Research Proposal

For the Construction Input-Output Model as an Instrument for Estimation the Impact of the Extractive Industry Sector on the Economy of Tajikistan¹

Working Paper

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¹ This research paper is part of PhD dissertation on “The Impact of the Extractive Industry on the Economy of Tajikistan” and presents its methodological part. (TAJSTAT, 2018)
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Input-Output Approach as an Instrument for Estimation the Impact of the Extractive Industry Sector on the Economy of Tajikistan

Abstract

The Republic of Tajikistan, a part of the former Soviet Union, is a mountainous landlocked country in Central Asia. Economic advantages of Tajikistan include, among others, abundant water resources, favorable climate conditions and cheap labor. Cotton growing, viticulture, horticulture and aromatic plants growing are the major sources of livelihood. Industrial products in the Republic of Tajikistan include metallurgy, mechanical engineering, and production of durable consumer goods, light industry, and food industry.

Tajikistan is extremely rich in natural resources, but there are major of challenges in converting these resources into wealth. Natural resources of Tajikistan have been explored, and partially excavated for industrial purposes. It is estimated that more than 600 deposits and 800 potential sites for excavation are found in Tajikistan. These minerals are iron, zinc, antimony, uranium, coal, oil, natural gas, mercury, gold, silver, precious/semiprecious stones, building materials, hydropower potential etc.; Although the mining sector has enormous potential for Tajikistan, its impact is still limited. It represents only 3.07% of the gross domestic product (GDP), 4.2% of the national budget, 5.18% of employments and 23.25% of exports (EITI-Tajikistan, 2015).
Tajikistan economy is still in a transitional stage. Government is struggling with various challenges that have occurred after independence but is not yet ready to deal with current global challenges. The government has failed in converting the country’s potential into real national wealth.

The main challenge facing Tajikistan is to find new sources of revenue for national budget from which government will be able to tap additional tax revenue. The extractive industry may provide a good opportunity for this new source of revenue.

If Tajikistan wishes to use extractive industry as a main source of revenue, it is necessary to establish strong governance underpinned by a transparent and accountable management system of natural resources. Lack of good governance in the extractive industry may lead to corruption with negative impact on the economy. If Tajikistan would be able to establish good governance, it would help the economy to grow in a more sustainable manner.

At the present time Tajikistan does not have Input-Output Tables which is one of the most important macroeconomic policy tools. This research is going to develop Input-Output Tables which would be applied to macro-economic analysis of Tajikistan with a significantly contribution to the national economy. Therefore, the overall objective of research is to ensure extractive industry to be developed and managed in a transparent and accountable manner so that all segments of the society can benefit from prospective expansion of the extractive industry. This would require: (1) grasping the potential magnitude of the impact that the extractive industry of Tajikistan can bring (through Input-Output Analysis) and also (2) instituting stronger governance for the extractive industry sector.
In order to achieve the above objective, the following research questions have been developed to guide the entire process of the research:

(1) What impact will the growth of the extractive industry bring to the economy of Tajikistan?

(2) What government policies should be adopted in order to ensure the extractive industry sector to be managed in a transparent and accountable manner so that the extractive industry can contribute to the sustainable development of the Tajikistan’s economy?

This research would be primarily carried out with the use of quantitative analysis, which would be supplemented later by the qualitative analysis. *Quantitative analysis* is primarily designed for assessing the impact of the extractive industry on the economic growth with use of an Input-Output table model. The quantitative method would be supplemented by the *qualitative analysis*, which would gather expert’s views on the effective ways of managing the extractive industry.

Keywords: Tajikistan, extractive industry, economic growth, impact on economy, Input-Output Tables.

1. Introduction

The Republic of Tajikistan is a mountainous landlocked country with absolute altitudes from 300 up to 7495 meters, located in Central Asia. About 93% of the territory of Tajikistan (a total area is 142,6 thousand km²) is covered by mountains (MFA of Tajikistan, 2018). Tajikistan has a population of 8,8 million (2017), and about two-third of the population in Tajikistan lives in rural areas (TAJSTAT, 2018).
Tajikistan is an agrarian country and a large population depends in agriculture and animal husbandry as the major source of livelihood. Additionally, Tajikistan is rich in many natural and underground mineral resources. Natural resources of Tajikistan have been identified, explored, and partially used for industrial purpose. It is estimated that more than 600 deposits and 800 occurrences of minerals lay under the Tajik soil (EITI-Tajikistan, 2015). Moreover, the high mountains in Tajikistan are major sources of modern glaciation, where some of the main waterways in Central Asia come from. In general, hydroelectric power resources have been of great importance in the country’s development, not only for its agricultural use, but also for its industrial use, especially the extractive and processing sectors (Baratov, 1999).

Today, Tajikistan faces numerous challenges in social and economic aspects. The country was devastated during the 1992–1997 civil war that has set back the development of its economy and infrastructure to a ten-year period of recovery. Among the five Central Asian Commonwealth Independent States (CIS) countries, Tajikistan remains far behind them with the transition economy. Over the past 26 years, the country has the lowest gross domestic product (GDP) per capita and the highest poverty rate. According to the present World Bank classification, Tajikistan remains in the group of countries with a low per capita income³ (Shukuropat, 2014).

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³ Countries are divided into the following groups (according to the value of GNI per capita in 2012, calculated in the base of World Bank Atlas method): low-income countries (US $ 1,035 or less); lower middle income (from US $ 1,036 up to 4,085); with a high average income (from US $ 4,086 to 12,615) and high income (from US $ 12,616 or more).
Despite, Tajikistan`s economic growth has been robust since the end of the civil war in 1992-1997. Five years civil war severely damaged the already weak economic infrastructure and caused the sharp decline in whole vital areas of country economic. By 1996, Tajikistan had lost more than two-thirds of its GDP and more than half of its budget revenues as a percentage of GDP; economic growth plunged 17.2% per year on average during 1991-1996. On top of much-needed, transition-related policy reforms, the country had to reestablish social cohesion and rebuild infrastructure. Moreover, the civil war in Tajikistan destroyed production capacity and infrastructure with an estimated replacement cost at that time of 10 billion US $. And in the present time Tajikistan is riding on post conflict recovery, significant increases in remittance inflows, and a favorable external environment largely brought about by economic development in Russia (mostly remittances are coming from Russia) (ADB, 2016).

Looking at the macroeconomic condition of the country, despite of external and internal economic socio-political situations, the negative impact of the global economic and financial crisis (2007-2012), Tajikistan was able to make its gradual progress in the national economy during the 10 years’ period (from 2007 till 2017). Annual GDP growth was 8-10% before the impact of the global financial crisis and Russian sanction, but in the present time it is declining to 6-7%. Nominal GDP grew from 23.4 billion somoni in 2007 up to 61.1 billion somoni in 2015. The GDP per capita increased from $523.6 to $1026, or 3.4 times during the period of 2007-2013, however, GDP has sharply started decline from 2014 up to present see below Figure 1.1(Eromenko&Saidahmadzoda, 2015; NBT,2018).
Tajikistan economy is still in a transitional stage. Government is struggling with various challenges that have occurred after independence but is not yet ready to deal with current global challenges. The government has failed in converting the country’s potential into real national wealth.

The main challenge facing Tajikistan is to find new sources of revenue for the national budget from which government will be able to tap additional tax revenue. The extractive industry may provide a good opportunity for this new source of revenue.

If Tajikistan wishes to use extractive industry as a main source of revenue, it is necessary to establish strong governance underpinned by a transparent and accountable management system of natural resources. Lack of good governance in the extractive industry

Figure 1.1 GDP Per Capita
Source: National Bank of Tajikistan, 2018;
may lead to corruption with negative impact on the economy. If Tajikistan would be able to establish good governance, it would help the economy to grow in a more sustainable manner.

The objective of this working paper is to present the methodological framework on development Input-Output Tables which would be applied to analyze the impact of the extractive industry on economic growth. Furthermore, a developed Input-Output table would help to conduct macro-economic analysis of Tajikistan with a significantly contribution to the national economy.

For the general research study (PhD Dissertation) is aimed to ensure the extractive industry to be developed and managed in a transparent and accountable manner so that all segments of the society can benefit from prospective expansion of the extractive industry. This would require: (1) grasping the potential magnitude of the impact that the extractive industry of Tajikistan can bring and also (2) instituting stronger governance for the extractive industry sector.

In order to achieve the above objective, the following research questions have been developed to guide the entire process of research:

(1) What impact will the growth of the extractive industry bring to the economy of Tajikistan?
(2) What government policies should be adopted in order to ensure the extractive industry sector to be managed in a transparent and accountable manner, so that the extractive industry can contribute to the sustainable development of the Tajikistan’s economy?

2. Significance of the Study
While the extractive industry of Tajikistan has been generating significant level of national wealth from the extractive industry, its distribution of wealth has not yet been fully equitable. If transparent and accountable management of the extractive industry is in place, Tajikistan will be able to establish a stronger economy.

At the present time Tajikistan does not have Input-Output Tables which is one of the most important macroeconomic policy tools. This research is going to develop Input-Output Tables which would be applied to macro-economic analysis of Tajikistan with a significantly contribution to the national economy.

3. The Extractive Industry

The term of the extractive industry can be defined as:

“processes that involve different activities that lead to the extraction of raw materials from the earth (such as oil, metals, mineral and aggregates), processing and utilization by consumers. These processes take place within host countries and home countries of operating companies, as well as consuming markets” (Sigam & Garcia, 2012, p.2).

There are positive and negative evidences regarding extractive industry opportunity on economic growth and human development in the literatures. Some studies found that countries which are using extractive industry for their economic activities have a higher level of development and growth than countries without a substantial extractive industry. But others find that using extractive industry for development reduced economic growth and make people poorer (Davis, 2009). The given studies claim that using extractive industry as
a leading economic growth tool might bring problems such as “resource curse” or “Dutch diseases” environmental catastrophe and others.

The question “Why do some resource-rich countries succeed in utilizing their natural resources and got economic growth, while other resource-rich countries felt? The given question is still the hot topic of extensive debate of scholars and experts in present time. According to Sachs and Warner (1995), there is a negative relationship between the extractive export share of GDP and economic growth. They determined that resource abundance has connection with slow economic growth, and later the given association was named as “resource curse”. Others scholars like Alexeev and Conrad 2009; Brunnschweiler and Bulte 2006; Davis and Tilton 2005 and others through using different methods of analyses disputed of the being of the resource curse. Due to fact, that still nobody clearly knows about when and why resource curse occurs. Moreover, the existence of it is mostly debatable, but it is obvious that many resource-rich countries mostly from developing countries, despite to certain periods of getting high the oil and mineral prices have not been able to render national resource wealth into sustainable long-term growth (Halland et al. 2015). It can be concluded that it happens because of weak governance, lack of transparency and accountability in the management of natural resources, corruption, conflicts and others rather than that extractive industry could not be used as development aspect for economic growth and the possibility for ending of poverty.

4 The resource curse (the paradox of plenty) refers to the failure of many resource-rich countries to benefit fully from their natural resource wealth, and for governments in these countries to respond effectively to public welfare need (NRGI, 2015).
Extractive Industry in Tajikistan

Tajikistan is rich in many natural and underground mineral resources. The fields comprise of deposits like iron, zinc, antimony, uranium, coal, oil, natural gas, mercury, gold, silver, tin, tungsten, molybdenum, phosphates, salt, talc, precious and semiprecious stones, building materials, etc. more than 50 kinds (EITI-Tajikistan, 2015).

Tajikistan's extractive sector has enormous potential but current data shows that the given sector is relatively modest; it is representing only 3.07% of GDP, 4.2% of Budget revenues, 5.18% of employment and 23.25% of exports (see Figure 3.1). However, the total industrial output in 2015 amounted to 12,196 million somoni, of which 1,640 million somoni, or 13.5%, is the contribution of the mining industry. Concerning tax revenues from the extractive industry in 2014, the amount of tax revenues from the payments amounted to 489.5 million somoni, that was 4.2% of total revenues (EITI of Tajikistan, 2015).
The share of extractive in GDP  
The share of extractive products in total industrial production

Amount of 468.6 million somoni is related to tax and custom payments, and 21 million somoni are other obligatory payments made to the budget. Collection of taxes from natural resources use (a subscription bonus for geological exploration, extraction signature bonus, commercial discovery bonus, royalties for extraction, and royalties for water) amounted to 185.7 million somoni (see Figure 3.2) (EITI-Tajikistan, 2015).
Significant growth in the collection of taxes from natural resources use was a result of an increase in the volume of the extraction of natural resources and the strengthening of tax control and tax administration. In 2014, about 154 extractive industries were registered in Tajikistan and grew to 214 by 2015 (EITI of Tajikistan, 2015).

The government of Tajikistan is trying to make the extractive sector as driving power of industrial development, which will help attract large investments. To achieve the given goal, it is necessary to solve internal challenges like old technology in extractive industry, transparent-ness, corruption, poor management (Governance), poor infra-structures (like a road), unclear tax system and so on. Which are the serious barriers for extractive industry to prosper and to contribute to the country’s economic development.

4. Methodology

This research would be primarily carried out with the use of quantitative analysis, which would be supplemented later by the qualitative analysis.
Quantitative analysis is primarily designed for assessing the impact of extractive industry on the economic growth with use of an Input-Output table model.

Historically, Input-Output analysis is an analytical model developed by Wassily Leontief in the late 1930s. However, there was already an earlier version of the Input-Output model by Fran Francois Quesnay in 1758, introduced through his work published in Tableau Economique. Another contributor, Leon Walras in 1874 also introduced this version through Elements d’Economie Politique Pure. After this, the model has been brushed up not only by Leontief but also by many economic scholars (Gutierrez, 2008). Leontief, for his significant work, in 1973 received the Prize in Economic Sciences in Memory of Alfred Nobel “for the development of the Input-Output method and for its application to important economic problems” (Gutierrez, 2008).

The main objective of the Input-Output table is to study the interdependency among all sectors of the economy (Miller & Blair, 1985). And, in the present time Input-Output framework is representing not only as tool for implementing economic analysis but exceeded its role as a leading organizing principle for national accounts and a statistical tool for balancing supply and use of products in significant detail (Thijs ten Raa, 2010).

Construction of the Input-Output tables in terms of distribution of goods and services is an important instrument for the comprehensive analysis of the economy of Tajikistan by sector. During the Soviet period, statistical authorities on a regular basis carried out such work. The State Statistics Committee of the Union of Soviet Socialist Republics

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5 Retrieved from the Nobel Foundation official website: www.nobelprize.org

Input-Output tables of the Soviet Union Republics developed by their Central Statistical Agencies for the detailed scheme for the same "balance" years, except for 1959. In the enlarged scheme the Input-Output table was developed for separate "intermediate" years until 1989, and on 1990 the Input-Output table was drawn up only for Russia. And by 1987 it was achieved a very high level of quality of development of Input-Output tables (Sadriddinov, 2014).

In Tajikistan, the reporting Input-Output tables of production and distribution of goods in the national economy was developed by the Central Statistic Department of the Tajik Soviet Socialist Republic for 1966, 1972, 1977, 1982 years. And the last Input-Output tables was developed for 1987. The reporting Input-Output tables were developed on the base of extensive statistical information obtained as a result of a representative sample survey of enterprises in all sectors of the economy (Sadriddinov, 2014).

From 1993, Tajikistan established National Account System (SNA) under United Nations standards and it acted as a framework during 2014 for developing Supply and Use tables for the year 2011 under Deutsche Gesellschaft für Internationale Zusammenarbeit (GIZ) financial support. Even thought it was carried out for the first time, developing Supply
and Use tables went well, but regrettably Supply and Use tables creation was stopped as the country lacked finance and manpower.

Therefore, development of Input-Output tables in many terms became a very difficult and required lot of time and resources.

Since Tajikistan does not have applicable Input-Output tables, the research will start with reconstructing Input-Output tables which consist of Transaction Flows, Technical Coefficient and Inverse Matrix tables. The Input-Output Tables will be constructed from Supply and Use Tables for 2011 by taking the following steps:

(1) **Reconstruction of the Input-Output Tables from the Available Data Sources of Supply and Use Tables**

- Based on Supply and Use tables, the Transaction Flow table would be developed for 2015 which reflects the entire transactions of Tajikistan economy;
- Based on the Transaction Flow table, the Technical Coefficient table would be developed by dividing each relevant cells of matrix by the total amount of the production for each segment of the industry;
- Based on the Technical Coefficient table, an Inverse Matrix table would be developed. This Inverse Matrix table would enable to estimate the multipliers effect of one-unit change of production in one specific segment of the industry to the entire economy.
(2) Estimate of Future Growth of the Extractive Industry of Tajikistan

- This part of the exercise is primarily designed for finding out growth potential of the extractive industry with use of its elasticity of the extractive industry over the GDP;
- Obtain the government projection of GDP growth for the next 5 years;
- Apply the above elasticity of the extractive industry for the projection of the future growth of the extractive industry;

(3) Assessment of the Impact of the Increased Production of the Extractive Industry on the Economy of Tajikistan

- Having developed Inverse Matrix together with estimates of future potential growth of the extractive industry, the research would aim at estimating the magnitude of the impact of the increased production on the extractive industry on the entire economy of Tajikistan;
- The above would be carried out by inserting the estimated amount of the future growth of the extractive industry into the Inverse Matrix.

Above quantitative method would be supplemented by the qualitative analysis, which would gather expert’s views on the effective ways of managing the extractive industry.

4.1 Supply and Use Tables

The system of Supply and Use tables, proposed for development guideline by the United Nations on the National Accounts in 1993, includes a sequence of tables characterizing the formation of the country's resources (supplies), the direction of their use,
the formation of value added, the transformation of the cost of goods and services in basic prices into prices of buyers. The set of these tables consists of: Supply and Use tables; Symmetric tables of Supply and Use tables; Trade and Transport Margins Tables; Taxes and Subsidies on Products (Goods) Tables and the Use tables on Imported Products (Goods). Under the given structure was constructed Supply and Use tables of Tajikistan for 2011 by the Statistical Agency of Tajikistan in 2014. (Tajikistan Statistic Agency, 2014).

The Supply table consists of two parts. The first part of the table reflects the formation of resources (supplies) of goods and services through own production and imports. The second part gives a quantitative description of the main components of the market price of buyers: taxes (T); subsidies (S), trade and transport margins (TTN) (Tajikistan Statistic Agency, 2014).

The Use table is a logical extension of the Supply table. It provides a detailed description of the distribution of available resources in terms of use. An intermediate consumption (productive) and final use is determined. The Use table is created according to the general scheme of tables of Supply Use Tables, which is consist of three quadrants and represent as "industry x product" (Tajikistan Statistic Agency, 2014).

By using the above-mentioned structure of construction of Supply and Use tables for 2011 was developed the project version of Supply and Use tables for 2015. The data was taken from the National Account of Tajikistan for 2015, which was published in 2016.

Under the Classification of Products by Activity (CPA) Supply and Use tables of Tajikistan for 2015 (project version) includes 92 products with the total balance of
108923421.76 somoni was developed. The given Supply and Use tables for 2015 consists with the following matrixes:

- **Matrix of Production Outputs**;
- **Intermediate Consumption Matrix**;
- **Matrix Import of Goods and Services**;
- **Matrix of Export of Goods and Services**;
- **Final Consumption of Households**
- **Final Consumption of Government Organizations**
- **Final Consumption of Non-profit Institutions Serving Households (NPISH)**;
- **Gross Capital Formation**

**Transforming Supply and Use Tables into Symmetric Input-Output Tables**

Development of Input-Output tables is quite different from the construction of Supply and Use tables and is presented as an analytical step or transformation rather than a development process (UNSTATS, 2017). And based to the Draft Handbook on Supply, Use and Input-Output Tables with Extensions and Applications of 2017, the Supply and Use tables establish the fundamental basis for compiling Input-Output tables and it is recommended that the Supply and Use tables should be developed first, and then the Input-Output tables itself (UNSTATS, 2017, p.12.4 p.368).

There are four models which can be used for the transformation of Supply and Use tables to Input-Output tables and how the various types of Input-Output tables connected to the four (A, B, C and D) different model assumptions. For transforming the project version
of Supply and Use Tables of Tajikistan for 2015 into the Input-Output Tables of Tajikistan for 2015 was selected the Technology Assumptions process under the Model B based on the assumption that:

“Each industry has its own specific way of production, irrespective of its product mix. This assumption applies best to cases of by-products or joint products, since in these cases several products are produced in a single production process” (UNSTATS, 2017, section 12.58, p.385).

For selecting the above mention model and for further its implementation the consult and advises was received by Dr. Hiroshi KUWAMORI Senior Researcher of the Development Studies Center from the Institute of Developing Economies (IDE-JETRO).

Construction of a Symmetric Input-Output Table based on Industry-Technology Assumption (Product by Product)

The data contained in Supply and Use tables is rearranged in the Input-Output frameworks. The capital letters signify matrices and the small letters are the vectors. Transpose matrices are written as matrices (T). Vectors are written as column vectors and row vectors are written as transposed column vectors (T). In addition, the superscript ^ is used to represent the diagonalization of a vector (UNSTATS, 2017).
1. Framework of Product-by-Product Input-Output Table (Competitive-import Type)

<table>
<thead>
<tr>
<th></th>
<th>Intermediate Sector (Product)</th>
<th>Final Demand</th>
<th>Exports</th>
<th>Imports (Less)</th>
<th>Gross Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermediate Sector</td>
<td>( S )</td>
<td>( F )</td>
<td>( E )</td>
<td>( M )</td>
<td>( g )</td>
</tr>
<tr>
<td>(Product)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value Added</td>
<td>( W )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gross Input</td>
<td>( g^T )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Developed by Hiroshi Kuwamori, 2017.

Legends:

\( S \): Intermediate transaction matrix (product by product)

\( F \): Column vector of (product) final demand

\( E \): Column vector of (product) exports

\( M \): Column vector of (product) imports

\( W \): Column vector of gross value-added matrix

(components by industry)

\( g \): Column vector of (product) output

\( g^T \): Row vector of (product) output
2. Framework of Supply and Use Tables (Data for the product by product Input-Output table)

(1) Supply Table

<table>
<thead>
<tr>
<th>Intermediate Sector (Industry)</th>
<th>Output</th>
<th>Imports</th>
<th>Total Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermediate Sector (Product)</td>
<td>$V^T$</td>
<td>X</td>
<td>M</td>
</tr>
<tr>
<td>Output</td>
<td>$g^T$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Developed by Hiroshi Kuwamori, 2017.

Legends:

- $V^T$: Supply matrix (product by industry)
- $g^T$: Row vector of industry output
- $X$: Column vector of product output
- $M$: Column vector of imports
- $q$: Column vector of total product supply

(2) Use Table

<table>
<thead>
<tr>
<th>Intermediate Sector (Industry)</th>
<th>Final Demand</th>
<th>Imports</th>
<th>Total Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermediate Sector (Product)</td>
<td>U</td>
<td>F</td>
<td>M</td>
</tr>
<tr>
<td>Value Added</td>
<td>$Z$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>$g^T$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Developed by Hiroshi Kuwamori, 2017.
Legends:

\( \mathbf{U} \): Use matrix (product by industry)

\( \mathbf{Z} \): Row vector of gross (industry) value added

\( \mathbf{g}^\top \): Row vector of industry output

\( \mathbf{F} \): Column vector of (product) final demand

\( \mathbf{M} \): Column vector of (product) imports

\( \mathbf{X} \): Column vector of product output

**Transformation of Supply and Use Tables to the Product by Product Input-Output Table (Competitive-import Type)**

For concreteness, assume that the number of products is \( m \) and the number of industry is \( n \).

**STEP 1: Derivation of Transformation Matrix**

Transformation matrix \( \mathbf{T} \) under the industry-technology assumption is calculated as follows.

\[
\begin{equation}
\mathbf{T} = \mathbf{C}^\top = [\mathbf{V}^\top (\mathbf{g}^\top)^{-1}]^\top \quad (n \times m)
\end{equation}
\]

Where,

\[
\mathbf{V}^\top = 
\begin{pmatrix}
v_{11} & \cdots & v_{1n} \\
\vdots & \ddots & \vdots \\
v_{m1} & \cdots & v_{mn}
\end{pmatrix}; \quad \text{Supply matrix} \ (m \times n)
\[ \hat{g}^T = \begin{pmatrix} g_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_n \end{pmatrix} : \text{Diagonal matrix of industry output (} n \times n \text{)} \]

\[ (\hat{g}^T)^{-1} = \begin{pmatrix} \frac{1}{g_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{g_n} \end{pmatrix} : \text{Inverse matrix of } \hat{g}^T \text{ (} n \times n \text{)} \]

By using above expressions, Equation (1) can be written as

\[
T = C^T = [V^T (\hat{g}^T)^{-1}]^T = \begin{bmatrix}
\begin{pmatrix}
\begin{bmatrix}
v_{11} & \cdots & v_{1n} \\
\vdots & \ddots & \vdots \\
v_{m1} & \cdots & v_{mn}
\end{bmatrix}
& \\
s_{n \times m}
\end{pmatrix}
& \\
\begin{pmatrix}
1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \frac{1}{g_n} \\
s_{n \times n}
\end{pmatrix}
\end{bmatrix}
\]

\[= \begin{bmatrix}
\begin{pmatrix}
v_{11} & \cdots & v_{1n} \\
\vdots & \ddots & \vdots \\
v_{m1} & \cdots & v_{mn} \\
g_1 & \cdots & g_n \\
g_1 & \cdots & g_n \\
g_n & \cdots & g_n \\
g_n & \cdots & g_n \\
g_n & \cdots & g_n 
\end{pmatrix}
& \\
(m \times n)
\end{bmatrix}
\]

As is obvious from the elements, transposed matrix \(C^T\) is the product-mix matrix that indicates the share of each product in output of industry (Kuwamori, 2017).

**STEP 2: Deriving the Matrices of Intermediate Transaction and Gross Value Added**

By using the transformation matrix derived in STEP 1, matrices of intermediate transaction and gross value added can be calculated.

Intermediate transaction matrix is calculated from the following formula.
(2) \( S = UT \) \((m \times m)\)

Where,

\[
U = \begin{pmatrix}
    u_{11} & \cdots & u_{1n} \\
    \vdots & \ddots & \vdots \\
    u_{m1} & \cdots & u_{mn}
\end{pmatrix} : \text{ Use matrix } (m \times n)
\]

\[
T = \begin{pmatrix}
    v_{11} & \cdots & v_{m1} \\
    g_1 & \cdots & g_1 \\
    \vdots & \ddots & \vdots \\
    v_{1n} & \cdots & v_{mn} \\
    g_n & \cdots & g_n
\end{pmatrix} : \text{ Transpose of the product mix matrix } (n \times m)
\]

The element-by-element calculation of Equation (2) thus becomes

\[
S = UT = \begin{pmatrix}
    u_{11} & \cdots & u_{1n} \\
    \vdots & \ddots & \vdots \\
    u_{m1} & \cdots & u_{mn}
\end{pmatrix} \begin{pmatrix}
    v_{11} & \cdots & v_{m1} \\
    g_1 & \cdots & g_1 \\
    \vdots & \ddots & \vdots \\
    v_{1n} & \cdots & v_{mn} \\
    g_n & \cdots & g_n
\end{pmatrix}_{m \times n}^{n \times m} = \begin{pmatrix}
    \frac{v_{11}}{g_1} + \cdots + \frac{v_{1n}}{g_1} & \cdots & \frac{v_{m1}}{g_1} + \cdots + \frac{v_{mn}}{g_1} \\
    \vdots & \ddots & \vdots \\
    \frac{v_{11}}{g_n} + \cdots + \frac{v_{1n}}{g_n} & \cdots & \frac{v_{m1}}{g_n} + \cdots + \frac{v_{mn}}{g_n}
\end{pmatrix}_{m \times m}
\]

Obtained symmetric matrix \( S \) is the product-by-product intermediate transaction matrix.

Vector of gross value added is calculated from the following formula.

(3) \( W = ZT \) \((1 \times m)\)

Where,

\[
Z = (z_1 \cdots z_m) : \text{ Row vector of gross value added } (1 \times m)
\]

\[
T = \begin{pmatrix}
    v_{11} & \cdots & v_{m1} \\
    g_1 & \cdots & g_1 \\
    \vdots & \ddots & \vdots \\
    v_{1n} & \cdots & v_{mn} \\
    g_n & \cdots & g_n
\end{pmatrix} : \text{ Transpose of the product mix matrix } (n \times m)
\]

The element-by-element calculation of Equation (3) thus becomes
\[ W = ZT = (Z_1 \cdots Z_m) \begin{pmatrix} \frac{v_{11}}{g_1} & \cdots & \frac{v_{m1}}{g_1} \\ \vdots & \ddots & \vdots \\ \frac{v_{1n}}{g_n} & \cdots & \frac{v_{mn}}{g_n} \end{pmatrix} \begin{pmatrix} 1 & \cdots & 1 \\ v_{11} & \cdots & v_{mn} \\ \vdots & \ddots & \vdots \\ v_{1n} & \cdots & v_{mn} \end{pmatrix} \]

\[ = (Z_1 \frac{v_{11}}{g_1} + \cdots + Z_m \frac{v_{1n}}{g_n} \cdots Z_1 \frac{v_{m1}}{g_1} + \cdots + Z_m \frac{v_{mn}}{g_1}) (1 \times m) \]

**STEP 3: Obtaining Final Demand and Gross Output from Supply and Use Tables**

Final demand and gross output of the product-by-product input-output table is obtained from Supply and Use Tables.

(4) \( F = F (m \times k) \)

Where,

\[ F = \begin{pmatrix} f_{11} & \cdots & f_{1k} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mk} \end{pmatrix}: \quad \text{Matrix of final demand} (m \times k) \]

\( k \) is the number of final demand items

(5) \( g = g (m \times 1) \) and \( g^T = g^T (1 \times m) \)

where

\[ g = \begin{pmatrix} g_1 \\ \vdots \\ g_m \end{pmatrix}: \quad \text{Column vector of gross output} (m \times 1) \]

\[ g^T = \begin{pmatrix} g_1 \\ \cdots \\ g_m \end{pmatrix}: \quad \text{Row vector of gross output} (1 \times m) \]

**STEP 4: Balancing/Reconciliation**

Since all the components are prepared from the above 4 steps, the product-by-product input-output table shown in Section 1 can be constructed by stacking obtained pieces into appropriate place of the matrix.
However, there is no guarantee that the total of columns and rows exactly the same value with gross output since data is processed and is obtained from other sources. Specifically, the following balance equation should hold.

**Column balance:** \( S + W = g^T \)

**Row balance:** \( S + F + E - M = g \)

If discrepancy exists between RHS and LHS, it should be eliminated and realize the balances between row and column totals and the gross output by applying the balancing procedure such as the RAS (Hiroshi Kuwamori, 2017).

4.2 Development Transaction Flow and Technical Coefficient Tables

**Transactional Flow Table**

The primary data for development of an Input-Output model is included a Transaction Flow table which is the main part of an Input-Output analysis. The Transaction Flow table represents a picture of all the inter-sectoral transactions in the economy for the determined time period. The main condition for construction a Transaction Flow table is representing of the value of a particular sector’s output that is obtained (purchased) by other sectors as inputs (Leontief, 1986).

In the Transaction Flow table, the rows show the flows of an industry’s output of the whole economy and the columns denotes the consumption of inputs that is desired by a specific industry to produce its output. The given inter-industry flows of goods and services are represented by the blue part of the table that is presented in below Figure 4.2.1. For this
research let’s introduce that $x_{ij}$ can be an inter-industry transaction, where $i$ is the sector which produces the product and $j$ is the sector which purchases the product. The horizontal rows in the table denotes total amount of sales and the vertical columns show the inputs (purchases) of each sector in connection to the other sectors. Meanwhile, each number in any row should be also a number in a column, that means that the output of each sector is also an input in another sector. In the last four columns of the table include the Final Demand details. Which represent sales to households, investment, government and net exports expenditure. Practically, the output of sector $i$ may be utilized with the same sector $i$, furthermore, it can be sold as an input to sector $j$, or sold to the final demands elements (Masouman, 2014).

<table>
<thead>
<tr>
<th>PRODUCERS as Consumers</th>
<th>FINAL DEMAND</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>Mining</td>
</tr>
<tr>
<td>Agriculture</td>
<td></td>
</tr>
<tr>
<td>Mining</td>
<td></td>
</tr>
<tr>
<td>Construction</td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td></td>
</tr>
<tr>
<td>Trade</td>
<td></td>
</tr>
<tr>
<td>Transportation</td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VALUE ADDED</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Employees</td>
<td>Employees Compensation</td>
</tr>
<tr>
<td>Owners of Business and Capital</td>
<td>Profit: type income and capital consumption allowances</td>
</tr>
<tr>
<td>Government</td>
<td>Indirect business taxes</td>
</tr>
</tbody>
</table>

Gross Domestic Product

Figure 4.2.1 A basis Input-Output Transaction Table
Source: Miller, 1985
“If the economy is divided into m sectors and if we denote the total output of sector i by \( X_i \) production and the total final demand for sector i’s product by \( Y_i \), then we can write the following equation:

(4.2-1)

\[
X_i = x_{i1} + x_{i2} + \cdots + x_{ij} + \cdots + x_{im} Y_i
\]

The \( x \)-terms on the right side of the inter-industry sales by sector i, thus the entire right-hand side of the equation is the sum of all sector i’s inter-industry sales and its sales to the final demand. The equation above, shows the distribution of sector i’s output. Within, for each of the m sectors there will be the following equations” (Miller & Blair, 1985):

(4.2-2)

\[
X_1 = x_{11} + x_{12} + \cdots + x_{1j} + \cdots + x_{1m} + Y_1
\]

\[
X_2 = x_{21} + x_{22} + \cdots + x_{2j} + \cdots + x_{2m} + Y_2
\]

\[
\vdots
\]

\[
\vdots
\]

\[
X_i = x_{i1} + x_{i2} + \cdots + x_{ij} + \cdots + x_{in} + Y_i
\]

\[
\vdots
\]
\[ X_m = x_{m1} + x_{m2} + \ldots + x_{mj} + \ldots + x_{mm} + Y_m \]

Here the \( x \)'s term on the right side shows the sales to sector \( i \). It denotes \( i \)'s purchases of the producing sector products within economy. Furthermore, concerning the industrial intermedia transactions, each sector records the spending (expenditure) on outputs like labor and capital including purchasing other inputs such as inventoried items. All the given inputs are represented by value added in the transactional table. Also, imported goods can be recorded as inputs by sector \( i \) (Masouman, 2014)

**The Technical Coefficient Table**

The Transactions Flow table delivers a complete picture of the inter-sectoral structure of an economy and it only observes a tendency of inter-sectoral linkages over a given time period, within they are not very accurate for economic impact analysis (Moghadam & Ballard, 1988; West, 1995; Rey, 2000). Thus, it is requested to have a Technical Coefficient table in order to use Input-Output analysis to explore how production changes (adjustments) in each sector act in response to a change in final demand (Masouman, 2014).

The Technical Coefficients table denotes the production function for each producing sector in the economy. This table represents the monetary value of inputs purchased from \( m \) sectors within the economy per monetary unit of output in sector \( i \). For sector \( i \), Technical Coefficient denote the value of purchases from each of the \( m \) sectors of the economy that is
purchased by the sector \( i \) with the aim to generate one monetary unit worth of \( i \)'s output. Then, Technical Coefficients can be calculated by dividing all records of each sector’s column by the total value of purchases of that sector. So, it could be understood, if \( x_{ij} \) represents the value of sales from sector \( i \) to sector \( j \), and \( x_i \) represents to the total output of sector \( j \), the Technical Coefficients, symbolized by \( a_{ij} \) for each sector which is calculated by the following equation:

\[
(4.2-3) \\
a_{ij} = \frac{x_{ij}}{x_j}
\]

A Technical Coefficient matrix is rectangular table consisted of a comprehensive set of all sectoral input coefficients in an economy. The given coefficients can be regulated by the modifications in the intermediate demand for output of industry \( i \). According to Leontief (1986), the internal structure of an economy is presented by the quantitative snapshot of the Technical Coefficients table. The secondary demand on the output of \( m \) industries that delivers industry \( i \)'s suppliers can be calculated through the consecutive outputs in the Technical Coefficient matrix.

**Independence Coefficients Matrix and Inverse Matrix**

According to Miller and Blair (1985) and Leontief (1986), the central part of the Input-Output matrices for economic analysis issues is the Interdependence Coefficient Matrix. The independence coefficients determine the total (direct and indirect), that is required outputs produced by \( m \) sector’s in order for sector \( i \) to produce, or sell, one monetary unit to any of the elements in the final demand vector. Thus, measures the total
impact of a variation that occurs in final demand for the $i$’s output on the output of $m$ sectors within economy after the entire effects of output increases have been written (Masouman, 2014). The output flow structure is presented in the below:

\[(4.2-4)\]

\[X + M = A_x + F = A_x + f^c + f^g + f^i + f^v + f^{NE}\]

$X + M$ on the left-hand side, represents the total supply of commodities by a sector and the right side represents the total demand for outputs where:

$X$: Represents an $m$- vector of total sectoral output

$M$: Represents an $m$- vector of sectoral imports

$A_x$: Represents an $m \times m$ matrix of technical Coefficients, where $a_{ij}$ means the amount by which sector $i$’s output is used as input by sector $j$ per unit of output

$F$: Represents an $m$- vector of sectoral output used by final consumers

$f^c$: Represents private consumption which includes households and private non-for-profit institutions

$f^g$: Represents governmental expenditure

$f^i$: Represents gross fixed capital formation by production sector (investments)

$f^v$: Represents changes in inventories plus statistical error
\( f^{NE} \): Represents net exports (total exports-imports)

The equation (4.2-4) defines the total output produced in the whole economy with considering the level of total final demand for outputs, specifically private consumption, government expenditure, investment, changes in inventory, and net exports (Masouman, 2014). Also, the inter-sectoral relations in the economy that was determined in the equation (4.2-3) could be rearranged into the following equation:

(4.2-5)

\[ x_{ij} = x_i \times a_{ij} \]

The \( x_{ij} \) which is the level of sales from the sector \( i \) to sector \( j \) determines its dependence upon \( x_j \) and \( a_{ij} \) (which is the level of output in sector \( j \) and the Technical Coefficient of input supplies of sector \( j \) from sector \( i \)). For example, if Tajikistan economy contents only three (or hundred) producing sectors; the final demand vector is represented by \( F \); the Technical Coefficients matrix is represented by \( A \); and the sectoral output vector is represented by \( X \) (Miller & Blair, 1985); within the transactions of the producing sectors can be developed in a set of simultaneous equations:

(4.2-6)

\[
egin{align*}
x_{11} + x_{12} + x_{13} + F_1 &= X_1 \\
x_{21} + x_{22} + x_{23} + F_2 &= X_2 \\
x_{31} + x_{32} + x_{33} + F_3 &= X_3
\end{align*}
\]

The \( x_{ij} \) represents sales from sector \( i \) to sector \( j \); \( F_i \) represents sales from sector \( i \) to final demand; and \( X_i \) is the total output of sector \( i \).
Replacing the equation (4.2-5) into equation (4.2-6) and reorganizing them with the aim to investigate the producing sectors \(i = 1, \ldots, 3\) we get the following equation:

\[(4.2-7)\]

\[
a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + F_1 = X_1
\]

\[
a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + F_2 = X_2
\]

\[
a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + F_2 = X_2
\]

The above-mentioned equation (4.2-7) denotes sectoral interdependence because it shows the outcomes of an increase or decrease in the level of output in all sectors. In easy way it shows the interconnectedness of the economy. Similarly, it represents the relationships of the input requirements of each sector relevant to the level of its final demand. For example, \(F_i\) that is the final demand for sector \(i\), and it is exogenous to the producing sectors in the below equation:

\[(4.2-8)\]

\[
X_1 - a_{11}X_1 - a_{12}X_2 - a_{13}X_3 = F_1
\]

\[
-a_{12}X_2 + X_2 - a_{22}X_2 - a_{23}X_3 = F_2
\]

\[
-a_{31}X_1 + a_{32}X_2 + X_3 - a_{33}X_3 = F_3
\]

Factoring the \(X\)'s from the equation above, we got:

\[(4.2-9)\]
\((1 - a_{11})X_1 - a_{12}X_2 - a_{13}X_3 = F_1\)

\(-a_{21}X_1 + (1 - a_{22})X_2 - a_{23}X_3 = F_2\)

\(-a_{31}X_1 + a_{32}X_2 + (1 - a_{33})X_3 = F_3\)

It can be improved by the following equation in the matrix format:

\((4.2-10)\)

\[
\begin{bmatrix}
X_1 \\
X_2 \\
X_3
\end{bmatrix} = \begin{bmatrix}
(1 - a_{11}) & -a_{12} & -a_{13} \\
-a_{21} & (1 - a_{22}) & -a_{23} \\
-a_{31} & -a_{32} & (1 - a_{33})
\end{bmatrix} \times \begin{bmatrix}
F_1 \\
F_2 \\
F_3
\end{bmatrix}
\]

And the given expression can be represented by the following equation:

\((4.2-11)\)

\[(I - A) \times X = F\]

After, multiplying it by each side of the equation by \((I - A)^{-1}\), that gives us \(X\) sectorial output as a function of \(F\) final demand we receive the following equation:

\((4.2-12)\)

\[X = (I - A)^{-1} \times F\]

The (4.2.12) equation is the central Input-Output system through which it could be find out the effects (impacts) of changes in the final demand elements on the level of sectoral output. The \((I - A)^{-1}\) is well known as the Inverse Leontief matrix, which determine the direct and indirect output levels produced by each sector in the economy with considering of
the levels of final demand components. \((I - A)^{-1}\) is also represents the multiplier matrix because it indicates the direct and indirect supplies of input-output per unit of sectoral final demand. With using the results from equation (4.2-12), which is derived from an iterative process that indicates the changes in the output level relevant to the input supplies and the elements of final demand. Let’s improve it with adding an infinite number of inter-sectoral transactions:

\[
(4.2-13)
\]

\[
X = (I + A + A^2 + A^3 + \cdots + A^{n-1}) \times F
\]

Where \(I\) represents the direct production supplies with the aim to satisfy the elements of the \(F\) that is final demand vector; \(A\) signifies the direct production supplies which aimed to meet the \(AF\) – intermediate demand vector that is also requested for the production of vector \(F\) in the previous round; \(A^2 F\) represents the direct output supply for the intermediate consumption, which is required for the production of vector \(AF\) in the preceding round; and the same way continues until the sum of the rows meets to the multiplier matrix \((I - A)^{-1}\) (Miller & Blair, 1985; Masouman, 2014).

After construction Input-Output tables and development Inverse Matrix the estimation of future growth of the extractive industry of Tajikistan and calculation of assessment of the impact of the increased production of the extractive industry on the economy of Tajikistan will be implemented in the separate Data Analysis paper.
References


