

## The 3D GRAS balancing method: applications to multiregional input-output frameworks

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The GRAS is a bi-proportional adjustment method commonly used among input-output practitioners for matrix balancing. It can deal with positive and negative elements [Gómez-Senesen and Bates (1988), Junius and Oosterhaven (2003)]. The literature describes several refinements, such as improving the target function in order to avoid biases [Lenzen, Wood and Gallego (2007), Huang, Kobayashi and Konji (2008) or Lemelin (2009)], dealing with rows and columns with no positive elements [Temurshoev, Miller and Bouwmeester (2013)] or allowing more flexibility to find a compromise solution between inconsistent constraints [(Lenzen, Gallego and Wood, 2009)].

A huge variety of applications benefits of using the GRAS. However, sometimes rather than simply imposing constraints summing row-wise or column-wise all the elements of a matrix, it is necessary to split the matrix into sub-blocks and to impose also constraints on the totals of the sub-blocks. Constraints on a cell-wise sum of elements located in different sub-blocks of the initial matrix might be also necessary. This is the situation in the balancing of multiregional input-output frameworks, where the structure of data and its relationships hampers the direct application of the bi-proportional GRAS method: for instance, to balance a bilateral trade database of exports and imports trying to match not only the bilateral trade values but also overall trade estimates coming from national accounts, such as the use table of imports by countries.

Our method is a tri-proportional extension of the bi-proportional GRAS method for balancing a cube (tri-dimensional matrix). We show that this method is intimately connected to the problem described above of balancing a bilateral trade matrix; it is suited for solving it and can be applied to many similar situations in this multiregional context.