From input-output to macro-econometric model

- paper presented to the 27th International Input-Output Association Conference, 30th June to 5th July 2019, Glasgow, Scotland

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Abstract: This paper shows how a static input-output model can be extended to make it dynamic and include important feedback effects by including econometrically estimated equations to represent behavioural relationships within the model. Using the example of the E3ME macro-sectoral econometric model, we describe how these elements may be combined to create a set of simultaneous equations that build on the structure of the national accounting system to create a dynamic, empirical macroeconomic model. Extensions to the basic accounting system, for example to include the labour market and demands for energy and materials are included.

Keywords: Input-output, macro-sectoral, macro-econometric.

1. Introduction

This paper begins with the example of a static input-output model, notes its limitations, and then describes how these have been overcome in an empirical, global, macro-sectoral econometric model, E3ME, which is widely used for policy simulations.

2. Extending the static input-output model

The basic input-output model

Consider the simple static input-output model:

$$y = Ay + f \tag{1}$$

where

A	is a n x n industry x industry matrix of exogenous coefficients of intermediate consumption per unit of domestic industry output measured in basic prices
f	is a n x 1 vector of exogenous final demand for domestic industry output measured in basic prices
y	is a n x 1 vector of endogenous gross output by industry

In the usual way this is solved to find y using the Leontief inverse:

$$y = (I - A)^{-1} * f \tag{2}$$

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Other variables of interest are determined using exogenous coefficients per unit of gross output, for example:

$$ce = CE * \ y \tag{3}$$

where

- ce is a n x 1 vector of compensation of employees in each industry
- CE is a n x n diagonal matrix with exogenous coefficients of compensation of employees per unit of gross output on the diagonal

and

$$lab = L * y \tag{4}$$

where

- lab is a n x 1 vector of employment in each industry
- L is a n x n diagonal matrix with exogenous coefficients of employment per unit of gross output on the diagonal

and

$$m_y = M * y \tag{5}$$

where

- m_y is a n x 1 vector of intermediate imports classified according to each industry's output and measured in basic prices
- M_y is a n x n matrix with exogenous coefficients of imports of each industry type per unit of gross output

In principle, imports for final demand can be explained in a similar way, by multiplying the final demand vector by coefficients of imports per unit of final demand.

Household spending may be endogenized by distinguishing it from other final demand, augmenting A with a column of coefficients of household spending per unit of (aggregate) compensation of employees, and with a row of coefficients of compensation of employees per unit of gross output:

$$y = A^{*}y + f^{*} \tag{6}$$

$$y = \ (I - A^{*})^{-1} * f^{*} \tag{7}$$

where

- f^* is a n x 1 vector of exogenous (non-household) final demand for domestic industry output
- A^* is a (n+1) x (n+1) comprising the n x n matrix of exogenous coefficients of intermediate consumption per unit of domestic industry output augmented with a column of coefficients of household spending per unit of (aggregate) compensation of employees, and with a row of coefficients of compensation of employees per unit of gross output

Motivation for extending the basic input-output model

The limitations of the basic input-output model are well understood. They include the following:

- other elements of final demand will change if output and value added change, notably investment and inventory accumulation

- the assumption of linear proportional behavioural relationships in the model is crude, especially beyond the very short term; for example, empirical evidence shows that household consumption elements do not all have unit income elasticity, and employment in the various industries do not all have unit output elasticity, both of which the model assumes
- in particular, some of the elements of A are of direct interest in analysis of the circular economy and energy use and policy seeks to decouple these material inputs from output growth, breaking the proportional relationship between output and the demand for inputs
- there is no representation of prices or their impact
- imports are effectively treated as complements rather than substitutes: the model does not allow, for example, output to fall in response to greater import penetration
- key variables are defined as 'consumption of domestic industry output', which is not readily observable except in a year when a full input-output analysis is undertaken
- there are no dynamics in the model: it can only be used in an exercise of comparative statics
- there is no explanation of long-term growth and no role for R&D and innovation

Framework for extending the basic input-output model

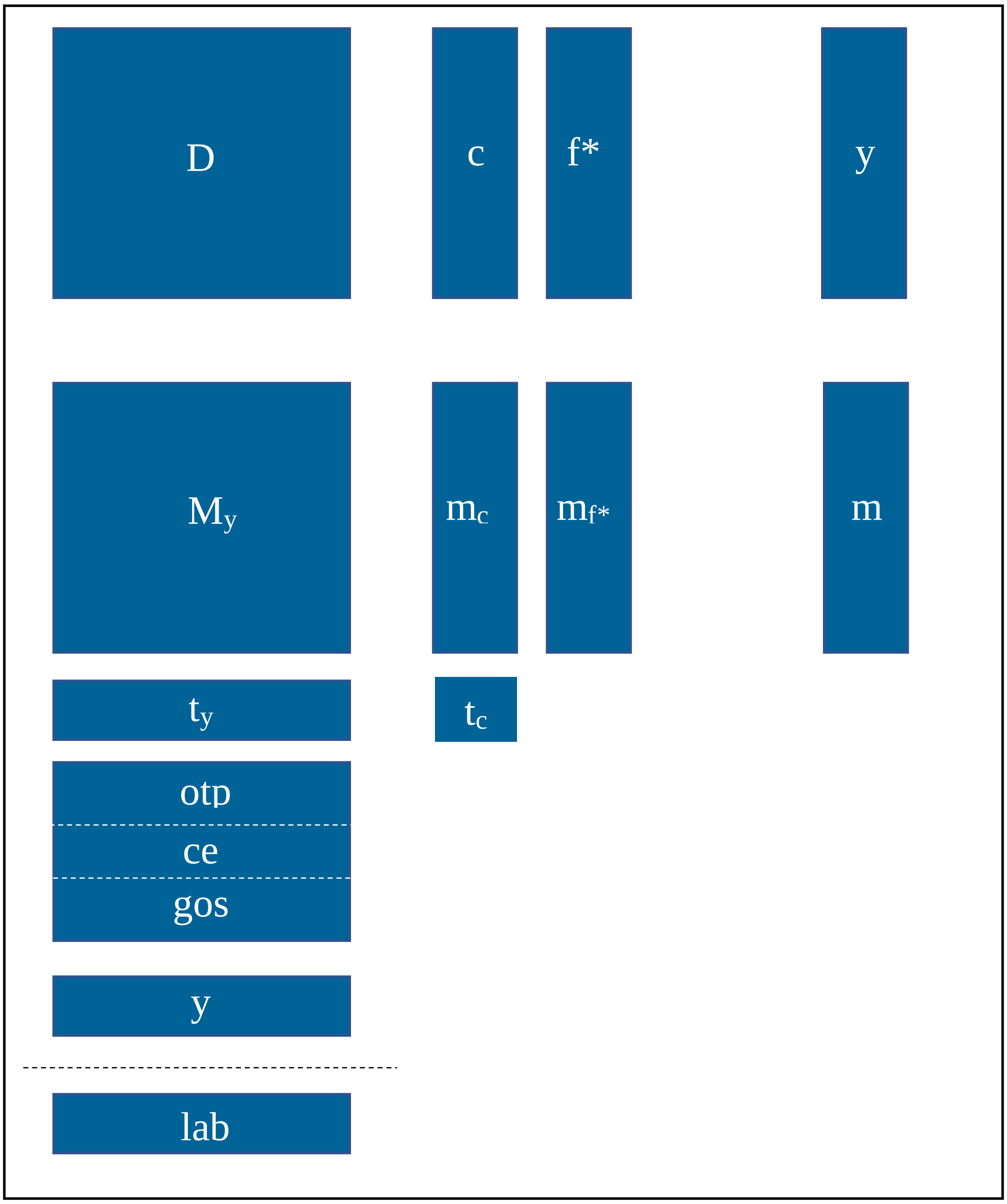
In this section we set out the framework for extending the model to address these limitations. Subsequent sections provide the detail for particular extensions.

Much of this thinking was developed originally for a model of the UK economy under the Cambridge Growth Project originally founded by Alan Brown and Richard Stone and subsequently directed by Terry Barker (Barker and Peterson 1987). This provided the foundation for the development of the global macro-sectoral econometric model E3ME[2] by a modelling team at Cambridge Econometrics.

[2] See www.e3me.com for more details.

Consider the national accounting framework underpinning the basic model set out in equations (1) - (3).

Figure 1: National accounting framework for basic input-output model



where

c	is a n x 1 vector of household consumption of domestic industry output measured at basic prices
ce	is a 1 x n vector of industry compensation of employees
D	is a n x n industry x industry matrix of intermediate consumption by industries of domestic industry output measured in basic prices
f^*	is a n x 1 vector of other (apart from household consumption) final use of domestic industry output measured in basic prices
gos	is a 1 x n vector of industry gross operating surplus

- l is a 1 x n vector of employment in each industry (this is not part of the national accounts as it is measured in numbers of jobs; it can be thought of here as a simple kind of 'satellite' account)
- m is a n x 1 vector of all imported products measured at basic prices
- m_c is a n x 1 vector of household consumption of imported products measured at basic prices
- m_{f*} is a n x 1 vector of other (apart from household consumption) final use of imported products measured at basic prices
- M_y is a n x n industry x industry matrix of intermediate consumption by industries of imported products measured at basic prices
- otp is a 1 x n vector of 'other' net taxes on production
- t_c is a scalar of net taxes on the products purchased by households for final consumption (in principle there may be product taxes levied on other final demand but in practice these are usually small in value)
- t_y is a 1 x n vector of net taxes on the products purchased by industries for intermediate consumption
- y is a n x 1 vector of endogenous gross output by industry

va, a 1 x n vector of gross value added by industries is identically equal to otp + ce + gos.

The following identities are satisfied:

$$y = i' * D + i'^{*} M_y + t_y + va \tag{8}$$

$$y = D * i + M_y * i + c + f^* \tag{9}$$

$$m = M_y * i + m_c + m_{f*} \tag{10}$$

where

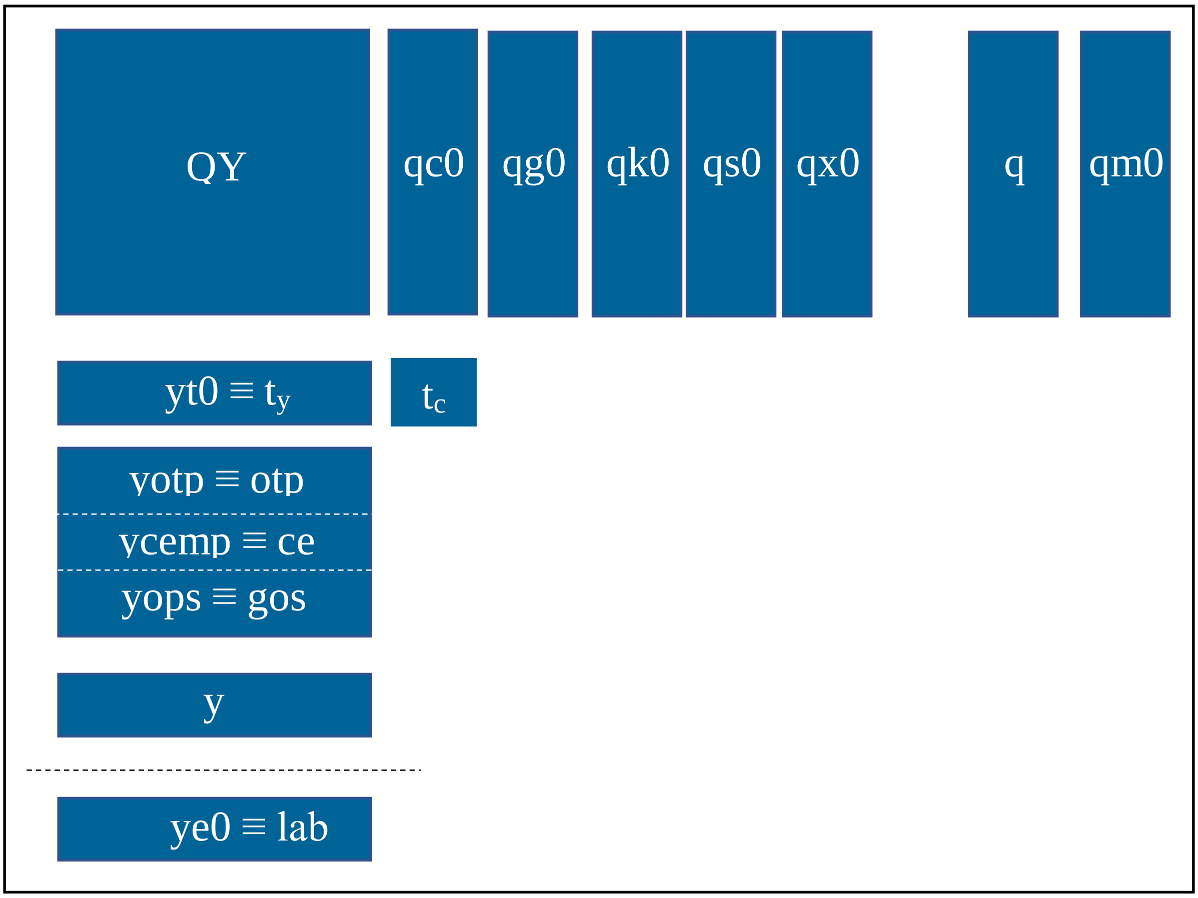
- i is a n x 1 vector of ones whose role is to aggregate matrices across one dimension

We make the following transformations to this framework:

- redefine purchases to be classified on a product rather than an industry basis, which is closer to what can readily be observed, and define the product classification to correspond 1-1 to the industry classification (i.e. the principal product of the firms that together form each industry); hence, for example, industry intermediate consumption is now defined as an n x n product x industries matrix
- define imports on the same product classification
- drop the treatment of imports that distinguishes different categories of purchaser (intermediate consumption, households, other final demand); treat all categories of demand as 'combined' (including both domestic and imported products); locate imported products within the supply and use framework
- expand the 'other final demand' vector to allow different kinds of explanation for its different elements

The resulting framework becomes:

Figure 2: Accounting framework for the extended model



where, in addition to the definitions set out above,

q   is a n x 1 vector of the supply of domestically-produced products measured at basic prices
qc0   is a n x 1 vector of household consumption of products (whether domestically-produced or imported) measured at basic prices
qg0   is a n x 1 vector of general government final consumption of products (whether domestically-produced or imported) measured at basic prices
qk0   is a n x 1 vector of the use of products for gross fixed domestic capital formation (whether domestically-produced or imported) measured at basic prices
qm0   is a n x 1 vector of the supply of imported products measured at basic prices
qs0   is a n x 1 vector of the use of products in inventory accumulation (whether domestically-produced or imported) measured at basic prices
qx0   is a n x 1 vector of the export of products (whether domestically-produced or imported) measured at basic prices
QY   is a n x n products x industries matrix of intermediate consumption of products (whether domestically-produced or imported) measured at basic prices

and some variables have been renamed from Figure 1 as shown in order to adopt a nomenclature in which the classification on which a variable is defined is indicated by its initial letter.

The following identities are satisfied:

$$y = i' * QY + t_y + va \tag{11}$$

$$q + qm0 = QY * i + qc0 + qg0 + qk0 + qs0 + qx0 \tag{12}$$

$$y = YQC * q \tag{13}$$

where

YQC is a n x n matrix of coefficients showing the proportion of each domestically-produced product that is supplied by each industry (formed from the 'make' matrix which maps product supply and industry production)

The key extensions with respect to modelling are then identified as:

- the development of time-series econometric equations to explain
  - household consumption of products (qc0)
  - industry investment and hence use of products (qk0)
  - exports (qx0)
  - imports (qm0)
  - industry employment (ye0)
- the development of a system to explain industry and product prices and average earnings which can be used as drivers in the econometric equations
- the development of more sophisticated methods than the application of exogenous input-output coefficients to determine key products for intermediate consumption, notably energy products because of their importance for climate change mitigation policy
- the determination of endogenous R&D and investment and their impact on growth and labour productivity
- the development of a linked multi-country treatment, in which economies interact through trade
- the introduction of dynamics through an explicit treatment of the process of adjustment over time
- an explicit representation of money and finance, and its relationship to the national accounts saving behaviour of the institutional sectors (households, non-financial companies, financial companies, government, rest of the world)

In practice, these extensions also imply that the empirical model must be solved iteratively as a series of simultaneous equations rather than through the simple application of the Leontief inverse in the basic model.

3. Time-Series Econometric Equations to Explain Demand for Products and Industry Employment

We begin with a brief discussion of our theoretical and empirical approaches and then proceed with a description of the econometric specification of examples of key equations sets.

Theoretical approach

The properties of the model are largely determined by the theoretical approach that guides and constrains the choice of specification for the behavioural equations and the econometric approach to its estimation.

Our modelling approach, which is embodied in the macro-sectoral econometric model E3ME, is post-Keynesian (King 2015) (Lavoie 2015) in nature. We regard the following features of economy and society as fundamental to its operation and therefore not something that should be abstracted from.

- Both households and firms are heterogeneous. We may find it helpful to group subsets together because we think that the members of a subset share characteristics that make them more similar, as when we define industries as groups of firms having the same principal product, but this does not wholly eliminate the heterogeneity from any particular group: an industry remains an aggregation of heterogenous firms, even if those firms have something in common.

- The fact of heterogeneity and the issues involved in aggregation across actors mean that the properties of any equation estimated for an aggregate variable (e.g. households' consumption spending, an industry's employment) cannot be identified with the parameters of a theoretical construct of the behaviour of a 'representative' individual. We assume that aggregation preserves normal broad economic behavioural features that are likely to be common across actors (a higher relative price deters consumption; a higher income increases consumption) but allow the econometric estimation to determine the 'average' strength of those relationships in the aggregate.
- Actors operate in an environment of radical uncertainty (Keynes 1921). Such uncertainty cannot be reduced to a known and well-behaved probability distribution in which the set of possible outcomes and the likelihood of each such outcome occurring are known with certainty.
- Actors nevertheless need to act on the basis of expectations about the future, but such expectations take the form of various heuristics to guide behaviour. Such expectations are not formally 'rational', in the sense of representing a mathematical expectation taken across the range of alternative outcomes given knowledge of the underlying model of the economy. Rather, the heterogeneity of actors extends also to their expectations and this insight can inform the modelling of behaviour when uncertainty is particularly important (for example, when an investment decision is being made for a long-lived asset).
- Any attempt to learn general lessons about the behaviour of the aggregate variables of interest from empirical analysis is necessarily at risk of a structural break due to a change in composition of the actors in the aggregate or due to the omission or inability to identify empirically an important explanatory variable (including, for example, a policy variable). The attempt to identify and estimate parameters that are invariant to such structural breaks by underpinning the aggregate explanation with strong theoretical microeconomic assumptions about the behaviour of a representative individual cannot be sustained when aggregation takes place over heterogeneous actors. Estimated empirical relationships for aggregate variables are therefore necessarily always work-in-progress. Evidence for structural breaks in the history can be reviewed using parameter stability tests and can shed light on the likelihood that such a break may apply in a future case.
- Prices, particularly at the level of industries and (aggregated) products do not move quickly to clear markets. In general, increasing marginal cost is not assumed, with the exception of the treatment of oil and gas extraction and the wage bargaining model of the labour market. Firms respond to unexpected changes in demand largely through quantity rather than price adjustments. In the longer term, in most industries prices are set to achieve a target profit margin above costs, although that margin may vary across industries depending on the degree of national and international competition.

Empirical approach

Equation sets are estimated for each country in the model (some 60 countries / regional blocs).

The econometric techniques used to specify the functional form of the equations are the concepts of cointegration and error-correction methodology, particularly as promoted by (Engle and Granger 2017) and (Hendry, Pagan and Sargan 1984). The process involves two stages. The first stage is a levels relationship, whereby an attempt is made to identify the existence of a cointegrating relationship between the chosen variables, selected on the basis of economic theory and a priori reasoning, e.g. for employment demand the list of variables contains real output, real wage costs, hours-worked, energy prices and the two measures of technological progress.

If a cointegrating relationship exists then the second stage regression is known as the error-correction representation, and involves a dynamic, first-difference, regression of all the variables from the first stage, along with lags of the dependent variable, lagged differences of the exogenous variables, and the error-correction term (the lagged residual from the first stage regression). Due to limitations of data size, however, only one lag of each variable is included in the second stage.

Stationarity tests on the residual from the levels equation are performed to check whether a cointegrating set is obtained. Due to the size of the model, the equations are estimated individually rather than through a cointegrating VAR. For both regressions, the estimation technique used is instrumental variables, principally because of the simultaneous nature of many of the relationships, e.g. wage, employment and price determination.

With regard to the nomenclature used, the parameters are given names that begin with B (as in 'beta'), followed by the name of the dependent variable. Individual elements of vectors, rows, columns or elements of matrices are denoted by replacing the dot by the appropriate number in the classification, e.g. BCR(.,11) is the 11th parameter in each region (the dot) in the disaggregated household consumption equation and relates to the long-term real income term.

Household consumption

The equation (one per country/region) relates (aggregate) household consumption to real personal disposable income, a measure of wealth for the sector, inflation and interest rates. Variables covering child and old-age dependency rates are also included in an attempt to capture any change in consumption patterns caused by an ageing population. The unemployment rate is used as a proxy for the degree of uncertainty in the economy and has been found to have significant effects on short-term consumption levels.

Due to a lack of available cross-country data on household wealth, cumulative investment in dwellings was used as a proxy for the housing stock (and there is no proxy for financial wealth). However, in line with other findings, E3ME's equations show only a modest link between household wealth and spending (very few studies find an elasticity greater than 0.1, and 0.02-0.03 is not uncommon).

Figure 3: Specification for aggregate household consumption spending

Co-integrating long-term equation:

LN(RSC)			[real consumers' expenditure]
	=	BRSC(11)	
	+	BRSC(12) * LN(RRPD)	[real gross disposable income]
	+	BRSC(13) * LN(RRLR)	[real rate of interest]
	+	BRSC(14) * LN(CDEP)	[child dependency ratio]
	+	BRSC(15) * LN(ODEP)	[OAP dependency ratio]
	+	BRSC(16) * LN(RVD)	[household wealth]
	+	ECM	[error]

Dynamic equation:

DLN(RSC)			[real consumers' expenditure]
	=	BRSC(1)	
	+	BRSC(2) * DLN(RRPD)	[real gross disposable income]
	+	BRSC(3) * DLN(RRLR)	[real rate of interest]
	+	BRSC(4) * DLN(CDEP)	[child dependency ratio]
	+	BRSC(5) * DLN(ODEP)	[OAP dependency ratio]
	+	BRSC(6) * DLN(RVD)	[household wealth]
	+	BRSC(7) * LN(RUNR)	[unemployment rate]
	+	BRSC(8) * DLN(RPSC)	[consumer price inflation]
	+	BRSC(9) * DLN(RSC(-1))	[lagged change in consumers' expenditure]
	+	BRSC(10) * ECM(-1)	[lagged error correction]

Identities:

RRLR	=	1 + (RLR–DLN(PRSC))/100	[real rate of interest]
RRPD	=	(RGDI*EX/PRSC)	[real gross disposable income]
CDEP, ODEP	=	CPOP/RPOP, OPOP/RPOP	[dependency ratios]

Restrictions:

BRSC(12) = 1	['life cycle hypothesis']
BRSC(2, 6, 16) >= 0	['right sign']
BRSC(3, 7, 8, 13) <= 0	['right sign']
0 > BRSC(10) > -1	['right sign']

Definitions

BRSC	is a matrix of parameters
RSC	is a vector of total consumers' expenditure for 61 regions, m euro at 2010 prices
RGDI	is a matrix of gross disposable income for 61 regions, m euro at current prices
RLR	is a matrix of long-run nominal interest rates for 61 regions
EX	is a vector of exchange rates, local currency per euro, 2010=1.0
RPOP	is a vector of regional population for 61 regions, in thousands of persons
CPOP	is a vector of child population for 61 regions, in thousands of persons
OPOP	is a vector of old-age population for 61 regions, in thousands of persons
RUNR	is a vector of unemployment rates for 61 regions, measured as a percentage of the labour force
PRSC	is a vector of consumer price deflator for 61 regions, 2010=1.0
RPSC	is a vector of consumer price inflation for 61 regions, in percentage terms
RVD	is the cumulative sum of investment in dwellings for 61 regions, m euro at 2010 prices

Figure 4: Specification for disaggregated household consumption spending

Co-integrating long-term equation:

LN(SHAR(.))			[consumers' budget share, logistic form]
	=	BCR(.,10)	
	+	BCR(.,11) * LN(RRPD)	[real gross disposable income]
	+	BCR(.,12) * LN(PRCR(.))	[relative price of consumption]
	+	BCR(.,13) * LN(RRLR)	[real rate of interest]
	+	BCR(.,14) * LN(PRSC)	[consumer price deflator]
	+	BCR(.,15) * LN(CDEP)	[child dependency ratio]
	+	BCR(.,16) * LN(ODEP)	[OAP dependency ratio]
	+	ECM	[error]

Dynamic equation:

DLN(SHAR(.))			[consumers' budget share, logistic form]
	=	BCR(.,1)	
	+	BCR(.,2) * DLN(RRPD)	[real gross disposable income]
	+	BCR(.,3) * DLN(PRCR(.))	[relative price of consumption]
	+	BCR(.,4) * DLN(RRLR)	[real rate of interest]
	+	BCR(.,5) * DLN(PRSC)	[consumer price deflator]
	+	BCR(.,6) * DLN(CDEP)	[child dependency ratio]
	+	BCR(.,7) * DLN(ODEP)	[OAP dependency ratio]
	+	BCR(.,8) * DLN(SHAR)(-1)	[lagged change in consumers' budget share]
	+	BCR(.,9) * ECM(-1)	[lagged error correction]

Identities:

SHAR	=	(VCR(.)/VCRT) / (1-(VCR(.)/VCRT))	[consumers' budget share, logistic form]
RRPD	=	(RGDI*EX/RPSC)/RPOP	[real gross disposable income]
PRCR	=	VCR(.)/CR(.)/PRSC	[real price of consumption]
RRLR	=	1+(RLR-DLN(PRSC))/100	[real rate of interest]
CDEP	=	CPOP/RPOP	[child dependency ratio]
ODEP	=	OPOP/RPOP	[OAP dependency ratio]

Restriction:

0 > BCR(.,9) > -1 ['right sign']

Definitions:

BCR	is a matrix of parameters
CR	is a matrix of consumers' expenditure for 43 commodities for 61 regions, m euro at 2010 prices
VCR	is a matrix of consumers' expenditure for 43 commodities for 61 regions, m euro at current prices
VCRT	is a vector of total consumers' expenditure for 61 regions, m euro at current prices
RGDI	is a matrix of gross disposable income for 61 regions, in m euro at current prices
RLR	is a matrix of long-run nominal interest rates for 61 regions
RPOP	is a vector of regional population for 61 regions, in thousands of persons
CPOP	is a vector of child population for 61 regions, in thousands of persons
OPOP	is a vector of old-age population for 61 regions, in thousands of persons
PRSC	is a vector of total consumer price deflator for 61 regions, in percentage terms
RPSC	is a vector of the real consumer price inflation for 61 regions, in percentage terms
EX	is a vector of exchange rates, local currency per euro, 2010=1.0

External trade

The modelling approach first models each country's imports by product, and then disaggregates by trade partner using a bilateral approach, similar in method to a Two Tier Armington model (Armington 1969). For EU Member States, trade within the EU is distinguished from trade external to the EU. The approach can be summarised in the following steps:

- solve the model equations for (total) imports of each product (split within and external to trading zones)
- solve the model equations for bilateral imports
- scale the bilateral trade results for consistency with the aggregate results
- derive total exports as the sum and inverse of bilateral imports

In the *aggregate* (across bilateral partners) *import equations,* activity is modelled by sales to the domestic market, while the three price effects are import price, price of sales to the domestic market and the relative price of the currency, i.e. the exchange rate. Aside from the restrictions on sign and significance, price homogeneity is imposed between the price of imports and price of sales to the domestic market. This has the effect of making the price relative, removing the long-term effect of the exchange rate variable. The technical progress measures are included to allow for the effects of innovations on trade performance. The specification of the internal and external import equations is very similar: in the internal imports equations there is an additional synthetic indicator for the development of the trading zone (e.g. the European single market). Figure 5 shows the specification for 'internal' (from within the EU) imports.

The *bilateral* trade data are defined for 43 products.. The other dimensions in the data are origin regions, destination regions and year (1995- latest year). Initial attempts were made to carry out time-series estimation at this level of detail (i.e. 53*53*43 equations) but this proved to be infeasible due to computation time and gaps in the data. The following adjustments were therefore made to the estimation procedure:

- the regions were aggregated to five global areas (Europe, US, China, North America and Rest of World)
- the 43 sectors were aggregated to 19
- only a levels based estimation was carried out

The equation specification allows the bilateral import share to be determined by export prices of the exporting region and technology in the exporting region. As the time series grow in length, additional explanatory factors (e.g. to take into account scale effects) will be added to the equation. The functional form will also be revisited.

The equations were estimated at the more aggregate level and then the parameters applied to each of the more disaggregated sectors. We plan to gradually expand the number of regions included in the estimation as the data are cleaned and further improved. Although the sectoral aggregation may seem quite severe, it has only a limited impact on the results because the sectors that are aggregated are principally non-traded ones (e.g. utilities, distribution/retail or public services) or thinly traded service sectors.

Given the results for bilateral imports, the model results for exports (either bilaterally or as a region's total) are relatively straightforward to derive; trade flows are reversed and aggregated to give regional totals. A further scaling 'calibration' exercise ensures that model outputs are consistent with historical figures for regional exports.

Figure 5: Specification for (aggregate) imports by product (internal)

Co-integrating long-term equation:

LN(QIM(.))			[internal import volume]
	=	BQIM(.,12)	
	+	BQIM(.,13) * LN(QRDI(.))	[home sales]
	+	BQIM(.,14) * LN(PQRM(.))	[import price]
	+	BQIM(.,15) * LN(PYH(.))	[price home sales by home producers]
	+	BQIM(.,16) * LN(EX)	[exchange rate]
	+	BQIM(.,17) * LN(YKNO(.))	[stock of knowledge]
	+	BQIM(.,18) * LN(YCAP(.))	[stock of capital]
	+	BQIM(.,19) * SVIM	[proxy for internal market programme]
	+	ECM	[error]

Dynamic equation:

DLN(QIM(.))			[change in internal import volume]
	=	BQIM(.,1)	
	+	BQIM(.,2) * DLN(QRDI(.))	[home sales]
	+	BQIM(.,3) * DLN(PQRM(.))	[import price]
	+	BQIM(.,4) * DLN(PYH(.))	[price home sales by home producers]
	+	BQIM(.,5) * DLN(EX)	[exchange rate]
	+	BQIM(.,6) * DLN(YKNO(.))	[stock of knowledge]
	+	BQIM(.,7) * DLN(YCAP(.))	[stock of capital]
	+	BQIM(.,8) * DSVIM	[proxy for internal market programme]
	+	BQIM(.,9) * LN(YYN(.))	[actual/normal output]
	+	BQIM(.,10) * DLN(QIM)(-1)	[lagged change in import volume]
	+	BQIM(.,11) * ECM(-1)	[lagged error correction]

Identity:

QRDI	=	QR(.) + QRM(.)	[home sales]
PYH	=	(VQR(.) - VQRX(.)) / (QR(.) - QRX(.))	[price home sales by home producers]

Restrictions:

BQIM(.,14) + BQIM(.,15) = 0	[price homogeneity]
BQIM(.,16) = BQIM(.,14) + BQIM(.,15)	[price and exchange rate symmetry]
BQIM(.,2 .,4 .,13 .,15) >= 0	[‘right sign’]
BQIM(.,3 .,5 .,6 .,7 .,14 .,16 .,17 .,18) <= 0	[‘right sign’]
0 > BQIM(.,11) > -1	[‘right sign’]

Definitions:

BQIM	is a matrix of parameters
QIM	is a matrix of internal imports for 69/43 industries and 61 regions, m euro at 2010 prices
PQRM	is a matrix of import prices for 69/43 industries and 61 regions, 2010=1.0, local currency
EX	is a vector of exchange rates, local currency per euro, 2010=1.0
QR	is a matrix of gross output for 69/43 industries and 61 regions, m euro at 2010 prices
QRM	is a matrix of imports for 69/43 industries and 61 regions, m euro at 2010 prices
QRX	is a matrix of exports for 69/43 industries and 61 regions, m euro at 2010 prices
YKNO	is a matrix of the knowledge stock for 69/43 industries and 61 regions, m euro at 2010 prices
YCAP	is a matrix of the capital stock for 69/43 industries and 61 regions, m euro at 2010 prices
YRKS	is a matrix of skills for 69/43 industries and 61 regions
SVIM	is an indicator of progress in the trade bloc
YYN	is a matrix of the ratio of gross output to normal output, for 69/43 industries and 61 regions
V-	is a matrix of parameters

Employment

The specification follows the work of (Lee, Pesaran and Pierse, Aggregation Bias in Labour Demand Equations for the UK Economy 1990) but also incorporates insights from the work on growth theory developed by (Scott 1989).

In the econometric representation in E3ME, employment is determined as a function of real output, real wage costs, hours worked, the oil import price (used as a proxy for energy prices) and measures of technological progress. This is shown in Figure 6. Industry prices are formed from sectoral unit costs and included in the wage term; higher energy prices within each sector therefore have a similar effect to reducing wage rates.

Figure 6: Specification for industry employment

Co-integrating long-term equation:

LN(YRE(.))			[total employment]
	=	BYRE(.,10)	
	+	BYRE(.,11) * LN(YR(.))	[real output]
	+	BYRE(.,12) * LN(LYLC(.))	[real wage costs]
	+	BYRE(.,13) * LN(YRH(.))	[hours worked]
	+	BYRE(.,14) * LN(PQRM(5,.))	[real oil price]
	+	BYRE(.,15) * LN(YKNO(.))	[stock of knowledge]
	+	BYRE(.,16) * LN(YCAP(.))	[stock of capital]
	+	ECM	[error]

Dynamic equation:

DLN(YRE(.))			[change in total employment]
	=	BYRE(,.1)	
	+	BYRE(,.2) * DLN(YR(.))	[real output]
	+	BYRE(,.3) * DLN(LYLC(.))	[real wage costs]
	+	BYRE(,.4) * DLN(YRH(.))	[hours worked]
	+	BYRE(,.5) * DLN(PQRM(5,.))	[real oil price]
	+	BYRE(,.6) * DLN(YKNO(.))	[stock of knowledge]
	+	BYRE(,.7) * DLN(YCAP(.))	[stock of capital]
	+	BYRE(,.8) * DLN(YRE)(-1)	[lagged change in employment]
	+	BYRE(,.9) * ECM(-1)	[lagged error correction]

Identity:

LYLC	=	(YRLC(.)/PYR(.)) / YREE(.)	[real labour costs]

Restrictions:

BYRE(.,2 .,11) >= 0	['right sign']
BYRE(.,3 .,4 .,12 .,13) <= 0	['right sign']
0 > BYRE(.,9) > -1	['right sign']

Definitions:

BYRE	is a matrix of parameters
YRE	is a matrix of total employment for 69/43 industries and 61 regions, in thousands of persons
YR	is a matrix of gross industry output for 69/43 industries and 61 regions, m euro at 2010 prices
YRH	is a matrix of average hours worked per week for 69/43 industries and 61 regions
YRLC	is a matrix of employer labour costs (wages plus imputed social security contributions) for 69/43 industries and 61 regions, local currency at current prices
YKNO	is a matrix of the knowledge stock for 69/43 industries and 61 regions, m euro at 2010 prices
YCAP	is a matrix of the capital stock for 69/43 industries and 61 regions, m euro at 2010 prices
PYR	is a matrix of industry output prices for 69/43 industries and 61 regions, 2010=1.0, local currency
YREE	is a matrix of wage and salary earners for 61 regions, in thousands of persons
PQRM	is a matrix of import prices for 69/43 industries and 61 regions, 2010=1.0, local currency
BYRE	is a matrix of parameters

Industry prices

The model of industry price formation implemented in Figure 7 was developed from (Lee, Labour Market Adjustment in a Disaggregated Model of the UK Supply Side 1988) having previously been derived from (Layard, Nickell and Jackman 1991).

The basis for price setting is a measure of unit costs, which is formed by summing material, labour and taxation costs, and dividing by industry output. Material costs are estimated using input-output coefficients and the relative prices in each sector that provides inputs. The degree to which cost increases are passed on in final product prices is determined by the level of competition in the sector.

Import prices of inputs to production are included in unit costs. Import prices for the industry's own product are added separately to allow international competition to influence domestic prices. In the long-term relationship, homogeneity is imposed between higher domestic and import cost effects, so that their combined impact is unitary. The equations also include the technology indices, as a higher quality product may command a higher price.

An important relationship in the short-term equation is the actual/normal output ratio. If actual output increases above expected/trend levels, this can cause prices to rise due to capacity constraints. However, if capacity increases (represented in the model by an increase in normal output) then prices can fall, leading to higher real incomes and economic growth.

Some sectors have a specific treatment of price and do not use the estimated equations. In particular, electricity prices have a special treatment which use the model's additional detail on electricity generation: prices are set by the average levelized cost of generation (so that higher-cost technologies lead directly to higher prices).

Figure 7: Specification for industry prices (domestic sales)

Co-integrating long-term equation:

LN(PYH(.))			[price of home sales by home producers]
	=	BPYH(.,9)	
	+	BPYH(.,10) * LN(YRUC(.))	[unit costs]
	+	BPYH(.,11) * LN(PQRM(.))	[import price]
	+	BPYH(.,12) * LN(YKNO(.))	[stock of knowledge]
	+	BPYH(.,13) * LN(YCAP(.))	[stock of capital]
	+	ECM	[error]

Dynamic equation:

DLN(PYH(.))			[change in price of home sales by home prods.]
	=	BPYH(.,1)	
	+	BPYH(.,2) * DLN(YRUC(.))	[unit costs]
	+	BPYH(.,3) * DLN(PQRM(.))	[import price]
	+	BPYH(.,4) * DLN(YKNO(.))	[stock of knowledge]
	+	BPYH(.,5) * DLN(YCAP(.))	[stock of capital]
	+	BPYH(.,6) * LN(YYN(.))	[actual/normal output]
	+	BPYH(.,7) * DLN(PYH)(-1)	[lagged change in price]
	+	BPYH(.,8) * ECM(-1)	[lagged error correction]

Identities:

PYH	=	(VQR(.) - VQRX(.)) / (QR(.) - QRX(.))	[price of home sales by home producers]
YRUC	=	YRUM(.) + YRUL(.) + YRUT(.)	[unit costs]
YRUM	=	SUM (QYC(.) * PQRD(.)) / YR(.)	[material input unit costs]
YRUL	=	YRLC(.) / YR(.)	[unit labour costs]
YRUT	=	YRT(.) / YR(.)	[unit tax costs]
PQRD	=	(VQR(.) + VQRM(.) – VQRX(.)) / (QR(.) + QRM(.) – QRX(.))	[price of sales to the domestic market]

Restrictions:

BPYH(.,10) + BPYH(.,11) = 1	[price homogeneity]
BPYH(.,2 .,3 .,4 .,5 .,6 .,10 .,11 .,12.,13) >= 0	['right sign']
0 > BPYH(.,8) > -1	['right sign']

Definitions:

BPYH	is a matrix of parameters
PQRM	is a matrix of import prices for 69/43 industries and 61 regions, m euro at 2010 prices
YR	is a matrix of gross industry output for 69/43 industries and 61 regions, m euro at 2010 prices
YKNO	is a matrix of the knowledge stock for 69/43 industries and 61 regions, m euro at 2010 prices
YCAP	is a matrix of the capital stock for 69/43 industries and 61 regions, m euro at 2010 prices
QR	is a matrix of gross output for 69/43 industries and 61 regions, m euro at 2010 prices
QRM	is a matrix of imports for 69/43 industries and 61 regions, m euro at 2010 prices
QRX	is a matrix of exports for 69/43 industries and 61 regions, m euro at 2010 prices
YYN	is a matrix of the ratio of gross output to normal output, for 69/43 industries and 61 regions
QYC	is an input-output coefficient matrix
YRLC	is a matrix of labour costs for 69/43 industries and 61 regions, local currency at current prices
YRT	is a matrix of net taxes for 69/43 industries and 61 regions, local currency at current prices
V-	indicates a current price version of the variable
BPYH	is a matrix of parameters

4. Innovation and endogenous technological progress

The conceptual framework for the way in which growth is driven by technological progress embodied in investment draws on (Scott 1989).

E3ME's technological indices

The approach to constructing the measure of technological progress in E3ME is originally adapted from that of (Lee, Pesaran and Pierse, Aggregation Bias in Labour Demand Equations for the UK Economy 1990). It has been further enhanced in the MONROE Horizon 2020[3] research project, which endogenised R&D expenditure in the model.

There are now two stock variables used in the equations. The first is the capital stock, which is accumulated investment, with a depreciation rate of 10%. The second is the knowledge stock, which is accumulated R&D expenditure, also with a depreciation rate of 10%.

Spillovers have also now been introduced to the model, based on patents data. Spillovers are treated as ‘virtual R&D’, i.e. as if the sector itself was carrying out the R&D. The indicator that is used in the accumulation of knowledge (YRDS) incorporates the spillover effects, both across regions and sectors.

Impacts of technological progress in the model

The measures of technological progress include both product and process innovation and this is represented in the various impacts on other parts of the model:

- a higher quality product could lead to higher levels of demand or command a higher price, so the technology indices feature in the model’s trade and price equations
- an improvement in efficiency increases capacity and potential supply, represented as ‘normal’ output in the model
- improved technological progress raises labour productivity, and so the technology indices feature in the employment equations

5. ‘Satellite accounts’ extensions - labour market and circular economy

Just as the national accounts can be extended to include non-financial indicators in so-called ‘satellite accounts’[4], the model can be extended with additional non-financial variables to incorporate important links commonly thought of as lying outside of the domain of economics.

Employment

Employment is, of course, included as an indicator in most economic models, and so it may not be considered a ‘satellite account’ concept, even though it lies outside the national accounts proper. Extensions using E3ME, for the EU countries, have involved disaggregating jobs by occupations and skill type (Cedefop and Eurofound 2018) in collaboration with colleagues from the Warwick Institute for Employment Research. E3ME also has a treatment of labour supply with labour force activity rates by gender and age-group.

[3] See https://www.monroeproject.eu/.
[4] See for example https://stats.oecd.org/glossary/detail.asp?ID=2385 and https://ec.europa.eu/eurostat/statistics-explained/index.php/Glossary:Satellite_account.

Energy and air emissions

E3ME includes the determination of indicators of energy use by different kinds of energy carriers (fuels) and different kinds of users (industries, but also for example in households and in the different transport modes). There is a detailed bottom-up treatment of the choice of technology in power generation using the 'Future Technology Transformations' approach (Mercure 2012) and a similar treatment of take-up of different kinds of energy using product for household heating (Knobloch, et al. 2017), and for road transport. For other types of fuel user, econometric equations are used to determine energy demand in aggregate and then disaggregated by type of fuel.

The feedbacks from the energy module assume a one-to-one correspondence between IEA energy balance data for fuel use in physical units and the entry for purchase of the relevant energy product in input-output tables. In practice there are often apparent inconsistencies between the two data sets and we make adjustments where necessary to deal with anomalies. Within the model, changes in fuel use are translated into changes in input-output coefficients to achieve consistency between the energy and economic indicators for energy use. There are also feedbacks from the energy module to household final demand.

Energy-related emissions to air (for a range of local and greenhouse gas pollutants) are determined by applying fixed coefficients to the use of each type of fuel.

Material consumption

The following material types are currently modelled:

- food
- feed
- forestry
- construction minerals
- industrial minerals
- ferrous ores
- non-ferrous ores

Compared with the fixed-coefficient input-output approach to modelling material consumption, E3ME follows an econometric treatment in which rates of material intensity are allowed to change in response to price and other economic factors. Material consumption (DMI per unit of output) is a function of economic activity, material prices and measures of technology.

6. Data

E3ME's historical database covers the period 1970-2016 and the model projects forward annually to 2050. The main data sources for European countries are Eurostat and the IEA, supplemented by the OECD's STAN database and other sources where appropriate. For regions outside Europe, additional sources for data include the UN, OECD, World Bank, IMF, ILO and national statistical agencies. Gaps in the data are estimated using customised software algorithms[5].

For data on national accounts, Eurostat data are used for European countries, combined with the OECD STAN data set for sectoral disaggregation. For non-European countries, the OECD STAN database is used as the primary data source, the Asian Development Bank has been used for

[5] CE has developed a software package to fill in gaps in any of the E3ME time series. This uses growth rates and shares between sectors and variables to estimate missing data points, both in cases of interpolation and extrapolation. See Section 3.3 of the E3ME technical manual at www.e3me.com

information on Asian countries, and for the remaining regions data have been collected from national sources.

The input-output tables in E3ME are derived where possible from Eurostat and OECD data. National sources have been used for remaining countries. All the input-output tables are expanded to the 70/43 E3ME sectors and moved to a base year of 2010 using RAS techniques[6]. The input-output tables include domestic production and imports. For projections, coefficients are based on logistic trends.

The primary data source for trade is COMTRADE for manufacturing sectors. Data for services are taken from the OECD for all member countries over the period 1995-2010 and expanded to include trade with non-OECD countries. The remaining values are estimated based on data that are available nationally and using share estimates. For projections, we hold shares fixed initially but allow them to vary in the economic equations.

Time-series data for CO2 emissions, disaggregated by energy user, are obtained from the EDGAR database. These are allocated to fuels using standard coefficients and then scaled to be consistent with the total. Non-CO2 emissions in E3ME include SO2, NOx, CO, methane (CH4), particulates (PM10 and PM2.5), volatile organic compounds (VOC), ammonia (NH3) and the other four greenhouse gases N2O, HFC, PFC, and SF6. These data are obtained from the EDGAR database. For projections, we use fixed coefficients.

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[6] The RAS method is a well-known method for data reconciliation. Its aim is to achieve consistency between the entries of some non-negative matrix and pre-specified row and column totals.

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