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"The Business Accounting Matrix: a proposal with an application"*

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1. Introduction

National accounts, the main basis of information for macroeconomic models, directly use the business accounts to calculate most of the data related to production at the economy-wide level (Vu Quang Viet, 2000). In fact, economic accounts and models offer a sound perspective for the economic simulation at the aggregate level.

However, economic models for the firm are normally based on a very limited portion of the information that the financial accounting framework is capable to provide. Even if we accept that economic models necessarily imply a simplification of reality, we probably expect them to try to exploit all relevant information available. This means the combination of economic and financial relationships both in the benchmark data and in the economic modeling structure.

The business accounting matrix (BAM) has been developed with the aim of presenting the most relevant information for the firm in a way that can be useful both for economic modelers and for decision makers at the firm level. It provides a clear link between the richness of information generated by financial accounting and the analytical methodology of economic models. It also offers a panoramic and structured view of all significant economic flows and the relationships between the different agents related to the firm's economic activity. The BAM highlights the main aggregates of the economic and financial activity of the firm.

In this paper a framework to develop business accounting matrices (BAM) is presented. It first shows the Value Added generated by the main operating activity of the company. Adding other sources of income and the results of the redistributive instruments of the public sector, aggregates related to the generated income can be defined. Incorporating financial and investment flows helps to establish the net financial needs of the firm.

Outside the use of a BAM as a representation of the benchmark data for potentially more complete business economic models, the BAM allows us to give a concise although global view of the most important economic variables and flows at the business level, facilitating the preparation of easy to use scorecards. The matrix structure of the BAM enables the analysis both for a single year and for the comparison of multiple periods.

The structure of this paper is as follows. Section 2 offers a a brief overview of the relations between matrix accounting and input-output accounting from the perspective of business Accounting. Section 3 describes the National Accounts Matrix (NAM), which has been the starting point for defining the BAM. Actually the BAM can be described as an effort to define a NAM at the firm level. Section 4 shows how the BAM is constructed, showing the main differences with the NAM and presenting a practical example. The last section proposes a set of potential uses of the BAM and the future lines of research and potential developments of the BAM.

2. On matrix-based accounting and input-output accounting

Literature on business accounting displays the existence of a stream of authors who, during the decades from 1950s to 1980s, try to avoid the limitations of the conventional accounting method and, thanks to the growing development of accounting information systems, proposed the implementation of a matrix-based accounting, following the wake of works as

Leontief (1936) on input-output accounting in the United States, Mattessich (1957) on axiomatic accounting and Mattessich (1958) on mathematical models applied to accounting. Although Mephram (1988) finds the origin of matrix-based business accounting in Augustus De Morgan who, in 1846, introduced a matrix framework for accounting in an appendix to the 5th edition of his *Elements of Arithmetic*, this approach will not appear in another paper until the 1950s and (according to Leech, 1986) in a textbook in 1971 by Mathews. Obviously, the predecessor of the input-output model is the widely known Quesnay's *tableau économique* (1760).

However, the first modern input-output model was initially proposed by Leontief in 1936 for the US economy, because it allowed the design of a planning and control tool that was able to state the general equilibrium of the economy, considered as a set of sectors, on the premise that the production of each sector is entirely distributed among the other sectors and the final demand. Since the use of an input-output matrix is based on the mathematical matrix concept itself, which was developed to facilitate the resolution of systems of simultaneous equations, each output corresponds to a set of inputs represented by a constant linear combination of that output. This linear combination is explained through a column vector of constants, so called "technical coefficients" or "intensity ratios".

Despite its potential advantages, the use of a matrix-based model must comply with certain restrictions derived from the formulation of deterministic models. Thus, labor processes and performance must be stable over time, as well as technology, organization or other structures of the company, technical coefficients and relative costs of factors must remain unchanged (Churruca, 1981, 42, 56, 58; Díaz, 1979, 94). Another limitation is that the results obtained are not necessarily the optimum and it also does not take into account the limitation of the resources available in the system, but this can be solved using models of linear programming, partial and general equilibrium models, whose inputs are taken from the IO matrix itself.

Although the input-output methodology was born and developed mainly in the field of general economics, its adaptation to business accounting soon began to be studied, assimilating accounting operations to a flow of transactions among the different departments of the company, the firm and the rest of the economic agents with which it interacts, or among this and the other companies that are members of the same business group. In this sense, the work of Mattessich (1956) highlights the convenience of building bridges between business accounting and national accounting.

However, on the field of business accounting, the initial objective was, according to Mattessich (1957), to find a clear and precise means of representing accounting transactions and their systems in the most general way possible. Thus, the operational basis of matrix-based accounting was based on the principle, pointed out by García (1972, 29), that "each phenomenon of accumulation (input or output) is represented by a column or row vector of the corresponding matrix and we know that every element of the matrix is an element that integrates one and only one row and one and only one column, which characterize the two contrary aspects of any phenomenon of circulation."

Mattessich (1957) proposes (although it was not the main purpose of his paper) a generalization of Leontief's model, noting that not only inter-industrial flows of input-output analysis could be represented by a square matrix, but also any accounting transaction. So the columns define the debit of the accounts and the rows the credit, while any intersection

between both defines the pair debit-credit representative of a transaction¹. Later Mattessich (1958) suggests a distinction between pure accounting models (those which are based on definitional equations only, supported by definitions, systems of accounts and measurements) and economic models (their purpose is to offer explanations about the causal factors involved in the corporate economic systems). So, "the unification of accounting identities with behavior equations, so far applied only in macroeconomic models promises also for business accounting a new field of experimentation and application" (p. 478).

For this reason, matrix-based Accounting go beyond mere substitution of the traditional double-entry bookkeeping based on the journal and the ledger. The use of matrices allowed know the interrelations between the different elements of the accounting system, being able to establish predictions about their behaviour, thanks to the relations of linearity observed among them, giving rise to functional accounting. Hicks (1960), one of the earliest authors that wrote on linear theory in Economics, pointed out that many questions treated in the field of microeconomic theory, such as joint production through so called "activity analysis", could be studied under this approach.

This feature determines the implementation of functional business accounting models (Calafell, 1967), where linear dependency relationships between inputs and outputs are defined. Thus Bueno (1970, 1971) proposed to use matrix accounting as an operational method for recording transactions with a watchful eye on the development that computing was experiencing at that time², in line with authors such as Richards (1960)³, Leech (1986), Babad & Balachandran (1988) or Mephram (1988).

Meanwhile Pinilla (1975) proposes to go further with a input-output financial business accounting model of income flows and wealth aggregates based on the national accounting model (production account, income account, etc.), starting from the distinction between functions (production, income formation, aggregation of wealth and foreign) and institutional sectors (productive sub-entities, financial sub-entities and economic units of consumption assigned to the entity). Although he defines a system input-output matrix he does not get to formalize it mathematically and does not offer a linear model. But, from the point of view of business accounting, his input-output methodology can allow the assimilation of production,

¹ Note that Mattessich follows the Leontief's convention (more properly, the de Morgan's convention), that is the opposite to the usual mapping of matrix transactions in business accounting: the debit in rows and the credit in columns (so called the Gomberg's convention) (Mattessich and Galassi, 2000, 209-210). According to Leech (1986, 331), Mattessich "concedes that the row-debit, column-credit convention may appeal to accountants because of the rule that the first subscript refers to the row and the second subscript refers to the column in a matrix", but its initial position does not change. So Mattessich (1964, 456) says that "We remind the reader that the first subscript refers to the row of a matrix or credit side of an account, whereas the second subscript refers to the column or debit side".

² In his paper of 1970, p.334, Bueno talks about how "transactions can be recorded in perforated cards - substitutive of "transaction matrices"- incorporating the sum of the transaction, the account number to be credited and the number of the account to be debited".

³ Richards (1960) proposes an input-output business accounting model that displays the flows into and out of the accounts, showing the change to the accounts as the result of transactions. This model, applied to a company for 1955, using five "aggregate" accounts (current and other non-fixed assets, fixed assets net, equities, a balance account and an operations account). Input-output coefficients are estimated as the simple means of coefficients computed for 1953, 1955, and 1957, which are used to estimate the effect on account balances from changes in other accounts. In this paper the presentation of accounting data in a matrix form is an example of the advantages of matrix-based planning models.

intermediate consumption, added value, income generated, current and capital transfers, gross and net capital formation, etc. to the different transactions that occur both within the company and in its interaction with the outside, in this case, the various economic agents with which the company is related: suppliers, customers, workers, lenders, government, etc. This shows how income is generated and how it is distributed.

Although in the eighties, many scholars glimpse a revolution in financial business Accounting practice and teaching, it eventually languished. According to Hughes (2016, 32) some explanations can be found in the strongly seated debit-credit model, established for over 500 years, the ever-faster computational technology (that allows operate under the double-entry bookkeeping so quick) and his opinion that "the biggest hurdle of the matrix accounting literature is that it was unreadable and incomplete" and the research was too abstract and showed no immediate use.

Notwithstanding the foregoing, matrix accounting had found a shelter in the field of management accounting. Thus Livingstone (1969) refers to the use of systems of simultaneous linear equations and linear algebra to deal with the problems of cost allocation between departments, starting from the idea that the "input-output analysis is a general model, of which the matrix cost allocation model is a special case". This question was raised, as we previously noted, by Hicks (1960) who uses matrix accounting to solve the problem of joint production, and Ijiri (1965) in the field of programming by objectives and management control. Churrua (1981, 319-320) notes its important applications in the field of business planning and budgeting, the analysis of the degree of utilization of production capacity and the use of coefficients of interrelationship for the quantitative measurement of quality production and sales effort. But, as Hughes (2016, 32) says, although Farag (1967) and Staubus (1971) built early matrix models of what came to be known as ABC after 1984 and made connections to linear programming, "the monographs were not well received, and cost accounting textbooks failed to make the connection between ABC and LP [linear programming]".

Despite its potential drawbacks (without intending to substitute the traditional debit-credit model), matrix-based accounting could have the advantage of presenting the accounting data in a mathematical format (thanks to formulas that can be logically reproduced, due to the characteristics of matrix algebra) and contribute to model economic relations, not only among the company and the rest of economic agents but inside the firm, contributing to an efficient use of resources and internal capacities. This could be a nexus among (professional and academic) financial and managerial accountants with other specialists like computer technicians, engineers and general economists.

3. The National Accounts Matrix (NAM)

A National Accounts Matrix (NAM) can be described as a representation of the National Accounts in matrix form at the highest level of aggregation. Each of the accounts appears both in a row and a column of the matrix, which hence has the same number of rows and columns. Each cell of the matrix shows a flow between the row and the column of the corresponding accounts. Columns reflect how an account is spending its resources while row accounts identify the origin of their resources. Unlike the T-accounts, where each transaction is normally presented twice, the NAM only needs to record them once in a single cell of the

matrix. We can represent each of these cells by a pair of numbers in parenthesis showing the row and the column of the cell.

Table 1, taken from the *Handbook on Social Accounting Matrices and Labor Accounts* (European Commission, 2003), shows the main structure of the NAM. The first two accounts exhibit what can be considered the primary activity of the economy, the provision of goods and services, which can be either produced or imported. The production process allows the generation of what is defined as primary income. The first account, goods and services, can be described as a virtual warehouse that stores all goods and services available in an economy.

The column of this account reflects the supply of goods and services while the row records the use of these goods and services by the rest of the economy and the rest of the World (ROW). The goods and services produced [2, 1] and imported [10, 1] are valued at basic prices (production costs plus net taxes on products), while the use of these supplies of goods and services are valued at purchase prices (basic prices plus trade and transport margins and net taxes on products). So, we therefore need to add the net taxes on products [4, 1] and the trade and transport margins [1, 1] to the value of imports [10, 1] and production [2, 1]. The first account of the NAM is balanced by construction.

The production account column shows the operating accounts of the economy. In order to be able to produce its output [2,1], the activities need to spend their resources consuming intermediate goods [1, 2]. The difference between the value of production and intermediate consumption is the **Gross Value Added**, which is the balance of the production account and one of the most important balances in the NAM. Since the output is valued at basic prices, the value added is also expressed in basic prices. If we deduct payments to labor from the value added we obtain the **Gross Operating Surplus**. Since we are leaving out the consumption of fixed capital [8, 2] we are actually defining the **Net Value Added** [3, 2].

Net value added is the main source of generation of primary income, the one generated through the process of domestic production. If we add the income generated by the labor factor in production activities located outside the local economy and we deduct the income paid by domestic producers to labor factor resident in the ROW, we obtain the **Net Generated Income**. Adding and subtracting other factor payments as the property income from and to the ROW together with net taxes on production, allow us to obtain the **Net National Income** as the balance. Property income flows include lease and royalties, interests, dividends etc. Property income paid among the local institutional sectors would not change the total value of net national income, but only the net national income of the individual institutional sectors.

If we now take into account the current transfers, we can calculate the **Net Disposable Income**. Current transfers may consist of insurance premiums, tax income, current donations or compensations, fines and penalties etc. Current transfers between local institutional sectors would generate no difference between the global values of net national income and net disposable income. It would just change these magnitudes among the different institutional sectors. However, if the amount of current transfers received from the ROW and the amount of current transfers paid to the ROW are different, net disposable income would differ from the value of net national income.

This net disposable income, plus the adjustment for the change in net equity of households on pension funds reserves for the ROW, minus the adjustment for the change in net equity of households on pension funds reserves to the ROW, define the amount of income that can either be consumed –final consumption expenditure– or saved –**Net Savings**.

If we add the net incurrence in liabilities and deduct the net acquisition of financial assets and the change in inventories, we would get the domestic amount of income that can be devoted to fixed investment. If we consider also capital transfers received from the ROW and we deduct the capital transfers paid to the ROW, we could finally obtain the total amount of funds available for fixed capital formation.

The relation of the local economy with the ROW can be analyzed both in terms of current and capital transactions and for each of them we can calculate a balance that summarizes these relationships. The **Current External Balance** [11.10] basically exhibits whether the current payments to the ROW are or not higher than the current payments received from the ROW. If we add the difference between the capital transfers paid to the ROW and deduct the capital transfers received from the ROW we can obtain the final balance, **Net Lending of the ROW** [9, 11] which allows us to identify the financing capacity or financing requirement of the total economy.

This simple matrix, the NAM, together with its main balances is perfectly able to describe the structure of an economy. This descriptive capability can be enhanced by disaggregating the number of elements of each account. For example, we use many activities and goods and services categories in the first two accounts. We could also define various primary input categories. These disaggregated accounts are normally named a Social Accounting Matrix (SAM) and serve as basis for most economy-wide multi-sector simulation models (e.g. input-output and computable general equilibrium models).

The main motivation of this paper consists in believing that the descriptive and modeling capacity of the NAM can be extended to the firm level. In fact, the BAM reflects the adaptation of this national accounts matrix to the environment of a company. The next section describes the main characteristics of the BAM.

Table 1. Aggregate National Accounts Matrix

ACCOUNT	Classifications	1. Goods and Services	2. Production industries	3. Generation of income	4. Allocation of primary income	5. Secondary distribution of income	6. Use of disposable income	7. Capital	8. Gross fixed capital formation	9. Financial	10. Rest of the world, current	11. Rest of the world, Capital	12.TOTAL
		Products	industries	primary input categories	institutional sectors	institutional sectors	institutional Sectors	institutional sectors	industries	Financial Assets			
1. Goods and services	products	Trade and Transport Margins	Intermediate consumption				Final consumption expenditure	Changes in inventories*	Gross fixed capital formation		Exports of goods and services		
2. Production	industries	Output (basic prices)											
3. Generation of income	primary input categories		NET VALUE ADDED (basic prices)								Compensation of employees from the ROW		
4. Allocation of primary income	institutional sectors	Taxes less subsidies on Products		GENERATED INCOME, NET (basic prices)	Property income						Property income and taxes less subsidies on production from the ROW		
5. Secondary distribution of income	institutional sectors				NATIONAL INCOME, NET	Current transfers					Current transfers from the rest of the world		
6. Use of disposable income	institutional sectors					DISPOSABLE INCOME, NET	Adjustment for the change in net equity of households on pension funds reserves				Adjustment for the change in net equity of households on pension funds reserves from ROW		
7. Capital	institutional sectors						SAVING, NET	Capital transfers**		Net incurrence of liabilities		Capital transfers from the ROW	
8. Gross fixed Capital formation	industries		Consumption of fixed capital					Net fixed capital formation					
9. Financial	financial assets							Net acquisitions of financial assets				NET LENDING OF THE ROW	
10. Rest of the world, current		Imports of goods and Services (c.i.f.)		Compensation of employees to the ROW	Property income and taxes less subsidies on production to the ROW	Current transfers to the ROW	Adjustment for the change in net equity of households on pension funds reserves to the ROW						
11. Rest of the world, capital								Capital transfers to the ROW			CURRENT EXTERNAL BALANCE		
12. TOTAL													

Source: European Commission (2003)

* Including acquisitions less disposals of valuables

** Including acquisitions less disposals of non-produced non-financial assets

4. The Business Accounting Matrix (BAM)

In describing the main aspects of the BAM we will proceed in two steps. First, we will compare the basic BAM with the aggregated NAM presented above. Then we will explain a more detailed BAM using a practical example that covers the standard information provided by the firm's financial accounts.

Table 2 highlights the basic aggregated BAM. Its structure is essentially the same as the one exhibited by the NAM in Table 1. However, there is an important characteristic that drives the main differences observed at this level of aggregation. For the economy as a whole, the rest of the world account refers to flows between the local economy and other foreign countries. The domestic flows, those framed with thick lines in Table 1, reflect transactions between many types of agents: producers, consumers, public sector, etc. In the BAM, the transactions between the firm and the rest of the agents, either domestic or foreign, are considered "external relations". The rest of the flows, those framed again with thick lines in Table 2, are purely the ones internal to the firm. Within this area, there is now no space for contents like the ones observed in Table 1 that reflect transactions between domestic agents which is the case of cells [4, 4], [5, 5], [6, 6] and [7, 7].

This is the reason why, instead of talking about industries we refer now to departments of the firm, and instead of being able to classify transactions according to the institutional sectors involved, we mainly refer to the corporation as the sole active agent. At the same time, the external world is full of agents that we may want to distinguish. Therefore we now need to classify the transactions with agents outside of the firm among the many different institutional sectors potentially capable of establishing links with the firm.

Another important difference is related to the valuation criteria. In the BAM, the most reasonable criteria seem for us to be valuing production at factor costs, therefore leaving net taxes on production as another external flow.

At the production level other important considerations have to be made. At the national level, cell [2,1] of NAM only considers the output produced by the different businesses in an economy. Distribution operations are mainly treated as trade margins (see cell [1,1] in NAM). In the description of our canonical firm, we have considered the fact that it can not only produce but also distribute goods and services produced by other firms. Therefore, BAM talks about sales instead of output, and considers not only the goods used as intermediate consumption but also the finished goods and services that have been acquired for resale.

The BAM also reflects the whole economic situation of the firm, as the NAM does for the economy as a whole, since it also covers financial transactions and its financing capacity/requirement. Naturally the BAM can extend its descriptive capacity by expanding its different accounts into their different elements. If the firm has a complete cost accounting, both accounts 1 and 2 can be used to describe the production, cost and value added structure at the level of as many goods, services and departments as required by the firm. The example we have prepared has expanded some of the accounts to give a more accurate description of the potentialities of the BAM.

Table 2. Business Accounting Matrix

ACCOUNT	1. Goods and services (G&S)	2. Production	3. Value added	4. Generation of income	5. Allocation of primary income	6. Secondary distribution of income	7. Use of disposable income	8. Capital	9. Use of change equity	10. Financial	11 & 12. Private & public sector (current)	13 & 14. Private sector (capital)	15. Balance	16. Total
1. Goods and services (G&S)	Intermediate consumption and marketed goods								Changes in inventories & gross fixed capital formation		Sales (taxes included)	Sales (at book value plus taxes)		
2. Production	Sales (factor prices)													
3. Value added		Net value added (agg.)												
4. Generation of income			Net value added (dissagg.)											
5. Allocation of primary income				Net operating surplus							Property income (taxes included)			
6. Secondary distribution of income					Net income						Current non repayable transfers received			
7. Use of disposable income						Net disposable income						Capital contributions		
8. Capital							Net savings	Changes in equity			Adjustments & operative gains	Capital transfers & adjustments & non operative gains		
9. Use of change equity								Changes in inventories & net fixed capital formation & excess of need of financing						
10. Financial		Fixed capital consumption								Change in financial assets				
11 & 12. Private & public sector (current)	Acquisitions of current G&S & taxes on current G&S			Labour costs & other production taxes	Property income paid (taxes included)	Current non repayable transfers paid	Dividend paid	Adjustments & operative losses					Balance of current transactions	
13 & 14. Private & public sector (capital)	Acquisitions of fixed capital goods & taxes						Returned capital contributions	Capital transfers & adjustments & non operative losses					Balance of capital transactions	
15. Balance										Balance of changes in financial accounts				
16. Total														

We have designed an example based on the standard financial accounts of a hypothetical non-financial company. Annex 1 shows the statement of income, the balance sheet, the current account and the capital and financial account of this fictitious firm. The conceptual BAM that we have prepared using the information contained in Annex 1, is showed in Annex 2. We will now proceed to explain the contents of each of the accounts of this expanded BAM.

4.1. The Goods and Services, Production and Generation of Income accounts

These are the two most important current accounts of the BAM and are shown in Table 3. In this table data from the financial accounting framework are showed with a grey background. The G&S (good and services) account shows the G&S available at the corporation. Columns of this account show essentially the origin of the G&S and the rows the destination of these G&S. This account is balanced by construction. The G&S account is divided into two main groups. The first one (1.1) gathers the G&S that are directly produced or resold by the corporation, and their subaccounts are "Merchandises" and "Finished goods and services". The second group (1.2) consists of goods and services originating outside the corporation considered as not own to it. This second group of G&S acquired by the corporation consists in intermediate G&S, merchandises acquired to be resold by the corporation and acquired fixed capital goods.

Cells [2.1, 1.1.1] and [2.1, 1.1.2] represent the sales of merchandises (10.520 units of account - ua) and own production (6.822.800 ua) respectively. They are valued at basic prices (i.e. they do include any net taxes on production). We assume these cells include all transport and trade margins, considered as intermediate goods. In order for this account to balance, rows and columns have to be valued by the same criteria. We therefore need to consider the net taxes on products and production which are registered in cells [12.2, 1.1.1] and [12.2, 1.1.2] to value these flows at purchase prices. Taxes on G&S are assumed to be a 7%, these payments are not part of the financial accounts (cells [12.2, 1.1.1] and [12.2, 1.1.2]).

Rows [1.1.1] and [1.1.2] show the destination of the merchandises resold and of the G&S produced by the corporation. In our example both merchandises and own production are sold to households and non-financial corporations (11.256 and 7.295.046 ua respectively). In the case of the own production, the part that is not sold ends up as increasing inventories (0 ua in our example in cell [1.1.2, 9.1]), while another part of the production was devoted to the elaboration of fixed capital goods (5.000 ua) in cell [1.1.2, 9.2].

Since the "Merchandises" account only registers resold G&S, change in inventories should not be considered as a potential destination of the goods in this case. Change in inventories in merchandises will be referred to by cell [1.2.1, 9.1].

Spending on other G&S is recorded in columns [1.2.1] and [1.2.2] (current and fixed capital ones, respectively). A portion of these current G&S is acquired to non-financial companies (row [11.1]), other to financial entities (row [11.2]), and the rest is paid to public sector (when it operates as a market supplier) (row [12.1]).

Cells [12.2, 1.1.1], [12.2, 1.1.2] and [12.2, 1.2.1] record the indirect taxes charged on G&S sold and supported and deductible indirect taxes paid on all current acquired goods, minus subsidies.

The uses given to these goods can be observed in rows [1.2.1] and [1.2.2]. They consist of using these goods and services and merchandises as untransformed merchandises resold by the corporation [1.2.1, 1.3.1.] intermediate consumption [1.2.1, 1.3.2]; and increasing inventories (1.2.1, 9.1) and acquired fixed capital goods [1.2.2, 9.2].

According to cost accounting rules, the costs of sold merchandises and intermediate consumption are assigned to the different departments of the company, namely cost and income centers as show in cells [1.3.1, 2.1] and [1.3.2, 2.1].

One of the most important balances of the BAM is the **value added** by the company (in our example, 3.508.321 ua, in cell [3.1, 2.1]). Value added can be defined in this context as the difference between the value of sales at factor prices and the value of all the intermediates used in the production process valued at purchase prices (not own G&S) and factor prices (own consumption) and the value of the merchandises bought by the corporation to be resold, also valued at purchase prices.

This value added can be described as the primary income directly generated by the corporation. It is intended to reward the factors of production of the firm that enable its primary activities: either the production or the mere distribution of G&S. It also includes the net taxes on production [4.2, 3.1]. The payments to the labor factor include not only wages and salaries [4.1.1, 3.1] but also all other payments related to the labor force like compensations [4.1.2, 3.1], social security [4.1.3, 3.1] and social benefits [4.1.4, 3.1]. The residual of this production account, the **operating surplus**, is the income directly accruing to the firm. The rest ends up out of the span of control of the corporation. Since we are also deducting the fixed capital consumption inputted to departments (460.430 ua, in cell [9.2, 2.1]), we are in fact considering the value added and the operating surplus in net terms (instead of in gross terms); in our example, -72.963 ua, in cell [4.3, 3.1].

To conclude the description of the G&S and product account, it is necessary to mention some operations with fixed capital goods. When these ones are acquired to non-financial companies, the amount is recorded in cell [13.1, 1.2.2]; in case of payments to financial entities, in cell [13.2, 1.2.2], and the rest, to the public sector when it operates as a market supplier, in cell [14.1, 1.2.2]. On the contrary, the sales of fixed capital goods are recorded in cells [1.2.2, 13.1], [1.2.2, 13.2] and [1.2.2, 14.1], depending on whether the buyer is a non-financial company, a financial entity or the public sector. In our example, the company has acquired an air conditioned system for 150.437 ua, and machinery has been sold with a book value and taxes of 75.600 ua. Also, cell [14.2, 1.2.2] records the indirect taxes charged on sold fixed capital goods and supported and deductible indirect taxes paid on all acquired ones, minus subsidies.

Table 4 describes the origin and destination of this primary income. The origin lies in all the different elements of the net value added. The destination depends on the institutional sector involved: households for salaries, compensations and social benefits (row 11.1), and the public sector for social security and production net taxes payments (row 12.2).

Table 3. The G&S and production accounts

			1. Goods and Services (G & S)						2. Production	3. Value Added	
			1.1		1.2		1.3		2.1	3.1	
			1.1.1	1.1.2	1.2.1	1.2.2	1.3.1	1.3.2			
			Merchandises (sold)	Finished products & services (sold)	Current goods and services (acquisitions)	Fixed capital goods	Merchandises (acquired)	Rest of goods & services (acquired)	Departments	Value added	
1. Goods and Services (G & S)	1.1	1.1.1	Merchandises (sold)								
		1.1.2	Finished products & services (sold)								
	1.2	1.2.1	Current goods and services (acquisitions)					5.653	2.858.916		
		1.2.2	Fixed capital goods								
	1.3	1.3.1	Merchandises (acquired)							5.653	
		1.3.2	Rest of goods & services (acquired)							2.858.916	
2. Production	2.1	Departments	10.520	6.822.800							
3. Value Added	3.1	Value added							3.508.321		
4. Generation of income	4.1	4.1.1	Wages & salaries							2.638.148	
		4.1.2	Compensations							50.005	
		4.1.3	Employer social security							653.523	
		4.1.4	Social benefits							34.790	
	4.2	Other production taxes								204.818	
	4.3	Net operating surplus								-72.963	
9. Use of change in equity	9.1	Inventories & valuables									
	9.2	Gross fixed capital formation							460.430		
	9.3	Financing capacity or requirement									
11. Private sector (current)	11.1	Households & non-financial companies			3.078.781						
	11.2	Financial companies			11.082						
12. Public sector (current)	12.1	Market operations			0						
	12.2	Public administration	736	477.246	-201.416						
13. Private sector (capital)	13.1	Households & non-financial companies				150.437					
	13.2	Financial companies				0					
14. Public sector (capital)	14.1	Market operations				0					
	14.2	Public administration				-4.242					
15. Balance	15.1	Balance									
16. Total	16.1	Total	11.256	7.300.046	2.888.448	146.195	5.653	2.858.916	6.833.320	3.508.321	

Table 3. The G&S and production accounts (cont.)

			9. Use of change in equity			11. Private sector (current)		12. Public sector (current)		13. Private sector (capital)		14. Public sector (capital)		15. Balance	16. Total	
			9.1	9.2	9.3	11.1	11.2	12.1	12.2	13.1	13.2	14.1	14.2	15.1	16.1	
			Inventories & valuables	Gross fixed capital formation	Financing capacity or requirement	Households & non-financial companies	Financial companies	Market operations	Public administration	Households & non-financial companies	Financial companies	Market operations	Public administration	Balance	Total	
1. Goods and Services (G & S)	1.1	1.1.1	Merchandises (sold)													
		1.1.2	Finished products & services (sold)	0	5.000											11.256
	1.2	1.2.1	Current goods and services (acquisitions)	23.879												7.300.046
		1.2.2	Fixed capital goods		70.595						75.600	0	0			2.888.448
	1.3	1.3.1	Merchandises (acquired)													146.195
1.3.2	Rest of goods & services (acquired)														5.653	
2. Production	2.1		Departments												2.858.916	
3. Value Added	3.1		Value added												6.833.320	
4. Generation of income	4.1	4.1.1	Wages & salaries												3.508.321	
		4.1.2	Compensations												2.638.148	
		4.1.3	Employer social security												50.005	
		4.1.4	Social benefits												653.523	
	4.2	Other production taxes													34.790	
4.3	Net operating surplus													204.818		
															-72.963	
9. Use of change in equity	9.1		Inventories & valuables												23.879	
	9.2		Gross fixed capital formation												75.595	
	9.3		Financing capacity or requirement												2.304.260	
11. Private sector (current)	11.1		Households & non-financial companies											1.421.071	7.362.789	
	11.2		Financial companies											-520.070	17.232	
12. Public sector (current)	12.1		Market operations											0	0	
	12.2		Public administration											-1.134.132	0	
13. Private sector (capital)	13.1		Households & non-financial companies											685.163	835.600	
	13.2		Financial companies											250.000	250.000	
14. Public sector (capital)	14.1		Market operations											0	0	
	14.2		Public administration											35.655	31.413	
15. Balance	15.1		Balance												737.686	
16. Total	16.1		Total	23.879	75.595	2.304.260	7.362.789	17.232	0	0	835.600	250.000	0	31.413	41.234.389	

Table 4. The generation of income account

	3. Value Added			4. Generation of income				15. Balance	16. Total			
	3.1			4.1				4.2	4.3	15.1	16.1	
	Value added	Wages & salaries	Compensations	Employer Social Security	Social benefits	Other production taxes	Net operating surplus	Balance	Total			
4. Generation of income	4.1	4.1.1	Wages & salaries						2.638.148			
		4.1.2	Compensations						50.005			
		4.1.3	Employer social security						653.523			
		4.1.4	Social benefits						34.790			
	4.2		Other production taxes						204.818			
	4.3		Net operating surplus						-72.963			
5. Allocation of primary income	5.1		Corporation					-72.963	756			
6. Secondary distribution of income	6.1		Corporation						-544.359			
7. Use of disposable income	7.1		Corporation						414.781			
8. Capital	8.1		Capital						339.781			
	8.2		Adjusted capital						381.194			
	8.3		Change in equity						376.731			
9. Use of change in equity	9.1		Inventories & valuables						23.879			
	9.2		Gross fixed capital formation						75.595			
10. Financial	9.3		Financing capacity or requirement						2.304.260			
	10.1		Operating						-428.006			
	10.2		Non-operating						2.732.266			
11. Private sector (current)	11.1		Households & non-financial companies	2.638.148	50.005		34.790		1.421.071	7.362.789		
	11.2		Financial companies						-520.070	17.232		
12. Public sector (current)	12.1		Market operations						0	0		
	12.2		Public administration			653.523		204.818	-1.134.132	0		
15. Balance	15.1		Balance						737.686			
16. Total	16.1		Total	3.508.321	2.638.148	50.005	653.523	34.790	204.818	-72.963	737.686	41.234.389

4.2. Allocation of primary income, secondary distribution of income, use of disposable income and capital and financial accounts

The net operating surplus [5.1, 4.3] is carried forward to the next account, the allocation of primary income, as shown again in Table 5. This surplus reflects the income generated (lost in this case) by productive factors directly used in the main activities of the firm. However, productive factors owned by the firm can be used by other agents generating rents (property income received) for the firm to be added to the operating surplus. On the other hand the firm can be using productive factors owned by other agents, therefore having to pay for them (property income paid) and reducing its own surplus.

In our example, we add 73.719 ua of property income from different institutions (households, and non-financial and financial companies) to the net operating surplus and we deduct 545.969 ua of property income paid to different agents. The remaining value, -544.359 ua, that includes supported and deductible taxes (-853 ua, in cell [12.2, 5.1]), is the **net income of the firm** [6.1, 5.1].

There may be other sources of income and expenses not linked to the use of production factors. This is the case of received and paid current transfers. The company could receive a non-repayable transfer from its business group (cells [6.1, 11.1] and [6.1, 11.2]), or from the public administration [6.1, 12.2] and it usually has to pay the profit tax to the public administration (cell [12.2, 6.1]). Once we deduct from the net income the transfers paid (40.859,80 ua) and we add the transfers received (0 ua) we can obtain the **net disposable income of the company** (cell [7.1, 6.1]).

Disposable income (-585.219 ua) is the income that the company has really available to either distribute among their shareholders or to save. If the company could have no access to financial markets, disposable income would put a maximum constraint to the fixed capital formation of the firm. However, financial markets allow corporations to either place their own financial resources or to obtain new funding possibilities. So, the company can obtain additional funds from its owners by means of a share capital increase or refund them with a share capital reduction (in both cases, they are types of capital transfers). In our example, private shareholders contribute with 1.000.000 ua (cells [7.1, 13.1] and [7.1,13.2]), but not the government (0 ua in cell [7.1, 14.2]), towards funding the company. If we deduct 75.000 ua of dividends paid (cells [11.1, 7.1], [11.2, 7.1] and [12.1, 7.1] we obtain the **net savings** of the company (339.781 ua) in [cell 8.1, 7.1].

Table 5. Allocation of primary income, secondary distribution of income and use of disposable income accounts

			4. Generation of income	5. Allocation of primary income	6. Secondary distribution of income	7. Use of disposable income	11. Private sector (current)		12. Public sector (current)		13. Private sector (capital)		14. Public sector (capital)		15. Balance	16. Total
			4.3	5.1	6.1	7.1	11.1	11.2	12.1	12.2	13.1	13.2	14.1	14.2	15.1	16.1
			Net operating surplus	Corporation	Corporation	Corporation	Households & non-financial companies	Financial companies	Market operations	Public administration	Households & non-financial companies	Financial companies	Market operations	Public administration	Balance	Total
5. Allocation of primary income	5.1	Corporation	-72.963				56.487	17.232	0							756
6. Secondary distribution of income	6.1	Corporation		-544.359			0	0		0						-544.359
7. Use of disposable income	7.1	Corporation			-585.219						750.000	250.000		0		414.781
8. Capital	8.1	Capital				339.781					0	0		0		339.781
	8.2	Adjusted capital														
	8.3	Change in equity														376.731
9. Use of change in equity	9.1	Inventories & valuables														23.879
	9.2	Net fixed capital formation														75.595
	9.3	Financing capacity or requirement														2.304.260
10. Financial	10.1	Operating														-428.006
	10.2	Non-operating														2.732.266
11. Private sector (current)	11.1	Households & non-financial companies		69.531	16.000	50.000									1.421.071	7.362.789
	11.2	Financial companies		476.438	24.781	25.000									-520.070	17.232
12. Public sector (current)	12.1	Market operations		0		0									0	0
	12.2	Public administration		-853	78										-1.134.132	0
15. Balance	15.1	Balance														737.686
16. Total	16.1	Total	-72.963	756	-544.359	414.781	7.362.789	17.232	0	0	835.600	250.000	0	31.413	737.686	41.234.389

Table 6 allows us to show the **capital, fixed capital formation and financial accounts** of the firm. The company could either obtain capital transfers in addition to capital increases abovementioned, i.e. grants, donations or bequests, via cells [8.1, 13.1], [8.1, 13.2] and [8.1, 14.2], or give capital transfers to other agents, through cells [13.1, 8.1] [13.2, 8.1] and [14.2, 8.1]. In our example no extra amount is obtained to finance investment activities. Therefore, the amount of **change in equity**, as an aggregate of capital transfers and net savings, is the same than the amount of net savings.

Some operations, like sales of fixed capital goods or financial instruments, among others, that generate non-operating gains and losses, or value and tax adjustments, can affect the amount of the change in equity, creating the **adjusted change in equity**. In our example, a benefit of 10.000 ua, obtained from the sale of a machine and recorded in cell [8.2, 13.1], and a tax effect of 31.413 ua, in the statement of recognized income and expense outside the income statement, originate a balance of 376.731 ua in cell [8.3, 8.2].

This amount is related to two economic variables: the change in inventories and valuables, and the net fixed capital formation. Their balance determines the **excess or need of financing**, as seen in cell [9.3, 8.3].

The value of **changes in inventories and valuables** is 23.879 ua, in cell [9.1, 8.3], and the net **fixed capital formation** have a negative balance of 384.834 ua, in cell [9.2, 8.3]. If we add the value of consumption of fixed capital, 460.430 ua in cell [9.2, 2.1], the gross fixed capital formation would amount to 75.595 ua; here 5.000 ua correspond to the fixed capital goods produced by the own company, while the remaining 70.595 ua would represent the acquired fixed capital goods. In our example, there is an excess of financing of 737.686 ua.

Financial accounts are divided into two parts: the financial flows related to the operating (cells [10.1, 9.3] and [9.3, 10.1]) and the non-operating activities (cells [10.2, 9.3] and [9.3, 10.2]).

In relation to operating activities, the firm has been able to obtain funds by increasing its liabilities in 1.973.089 ua while it has decreased its acquisition of financial assets in 428.006 ua. For accounting control, the balance of -2.401.096 ua is displayed in cell [15.1, 10.1].

In relation to the financial flows affecting non-operating activities, 406.516 ua were used to reduce the net financial liabilities, while 2.732.266 ua were obtained through the increase of net financial assets. For accounting control, the balance of 3.138.782 ua is displayed in cell [15.1, 10.2].

On the other hand, financial accounts displayed in cells [9.3, 10.1] and [9.3, 10.2] show the net incurrence in liabilities, +1.566.573 ua, while cells [10.1, 9.3] and [10.2, 9.3] show the net acquisitions of financial assets, i.e. +2.304.260 ua. These financial flows would result in increasing the resources available to finance capital spending in 737.686 ua. (financing capacity).

Table 6. Capital, fixed capital formation and financial accounts

			2. Production	7. Use of disposable income	8. Capital			9. Use of change in equity			10. Financial		13. Private sector (capital)		14. Public sector (capital)		15. Balance	16. Total	
			2.1	7.1	8.1	8.2	8.3	9.1	9.2	9.3	10.1	10.2	13.1	13.2	41.1	14.2	15.1	16.1	
Departments			Corporation	Capital	Adjusted capital	Change in equity	Inventories & valuables	Gross fixed capital formation	Financing capacity or requirement	Operating	Non-operating	Households & non-financial companies	Financial companies	Market operations	Public administration	Balance	Total		
1.1	1.1.1	Merchandises (sold)																	11.256
	1.1.2	Finished products & services (sold)					0	5.000											7.300.046
1.2	1.2.1	Current goods and services (acquisitions)					23.879												2.888.448
	1.2.2	Fixed capital goods						70.595											146.195
1.3	1.3.1	Merchandises (sold - consumption)	5.653																5.653
	1.3.2	Rest of goods & services acquired (consumption)	2.858.916																2.858.916
7.1		Corporation																	
8.1		Capital		339.781															414.781
8.2		Adjusted capital			339.781														339.781
8.3		Change in equity				376.731													376.731
9.1		Inventories & valuables					23.879												23.879
9.2		Gross fixed capital formation	460.430				-384.834												75.595
9.3		Financing capacity or requirement					737.686				1.973.089	-406.516							2.304.260
10.1		Operating									-428.006								-428.006
10.2		Non-operating									2.732.266								2.732.266
13.1		Households & non-financial companies		0	0	0												685.163	835.600
13.2		Financial companies		0	0	0												250.000	250.000
14.1		Market operations				0												0	0
14.2		Public administration		0	0	0												35.655	31.413
15.1		Balance									-2.401.096	3.138.782							737.686
16.1		Total	6.833.320	414.781	339.781	381.194	376.731	23.879	75.595	2.304.260	-428.006	2.732.266	835.600	250.000	0	31.413	737.686	41.234.389	

We can identify the financing capacity or requirements of the firm with respect to external agents to it by looking at the external relations accounts in Tables 7-1 and 7-2. If we analyze the current flows with external agents, in accounts 11 and 12, we can easily note that the firm receives more current flows than it pays from the households and non-financial firms (1.421.071 ua, in cell [11.1, 15.1]) while it pays more than it receives to financial entities (-520.070 ua in cell [11.2, 15.1]) and the public administration (-1.134.132 ua, in cell [12.2, 15.1]). If we add to this balances the capital transfers received and subtract the capital transfers paid we will end up with the global financing capacity/requirement of the firm in cell [16.1, 15.1] or in the opposite cell [15.1, 16.1], that are the same than cell [9.3, 8.3]: 737.686 ua.

In our example, the main surplus appears to be in relation to households and non-financial corporation for current flows and the highest deficit appears to be with the public sector. This distribution among institutions cannot be usually extracted from the financial accounts of a standard corporation; we would therefore need to get extra information for this information to be completed.

Table 7-1. External relations, current and capital flows (Rows)

1. Goods and Services (G & S)						4. Generation of income						5. Allocation of primary income	6. Secondary distribution of income	7. Use of disposable income		8. Capital		15. Balance	16. Total
1.1		1.2		1.3		4.1				4.2	4.3	5.1	6.1	7.1	8.1	8.2	15.1	16.1	
1.1.1	1.1.2	1.2.1	1.2.2	1.3.1	1.3.2	4.1.1	4.1.2	4.1.3	4.1.4	4.2	4.3	5.1	6.1	7.1	8.1	8.2	15.1	16.1	
Merchandise (sold)	Finished products & services (sold)	Current goods and services (acquisitions)	Fixed capital goods	Merchandise (acquired)	Rest of goods & services (acquired)	Wages & salaries	Compensations	Employer Social Security	Social benefits	Other production taxes	Net operating surplus	Corporation	Corporation	Corporation	Capital	Adjusted capital	Balance	Total	

11. Private sector (current)	11.1	Households & non-financial companies			3.078.781							2.638.148	50.005		34.790			69.531	16.000	50.000		4.463	1.421.071	7.362.789	
	11.2	Financial companies			11.082														476.438	24.781	25.000		0	-520.070	17.232
12. Public sector (current)	12.1	Market operations			0													0		0		0	0	0	0
	12.2	Public administration	736	477.246	-201.416																				
13. Private sector (capital)	13.1	Households & non-financial companies				150.437															0	0	0	685.163	835.600
	13.2	Financial companies				0															0	0	0	250.000	250.000
14. Public sector (capital)	14.1	Market operations				0																	0	0	0
	14.2	Public administration				-4.242															0	0	0	35.655	31.413
15. Balance	15.1	Balance																							737.686
16. Total	16.1	Total	11.256	7.300.046	2.888.448	146.195	5.653	2.858.916				2.638.148	50.005	653.523	34.790	204.818	-72.963	756	-544.359	414.781	339.781	381.194	737.686	41.234.389	

Table 7-2. External relations, current and capital flows (Columns)

				11. Private sector (current)		12. Public sector (current)		13. Private sector (capital)		14. Public sector (capital)		15. Balance	16. Total	
				11.1	11.2	12.1	12.2	13.1	13.2	14.1	14.2	15.1	16.1	
				Households & non-financial companies	Financial companies	Market operations	Public administration	Households & non-financial companies	Financial companies	Market operations	Public administration	Balance	Total	
1. Goods and Services (G & S)	1.1	1.1.1	Merchandises (sold)	11.256	0	0							11.256	
		1.1.2	Finished products & services (sold)	7.295.046	0	0								7.300.046
	1.2	1.2.1	Current goods and services (acquisitions)											2.888.448
		1.2.2	Fixed capital goods					75.600	0	0				146.195
	1.3	1.3.1	Merchandises (acquired)											5.653
		1.3.2	Rest of goods & services (acquired)											2.858.916
5. Allocation of primary income	5.1		Corporation	56.487	17.232	0							756	
6. Secondary distribution of income	6.1		Corporation	0	0		0						-544.359	
7. Use of disposable income	7.1		Corporation					750.000	250.000		0		414.781	
8. Capital	8.1		Capital					0	0		0		339.781	
	8.2		Adjusted capital	0	0	0		10.000	0	0	31.413		381.194	
	8.3		Change in equity										376.731	
11. Private sector (current)	11.1		Households & non-financial companies									1.421.071	7.362.789	
	11.2		Financial companies									-520.070	17.232	
12. Public sector (current)	12.1		Market operations									0	0	
	12.2		Public administration									-1.134.132	0	
13. Private sector (capital)	13.1		Households & non-financial companies									685.163	835.600	
	13.2		Financial companies									250.000	250.000	
14. Public sector (capital)	14.1		Market operations									0	0	
	14.2		Public administration									35.655	31.413	
15. Balance	15.1		Balance										737.686	
16. Total	16.1		Total	7.362.789	17.232	0	0	835.600	250.000	0	31.413	737.686	41.234.389	

5. Conclusions

In this paper we have shown the applicability of the national accounts structure at the company level. The different national accounts aggregates have a direct relevance for the analysis about the progress of the firm and therefore can be used as a tool for the decision making process.

However, it is obvious that a strict translation from the national accounting framework to the financial accounting model is not a reasonable approach to follow, due to aspects like valuation criteria, or the existence of flows at the national level that have no relevance at the firm level. We have therefore tried to adapt both frameworks in an equilibrated way, so as to preserve the basic financial accounting concepts untouched.

Any case, this research can be only considered to be a preliminary effort to combine both accounting approaches that, without solving all the possible inconsistencies, offers a sound perspective for anybody willing to obtain a complete picture of the situation of a certain company.

The matrix structure of the BAM also facilitates the mathematical treatment of the accounting information, in fields like trends identification, cost analysis, decomposition of the origin of changes in the main aggregates, etc.

On the other hand, the availability of the BAM provides a benchmark representation of the firm that could be used for economic simulations. The main accounting balances would be the basis for the equilibrium identities of simulation these models.

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Annex 2-1. Descriptive BAM (example)

			1. Goods and Services (G & S)				2. Production	3. Value Added			
			1.1		1.2		1.3		2.1	3.1	
			1.1.1	1.1.2	1.2.1	1.2.2	1.3.1	1.3.2			
			Merchandises (sold)	Finished goods & services (sold)	Current goods and services (acquisitions)	Fixed capital goods	Merchandises (acquired)	Intermediate goods	Departments	Value added	
1. Goods and Services (G & S)	1.1	1.1.1	Merchandises (sold)								
		1.1.2	Finished goods & services (sold)								
	1.2	1.2.1	Current goods and services (acquisitions)					Cost of sold merchandises	Intermediate consumption		
		1.2.2	Fixed capital goods								
	1.3	1.3.1	Merchandises (acquired)							Cost of sold merchandises	
		1.3.2	Intermediate goods							Intermediate consumption	
2. Production	2.1		Departments	Sales of merchandises	Sales of production						
3. Value Added	3.1		Value added						NET VALUE ADDED		
4. Generation of income	4.1	4.1.1	Wages & salaries							Wages and Salaries	
		4.1.2	Compensations							Compensations	
		4.1.3	Employer social security								Employer social security
		4.1.4	Social benefits								Social Benefits
	4.2		Other production taxes								Other Production Taxes
4.3		Net operating surplus								NET OPERATING SURPLUS	
5. Allocation of primary income	5.1		Corporation								
6. Secondary distribution of income	6.1		Corporation								
7. Use of disposable income	7.1		Corporation								
8. Capital	8.1		Capital								
	8.2		Adjusted capital								
	8.3		Change in equity								
9. Use of change in equity	9.1		Inventories & valuables								
	9.2		Gross fixed capital formation						Fixed capital consumption		
	9.3		Financing capacity or requirement								
10. Financial	10.1		Operating								
	10.2		Non-operating								
11. Private sector (current)	11.1		Households & non-financial companies			Acquisition of goods & services (taxes included)					
	11.2		Financial companies			Payments to financial corporations (operation - goods & services)					
12. Public sector (current)	12.1		Market operations			Acquisition of goods & services (taxes included)					
	12.2		Public administration	Net taxes on sold merchandises (sales tax minus subsidies)	Net taxes on sold products & services (sales tax minus subsidies)	-Supported and deductible indirect taxes (goods & services)					
13. Private sector (capital)	13.1		Households & non-financial companies			Acquisition of fixed capital goods (taxes included)					
	13.2		Financial companies			Payments to financial corporations (fixed capital goods)					
14. Public sector (capital)	14.1		Market operations			Acquisition of fixed capital goods (taxes included)					
	14.2		Public administration			Net taxes on fixed capital goods (output tax minus subsidies & supported and deductible indirect taxes)					
15. Balance	15.1		Balance								
16. Total	16.1		Total								

Annex 2-1. Descriptive BAM (example) (cont.)

			4. Generation of income						5. Allocation of primary income	6. Secondary distribution of income	7. Use of disposable income	
			4.1				4.2	4.3	5.1	6.1	7.1	
			4.1.1	4.1.2	4.1.3	4.1.4						
			Wages & salaries	Compensations	Employer Social Security	Social benefits	Other production taxes	Net operating surplus	Corporation	Corporation	Corporation	
1. Goods and Services (G & S)	1.1	1.1.1	Merchandises (sold)									
		1.1.2	Finished goods & services (sold)									
	1.2	1.2.1	Current goods and services (acquisitions)									
		1.2.2	Fixed capital goods									
	1.3	1.3.1	Merchandises (acquired)									
		1.3.2	Intermediate goods									
2. Production	2.1		Departments									
3. Value Added	3.1		Value added									
4. Generation of income	4.1	4.1.1	Wages & salaries									
		4.1.2	Compensations									
		4.1.3	Employer social security									
		4.1.4	Social benefits									
	4.2		Other production taxes									
	4.3		Net operating surplus									
5. Allocation of primary income	5.1		Corporation					NET OPERATING SURPLUS				
6. Secondary distribution of income	6.1		Corporation						NET INCOME			
7. Use of disposable income	7.1		Corporation							NET DISPOSABLE INCOME		
8. Capital	8.1		Capital								NET SAVINGS	
	8.2		Adjusted capital									
	8.3		Change in equity									
9. Use of change in equity	9.1		Inventories & valuables									
	9.2		Gross fixed capital formation									
	9.3		Financing capacity or requirement									
10. Financial	10.1		Operating									
	10.2		Non-operating									
11. Private sector (current)	11.1		Households & non-financial companies	Wages and Salaries	Compensations		Social benefits			Property income paid (taxes included)	Current non-repayable paid transfers	Dividends paid
	11.2		Financial companies							Property income paid (taxes included)	Current non-repayable paid transfers	Dividends paid
12. Public sector (current)	12.1		Market operations							Property income paid (taxes included)		Dividends paid
	12.2		Public administration			Employer Social Security		Other Production Taxes		-Supported and deductible indirect taxes (property income)	Current non-repayable paid transfers (included income tax)	
13. Private sector (capital)	13.1		Households & non-financial companies									Returned capital contribution (capital & reserves)
	13.2		Financial companies									Returned capital contribution (capital & reserves)
14. Public sector (capital)	14.1		Market operations									
	14.2		Public administration									Returned capital contribution (capital & reserves)
15. Balance	15.1		Balance									
16. Total	16.1		Total									

Annex 2-1. Descriptive BAM (example) (cont.)

			8. Capital			9. Use of change in equity			10. Financial		
			8.1	8.2	8.3	9.1	9.2	9.3	10.1	10.2	
			Capital	Adjusted capital	Change in equity	Inventories & valuables	Gross fixed capital formation	Financing capacity or requirement	Operating	Non-operating	
1. Goods and Services (G & S)	1.1	1.1.1	Merchandises (sold)								
		1.1.2	Finished goods & services (sold)			Change in inventories (production)	Gross fixed capital formation (self-built)				
	1.2	1.2.1	Current goods and services (acquisitions)			Change in inventories (goods & services acquired)					
		1.2.2	Fixed capital goods				Gross fixed capital formation (acquired)				
	1.3	1.3.1	Merchandises (cost of sales)								
		1.3.2	Intermediate goods								
2. Production	2.1		Departments								
3. Value Added	3.1		Value added								
4. Generation of income	4.1	4.1.1	Wages & salaries								
		4.1.2	Compensations								
		4.1.3	Employer social security								
		4.1.4	Social benefits								
	4.2		Other production taxes								
4.3		Net operating surplus									
5. Allocation of primary income	5.1		Corporation								
6. Secondary distribution of income	6.1		Corporation								
7. Use of disposable income	7.1		Corporation								
8. Capital	8.1		Capital								
	8.2		Adjusted capital	CHANGE IN EQUITY							
	8.3		Change in equity		ADJUSTED CHANGE IN EQUITY						
9. Use of change in equity	9.1		Inventories & valuables			Change in inventories & valuables					
	9.2		Gross fixed capital formation			Net fixed capital formation					
	9.3		Financing capacity or requirement			EXCESS OR NEED OF FINANCING			Net borrowing (operating)	Net borrowing (non-operating)	
10. Financial	10.1		Operating					Change in financial assets (operating)			
	10.2		Non-operating					Change in financial assets (non-operating)			
11. Private sector (current)	11.1		Households & non-financial companies		Adjustments & Operating losses						
	11.2		Financial companies		Adjustments & Operating losses						
12. Public sector (current)	12.1		Market operations		Adjustments & Operating losses						
	12.2		Public administration								
13. Private sector (capital)	13.1		Households & non-financial companies	Capital transfers paid	Adjustments & Non-operating losses						
	13.2		Financial companies	Capital transfers paid	Adjustments & Non-operating losses						
14. Public sector (capital)	14.1		Market operations		Adjustments & Non-operating losses						
	14.2		Public administration	Capital transfers paid	Tax adjustments						
15. Balance	15.1		Balance						Balance of changes in financial accounts (operating)	Balance of changes in financial accounts (non operating)	
16. Total	16.1		Total								

Annex 2-1. Descriptive BAM (example) (cont.)

				11. Private sector (current)		12. Public sector (current)		13. Private sector (capital)		14. Public sector (capital)		15. Balance	16. Total	
				11.1	11.2	12.1	12.2	13.1	13.2	14.1	14.1	15.1	16.1	
				Households & non-financial companies	Financial companies	Market operations	Public administration	Households & non-financial companies	Financial companies	Market operations	Public administration	Balance	Total	
1. Goods and Services (G & S)	1.1	1.1.1	Merchandises (sold)	Sales of merchandises (taxes included)	Sales of merchandises (taxes included)	Sales of merchandises (taxes included)								
		1.1.2	Finished goods & services (sold)	Sales of finished goods & services (taxes included)	Sales of finished goods & services (taxes included)	Sales of finished goods & services (taxes included)								
	1.2	1.2.1	Current goods and services (acquisitions)											
		1.2.2	Fixed capital goods					Sales of fixed capital goods (at book value plus taxes)	Sales of fixed capital goods (at book value plus taxes)	Sales of fixed capital goods (at book value plus taxes)				
	1.3	1.3.1	Merchandises (cost of sales)											
		1.3.2	Intermediate goods											
2. Production	2.1		Departments											
3. Value Added	3.1		Value added											
4. Generation of income	4.1	4.1.1	Wages & salaries											
		4.1.2	Compensations											
		4.1.3	Employer social security											
		4.1.4	Social benefits											
	4.2		Other production taxes											
4.3		Net operating surplus												
5. Allocation of primary income	5.1		Corporation	Property income received (incl. financial investments) (taxes included)	Property income received (incl. financial investments) (taxes included)	Property income received (incl. financial investments) (taxes included)								
6. Secondary distribution of income	6.1		Corporation	Current non-repayable transfers received	Current non-repayable transfers received		Current non-repayable transfers received							
7. Use of disposable income	7.1		Corporation					Capital contribution (capital & reserves)	Capital contribution (capital & reserves)		Capital contribution (capital & reserves)			
8. Capital	8.1		Capital					Capital transfers received	Capital transfers received		Capital transfers received			
	8.2		Adjusted capital	Adjustments & operating gains	Adjustments & operating gains	Adjustments & operating gains		Adjustments & non-operating gains	Adjustments & non-operating gains	Adjustments & non-operating gains	Tax adjustments			
	8.3		Change in equity											
9. Use of change in equity	9.1		Inventories & valuables											
	9.2		Gross fixed capital formation											
	9.3		Financing capacity or requirement											
10. Financial	10.1		Operating											
	10.2		Non-operating											
11. Private sector (current)	11.1		Households & non-financial companies									Balance of current transactions (households and non financial companies)		
	11.2		Financial companies									Balance of current transactions (financial companies)		
12. Public sector (current)	12.1		Market operations									Balance of current transactions (market operations of public administration)		
	12.2		Public administration									Balance of current transactions (public administration)		
13. Private sector (capital)	13.1		Households & non-financial companies									Balance of capital transactions (households and non financial companies)		
	13.2		Financial companies									Balance of capital transactions (financial companies)		
14. Public sector (capital)	14.1		Market operations									Balance of capital transactions (public administration)		
	14.2		Public administration									Balance of capital transactions (public administration)		
15. Balance	15.1		Balance											
16. Total	16.1		Total											