# Transforming a Rectangular Input-Output Model into the Coordinates with Respect to Eigenbasis

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A rectangular input-output table with N products and M industries is defined by production matrix X and intermediate consumption matrix Z with N rows and M columns both. The square matrices FF'= (Xâ€“Z)(Xâ€“Z)' of order N and F'F=(Xâ€“Z)'(Xâ€“Z) of order M are symmetric and have the same spectrum of nonzero real eigenvalues.
The eigenvectors of matrix FF' form an orthonormal basis of N-dimensional vector space that could be considered as eigenbasis for rectangular input-output model at N>M (i.e., the number of products exceeds the number of industries as it often happens in statistical practice). Being transformed with respect to the eigenbasis, matrices X and Z have Nâ€“M lower rows coincided between each other (with zero final demand for the last Nâ€“M products). This property allows employing rectangular input-output table written in the coordinates with respect to the eigenvectors of matrix FF' as operational demand-driven input-output model in which M components of final demand are exogenous variables and the other Nâ€“M components are set to zero.
In turn, the eigenvectors of matrix F'F make an orthonormal basis of M-dimensional vector space that could serve as eigenbasis for rectangular input-output model at M>N (i.e., the number of industries exceeds the number of products). Being transformed with respect to this basis, matrices X and Z have Mâ€“N right columns coincided between each other (with zero value added in the last Mâ€“N industries). Thus, rectangular input-output table written in the coordinates with respect to the eigenbasis can be used as operational supply-driven input-output model in which N components of value added are exogenous variables and the other Mâ€“N components are set to zero.
The analytical opportunities of practical applying the proposed models are slightly limited because of explicit shortage of exogenous variables. Nevertheless, It is shown that the models appear to be a useful additional toolbox to regular computational schemes of input-output analysis. Their main advantage is direct handling the initial rectangular input-output table without obvious data distortion being entailed by transformations the table to symmetric format under various assumptions.