life of the *j^th^* machine in the *i^th^* industry. The notation $F^T(r)$ has been used to suggest that the elements of F^T are functions of the rate of profit. It is reasonable to expect in real world production processes that the cells of the F^T matrix of the F^T matrix for which $f_{ji} > 0$ would be cells in which s_{ji}, $a_{ji} = 0$. However, a great majority of cells for which $s_{ji} > 0$ would also be cells for which $a_{ji} > 0$ except for non-storable inputs for which $a_{ji} > 0$ but s_{ji}, $f_{ji} = 0$. At $w = 0$ equation (22) gives the solution for the maximum rate of profit *R* which is the reciprocal of the dominant eigenvalue of the matrix $(I - A^T)^{-1}[F^T(r) + S^T]$. The prices in (22) are amenable to being reduced to dated labour terms in the usual way.

$$P = [I - (I - A^T)^{-1}(F^T(r)r + S^T r)]^{-1}(I - A^T)^{-1} wL \tag{24}$$

When production is carried out by means of working capital alone the long-run rates of gross and net profit on capital stock are one and the same. This is not true when fixed capital is used. The rates of gross profit on total capital will differ between industries depending upon the use of fixed capital in relation to working capital even when the rate of net profit is equalized across industries. Thus,

$$\begin{aligned} r_i &= \frac{(F^T(r)Pr + S^T r)P_i}{(F^T P + S^T P)_i} \\ &= \left[\frac{(P - A^T P - wL)_i}{(A^T P + wL)_i}\right]\left[\frac{(A^T P + wL)_i}{(F^T P + S^T P)_i}\right] \\ &= m_i T_i \end{aligned} \tag{25}$$

Where m_i is the on-cost gross profit markup and T_i is the assets turnover ratio and the subscript i on the right hand side denotes the i^{th} component of vectors representing the i^{th} industry.

The dual of the price system (22) can be expressed as,

$$F(g)Xg + SXg + AX + C = X \tag{26}$$

where the elements of F are $F_{ij}\Omega_{ij}$ or 0 depending on whether i is used for more than one year or not and

$$\Omega_{ij} = \frac{g(1+g)^{k_{ji}}}{(1+g)^{k_{ji}} - 1} \tag{27}$$

For C=0 equation (26) solves for the maximum rate of growth *G* which is the reciprocal of the dominant eigenvalue of the matrix $(I-A)^{-1}\,[F(g)+S]$ which must equal R. In a state of balanced growth the output of new machines is given by

$$x_i = \Sigma f_{ij}\, x_j \Omega_{ij} \qquad i = 1 \ldots F \tag{28}$$

and in each industry there will prevail an age distribution of machines satisfying,

$$f_{ij} = x_{ij} + \frac{x_{ij}}{1+g} + \frac{x_{ij}}{(1+g)^2} + \ldots + \frac{x_{ij}}{(1+g)^{k_{ji}-1}} \tag{29}$$

In expression (29) x_{ij} represents the number of new machines and the successive terms the number of one-year older machines. The stock of machines that will be used in the following year shall be $(1+g)f_{ij}$ in industry i. Every year the machines of age $k_{ij}-1$ will be retired and gf_{ij} new machines will be added so that the number of machines in each group will increase by the factor $1+g$ as compared to the earlier period. To obtain a numerical idea suppose the growth rate is 10%. Suppose that is in an industry the stock of machines in use are 3.63 new machines, 3.3 one-year old machines and 3 two-year old machines, i.e. 9.93 machines is all. Then $\Omega = g(1+g)^h/(1+g)^{h-1} - 1 = 0.40211$ and 9.93 Ω = 3.993 = (1.1) (3.63). In the following year the stock of machines in use shall be 3.993 new machines, 3.63 one-year old machines and 3.3 two-year old machines i.e. a total of 10.923 machines. Symmetrically with equation (25) it is now possible to devise a formula for the rate of growth,

$$\begin{aligned} g &= \frac{(F(g)X + SX)_i}{(FX + SX)_i} \\ &= \left[\frac{(X - AX - C)_i}{(AX + C)_i}\right]\left[\frac{(AX + C)_i}{(FX + SX)_i}\right] \\ &= s_i\,\tau_i \end{aligned} \tag{30}$$

where s_i is the ratio of the physical surplus of commodity i produced over and above the quantity of it that is used up during the year (as inputs in industries and for final consumption) and τ_i is the ratio of the quantity of i used up during the year to the total stock carried through the year. These ratios are the physical counterparts of the markup factors and asset turnover rates appearing in price equations (25).

The formula (30) applies to intermediate goods part of which are used up and part of which are ploughed back. And if i represent a durable machine then the quantity of it retired in the last year of its life does not appear in the expression for the quantity used up because it stands replaced by machines of the previous age. If the commodity *i* is it is obviously not carried as a stock – it is only used up and its gross output equals its intermediate a non-storable good use so that $s_i = 0$. Its production and its use grow at the rate g over time.

7. Notes on Fixed Capital Systems

The technique of reducing all old machines to their new machine equivalents by applying to them their book values allows the system to handle the problems of joint utilization and transferability of any number of machines in any number of industries. For example, if trucks are used half the time to transport bread and the remaining time to transport medicines then 1/2 $F_{ji}\psi_{ji}p_j$ would be the annual value charged in the bread and pharmaceutical industries respectively. Also old machines can be traded and transferred between industries. If a uniform rate of profit prevails the book-values should be the market prices otherwise either the buyer or the seller would stand to lose by trading it any other price. If the rate of profit is not uniform, the demand prices for machines that industries enjoying a higher rate of profit would be willing to pay stand at a higher level than the book-values ascribed to them by low profit industries and there is scope for mutually profitable trade. The industry that buys the old machine would of course apply to it the annuity charge for its remaining life calculated at the higher rate of profit that it is enjoying.

Machines that work with changing efficiency over their lifetimes pose a problem. For instance if a machine works with decreasing efficiency over its life then the technical coefficient corresponding to its use must rise over time. This has somewhat uneasy implications. If the technical coefficient rises over time so does the cost of production and therefore the price. Surely this is counterintuitive and counterfactual. For instance if there are two firms producing an identical product but one firm is using older machines whose efficiency has fallen with age than the other then the price it charges would be higher and it would be competed out of the market. And the opposite would hold if the machines work with increasing efficiency over their life. Surely this does not happen. A method is required to 'average' out the costs over the life time of the machine so as to yield a uniform price of the product. Sraffa (1960) proposed the detailed joint-products approach for a treatment of this case. However, the annuity method too can be suitably adapted for application. The method for doing so is as follows. As an example consider a machine that has a life of 3 years. It works with full efficiency when it is brand new, but produces 80% of the output when it is 1 year old and 75% when it is 2 years old. Then the production equations for the three processes are

$$(p_{f0})(1+r) + (\sum A_{j1}p_j)(1+r) + wL_1 = p_1B_{10} + p_{f1}$$

$$(p_{f1})(1+r) + (\sum A_{j1}p_j)(1+r) + wL_1 = p_1B_{10}\left(\frac{4}{5}\right) + p_{f2}$$

$$(p_{f2})(1+r) + (\sum A_{j1}p_j)(1+r) + wL_1 = p_1B_{10}\left(\frac{3}{4}\right) + 0$$

Using the book values from (19) to eliminate the old machines and adding up the equations would give

$$\left(\frac{4}{3}+\frac{5}{4}+1\right)\psi p_{f0}+\left(\frac{4}{3}+\frac{5}{4}+1\right)\sum A_{ji}\,p_j(1+r)+\left(\frac{4}{3}+\frac{5}{4}+1\right)\ wL_1=(p_1B_{10}) \qquad (31)$$

i.e. $(1.1944)\psi p_{f0}+(1.1944)\sum A_{j1}\,p_j(1+r)+1.1944wL_1=p_1B_{10}$

In this manner all the individual processes belonging to different ages of the machines can be reduced to a single average process over the lifetime of the machine, and all firms belonging to an industry and producing an identical product would charge a uniform price irrespective of the age distribution of machines in the particular firms.

When several types of machines of differing ages and therefore differing efficiencies work side by side the situation becomes complicated. However, if it is supposed that there exists a method for ascertaining the output that is realisable for every machine-age-efficiency combination in relation to the output that is realized in some ideal situation, for example, when all the machines used are brand new, then the method of equations (21) and (31) can be jointly used to arrive at the viable solution.

8. Empirical Considerations

Equation (23) which specifies the long run relationship between the rate of profit earned on total assets, the on-cost markup rate and the rate of assets turnover is an idealization based on the assumptions that total assets consist only of inventories and fixed capital items and that these are acquired exclusively by means of equity capital. In reality industrial firms carry a variety of assets other than inventories and plant & machinery including bank balances, receivables, advance payments, financial investments, etc. The assets are acquired by means of debts from several sources carrying different interest rates across different maturities besides equity capital. So the idea of profit earned only on inventories and fixed assets that appears in the idealized formulae ceases to have a tangible empirical counterpart. It should be replaced with gross profit that is to say, profit before interest, depreciation and income tax earned on total assets and net profit (i.e. profit after organizational overheads, interest, depreciation and income tax) earned on owners' capital or net worth. The gross profit markup rate would then be defined as

$$m_i=\frac{Net\ Sales}{Cost\ of\ Goods\ Sold}-1 \qquad (32)$$

where net sales denotes sales less indirect taxes and the total assets turnover rate would be defined as

$$T_i=\frac{Cost\ of\ Goods\ Sold}{Total\ Assets} \qquad (33)$$

In the circumstances there is little point in expecting that the product m_i and T_i estimated from balance sheet data would tend to be equal to a uniform scalar across all industries because different industries will operate with different asset and liability compositions.

Nevertheless if the industries and the firms belonging to them are to be regarded as being competitive (albeit to varying extents) it is to be minimally expected that the relationship between m_i and T_i is inverse. In other words, increases in sales without proportionate increases in the assets are achievable at least in part by reductions in prices brought about by reductions in the markup rates. And vice versa, price reductions brought about by reductions in markup rates allow sales to increase without requiring proportionate increases in total assets. For the same reasons it is to be expected that the net profit margin n_i be inversely related to the net worth turnover ratio. In what follows some empirical results on these relationships have been reported based on data for Indian corporates covering the 10 - year period from 2005-2015 across 15 industry groups drawn from CMIE's Prowess Database. Table 1 gives an idea of the basic data on the magnitudes of the variables and Tables 2 and 3 present the summary results of the following regressions across companies *i* belonging to industry *j*.

$$\log m_{ij} = a_j + b_j \log T_{ij} \tag{34}$$

$$\log n_{ij} = \alpha_j + \beta_j \log W_{ij} \tag{35}$$

where m_{ij} and T_{ij} are the gross profit markup rate and total assets turnover rate of company *i* belonging to industry *j* and n_{ij} and W_{ij} are the net profit margins and net worth turnover rates respectively. Considering that the Leontief-Sraffa systems pertain to the long run the regressions have been carried out on successively cumulated data across the time period 2005-2015.

The notions of "short" and "long runs" pertain to logical rather than historical time. Every historical slice of time belongs at once to the short, medium and long runs depending upon the degree of variabilities of the factors of production and presence and magnitudes of disturbances of all kinds. Even a very short slice of historical time can be said to belong to the long run if firms have complete control over their schedules for purchases, production and sales. Of course it always more likely that over longer periods of historical time the limitationalities and disturbances that loom large in their impact in short periods would have a muted influence. This consideration has a special force when the object is to study the behavior of mutually interacting industries. A spell of bad weather may affect one group of industries but not others directly. The others will be affected to varying extents in the course of their interactions with the directly affected industries. But during this time the performance of all the industries can hardly be compared. And waiting or a searching for a more propitious interval of time may be futile – it may never come. Cumulation over long periods of time ensures that disturbances affecting industries in each short period are subsumed under the weight of the data belonging to other periods. Longer runs are therefore understood to mean longer *units* of time over which the behavior of the variables is observed; a year instead of a quarter, a decade instead of a year, and so on.

Table 1 gives an idea of the great observed variabilities in the data relating to $m_{ij},\ T_{ij},\ n_{ij},\ and\ W_{ij}$ for 15 industry groups during the period 2005-2015.

Table 1

		Gross profit margin	Asset turnover ratio	Net profit margin	Networth turnover ratio
Manufacturing	min	0.00014	0.00004	0.00003	0.00105
	max	2482.19444	51.86364	105.20968	21442
Retail Trade	min	0.02239	0.34236	0.00197	0.69269
	max	1.70078	6.68585	0.17345	49.25490
Transport Services	min	0.00058	0.00044	0.00011	0.01473
	max	24.00000	7.90436	8.04124	96.33217
Food Product	min	0.00415	0.00248	0.00006	0.02317
	max	8.34211	19.07697	2.95870	3745.43750
IT	min	0.00051	0.00100	0.00007	0.00481
	max	79.71429	125.73564	16.11765	9354.14286
Metal and Metal Products	min	0.00014	0.00106	0.00010	0.00288
	max	19.00000	9.78698	26.52235	21442
Mining	min	0.00498	0.00013	0.00044	0.00074
	max	7.89744	3.94615	1.47070	116.67857
Real Estate	min	0.00044	0.00016	0.00022	0.00252
	max	2693.00000	4.99087	6.15248	1082.57143
Telecom	min	0.03926	0.00021	0.00168	0.00027
	max	7.72620	2.77932	1.34845	455.12500
Cement	min	0.03498	0.10571	0.00074	0.14064
	max	1.73299	1.72403	0.29603	228.69560
Communication Services	min	0.03926	0.00021	0.00168	0.00027
	max	5.00000	2.77932	1.34845	455.12500
Drugs and Pharma	min	0.00331	0.02907	0.00039	0.05067
	max	2.84716	9.22887	1.22353	101
Consumer Goods	min	0.00082	0.00664	0.00005	0.01493
	max	2.06306	22.47231	3.25000	372.50647
Chemical	min	0.00247	0.00384	0.00007	0.02734
	max	2.84716	9.22887	9.25000	1129.42857
Automobile	min	0.00058	0.00044	0.00011	0.01473
	max	24.00000	7.90436	8.04124	96.33217

Tables 2 and 3 present the summary results of the regressions (34) and (35) respectively. The size of the slope estimate for regression (34) under the idealized condition of a uniform rate of profit for all firms and industries is supposed to be – 1. But surely we cannot expect that to hold for the sample data for several reasons. Firstly, the data for the regressions pertains to listed companies many of which are diversified but have been classified into an industry group depending upon the proportion of sales revenue earned by its dominant product. The industries do not contain firms that produce and sell an identical products as required by the theory. Secondly, some industries have very few firms, e.g. telecom, automobiles, information technology and communication services. Besides, these industries sell services in which there are substantial possibilities for product and price discrimination. Thirdly some industries like cement being homogenous oligopolies they are amenable to the formation of cartels. We should expect the slope coefficients for these industries to be lower than 1 in absolute value which is indeed so. On the other hand industries like manufacturing, transport, food, consumer goods, and chemicals, drugs and pharmaceuticals have large numbers of companies selling closely competing products. Their slope coefficients are greater (in absolute value) than those of the former set of industries and are closer to 1. It has been customary to suppose that the rate of return on total assets (inventories and fixed capital) is uniform across industries in the long run. The data does not support this. The standard deviation of the return on total assets across industry groups ranges from 0.14 to 0.12 for short and long runs respectively. However the returns on net worth tend to move towards equality with a long-run standard deviation of 0.02.

Table 2

Gross Profit Margin and Total Assets Turnover

GPM on ATR	**Intercept**		**Slope**		**Rsq**		**N**	
Sector	**Min**	**Max**	**Min**	**Max**	**Min**	**Max**	**Min**	**Max**
Manufacturing	-1.616951	-1.276504	-0.81706	-0.603016	0.339972	0.394183	3638	4042
Retail Trade	-1.576365	-1.474346	-0.69886	-0.4897364	0.360207	0.584272	13	16
Transport Services	-1.596857	-1.347732	-0.76172	-0.6526926	0.494214	0.689609	145	198
Food Product	-1.689571	-1.630291	-0.73505	-0.6587441	0.548332	0.614594	384	426
IT	-0.918859	-0.592468	-0.44974	-0.3178418	0.146452	0.221971	280	356
Metal and Metal Products	-1.853344	-1.733277	-0.7541	-0.682106	0.400896	0.525226	501	542
Mining	-1.285816	-1.094703	-0.54715	-0.4487519	0.263941	0.363092	69	77
Real Estate	-2.60723	-1.916715	-0.82209	-0.6167367	0.362066	0.515086	106	175
Telecom	-0.930547	-0.530134	-0.3715	-0.0426844	0.006258	0.19482	29	41
Cement	-1.053325	-0.698485	-0.74956	-0.3324305	0.060423	0.463989	34	55
Communication Services	-0.932857	-0.734631	-0.41201	-0.121021	0.053343	0.230489	35	48
Drugs and Pharma	-1.327673	-1.26005	-0.71801	-0.4309376	0.120687	0.276722	230	242
Consumer Goods	-1.729171	-1.685815	-0.68998	-0.5281386	0.182618	0.236672	204	227
Chemical	-1.517183	-1.471212	-0.63474	-0.5307254	0.241971	0.330285	780	875
Automobile	-2.004256	-0.772946	-0.33987	0.0493302	1.53E-06	0.09533	12	17

The results presented in Table 2 and 3 are summaries of the outputs of 150 cross-sectional regressions (across companies) for the 15 industry groups for each of 10 years' cumulated results, i.e. they are drawn from 150 estimates of the intercept, slope and other statistics. It can be readily observed that the parameter estimates of a, b, ∝, β are fairly range bound and are invariably of the correct sign excepting for those industries for which the number of observations (companies) is exceedingly small.

Table 3

Net Profit Margin and Net Worth Turnover

NPM on NWTR	Intercept		Slope		Rsq		N	
Sector	**Min**	**Max**	**Min**	**Max**	**Min**	**Max**	**Min**	**Max**
Manufacturing	-2.708323	-2.177922	-0.86435	-0.6035544	0.216081	0.39947	3638	4042
Retail Trade	-3.51179	-3.005229	-0.77071	-0.2964146	0.081359	0.545832	13	16
Transport Services	-2.403491	-2.122042	-0.89029	-0.7279843	0.455834	0.59896	145	198
Food Product	-2.816319	-2.602121	-0.85955	-0.672939	0.337509	0.418123	384	426
IT	-2.605485	-2.256721	-0.41592	-0.2819246	0.09966	0.183398	280	356
Metal and Metal Products	-2.730373	-2.401974	-0.76663	-0.6478191	0.252093	0.362692	501	542
Mining	-2.581373	-2.398414	-0.55909	-0.3539264	0.11183	0.28908	69	77
Real Estate	-2.397184	-1.901338	-0.50639	-0.3414478	0.204362	0.306597	106	175
Telecom	-2.268705	-1.874936	-0.55451	-0.1361961	0.013254	0.574596	29	41
Cement	-3.002117	-1.709412	-0.66611	0.2487805	0.011682	0.38818	34	55
Communication Services	-2.351086	-1.935371	-0.70947	-0.3038221	0.087299	0.631986	35	48
Drugs and Pharma	-2.610575	-2.43329	-0.62669	-0.4938478	0.11269	0.215099	230	242
Consumer Goods	-2.863938	-2.647172	-0.71811	-0.5619491	0.19787	0.346716	204	227
Chemical	-2.727576	-2.531272	-0.6832	-0.5566653	0.154529	0.236672	780	875
Automobile	-3.374743	-0.700485	-1.57454	-0.1976879	0.003639	0.708838	12	17

In the results reported in Table 2 (equation 34) all the intercept terms are found significantly different from zero (at 5% level of significance) excepting 26 cases in which they are insignificant. All the slope coefficients have the expected negative sign and are significant excepting one instance, viz. automobiles for the period 2005-15. In all cases where the coefficients are significant the t values exceed 2.25 and the F values exceed 4.5. The results in Table 3 (equation 35) follows a broadly similar pattern. Insignificant intercepts occur in 18 out of 150 estimates (F values also insignificant) and the slope coefficients are insignificant in only two instances, viz. automobiles for the period 2005-15 and cement in 2005-09. The industries that are found to "misbehave" between the two tables are common; retail trade, telecom, communication services, cement and automobiles. It is at once apparent from the tables that these very industries have a much smaller number of (listed) companies than the other industries. Further, the cement and telecom industries are homogenous oligopolies having cartels and the automobile industry is a heterogeneous oligopoly. Consequently, the magnitudes of their slope coefficients are seen to be lower than for industries with numerous companies; they can achieve higher asset and net worth turnover rates without as much downward pressure on their margins than the more competitive industries having larger numbers of companies. In effect the size of the slope parameter serves as measure of the degree of monopoly of the industry – the lower it is (in absolute value) the greater the degree of monopoly.

9. Static Representation of the Dynamic System

The remarkable empirical relationship between markup rates, asset-turnover rates and the return on total assets, while it does not actually solve the problem of determining the distributive variables, suggests a method for considerably improving the empirical performance of the Leontief dynamic price system and providing a partial solution to the Sraffa problem even when data on capital coefficients (both stocks and fixed capital) are not available. Indeed the method goes further to automatically incorporate assets other than stocks and fixed capital items such as cash and financial and intangible assets and sources of other than owners' capital that are customarily omitted by input-output systems. The method is to estimate gross profit $(1 + m_i)$ for each industry using equation (34). For reasons earlier mentioned it is advisable to do so from cumulative balance sheet data over longish periods of time. The dynamic open price system that includes all types of assets can then be represented by a convenient static formulation,

$$\sum(a_{ji}p_j + wl_i)(1 + m_i) = p_i$$

which, in matrix notation is

$$A_m^T P + wL\,(1 + m_i) = P$$

having the solution

$$P = (I - A_m^T)^{-1} wL_m \qquad (36)$$

where A_m^T is the matrix containing elements $a_{ji}(1 + m_i)$ and L_m is the vector containing $l_i(1 + m_i)$. Viable solutions obtain only if $(I - A_m^T)$ satisfies the Hawkins-Simon conditions. If the objective is to estimate prices inclusive of indirect taxes the markup factors of equation (32) can be estimated using gross instead of net sales. The solution would more accurately approximate the empirically observed average market prices. Although it is not even remotely adequate from the theoretical viewpoint, equation (36) provides a practical empirical method for the determination of the real wage rate and relative prices in terms of any desired commodity or basket of commodities. Moreover the price system (36) incorporates imperfect competition and makes it possible to study its implications for the real wage. The corresponding dual system for the determination of outputs would be derived from equation (30),

$$\sum(a_{ij}x_i + C_i)(1 + s_i) = x_i$$

so that

$$X = (I - A_s)^{-1} \ C_s \tag{37}$$

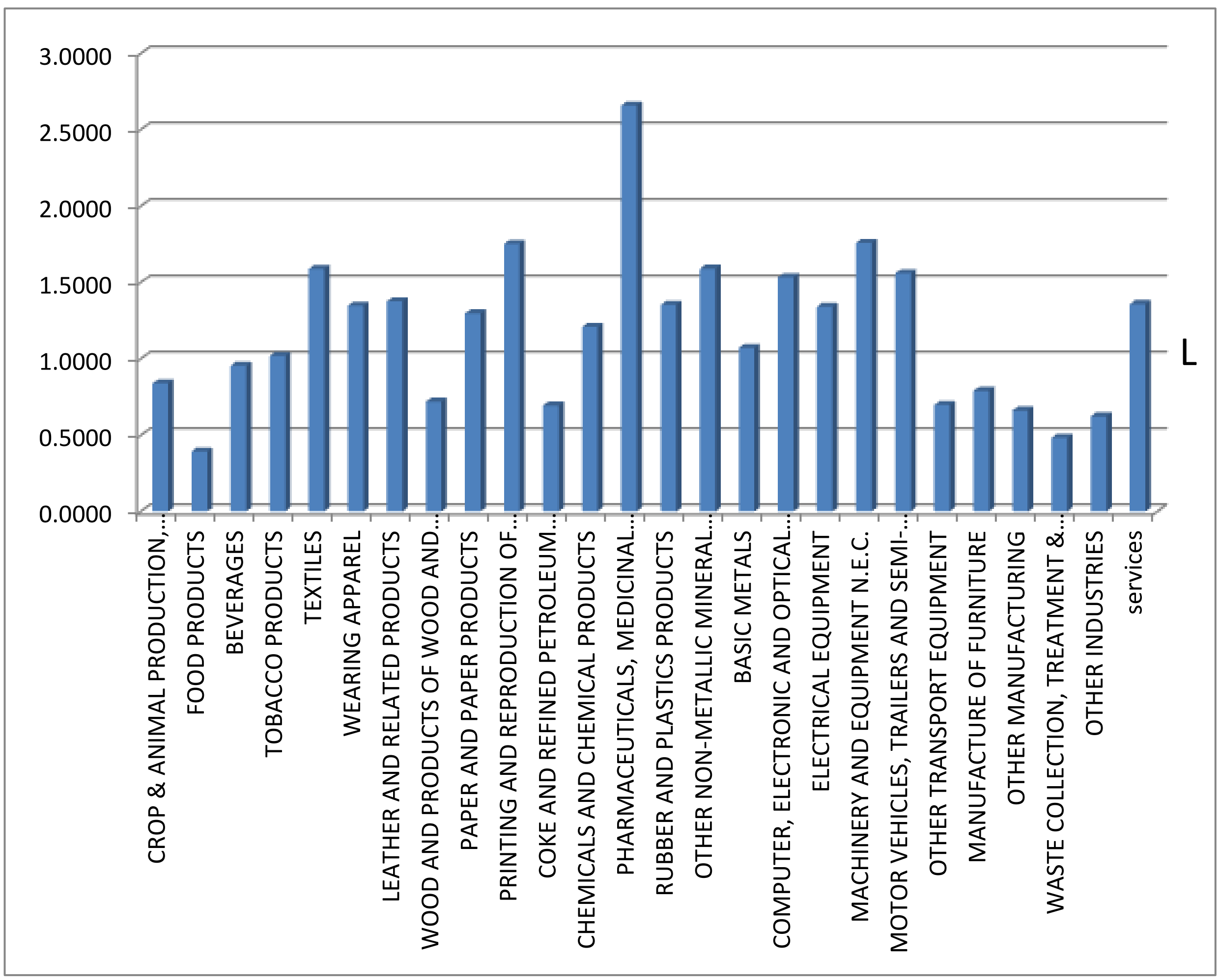
Where A_s has as elements $a_{ij}(1 + s_i)$ and C_s has as elements $C_i(1 + s_i)$. Unfortunately there is no separate data base for estimating s_i other than the dynamic input-output data base itself.

The graph below gives an idea of the results that obtain if prices are estimated from equation (36). These have been compared with the usual method of solving prices from input-output matrices,

$$P = (I - A^T)^{-1} V \tag{38}$$

Where V is the vector of value-added per unit of output. These have been normalized to unity by a change of units. The horizontal L line shows the normalized Leontief prices in the graph. The A^T matrix has been estimated by the CSO for India for the year 2007-08. The markups are estimated from the data reported in the Annual Survey of Industries (2010-2015) and are available for 24 industry groups. Accordingly the 130 x 130 matrix has been aggregated into a 24 x 24 commodity x commodity 1-0 matrix.

Cost plus prices compared to normalized Leontief prices 2009-10



It is evident from the graph that there are marked departures between the prices estimated by the two methods. Specifically, cost plus prices reflect the marked influence of inventories and fixed capital and the manner in which the markups are charged in various industries.

10. Concluding Remarks

This paper began by seeking a generalization of the Sraffa system to incorporate continuous industrial production. The generalization turns out to be formally identical with Leontief's dynamic open price system. Due to the identity between the generalized Sraffa system and Leontief's dynamic system all the essential properties of the Sraffa system such as possibility of reswitching of the techniques, possibility of constructing an invariable measure of value, impossibility of aggregating periods of production to measure capital independently of distribution and prices, etc. are seen to belong to the Leontief model as well, even though they have never been discussed in the context of that model. The Sraffa and Leontief systems stand unified. Further generalization to incorporate fixed capital items has also been made. The resulting system gives a better description of the technology in terms of the usual input-output and labour coefficients as well as stock and fixed-asset turnover ratios. In so doing a more realistic description of the pricing of industrially produced commodities is obtained in terms of the role played by the turnover ratios in determining the on-cost gross and net profit markup rates. Empirical examination of the relation between gross profit markups and the assets turnover rates even when assets other than physical stocks and machinery are included is found to give fairly robust results. These empirical relations have been used to formulate a static version of the Leontief-Sraffa dynamic price system that represents a direct application of the widely prevalent cost-plus method that is used for industrial pricing. This price system automatically incorporates imperfectly competitive industries and permits an analysis of their consequences on the economic outcomes. As an illustration this system has been used to estimate prices based on the Indian input-output matrix and contrast it with the prices obtained by the usual method. The marked difference in prices estimated by the respective methods deserves further attention.

NOTES

1. A caveat is in order. The relevant relationship is not equation (30). Instead it is the one known as the Du Pont System in the literature on strategic financial management;

$$RONW = \left[\frac{EBIT - Interest}{EBIT}\right]\left[1 - \frac{Tax}{EBIT - Interest}\right]\left[\frac{EBIT}{Sales}\right]\left[\frac{Sales}{Assets}\right]\left[\frac{Assets}{NetWorth}\right]$$

   Where RONW is the rate of return on owners' net worth, and EBIT is earnings/profit before interest and taxes. Total Assets includes cash balances and other assets not only inventories and fixed capital considered in the Leontief-Sraffa models. The idea is that owners' return is a product of the operating efficiency (profit margin), asset use efficiency (asset turnover ratio), financial leverage (debt to equity ratio) and efficiency in tax management (tax/taxable profit ratio). This last efficiency depends also upon allowable depreciation rates and tax shelters in different industries. So far as the former concerned in India two sets of depreciation rates are applicable, i) that specified in the Income Tax Act for the computation of taxable profit and ii) that specified by the Companies Act for the computation of distributable profit; the rates allowable under (ii) are lower than those under (i).

2. The inverse relationship between the on-cost markup rate and the capital turnover ratio has been completely ignored in standard price theory. For perfectly competitive firms the markup rate is not determined at all but is tactily supposed to be such as to enable firms to earn "normal competitive profits" whose sizes are not clearly specified. For imperfectly competitive firms the profit maximizing prices are determined as $\mathrm{p_i} = u_{\mathrm{i}}(1 + \mathrm{m_i})$ where u_i is the marginal cost and the "degree of monopoly" $1 + \mathrm{m_i} = \mathrm{e_i}/(\mathrm{e_i} - 1)$ where $\mathrm{e_i}$ the elasticity of demand,. The capital turnover ratio is not allowed to play any role in determining the markup rate.

3. Long-run values are not merely averages over a succession of short-runs. Consider an example. A certain variable, say Net Sales of a firm rises from 10 to 25 in two years. The average annual rate of growth over the two-year period is 58.11%. In reality the paths by which the level of 25 was reached from the initial value of 10 may have been 10-24-25 or 10-5-25 having annual growth rates 140% and 4.166% in the first and second years for the first path and -50% and 400% for the second path with their annual averages being 72.05% and 175% respectively. None of these averages can be said to represent the long-term average. The point is that the annualized growth rate of 58.11% over a two-year time unit is arrived at by ignoring the annual growth rates. This is the sense in which the terms long and short runs have been employed.

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