Multisectoral Aggregate Supply- Aggregate Demand Analysis

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Abstract

Aggregate supply-aggregate demand (AS-AD) analysis is the core analytic approach in modern macroeconomics. conventional AS-AD analysis has two features, one is that its analytical indexes are aggregate, such as gross domestic product (GDP), aggregate consumption, total export and import, total price level, etc., and another is that intermediate demand is not considered. Compared with the conventional AS-AD analysis, the input-output analysis takes the sectoral structure of economic production and consumption as its frame, and forms a multisectoral analytical system through the links of intersectoral demand relations. This paper sets up a model system of AS-AD analysis based on the input-output structure, and it carefully builds the relations between final uses demand and primary inputs. The paper starts the analysis from a simple model with a single production sector to make clear about the demand and supply meanings of the elements in a input-output model. The generally multisectoral AS-AD analytical system is set up with a closed input-output table. The paper shows what information is needed to get the equilibrium gross output and price, and the differences from the AS-AD model in conventional macroeconomics.

Keywords: AS-AD analysis； input-output； multisectoral model

Leontief has ever said that input-output analysis (IOA) studies the quantitatively interdependent relations among various complicate economic activities based on the general equilibrium theory of neoclassical school[①]. Theoretically, general equilibrium theory takes every consumer, labourer, producer and good as its object of study, and this theory cannot be applied practically for a real economy. By macroeconomics, equilibrium analysis is also used, but all consumers, all labourers, all producers and all goods are aggregated into one quantity respectively as aggregate consumption, aggregate labour, aggregate production, and there is also a macro price level. IOA is more detailed than macroeconomics, but it is also practical. IOA aggregate many kinds of goods which have similar input structure of production into one sector, all producers are classified into a limited number of production sectors.

In conventional IOA, final uses are exogenous, which is not like the way to do AS-AD analysis in mainstream economics. We have set up a system of AS-AD analysis in single sector with IOA frame[②] and applied it in Chinese economy[③]. This paper will discuss a way to set up

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① Leontief W. Input-Output Economics, New York: Oxford University Press, 1966

② Liu X. A New Model for AS-AD Analysis Based on Input-Output Frame . Modern Economy, 2011 , 2 (3) :203-212

③ Wang D, Song H, Liu X. The AS-AD Approach of China Economy Based on Input-Output Structure. Journal of Quantitative and Technical Economics Research, 2012 (6) :152-160 .

multisectoral AS-AD analysis system with IOA frame.

1. The AS-AD analysis characteristics of conventional IOA

conventional IOA is an open model system which takes final use as an exogenous variable, so it is more similar to Keynesian economics.

1.1 Observation based on a single sector model of IOA

The economic meaning can be seen more clearly with very simple model. Suppose there is an economy which consists of one production sector and one consumption sector without international trade and investment activity. In one accounting period, its gross output is q, consumption in production is x, consumption by residents is c, economic value added *i.e.* GDP is z, and the price level is p, then there are accounting balance equations:

$$x + c = q \tag{1}$$

$$px + z = pq \tag{2}$$

The variables in Eq. (1) are real quantities, and z in Eq.(2) is nominal.

From the perspective of accounting, $z = pc$ is identical equation, but from the perspective of economic analysis, it is possible that the value of resident consumption is not equal to the income because the distribution of income and the consumption of residents are carried out separately, such that disequilibrium may appear.

By macroeconomics, q in Eq. (1) should be aggregate demand, note it as q^d, q in Eq. (2) should be aggregate supply, note it as q^s, then Eq. (1) and (2) become

$$x + c = q^d \tag{3}$$

$$px + z = pq^s \tag{4}$$

Suppose the nominal value of consumption c is a function of income, *i.e.* $pc = f(z)$. When the economy is in equilibrium, there should be

$$q^s = q^s = q\,, \quad z = pc = f(z) \tag{5}$$

From Eq. (5), equilibrium z can be solved. Let a be the intermediate consumption per unit of output in production, then

$$q = \frac{z}{p(1-a)} \tag{6}$$

It is evident that price variable p can not be got from those equations above. Actually, price is only the money present of income, and it will give different price level if different unit of money is used, as is decided by money offer.

1.2 Multisectoral AS-AD model of conventional IOA

A simple conventional IO table is shown as following

$$\begin{pmatrix} X & Y \\ Z & 0 \end{pmatrix} \begin{matrix} Q \\ \widetilde{Z} \end{matrix} \tag{7}$$

where X is the intermediate consumption matrix of an economy, Y is a column vector of the final use, Z is a row vector of added values, $\widetilde{Z}$ equals the sum of Z 's elements *i.e.* GDP, and

Q is a column vector of gross outputs. X , Y and Q are measured in real quantity, and Z is measured in nominal quantity.

Defining direct input coefficient matrix as[4] $A = X\hat{Q}^{-1}$, By perspective of economics, aggregate demand of multisectoral system is

$$AD = Q^d = Xe + Y = AQ^s + Y \tag{8}$$

where e is a column vector with unitary elements, Q^s is the supply column vector. when it is equilibrium, $Q^s = Q^d = Q$, such that

$$Q = (1 - A)^{-1} Y \tag{9}$$

Eq.(9) is the solution of conventional IOA, where Number "1" representative unit matrix. It can be seen that it is similar with classical Keynesian theory which says the equilibrium quantity is decided by demand and the demand is a function of income. But in IOA, Y is exogenous so that the problem of equilibrium is not solved thoroughly.

AS-AD analysis is a more general frame than classical Keynesian theory. It has not only aggregate demand function but also aggregate supply function, and equilibrium price can be solved, too.

Let row vector P is the price level, then it has

$$PA\hat{Q} + Z = P\hat{Q} \tag{10}$$

and when it is equilibrium, there is

$$\widetilde{Z} = Ze = PY \tag{11}$$

It is evident that the equilibrium point (P , Q) cannot be solved only with the information given by the above equations. In order to get (P , Q) , Either Y or Z should be known and the relations between them also should be set up, but there is not specific constraints for them.

In fact, the essence of economic equilibrium is proportional coordination. By defining $z = Z(\hat{P}\hat{Q})^{-1}$ which means the rate of value added on nominal gross output, then it can be got from Eq.(10) that

$$P(1 - A - \hat{z}) = 0 \tag{12}$$

If P is known, Eq.(12) gives

$$z = P(1 - A)\hat{P}^{-1} \tag{13}$$

However If z is known, the solution of P will not be unique because the matrix $(1 - A - \hat{z})$ is singular; the price of one sector must be given in advance, then the prices of the other sectors are determined, which means that the absolute price level of the economic system is not unique, and the relative price level is unique. This situation is similar to the scenario in single sector case

[4] Let r be a vector, then $\hat{r}$ is a diagonal matrix which has the elements of r as its main diagonal elements.

where the price level is determined by the monetary system. z and P represent the distribution system of income on production factors of the sectors. After the total price level is determined by monetary system, the price level of each sector is determined by the distribution system which depends on market system and national financial system.

Even if P is known, Q is still not possible to get by IO system without the determine of Y. Suppose the value structure of Y is α, then based on Eq.(11), there should be

$$Y = \hat{P}^{-1}(\hat{P}Y) = \hat{P}^{-1}\alpha\widetilde{Z} \tag{14}$$

Substitute Eq.(14) into Eq. (9), it has

$$Q = (1-A)^{-1}\hat{P}^{-1}\alpha\widetilde{Z} \tag{15}$$

And there is also that

$$\widetilde{Z} = PQ - PXe = P(1-A)Q \tag{16}$$

Substitute Eq.(16) into Eq.（15）, it has

$$[1-(1-A)^{-1}\hat{P}^{-1}\alpha P(1-A)]Q = 0 \tag{17}$$

Eq.(17) shows that the equilibrium gross output is still not unique when input structure A, final consumption structure α and distributional system P is given. Only was the gross output of one sector determined, the prices of the other sectors could be set. At this point, it is different from the single sector system discussed previously where equilibrium gross output is determined as only as price level is known . The reason is that we has not considered the market mechanism between Y and Z like Eq.(5). For multisectoral system, that kind of balance equation can only be made in aggregate level. When it is equilibrium, suppose an aggregate final demand function as

$$PY = f(\widetilde{Z}) = \widetilde{Z} \tag{18}$$

When $f(\widetilde{Z}) \neq \widetilde{Z}$, disequilibrium will appear. If Eq.(18) is hold, then equilibrium $\widetilde{Z}$ can be solved, and when P is solved by Eq.(12), equilibrium Q will be got from Eq.(15).

It can be seen that Eq.(18) is a simple Keynesian model, what the IOA contributes is to be able to calculate the intermediate consumption matrix and the sectoral structure of final consumption. Additionally, what a difference from general macroeconomics is that labour market is out of and the price variables is directly decided by input structure and money supply, which is nothing to do with the influence of demand.

2. The model with a closed IO table and without price variable

It is should noticed that a closed IO table is different from a closed economy. A closed IO table just means that the IO table is a square and full of numbers. In normal IO table, the fourth quadrant is empty.

Classical Keynesian model is easy because of fixed price level. In this section we hope to set up a closed equilibrium system similar to Keynesian model without price variable.

The normally practical IO table takes the shape as following:

$$\begin{pmatrix} X & C & G & F & NX \\ D & & & & \\ W & & & & \\ T & & & & \\ M & & & & \end{pmatrix} Q$$

In order to get a closed system, we need to fill the empty unit in the 4th quadrant, see the following

$$\begin{pmatrix} X & EX & C & G & F \\ X^I & 0 & C^I & G^I & F^I \\ W & W_o & 0 & W_G & 0 \\ T & T_o & T_C & 0 & 0 \\ M+D & EX_S & C_S & G_S & F_S \end{pmatrix} \begin{matrix} Q \\ IM \\ \widetilde{W} \\ \widetilde{T} \\ \widetilde{M} \end{matrix} \tag{19}$$

In Eq.(19), X —intermediate input matrix with domestic goods and services; EX —column vector of export; C —column vector of residential consumption with domestic goods and services; G —column vector of public consumption with domestic goods and services; F —column vector of capital formation with domestic goods and services; Q —column vector of gross output ; X^I —intermediate input row vector with import goods and services; C^I —residential consumption with import goods and services; G^I —public consumption with import goods and services; F^I —capital formation with import goods and services; IM —total import; W —column vector of labour wages; W_O —Labor remuneration and transfer payments that home residents get from abroad; W_G —transfer payments that home residents get from government; $\widetilde{W}$ —total income of residents ; T —row vector of net taxes on production; T_O —Taxes and fees obtained by the government from imported products（suppose that the taxes obtained by government are accounted in taxes on production）; T_C —taxes and fees government get from residents; $\widetilde{T}$ —total income of government; M —row vector of operational surplus; D —row vector of discount fund; EX_S —save by abroad which equals total import minus total export， and minus($W_o + T_o$); C_S —save by residents; G_S —public save which equals total income of government minus total public consumption, and minus W_G ; F_S =0，it is equivalent to the identity of national income accounts：total investment=total save，the equilibrium condition of an economy； $\widetilde{M}$ —the sum of the final line which is equivalent to total save.

Let

$$\tilde{X} = \begin{pmatrix} X & EX & C & G & F \\ X^I & 0 & C^I & G^I & F^I \\ W & W_o & 0 & W_G & 0 \\ T & T_o & T_C & 0 & 0 \\ M+D & EX_S & C_S & G_S & F_S \end{pmatrix}, \quad \tilde{Q} = \begin{pmatrix} Q \\ IM \\ \tilde{W} \\ \tilde{T} \\ \tilde{M} \end{pmatrix}$$

and $\tilde{A} = \tilde{X}\hat{\tilde{Q}}^{-1}$, then it has

$$(1-\tilde{A})\tilde{Q} = 0 \tag{20}$$

For the economy represented by Eq. (19), Eq. (20) is a closed model and also the general equilibrium condition. For a practical economy, $\tilde{Q} \neq 0$, so that $(1-\tilde{A})$ should be a singular matrix. If β is a solution of $\tilde{Q}$ in Eq.(20), then $k\beta$ must be a solution where $k \neq 0$ and a real number. This means that the equilibrium solution of the economy is not unique for a given $\tilde{A}$.

Let

$$X^* = \begin{pmatrix} X & EX & C & G \\ X^I & 0 & C^I & G^I \\ W & W_o & 0 & W_G \\ T & T_o & T_C & 0 \end{pmatrix}, \quad F^* = \begin{pmatrix} F \\ F^I \\ 0 \\ 0 \end{pmatrix}, \quad Q^* = \begin{pmatrix} Q \\ IM \\ \tilde{W} \\ \tilde{T} \end{pmatrix}, \quad M^* = (M+D \quad EX_S \quad C_S \quad G_S)$$

and $A^* = X^* \hat{Q}^{*-1}$, then it has

$$Q^* = (1-A^*)^{-1}F^* \tag{21}$$

Eq. (21) is given the name of partial closed model as partial final uses are endogenetic.

As[5]

$$M^* = (e' - e'A^*)\hat{Q}^* \text{ 或 } (M^*)' = \langle e' - e'A^* \rangle Q^* \tag{22}$$

Substitute Eq. (21) into Eq. (22), it has

$$(M^*)' = \langle e' - e'A^* \rangle (1-A^*)^{-1}F^* \tag{23}$$

According to Eq. (23), a closed IOA model makes a linear relation assumption between F^* and M^*. This relation still holds when all the final uses were aggregated into one column vector Y.

In order to solve Eq. (21), it is still necessary to have a determine mechanism of F^*.

3. A closed general equilibrium model including price

In a general AS-AD model system, price is a indispensable variable. In order to show the methodology clearly, abroad sector is omitted and thus Eq. (19) is changed into

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[5] If a vector expression such as WXF is long, then $\langle WXF \rangle$ represents its corresponding diagonal matrix.

$$\begin{pmatrix} X & C & G & F \\ W & 0 & W_G & 0 \\ T & T_C & 0 & 0 \\ M+D & C_S & G_S & F_S \end{pmatrix} \begin{matrix} Q \\ \widetilde{W} \\ \widetilde{T} \\ \widetilde{M} \end{matrix} \tag{24}$$

In Eq. (24), gross output Q and final uses $(C \quad G \quad F)$ are reckoned as real quantities, primary inputs $(W \quad T \quad M+D)$ are reckoned as nominal quantity; intermediate input coefficients are calculated with real quantity, and primary input coefficients are calculated with nominal quantity. Based on the above information, there are the following row balance equations,

$$Xe + (C + G + F) = Q \tag{25}$$

$$We + 0 + W_G + 0 = \widetilde{W} \tag{26}$$

$$Te + T_C + 0 + 0 = \widetilde{T} \tag{27}$$

$$(M + D)e + C_S + G_S + 0 = \widetilde{M} \tag{28}$$

The various input coefficients are defined as follows:

$$A = X\hat{Q}^{-1}, \quad A_w = W(\hat{P}\hat{Q})^{-1}, \quad A_T = T(\hat{P}\hat{Q})^{-1}, \quad A_M = (M + D)(\hat{P}\hat{Q})^{-1} \tag{29}$$

then there are

$$X = A\hat{Q}, \quad W = A_w(\hat{P}\hat{Q}) = P\hat{A}_w\hat{Q}, \quad T = A_T(\hat{P}\hat{Q}) = P\hat{A}_T\hat{Q}, \quad (M + D) = A_M(\hat{P}\hat{Q}) = P\hat{A}_M\hat{Q} \tag{30}$$

Let $\widetilde{C}$, $\widetilde{G}$ and $\widetilde{F}$ are respectively the total residential consumption, total public consumption and total capital information in nominal terms; c, g and f are respectively the nominally proportional structure of residential consumption, public consumption and capital information, then it has

$$\widetilde{C} = PC = c\widetilde{C}, \quad \widetilde{G} = PG = g\widetilde{G}, \quad \widetilde{F} = PF = f\widetilde{F} \tag{31}$$

such that

$$C = \hat{P}^{-1}c\widetilde{C}, \quad G = \hat{P}^{-1}g\widetilde{G}, \quad F = \hat{P}^{-1}f\widetilde{F} \tag{32}$$

The key of analyzing equilibrium is to set up the corresponding relations between incomes and demands for output, and it is to set up the relations of three final uses with three types of incomes for the economy of Eq.(24).

The total income of residents consists of two parts, one is labour wages and transfer payments *i.e.* $\widetilde{W}$, the other is property income which comes from corporate dividend *i.e.* M. The resident consumption demand is mainly decided by disposable income which is equal to total income minus taxes and fees payment. If let residential save rate is s, the distribution ratio of surplus to shareholders is vector θ, and the private share ratio of total capital is vector $¢$, then the nominally residential consumption demand will be

$$\widetilde{C}=(1-s)(\widetilde{W}+M\hat{\Phi}\theta-T_C) \tag{33}$$

Let the tax rate of Labor remuneration be t_w, the dividend tax rate be t_M, and there is not tax for government transfer payment, then

$$T_C=t_wWe+t_MM\hat{\Phi}\theta \tag{34}$$

and the resident consumption balance equation is

$$PC=(1-s)[(1-t_w)We+W_G+(1-t_M)M\hat{\Phi}\theta] \tag{35}$$

Suppose The total tax revenue of government is made up of production tax and resident income tax, then

$$\widetilde{T}=A_T'\hat{P}Q+t_wWe+t_MM\hat{\Phi}\theta=P'(\hat{A}_T+t_w\hat{A}_w)Q+t_MM\hat{\Phi}\theta \tag{36}$$

Suppose the total revenue of the government is made up of taxes and the dividends which come from state-owned capital, then the total public income is $\widetilde{T}+\theta(1-\hat{\Phi})M$. The total fiscal payment is $PG+W_G$. Because the modern finance is generally a deficit finance, if let deficit rate is h, the public finance balance condition is

$$PG+W_G=(1+h)(\widetilde{T}+M(1-\hat{\Phi})\theta) \tag{37}$$

If $h<0$, it means there is fiscal surplus.

The investment balance *i.e.* general equilibrium condition is

$$(M+D)e+C_S+G_S+0=PF=\widetilde{F} \tag{38}$$

where

$$M+D=P\hat{Q}-PX-W-A_T\hat{P}\hat{Q}=P(1-A-\hat{A}_w-\hat{A}_T)\hat{Q} \tag{39}$$

$$\begin{aligned}C_S&=\widetilde{W}-P'C-T_C\\&=(We+W_G)-(1-s)[(1-t_w)We+W_G+(1-t_M)M\hat{\Phi}\theta]-(t_wWe+t_MM\hat{\Phi}\theta)\\&=[1-(1-s)(1-t_w)-t_w]We-[(1-s)(1-t_M)+t_M]M\hat{\Phi}\theta+sW_G\end{aligned} \tag{40}$$

$$G_S=\widetilde{T}-P'G-W_G=-h\widetilde{T}-(1+h)M(1-\hat{\Phi})\theta \tag{41}$$ [⑥]

If let $D=\hat{\delta}\hat{P}\hat{Q}$ where δ is the ratio vector of the depreciation fund over gross output (it is different in different industries), then

$$\begin{aligned}M&=P(1-A-\hat{A}_w-\hat{A}_T)\hat{Q}-D=P(1-A-\hat{A}_w-\hat{A}_T)\hat{Q}-P\hat{\delta}\hat{Q}\\&=P(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{Q}\end{aligned} \tag{42}$$

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⑥ Because the dividend of state-owned capital as fiscal revenue does not appear in the IO table Eq.(24), G_s does not mean fiscal surplus.

Substitute Eq.(42) into Eq.(40), it has

$$\begin{aligned} C_S &= [1-(1-s)(1-t_w)-t_w]P\hat{A}_w Q - [(1-s)(1-t_M)+t_M]P(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{Q}\hat{\phi}\theta + sW_G \\ &= [1-(1-s)(1-t_w)-t_w]P\hat{A}_w Q - [(1-s)(1-t_M)+t_M]P(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{\phi}\hat{\theta}Q + sW_G \\ &= P\{[1-(1-s)(1-t_w)-t_w]\hat{A}_w - [(1-s)(1-t_M)+t_M](1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{\phi}\hat{\theta}\}Q + sW_G \end{aligned} \quad (43)$$

Substitute Eq.(36) and (42) into Eq.(41), it has

$$\begin{aligned} G_S &= -h\tilde{T} - (1+h)M(1-\hat{\phi})\theta \\ &= -h[P(\hat{A}_T+t_w\hat{A}_w)Q + t_M M\hat{\phi}\theta] - (1+h)M(1-\hat{\phi})\theta \\ &= -hP(\hat{A}_T+t_w\hat{A}_w)Q - P(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{Q}(ht_M\hat{\phi}\theta + (1+h)(1-\hat{\phi})\theta) \\ &= -P\{h(\hat{A}_T+t_w\hat{A}_w) + (1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})(ht_M\hat{\phi} + (1+h)(1-\hat{\phi}))\hat{\theta}\}Q \end{aligned} \quad (44)$$

By Eq.(37), it holds

$$\tilde{G} = PG = (1+h)(\tilde{T} + M(1-\hat{\phi})\theta) - W_G \quad (45)$$

Substitute Eq.(36) into (45), it has

$$\begin{aligned} \tilde{G} &= PG = (1+h)[P(\hat{A}_T+t_w\hat{A}_w)Q + t_M M\hat{\phi}\theta + M(1-\hat{\phi})\theta] - W_G \\ &= (1+h)\{P(\hat{A}_T+t_w\hat{A}_w)Q + M[t_M\hat{\phi} + (1-\hat{\phi})]\theta\} - W_G \end{aligned} \quad (46)$$

Substitute Eq(42) into Eq. (46), it has

$$\begin{aligned} \tilde{G} &= (1+h)\{P(\hat{A}_T+t_w\hat{A}_w)Q + P(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{Q}[t_M\hat{\phi} + (1-\hat{\phi})]\theta\} - W_G \\ &= (1+h)P\{(\hat{A}_T+t_w\hat{A}_w) + (1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})[t_M\hat{\phi} + (1-\hat{\phi})]\hat{\theta}\}Q - W_G \end{aligned} \quad (46)$$

Let

$$H_G = (\hat{A}_T+t_w\hat{A}_w) + (1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})[t_M\hat{\phi} + (1-\hat{\phi})]\hat{\theta} \quad (47)$$

then

$$\tilde{G} = (1+h)P'H_G Q - W_G \quad (48)$$

Substitute the definition of A_w and Eq.(42) into Eq.(35), it has

$$\begin{aligned} \tilde{C} &= PC = (1-s)[(1-t_w)A_w\hat{P}Q + W_G + (1-t_M)P(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{Q}\hat{\phi}\theta] \\ &= (1-s)[(1-t_w)P\hat{A}_w Q + (1-t_M)P'(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{\phi}\hat{\theta}Q] + (1-s)W_G \\ &= (1-s)P[(1-t_w)\hat{A}_w + (1-t_M)(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{\phi}\hat{\theta}]Q + (1-s)W_G \end{aligned} \quad (49)$$

Let

$$H_C = (1-t_w)\hat{A}_w + (1-t_M)(1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{\phi}\hat{\theta} \quad (50)$$

then

$$\tilde{C} = (1-s)P'H_C Q + (1-s)W_G \quad (51)$$

Substitute Eq.(39), (43) and (44) into Eq.(38), it has

$$\begin{aligned} \tilde{F} &= (M+D)e + C_S + G_S + 0 \\ &= P(1-A-\hat{A}_w-\hat{A}_T)Q + P\{[1-(1-s)(1-t_w)-t_w]\hat{A}_w - [(1-s)(1-t_M)+t_M](1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{\phi}\hat{\theta}\}Q + sW_G \\ &\qquad - P\{h(\hat{A}_T+t_w\hat{A}_w) + (1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})[ht_M\hat{\phi} + (1+h)(1-\hat{\phi})]\}\hat{\theta}Q \\ &= P\{(1-A-\hat{A}_w-\hat{A}_T) + [1-(1-s)(1-t_w)-t_w]\hat{A}_w - [(1-s)(1-t_M)+t_M](1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})\hat{\phi}\theta \\ &\qquad - [h(\hat{A}_T+t_w\hat{A}_w) + (1-A-\hat{A}_w-\hat{A}_T-\hat{\delta})[ht_M\hat{\phi} + (1+h)(1-\hat{\phi})]]\hat{\theta}\}Q + sW_G \end{aligned}$$

$$(52)$$

Let
$$H_F = (1 - A - \hat{A}_w - \hat{A}_T) + [1 - (1-s)(1-t_w) - t_w]\hat{A}_w - [(1-s)(1-t_M) + t_M](1 - A - \hat{A}_w - \hat{A}_T - \hat{\delta})\hat{\Phi}\hat{\theta} \\ - [h(\hat{A}_T + t_w\hat{A}_w) + (1 - A - \hat{A}_w - \hat{A}_T - \hat{\delta})[ht_M\hat{\Phi} + (1+h)(1-\hat{\Phi})]]\hat{\theta} \tag{53}$$

then
$$\widetilde{F} = PH_FQ + sW_G \tag{54}$$

Substitute the definition express of A and Eq.(32) into Eq.(25) , it has

$$(1 - A)Q = \hat{P}^{-1}(c\widetilde{C} + g\widetilde{G} + f\widetilde{F}) \tag{55}$$

Substitute Eq.(48), (51) and (54) into Eq.(55), it has

$$(1 - A)Q = \hat{P}^{-1}\{c[(1-s)PH_CQ + (1-s)W_G] + g[(1+h)PH_GQ - W_G] + f[P'H_FQ + sW_G]\} \tag{56}$$

i.e.

$$\{\hat{P}(1-A) - c(1-s)PH_C - g(1+h)PH_G - fPH_F\}Q = c(1-s)W_G - gW_G + fsW_G = [c(1-s) - g + fs]W_G \tag{57}$$

It is necessary to know W_G (fiscal transfers) and price level P in order to get equilibrium quantity Q.

(1) Determining of W_G

By Eq.(37), there is a balance relation among public consumption PG, fiscal transfers W_G and deficit rate h. If PG is the function of GDP and h is exogenous by government control, then W_G can be calculated by Eq.(37). If h and W_G are decided by government, then PG will be endogenous and can be calculated.

(2) Determining of P

For the system Eq. (24), its column balance relation is

$$e\hat{P}X + W + T + M + D = P\hat{Q} \tag{58}$$

Substitute Eq.(30) into Eq.(58), it has

$$PA\hat{P}^{-1} + A_w + A_T + A_M = e \tag{59}$$

then it holds

$$P(1 - A - \hat{A}_w - \hat{A}_T - \hat{A}_M) = 0 \tag{60}$$

It is easy to see that the equilibrium P determined by Eq.(60) is relative, and the determining of the absolute value of P needs some special mechanism on A_w, A_T or A_M. For example, let wage rate be vector w, total labour employed be vector L and the employment coefficient of labour be vector l, then

$$P\hat{A}_w = P\hat{W}(\hat{P}\hat{Q})^{-1} = P\hat{W}\hat{Q}^{-1}\hat{P}^{-1} = P\hat{w}\hat{L}\hat{Q}^{-1}\hat{P}^{-1} = P\hat{w}\hat{l}\hat{P}^{-1} = w\hat{l} \tag{61}$$

Substitute Eq.(61) into Eq. (60), then

$$P = w\hat{l}(1 - A - \hat{A}_T - \hat{A}_M)^{-1} \tag{62}$$

Eq.(62) means that the absolute value of P is determined by the wage level. In conventional macroeconomics, wage level is determined by labour market.

4. Conclusion and suggestions

We have made an exploration of building multisectoral AS-AD model with input-output frame[7]. Our approach reveals the differences of AS-AD models between IOA frame and conventional macroeconomics. The different models in this paper all show that the absolute value of total price level is determined by money supply or by one nominal primary input level, and the essence of economic equilibrium is proportionally coordinated. This discovery suggests that the government may control some primary input level such as tax or wage to control inflation. This exploration may be deepen by investigating an real economy.

[7] It might be thought that our AS-AD with IOA has nothing the same as CGE approach. Indeed the two approaches——IO frame and CGE frame——are essentially different. CGE focuses on the estimations of functions which link different structural variables, e.g. consumption between domestic and import goods, but AS-AD with IO frame focuses on the economic mechanism, so that to build CGE models usually needs long time series of economic data. However as a short-run approach, the equations should be sensitive with instantaneous scenarios, to build equations with short series is appropriate. The IO frame makes distinction between productive government and fiscal government. the former belongs to the first quadrant in IO table and the later belongs to the final use and primary input. Anyway, CGE is based on SAM (social accounting matrix) and AS-AD with IOA is based on the closed IO table (CIOT).