A supply-driven model able to endogenise simultaneous homogeneous and heterogeneous primary inputs: overcoming the Ghosh model's limitation

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Abstract

The Ghosh model, known as the supply-driven model, has been put aside by the input-output community because its interpretation is not clear. The main issue is that, by applying an supply-driven model to the economic system, the primary input increase of a specific sector is pushed forward to all sectors linked to it, without any increase in the primary inputs of these other sectors. This outcome is troubling since it has been interpreted as if the other sectors could produce some output without actually using any input themselves. Analytically, the issue is that the Ghosh model is not able to deal with simultaneous primary inputs of the same type (i.e. homogeneous, from the same row) and different type (i.e. heterogeneous, from different rows).

This paper develops a new supply-driven model that overcomes this issue. In particular, the new model is able to push an increase of a primary input through the economy and calculate all required primary inputs of the other sectors, both homogeneous and heterogeneous, associated to that increase in activity.

The structure of this paper is as follows. First, a literature review illustrating the evolution of the Ghosh model interpretation and criticisms is provided. Second, the new supply-driven model able to endogenise and calculate simultaneously the homogeneous and heterogeneous primary inputs is theoretically developed, highlighting the difference with the traditional Ghosh model. Third, a case study is provided by applying the new model to a conventional monetary input-output table (MIOT). Fourth, for completeness, the traditional Ghosh model is also applied to the same MIOT. It is found that the new

model provides very different results than the traditional Ghosh model (different primary inputs, intermediate flows and final outputs). It is demonstrated that this is because both models answer different questions: the new model reveals what is the new level of overall activity due to the primary input push of a single sector while the traditional Ghosh model reveals where does a unit of primary input end up (i.e. in which final demand it is embedded).

To sum up, this paper develops a new supply-driven model overcoming the inherent limitations and associated criticisms of the traditional Ghosh model. The analytical interest on supply-driven models might consequently be rekindled, especially since supply-driven models provide a different view on the economic structure, answering different questions than the Leontief model (e.g. what primary inputs are complementary to each other or how primary inputs are allocated within the economic system).

Contents

1 Introduction

2 Literature review

2.1 On homogeneous and (multiple related) heterogeneous primary inputs and final outputs

Input-Output Tables represent the primary inputs and final outputs that the productive system requires and produces, and the intermediate production and demand between the different economic sectors. Such flows can be homogeneous, heterogeneous and multiple-related:

- *Homogeneous goods*[1] refers to goods that have (or are assumumed to have) the same physical composition or properties, and, thus, same input requirements.
- *Heterogeneous goods* are goods that differ from each other. This poses issues when applying traditional input-output models to input-output tables that have heterogeneous outputs (discussed in section 2.3) and inputs (discussed in section 2.4).
- *Multiple related goods* are heterogeneous goods whose consumption or production are tightly related to each other by the very production struture. For example, all sectors produce several emissions while producing intermediate and final goods. By emissions it is meant all kinds of simultaneous production, such as waste, sewage, etc., i.e. pollutant and non-pollutant emissions in the broad sense. The amount of heterogeneous emissions generated is related to the amount of (homogeneous[2]) intermediate and final good produced; in particular, it can be assumed that emission generation is linearly related to total production (Altimiras-Martin, 2014; Xu and Zhang, 2009), i.e. the final goods and different emissions are multiple related heterogeneous final outputs. Also, primary inputs can be considered to be related to each other, as will be discussed in the section below.

2.1.1 In MIOTs

The underlying flows of Input-Output Tables are constituted of heterogeneous goods due to the level of aggregation of sectoral data. In other words, different (but similar) economic activities are put together under the same hood although their products have different physical composition and prices, e.g. carrots and parnsnips are usually aggregated under the agricultural sector, among others.

In fact, since Leontief (1941) used the homogeneous goods assumption when building the quantity output-driven model, it is common practice to consider that each sector within a Monetary Input-Ouput Table produces homogeneous goods, i.e. the same type of goods.

2.1.2 In PIOTs

PIOTS: multiple related final output between columns (not within columns), multiple related inputs within and between rows

[1]Goods also entail services.

[2]It is usually considered that intermediate and final goods are homogeneous but these homogeneous can also be considered heterogeneous when compared to the emissions.

	Sector 1	...	Sector n	Final demand	Total outputs
Sector 1					
$\vdots$		$\mathbf{Z}$		$\mathbf{f}$	$\mathbf{x}$
Sector n					
Primary input 1		$\mathbf{v_1}'$			
$\vdots$		$\vdots$			
Primary input m		$\mathbf{v_m}'$			
Total inputs		$\mathbf{x}'$			

Table 1: Structure of an IOT with a single final output $\mathbf{f}$ and m primary inputs $\mathbf{v_i}'$ representing a traditional MIOT, all its components are in monetary units.

	Sector 1	...	Sector n	Final demand	Emissions	Total outputs
Sector 1						
$\vdots$		$\mathbf{Z}$		$\mathbf{f}$	$\mathbf{w}_1$ ... $\mathbf{w}_k$	$\mathbf{x}$
Sector n						
Primary input 1		$\mathbf{v_1}'$				
$\vdots$		$\vdots$				
Primary input m		$\mathbf{v_m}'$				
Total inputs		$\mathbf{x}'$				

Table 2: Structure of an IOT with $k+1$ final outputs (final goods $\mathbf{f}$ and k emissions $\mathbf{w_j}$) and m primary inputs $\mathbf{v_i}'$ representing a PIOT, all its components are in physical units.

2.2 The traditional Leontief model, i.e. for IOTs with a single final output

Leontief: each sector intermediate inputs are proportional to the sector total outputs

2.3 Output-driven method and model to deal with IOTs with multiple related heterogeneous final outputs

Put a PIOT with Z, f (one column), w (l columns), v primary inputs (m in total), total outputs x (underscore bc heterogeneous outputs)

2.3.1 Changing the IOT units to use the traditional Leontief model

emphasis: this is a method to use the traditional leontief model

2.3.2 Using a quantity output-driven model able to endogenise multiple related final outputs

this is an different model than traditiounal Leontief

2.3.3 Structural differences

2.4 The traditional Ghosh model

2.4.1 The (Ghosh) input-driven quantity model

The traditional quantity input-driven was developed by Ghosh (1958) and it is based on the same premises as the traditional quantity output-driven developed by Leontief (1941) but from the "input" perspective.

Considering the notation of table 1, aggregating its primary inputs, and $\mathbf{i}$ a vector of ones, the model is established from the following input relationship:

$$\mathbf{x}' = \mathbf{v}' + \mathbf{i}' \cdot \mathbf{Z} \tag{1}$$

Then, it is assumed that the intermediate production of each sector is proportional to the sector's total inputs, i.e.:

$$\mathbf{Z} = \langle \mathbf{x} \rangle \, \mathbf{B} \tag{2}$$

$$\langle \mathbf{x} \rangle^{-1} \mathbf{Z} = \mathbf{B} \tag{3}$$

By using eq. (2) in eq. (1):

$$\mathbf{x}' = \mathbf{v}' + \mathbf{x}' \cdot \mathbf{B} \tag{4}$$

$$\mathbf{x}' = \mathbf{v}' \cdot (\mathbf{I} - \mathbf{B})^{-1} \tag{5}$$

$$\mathbf{x}' = \mathbf{v}' \cdot \mathbf{G} \tag{6}$$

$$\mathbf{x} = \mathbf{G}' \cdot \mathbf{v} \tag{7}$$

where $\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1}$ is traditionally known as the Ghosh inverse matrix.

Finally, given that the IOT data is in monetary flows, the underlying quantities and prices are unknown, so the unitary price assumption is used as in the Leontief model to consider that the monetary flows are quantity flows. In the Leontief model, for practical reasons, it is also assumed that each sector produces a single homogeneous good; however, such assumption is not strictly required for the models to work since the unitary price assumption can absorb transparently any difference in price — and thus homogeneity — of the underlying flows (Altimiras-Martin, 2016).

2.4.2 Criticisms to the Ghosh quantity model

main difference with Leontief: leontief does not need to calculate simultaneous outputs because it is assumed the economy works "independently" to produce each final product and what we see is the agregate (i.e. the sum of each product-based structures (provide the algebraic notation/demosntration);

however, the Ghosh model needs to deal with simultaneous inputs since all economic sectors require inputs to produce outputs.

- difference with output-driven approach: all inputs are required at the same time but Ghosh model does not consider simultaneous related inputs.
- Other issues: ghosh as price model > this interpretation is not necessary given the robustness of model definition and development of this paper.

3 New method to calculate endogenously the total amount of primary resources associated to an exogenous change in primary resources: changing the IOT units and using the traditional Ghosh model

3.1 Theory

As explained in section 2.4, the traditional input-driven quantity model can calculate the final production associated to a newly (exogenously) defined primary input. However, the issue is that, as seen in section 2.1, both MIOTs and PIOTs have multiple related inputs within and between input rows, so an increase in a primary input should induce an increase in all other related primary inputs. An easy but non-trivial workaround is to remove all primary inputs that should be calculated endogenously and place them as final outputs, so that they can be calculated by using traditional output coefficients, in the same fashion imports are sometimes placed as final outputs to create a final output column of net exports.

The reallocation of primary inputs as final outputs by substracting the primary input rows that are to be endogenised implies a change of units, as follows:

$$\mathbf{x}' = \underline{\mathbf{x}}' + \mathbf{v}^{endo\prime} \tag{8}$$

$$\mathbf{x}' - \mathbf{v}^{endo\prime} = \underline{\mathbf{x}}' \tag{9}$$

Before the unit change, the total outputs or inputs of the IOT $\mathbf{x}$ included the primary inputs that the economy requires, i.e. total units required by the economy; after the change of units, the total outputs or inputs of the IOT $\underline{\mathbf{x}}$ do not include the primary inputs anymore except for the ones that are exogenously determined. Although this might seem counter-intuitive, it holds mathematically and the results of this section match the results of section 4, which uses a model endogenising directly these primary inputs within the total requirements matrix. Also, as it will be explained in section 8.3, this is the same procedure used when substracting imports from the primary inputs so as to get net exports as final outputs. Note that the underlined variables in this section

Then, the components of the Ghosh model that are calculated from $\underline{\mathbf{x}}$ need to be recalculated as follows; the underline highlights the difference with the values calculated before changing units. Equations (1) to (7) become:

$$\mathbf{Z} = \langle \underline{\mathbf{x}} \rangle \, \underline{\mathbf{B}} \tag{10}$$

$$\langle \underline{\mathbf{x}} \rangle^{-1} \mathbf{Z} = \underline{\mathbf{B}} \tag{11}$$

By using eq. (2) in eq. (1):

$$\underline{\mathbf{x}}' = \mathbf{v}' + \underline{\mathbf{x}}' \cdot \underline{\mathbf{B}} \tag{12}$$

$$\underline{\mathbf{x}}' = \mathbf{v}' \cdot (\mathbf{I} - \underline{\mathbf{B}})^{-1} \tag{13}$$

$$\underline{\mathbf{x}}' = \mathbf{v}' \cdot \underline{\mathbf{G}} \tag{14}$$

$$\underline{\mathbf{x}} = \underline{\mathbf{G}}' \cdot \mathbf{v} \tag{15}$$

$$\text{with} \quad \underline{\mathbf{G}} = (\mathbf{I} - \underline{\mathbf{B}})^{-1} \tag{16}$$

where $\underline{\mathbf{G}}$ is the Ghosh inverse matrix in the new unit system.

We will usually need to proceed in two steps due to the particularity that all components of any primary input row are related to each other, so to be able to place the endogenous components we will need to disaggregate the row that contain the component that is to be exogenously determined from the others.

Consider an IOT with n sectors with two primary inputs rows $\mathbf{v_1}$ and $\mathbf{v_2}$ and we would like to know the primary inputs required due to $v_{1,1}$, we have choosen which primary input of which sector will drive the model. We will proceed in two steps. First, we'll need to disaggregate the row which contains the exogenously defined primary input between the exogenously defined primary input itself, i.e. $\mathbf{v_1}^{exo} = (v_{1,1}, 0, \ldots, 0)$, and the rest which will be endogenously determined $\mathbf{v_1}^{endo}$. In this case, it will be $\mathbf{v_1} = \mathbf{v_1}^{exo} + \mathbf{v_1}^{endo}$.

From $\mathbf{v_1}$, only $\mathbf{v_1}^{endo}$ will be subtracted to the IOT. Second, the other primary input rows will also be be subtracted from the IOT, only $\mathbf{v_2}$ in this case but as many as required if there are more than 2 primary input vectors. Using eq. (8), table 1 becomes table 3.

	Sector 1	...	Sector n	Final demand	Primary inputs		Total outputs
Sector 1							
$\vdots$		$\mathbf{Z}$		$\mathbf{f}$	$-\mathbf{v_1}^{endo}$	$-\mathbf{v_2}$	$\underline{\mathbf{x}}$
Sector n							
$\mathbf{v_1}^{exo\prime}$		$\mathbf{v_1}^{exo\prime}$					
Total inputs		$\underline{\mathbf{x}}'$					

Table 3: IOT where the primary inputs that are to be endogenously determined by the model have been substracted to total outputs following eq. (8).

Then, the output coefficients related to the negative primary inputs can be calculated as follows:

$$\mathbf{c}(\mathbf{v_j}) = \langle \underline{\mathbf{x}} \rangle^{-1} \cdot \mathbf{v_j} \tag{17}$$

So, through the unit change, the primary inputs can be calculated endogenously as follows:

$$\mathbf{v_j}' = \mathbf{v}^{exo\prime} \cdot \underline{\mathbf{G}} \cdot \langle \mathbf{c}(\mathbf{v_j}) \rangle \tag{18}$$

3.2 Numerical example

The numerical example is based on table 1, a hypothetical three sector IOT with two primary input rows. The numerical example is based on table 1, a hypothetical three sector IOT with two primary input rows.

	Sector 1	Sector 2	Sector 3	$\mathbf{f}$	$\mathbf{x}$
Sector 1	150	500	250	800	1700
Sector 2	200	100	300	1400	2000
Sector 3	100	300	125	700	1225
$\mathbf{v_1}$	350	650	300		
$\mathbf{v_2}$	900	450	250		
$\mathbf{x}'$	1700	2000	1225		

Table 4: Hypothetical three sector IOT with two primary input rows

The idea is to calculate the primary input requirements due to a change in $v_{1,1}$, i.e. the first column value of $\mathbf{v_1}$. Thus, $\mathbf{v_1}^{exo} = (350, 0, 0)$ and $\mathbf{v_1}^{endo} = (0, 650, 300)$. Applying eq. (8) — the unit change—, table 1 becomes table 5.

	Sector 1	Sector 2	Sector 3	$\mathbf{f}$	$\mathbf{v_1}^{endo}$	$\mathbf{v_2}$	$\underline{\mathbf{x}}$
Sector 1	150	500	250	800	0	−900	800
Sector 2	200	100	300	1400	−650	−450	900
Sector 3	100	300	125	700	−300	−250	675
$\mathbf{v_1}^{exo\prime}$	350	0	0				
$\underline{\mathbf{x}}'$	800	900	675				

Table 5: Hypothetical three sector IOT with two primary input rows reallocated as negative final outputs.

Using eq. (11) and eq. (16), $\underline{\mathbf{B}} = \begin{pmatrix} 0{,}19 & 0{,}63 & 0{,}31 \\ 0{,}22 & 0{,}11 & 0{,}33 \\ 0{,}15 & 0{,}44 & 0{,}19 \end{pmatrix}$ and $\underline{\mathbf{G}} = \begin{pmatrix} 2{,}29 & 2{,}57 & 1{,}93 \\ 0{,}91 & 2{,}44 & 1{,}35 \\ 0{,}91 & 1{,}8 & 2{,}31 \end{pmatrix}$.

Then, using eq. (18), we can endogenously calculate the new primary inputs $\mathbf{v_1}^{endo}$ and $\mathbf{v_2}$ required to process a newly exogenously defined primary input vector: $\mathbf{v_1^*}^{exo} = (175, 0, 0)$ — all noted with a superscripted * to denote the new state: $\mathbf{v_1^*}^{endo\prime} = (0, 325, 150)$ and $\mathbf{v_2^*}' = (450, 225, 125)$.

It can be observed that trivial results are generated, i.e. all primary inputs vary in the same proportion as the newly exogenously defined primary input. In this case, a value of 50% of $v_{1,1}$ was used and, thus, all primary inputs are 50% of their original values. This result will be further discussed in section 5.

4 New quantity input-driven model able to endogenise multiple related primary inputs to calculate the total amount of primary resources associated to an exogenous change in primary resources

4.1 Theory

In this case, we will directly assume that all primary inputs are consumed proportionally to total inputs, except for the primary input that will be exogenously determined. First, we need to disaggregate the primary input that will drive the model. To do that, we need to divide the chosen row between its exogenous and endogenous components, as follows:

$$\mathbf{v_j}' = \mathbf{v_j}^{exo\prime} + \mathbf{v_j}^{exo\prime} \tag{19}$$

Then, using eq. (19) with j=1 in table 1, we get the following input relationship:

$$\mathbf{x}' = \mathbf{v_1}^{exo\prime} + \mathbf{v_1}^{endo\prime} + \mathbf{v_2}' + \ldots + \mathbf{v_m}' + \mathbf{i}' \cdot \mathbf{Z} \tag{20}$$

Assuming that the primary inputs will be endogenously determined by assuming they are proportional to total inputs (except for the one exogenously determined driving the model) implies that input coefficients can be established for these inputs. In particular, these input coefficients need to be in matricial form ($\mathbf{\Phi}$) in order to fit eq. (20), as follows:

$$\mathbf{v_j}' = \mathbf{x}' \cdot \mathbf{\Phi_j} \tag{21}$$

where

$$\mathbf{\Phi_j} = \langle \mathbf{v_j} \rangle \cdot \left\langle \mathbf{x}^{-1} \right\rangle \tag{22}$$

Using eq. (21) and eq. (2) in eq. (20):

$$\mathbf{x}' = \mathbf{v_1}^{exo\prime} + \mathbf{x}' \cdot \mathbf{\Phi_1}^{endo} + \mathbf{x}' \cdot \mathbf{\Phi_2} + \ldots + \mathbf{x}' \cdot \mathbf{\Phi_m} + \mathbf{x}' \cdot \mathbf{B} \tag{23}$$

$$\mathbf{v_1}^{exo\prime} = \mathbf{x}' - \mathbf{x}' \cdot \mathbf{\Phi_1}^{endo} - \mathbf{x}' \cdot \mathbf{\Phi_2} - \ldots - \mathbf{x}' \cdot \mathbf{\Phi_m} - \mathbf{x}' \cdot \mathbf{B} \tag{24}$$

$$\mathbf{v_1}^{exo\prime} = \mathbf{x}' \cdot \left(\mathbf{I} - \mathbf{B} - \mathbf{\Phi_1}^{endo} - \mathbf{\Phi_2} - \ldots - \mathbf{\Phi_m}\right) \tag{25}$$

$$\mathbf{x}' = \mathbf{v_1}^{exo\prime} \cdot \left(\mathbf{I} - \mathbf{B} - \mathbf{\Phi_1}^{endo} - \mathbf{\Phi_2} - \ldots - \mathbf{\Phi_m}\right)^{-1} \tag{26}$$

$$\mathbf{x}' = \mathbf{v_1}^{exo\prime} \cdot \mathbf{\Gamma} \tag{27}$$

$$\mathbf{x} = \mathbf{\Gamma}' \cdot \mathbf{v_1}^{exo} \tag{28}$$

where the total requirements matrix endogenising all primary inputs except the one driving the model is:

$$\mathbf{\Gamma} = \left(\mathbf{I} - \mathbf{B} - \mathbf{\Phi_1}^{endo} - \mathbf{\Phi_2} - \ldots - \mathbf{\Phi_m}\right)^{-1} \tag{29}$$

Therefore, this formulation consitutes a new input-driven quantity model able to endogenise primary input requirements within the total requirement matrix so that the new primary inputs associated to the new exogenously determined primary inputs can be directly calculated.

In particular, by using eq. (27) in eq. (21), the j primary inputs associated to a new (exogenously) determined primary input ($\mathbf{v}^{*exo\prime}$) can be calculated as follows (the superscripted * denotes the new state):

$$\mathbf{v_j^*} = \mathbf{v}^{*exo\prime} \cdot \mathbf{\Gamma} \cdot \mathbf{\Phi_j} \tag{30}$$

4.2 Numerical example

Using table 4 and using $v_{1,1}$ as the primary input driving the model, $\mathbf{v_1}^{exo} = (350, 0, 0)$ and $\mathbf{v_1}^{endo} = (0, 650, 300)$.

Using eq. (22), $\mathbf{\Phi_1}^{endo} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0{,}33 & 0 \\ 0 & 0 & 0{,}24 \end{pmatrix}$ and $\mathbf{\Phi_2} = \begin{pmatrix} 0{,}53 & 0 & 0 \\ 0 & 0{,}23 & 0 \\ 0 & 0 & 0{,}2 \end{pmatrix}$.

Using eq. (3) and eq. (29): $\mathbf{B} = \begin{pmatrix} 0{,}09 & 0{,}29 & 0{,}15 \\ 0{,}1 & 0{,}05 & 0{,}15 \\ 0{,}08 & 0{,}24 & 0{,}1 \end{pmatrix}$ and $\mathbf{\Gamma} = \begin{pmatrix} 4{,}86 & 5{,}71 & 3{,}5 \\ 1{,}94 & 5{,}43 & 2{,}45 \\ 1{,}94 & 4 & 4{,}2 \end{pmatrix}$

Note that these are different from $\underline{\mathbf{B}}$ and $\underline{\mathbf{G}}$ found in the numerical example using the unit change (section 3.2).

Then, using eq. (30), we can endogenously calculate the new primary inputs $\mathbf{v_1}^{endo}$ and $\mathbf{v_2}$ required to process a newly exogenously defined primary input vector: $\mathbf{v_1^*}^{exo} = (175, 0, 0)$: $\mathbf{v_1^*}^{endo\prime} = (0, 325, 150)$ and $\mathbf{v_2^*}' = (450, 225, 125)$.

These results correspond to ones found in section 3.2, demonstrating the equivalence of both methods.

5 On the triviality of the results and choice of exogenous primary inputs

After developing one method to endogenously calculate all primary inputs using the traditional Ghosh model and developing a new input-driven quantity model able to directly endogenise the primary inputs within the total requirements matrix, a reason must be given for the triviality of the results, i.e. of the fact that the new state can be found by scaling the whole input-output table with

$$\theta = \frac{v_{1,1}^{*\ exo}}{v_{1,1}{}^{exo}}. \tag{31}$$

In fact, the reason becomes apparent after setting explicitly the assumption that primary inputs will also be consumed proportionally to total inputs: considering a fixed structure, i.e. constant direct and total requirement matrices ($\underline{\mathbf{B}}$ and $\underline{\mathbf{G}}$ or $\mathbf{B}$ and $\mathbf{\Gamma}$) and the fact that all variables (including endogenouly determined primary inputs) are linearly related, it follows that the primary input structure must also be maintained.

So, instead of using the theoretical developments of section 3.1 and section 4.1, one can directly use the scalar from eq. (31) to scale the whole input-output table to the new state. However, despite the triviality of finding the new state, the structural analyses are non-trivial. In particular, since the direct and total requirement matrices of both models are different. This aspect will be discussed in section 6.

Another key question is whether the exogenously determined primary input can only be determined for a single sector (as suggested in section 3.1

and section 4.1) or whether we can determine exogenously the primary inputs of several sectors simultaneously. The answer lays with the explation given above in this section. If we set two or more primary inputs exogenously, they will not match the primary input structure that should fixed since the main assumption of the input-driven model is that the primary inputs and intermediate production/consumption are linearly related to total inputs/outputs.

In fact, setting two or more primary inputs exogenously would be the linear composition of two models: applying the input-driven quantity model with two or more exogenous primary inputs as presented either in section 3.1 or section 4.1 and applying the traditional Ghosh model to cover the difference between the total primary inputs found setting endogenously two or more primary inputs and their actual value if the whole IOT was scaled using the scalar from eq. (31).

6 Explaining and exemplifying the structural difference between the two models

Forward linkage analysis on both structures

7 On price and quantity models

8 Discussion and conclusion

8.1 On the four quantity input-output models

8.2 On the triviality of the input-driven quantity model able to endogenously calculate all primary inputs

8.3 On the structural differences when changing the IOT units

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