AN INPUT-OUTPUT BASED METHODOLOGY FOR CALCULATING THE IMPACT OF FINAL DEMAND CHANGES BY DEMANDED PRODUCTS AT PURCHASERS' PRICES

(FIRST DRAFT PAPER – TO BE COMPLETED!)

Ana Maria Dias (ana.dias@sg.mate.gov.pt)

Prospective and Planning Services, General Secretary of the Ministry of Environment and Energy Transition, Portugal

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Abstract

Input-Output manuals normally present the calculation of output, primary inputs, employment and joint products (*eg.* pollution variables) multipliers using the so-called Leontief Inverse, pre-multiplied by the respective vector (or matrix) of coefficients (per unit of output). However, this method only estimates the impact of a unit change of final demand for each product if this demand is totally addressed to domestic output and expressed at basic prices, which is far from being realistic. In fact, the direct import content of final demand has been increasing over time and it is relatively high in small open economies. On the other hand, trade (and transport) margins and taxes on products also represent often an important (direct) share of the value of final demand.

The methodology presented in this paper incorporates matrices for direct unit contents of imports, domestic output and taxes on products, as well as trade and transport margin rates in order to estimate the direct and indirect effect of a change in final demand for each product and demand category, at purchasers' prices and considering a direct import content for this demand. Its application requires the existence of a system of input-output tables (for domestic output, imports, taxes, subsidies and trade and transport margins). However, an alternative version of this methodology was also conceived and is presented in this paper for the case when only I-O tables at basic prices are available.

This methodology was first developed and applied to the Portuguese economy by the author in 2008 and it was presented in a paper to the 19th IIOC (Alexandria, USA, 2011). This methodology started also to be applied by the Portuguese Statistical Office in 2017 (to the input-output matrices for 2013 and, more recently, to 2015 matrices). This paper presents an updated and extended version of the methodology, comparing it with the traditional (abovementioned) methodology, showing its advantages for the impact evaluation of final demand changes in each demanded product or for total final demand when there is a change in its structure.

1. INTRODUCTION

The purpose of this paper is to present an updated and improved version of the methodology (developed by the author) to calculate primary input unit contents of final demand at purchasers' prices by demanded products and categories of demand and to compare it with the, traditionally used, primary input multipliers.

The first version of this methodology was described (and applied to the Portuguese economy for 2005) in Dias (2010) and in a paper (Dias, 2011) presented to the 19th International Input-Output Conference (Alexandria, Virginia, USA, June 2011). In 2016 this methodology was improved and updated and it was applied to Portuguese data for 2008 (Dias, 2016).

Since this methodology requires the existence of data for trade and transport margins (disaggregated by trade and transport supplying sectors), as well as of taxes and subsidies, applied to each product, by final demand category, and alternative version of the methodology is also presented in this paper, when such an information is not available, applicable to final demand at basic prices.

Section 2 describes the main methodology, starting with the traditional output and primary input multipliers (which are components of the formulas used) and evidencing the advantages of it compared to the traditional approach.

Section 3 presents the alternative version for the methodology, applicable to final demand at basic prices, showing its advantages compared to the simple use of primary input multipliers and its shortcomings compared to the main methodology (applied to final demand at purchasers' prices).

Section 4 presents some examples of comparison of the different methodologies (using Portuguese data) to evaluate the impact of a change in final demand on total economy and on the various economic sectors. Finally, section 5 presents some concluding remarks.

Methodological details are presented in Appendix 1.

2. METHODOLOGY DESCRIPTION

2.1 General features

Final demand for each product can be satisfied either by domestic output or by imports, generating also taxes and subsidies on the demanded products (direct effects).

This domestic output can itself be decomposed into domestically produced intermediate inputs and the so-called primary inputs (because they are not domestically produced): imported inputs, value added, taxes and subsidies on inputs.

Domestically produced inputs can, again, be decomposed into intermediate and primary inputs, so that, at the end, the value of final demand can be totally decomposed into these primary inputs: imports (direct and indirect), value added (and its components), taxes and subsidies on products (direct and indirect contents).

2.2 Output and primary input multipliers

The core of the quantity Leontief input-output model is a matrix known as the **Leontief inverse,** that we will designate by B:

$B = (I-AN)^{-1}$ - Matrix of output multipliers

The element of order (i,j) of this matrix (b_{ij}) represents the quantity of domestic output of product i necessary to satisfy one unit of final demand for domestically produced product j. Therefore, this matrix is also known as the matrix of output multipliers.

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APPENDIX 1

METHODOLOGICAL DETAILS

1. GENERAL FEATURES

The calculation of **primary input contents of final demand at purchasers' prices** can be made using the following symmetric input-output tables:

- FT: Total Flows at purchasers' prices;
- PN: Domestic Output;
- M : Imports CIF;
- TS: Taxes, net of Subsidies, on products;
- MCi: Trade Margins of type i, for i= trade sectors;
- MTNi: Transport Margins of type i, satisfied by domestic output;
- MTMi: Transport Margins of type i, satisfied by imports.

The matrix of Total Flows at purchasers' prices is equal to the sum of all the other matrices:

(1) FT = PN+M+TS+MC+MTN+MTM

where:

MC = $\sum$ MCi for i = trade sectors;

MTN = $\sum$ MTNi for i = transport sectors (land and water);

MTM = $\sum$ MTMi for i = transport sectors (land and water).

Elements of order (i,j) and (i,F) of each of the abovementioned matrices (MAT_{ij} e MAT_{iF} , using MAT as a generic designation of those matrices), represent, respectively, intermediate consumption of product i by the homogeneous branch j and final demand of category F for product i.

Let X_j represent domestic output of product j and Ftot represent total final demand of type F.

Technical coefficients are calculated using the following formulas:

$a_{ij} = FT_{ij}/X_j$ Total technical coefficient of order (i, j), representing the quantity of product *i* (at purchasers' prices) necessary to produce one unit of product *j* (at basic prices);

$a_{iF} = FT_{iF}/Ftot$ Share of product *i* (at purchasers' prices) in total final demand of type F (at purchasers' prices);

$an_{ij} = PN_{ij}/X_j$ Quantity of domestically produced good *i* (at basic prices) used to produce one unit of product *j* (at basic prices);

$an_{iF} = PN_{iF}/Ftot$ Share of domestically produced good *i* (at basic prices) in total final demand of type F (at purchasers' prices);

$am_{ij} = M_{ij}/X_j$ Quantity of imported product *i* (CIF) used to produce one unit of product *j* (at basic prices);

$am_{iF} = M_{iF}/Ftot$ Share of imported product good *i* (CIF) in total final demand of type F (at purchasers' prices);

$ats_{ij} = TS_{ij}/X_j$ Taxes on products (net of subsidies) included in the input of product *i* necessary to produce one unit of product j;

$ats_{iF} = TS_{iF}/Ftot$ Share of taxes on products (net of subsidies) paid for product *i* in total final demand of type F (at purchasers' prices);

$av_j = VAB_j/X_j$ Product transformation coefficient for product *j* (share of GrossValue Added in the value of domestic output of product *j* at basic prices);

From these calculations we create the following square matrices of coefficients (n×n), for n = number of products/sectors considered, identified with the corresponding generic element of order (i,j):

- $A = [a_{ij}]$ - Matrix of total technical vertical coefficients;
- $AN = [an_{ij}]$ - Matrix of vertical coefficients for domestically produced inputs;
- $AM = [am_{ij}]$ - Matrix of vertical coefficients for imported inputs;
- $ATS = [ats_{ij}]$ - Matrix of vertical coefficients for taxes (net of subsidies) on inputs;
- diag(AV) - Diagonal matrix for value added (av_j) coefficients;
- diag(F) - Diagonal matrix for Final Demand of category F (total flows) by products, at purchasers' prices (see definition of F below).

The following column vectors (n×1) are also defined:

- $X = [X_i]$ – Domestic Output by products, at basic prices;
- $F = [FT_{iF}]$ - Final Demand of category F (total flows) by products, at purchasers' prices;
- $FN = [PN_{iF}]$ - Final Demand, of type F, for domestically produced goods, by products, at basic prices;
- $FM = [M_{iF}]$ - Final Demand, of type F, for imported goods (CIF), by products;
- $FTS = [TS_{iF}]$ – Taxes, net of subsidies, on Final Demand of type F, by products;
- $VAB = [VAB_j]$ - Gross Value Added by sectors (products);
- $MT = [M_i]$ - Total Imports (CIF) by products;
- $TST = [TS_i]$ - Total taxes, net of subsidies, applying to each product;
- $AF = [a_{iF}]$ - Structure of distribution, by products, of final demand of type F.

2. DIRECT CONTENTS

We can define the following square matrices (n×n) representing direct unit contents of final demand in domestic output (QNF), imports (QMF) and taxes (net of subsidies) on products (QTSF), for the various types of final demand. These matrices are calculated in order to meet the following identities:

(2) $FN = QNF \times F$

(3) $FM = QMF \times F$

(4) $FTS = QTSF \times F$

Each element of these matrices, qnf_{ij}, qmf_{ij}, $qtsf_{ij}$, represents, respectively, domestic output, imports and taxes (net of subidies) regarding product i, for each unit of final demand, of type F, for product j (direct contents).

Note that QNF and QMF are not diagonal matrices because, for each demanded product (with demand evaluated at purchasers' prices), there is (for most of the products) a direct content of trade and transport margins, the supply of which is made by the trade and transport sectors (off-diagonal).

Therefore, the elements of QNF were calculated as follows:

$qnf_{ii} = PN_{iF}/FT_{iF}$ for $i \neq$ trade and transport (land and water) sectors

$qnf_{ij} = 0$ for $i \neq j$ and $i \neq$ trade and transport (land and water) sectors

$qnf_{ii} = (PN_{iF} + MCi_{iF})/FT_{iF}$ for i = trade sectors (contents of domestic output of final demand addressed to trade sectors which do not correspond to trade margins[1])

$qnf_{ij} = MCi_{jF}/FT_{jF}$ (trade margin rate, of type i, on final demand of type F, for product j), for $i \neq j$ and i = trade sectors

$qnf_{ii} = (PN_{iF} + MTNi_{iF})/FT_{iF}$ for i = transport sectors (land and water) (contents of domestic output of final demand addressed to land and water transport sectors which do not correspond to transport margins[2])

$qnf_{ij} = MTNi_{jF}/FT_{jF}$ (transport margin rate, of type i, satisfied by domestic output, on final demand of type F, for product j), for $i \neq j$ and i = transport sectors (land and water).

In the same way, considering that part of the imports of land and water transport servives result from transport margins (satisfied by imports) applied to products of the remaining, the elements of QMF were calculated as follows:

$qmf_{ii} = M_{iF}/FT_{iF}$ for $i \neq$ transport (land and water) sectors

$qmf_{ij} = 0$ for $i \neq j$ and $i \neq$ transport (land and water) sectors

$qmf_{ii} = (M_{iF} + MTMi_{iF})/FT_{iF}$ for i = transport sectors (contents of imports of final demand addressed to land and water transport sectors which do not correspond to imported transport margins[3])

$qmf_{ij} = MTMi_{jF}/FT_{jF}$ (transport margin rate, of type i, satisfied by imports, on final demand of type F, for product j), for $i \neq j$ and i = transport sectors (land and water).

QTSF (the matrix of direct unit contents in taxes, net of subsidies, on products, for final demand of type F) is a diagonal matrix which elements of the principal diagonal were calculated as follows:

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[1] Note that MCi_{iF} and $MTNi_{iF}$ have negative values when i = trade/transport sectors, which are equal to the symmetric of the total value of the respective margins applied to the various products (*vide* Dias, 2009, page 4, 3rd paragraph). Therefore the sums $(PN_{iF} + MCi_{iF})$ and $(PN_{iF} + MTNi_{iF})$ represent the part of sector i's domestic output that does not correspond to margins of type i.

[2] *Vide* previous note.

[3] Note that $MTMi_{iF}$ has a negative value when i = land and water transport sectors, which is equal to the symmetric of the total value of the respective transport margins (satisfied by imports) applied to the various products (*vide* Dias, 2009, page 4, 3rd paragraph). Therefore the sum $(M_{iF} + MTM_{iF})$ represents the part of sector i's imports that does not correspond to imported transport margins of type i.

$qtsf_{ii} = TS_{iF}/FT_{iF}$

Total direct unit contents in domestic output, imports and taxes (net of subsidies) on products, regarding final demand of type F, for product j, called, respectively, qnf_j, qmf_j and $qtsf_j$, are given by:

$$(5)\ qnf_j = \sum_i qnf_{ij} = (PN_{jF} + MC_{jF} + MTN_{jF})/FT_{jF}$$

$$(6)\ qmf_j = \sum_i qmf_{ij} = (M_{jF} + MTM_{jF})/FT_{jF}$$

$$(7)\ qtsf_j = \sum_i qtsf_{ij} = TS_{jF} / FT_{jF}$$

Given identity (1), we verify that the sum of these three direct contents is equal to one, for each demanded product and type of demand:

$$(8)\ qnf_j + qmf_j + qtsf_j = (PN_{jF} + MC_{jF} + MTN_{jF} + M_{jF} + MTM_{jF} + TS_{jF})/FT_{jF} = FT_{jF}/FT_{jF} = 1$$

3. INDIRECT AND TOTAL CONTENTS

Product i's domestic output may be used as an intermediate input by the various domestic productive sectors and also to satisfy the various types of final demand for domestically produced good i ($FN_{i,}$), which may be expressed by the following equation:

$$(9)\ X_i = \sum_j an_{ij} \times X_j + \sum_F FN_i$$

Expressing this equation in matrix notation yields:

$$(10)\ X = AN \times X + \sum_F FN$$

from which we can deduct that (with I representing the *unit* or *identity* matrix):

$$(11)\ X = (I-AN)^{-1} \times (\sum_F FN) = \sum_F [(I-AN)^{-1} \times FN]$$

Matrix $(I\text{-}AN)^{-1}$ is the so-called Leontief inverse. The element of order (i,j) of this matrix (b_{ij}) represents the quantity of domestic output of product i necessary to satisfy one unit of final demand for domestically produced product j. Therefore, this matrix is also known as the matrix of output multipliers.

Combining equations (2) and (11) we can write the equation for X as a function of total final demand:

$$(12)\ X = \sum_F [(I-AN)^{-1} \times QNF] \times F$$

Given that :

$$(13)\ VAB = diag(AV) \times X$$

We have, combining (12) with (13):

$$(14)\ VAB = \sum_F [diag(AV) \times (I-AN)^{-1} \times QNF] \times F$$

Matrix (n×n) resulting from the operations inside straight brackets in (14) represents total unit contents of Gross Value Added for final demand of type F. Element of order (i,j) of this matrix represents Gross Value Added generated in sector i per unit of final demand, of type F, for product j.

Imports of each product are made in order to satisfy intermediate and final demand, which may be expressed in the following equation:

$$(15)\quad M_i = \sum_j am_{ij} \times X_j + \sum_F FM_i$$

In matrix notation we have:

$$(16)\quad MT = AM \times X + \sum_F FM$$

Combining (16) with (3) and (12) we have:

$$(17)\quad MT = \sum_F [AM \times (I-AN)^{-1} \times QNF + QMF] \times F$$

Matrix (n×n) resulting from the operations inside straight brackets in (17) represents unit contents of Imports (direct+indirect) for final demand of type F. Element of order (i,j) of this matrix represents imports of product i, per unit of final demand, of type F, for product j. This matrix has two components, representing, respectively direct (QMF) and indirect $[AM \times (I\text{-}AN)^{-1} \times QNF]$ unit contents of imports.

Concerning taxes, net of subsidies, on products, we have the following matrix equation:

$$(18)\quad TST = ATS \times X + \sum_F FTS$$

Combining (18) with (4) and (12) we have:

$$(19)\quad TST = \sum_F [ATS \times (I-AN)^{-1} \times QNF + QTS] \times F$$

Matrix (n×n) resulting from the operations inside straight brackets in (19) represents unit contents of taxes (net of subsidies) on products (direct+indirect) for final demand of type F. Element of order (i,j) of this matrix represents net taxes on product i, per unit of final demand, of type F, for product j. This matrix has two components, representing, respectively direct (QTS) and indirect $[ATS \times (I\text{-}AN)^{-1} \times QNF]$ unit contents of taxes (net of subsidies) on products.

4. SUMMARY

4.1. Matrices of unit contents of final demand: UC (n×n):

Table 1 presents a summary of matrices (n×n) of direct, indirect and total contents of final demand of type F at purchasers' prices, by types of contents. The element of order (i,j) of each one of these matrices (generically denoted by UC), uc_{ij}, represents a certain type of content (imports, taxes, domestic output, GVA), direct, indirect or total, of product i, per unit of final demand of type F, addressed to product j.

Table 1 – Matrices of unit contents of final demand of category F, at purchasers' prices

Type of contents :	Direct	Indirect	Total
Imports	QMF	$AM\ (I-AN)^{-1}\ QNF$	$QMF+AM\ (I-AN)^{-1}\ QNF$
Taxes, net of subsidies, on products	QTSF	$ATS\times(I-AN)^{-1}\times QNF$	$QTSF+ ATS\ (I-AN)^{-1}\ QNF$
Domestic Output	QNF		
Gross Value Added (GVA)			$diag(AV)\ (I-AN)^{-1}\ QNF$

4.2 Vectors of unit contents for all products, by demanded product: UCT (1×n)

When we calculate these matrices' column sums, we obtain row-vectors, generically denoted by UCT (1×n), which elements of order j represent contents in all (produced/imported) goods, per unit of final demand (of type F), for product j. Denoting a column vector (n×1), with all elements equal to 1, by **i** and by **i'** its transposed (row vector) we have, in matrix notation:

(20) $UCT = \mathbf{i}'\ UC$

Given that the generic elements of order j of $\mathbf{i}'\times QNF$, $\mathbf{i}'\times QMF$ e $\mathbf{i}'\times QTSF$ are, respectively, qnf_j, qmf_j e qts_j, previously defined by equations (5), (6) and (7), and given the identity presented in (8), we verify that the sum of these three vectors is equal to a row vector with all elements equal to 1 ($\mathbf{i}'$):

(21) $QNFT+QMFT+QTSFT = \mathbf{i}'QNF + \mathbf{i}'QMF + \mathbf{i}'QTSF = \mathbf{i}'$

On the other hand, given that, for each sector j, the sum of all inputs per unit produced (intermediate and GVA) is equal to 1:

(22) $\sum_i an_{ij} + \sum_i am_{ij} + \sum_i ats_{ij} + av_j = 1$

or, equivalently:

(23) $\sum_i am_{ij} + \sum_i ats_{ij} + av_j = 1 - \sum_i an_{ij}$

we verify that the sum of the row-vectors of indirect contents of imports and of taxes (net of subsidies) on products with the row-vector for total GVA contents is equal to the row-vector for direct contents of domestic output. In fact, it follows from (23) that:

(24) $\mathbf{i}'\times[AM+ATS+diag(AV)] = \mathbf{i}'\times(I-AN)$

Therefore we can deduct that:

(25) $\mathbf{i}'[AM\times(I-AN)^{-1}\ QNF]+\mathbf{i}'[ATS\ (I-AN)^{-1}\ QNF]+\mathbf{i}'[diag(AV)(I-AN)^{-1}\ QNF] =$

$= \mathbf{i}'\times[AM+ATS+diag(AV)](I-AN)^{-1}\ QNF = \mathbf{i}'(I-AN)(I-AN)^{-1}\ QNF = \mathbf{i}'\times QNF$

Combining the results of equations (21) and (25) we conclude that **the sum of total unit contents of imports, taxes (net of subsidies) on products and value added** for each category of final demand and demanded product **is equal to 1**. Additionally, we defined GDP contents of final demand by demanded products as the sum of value added contents with total contents of taxes (net of subsidies) on products. Therefore, **GDP unit contents are equal to 1 minus total import contents**, for final demand of each demanded product.

4.3. Unit contents by products (and for all products), per composite unit of final demand

We can also calculate vectors of contents, by (produced/imported) products, of a composite unit of final demand, *i.e*., for each unit of final demand containing a combination of various demanded products, considering, for example, the observed structure of final demand from National Accounts. The column-vector (n×1), for each type of contents, obtained from UC, which will be denoted by UCC, is obtained by the formula:

(26) $UCC = UC \times AF$

where AF is a column-vector (n×1) representing the structure, by demanded products, of final demand of category F.

Finally, we can calculate scalars (1×1) representing the contents, in all produced/imported goods, per composite unit of final demand F (UCTC), through the formula:

(27) $UCTC = \mathbf{i}' \times UC \times AF = \mathbf{i}' \times UCC$

4.4. Contents in value (CV)

Multiplying unit contents by the respective value of (observed, projected or simulated) final demand (expressed, for example, in million euros), we obtain the value of these contents in the same monetary units. In matrix notation, we can obtain CV (n×n) matrices, for each type of unit content and final demand, through the formula:

(28) $CV = UC \times \text{diag}(F)$

5. CALCULATION OF PRIMARY INPUT CONTENTS OF FINAL DEMAND AT BASIC PRICES

5.1 General features

The calculation of **primary input contents of final demand at basic prices** can be made using the following input-output tables:

- FTB: Total Flows at basic prices;
- PN: Domestic Production;
- M : Imports CIF;

The matrix of Total Flows at basic prices is equal to the sum of Domestic Output and Imports matrices:

(1b) $FTB = PN + M$

With these I-O tables we calculate the following additional **technical coefficients**:

$atst_j = TST_j/X_j$ Share of total taxes (net of subsidies) on inputs used to produce product j, in the value of domestic output of product *j* (at basic prices);

$amt_j = MT_j/X_j$ Share of total imported inputs(CIF) used to produce product j, in the value of domestic output of product *j* (at basic prices).

Using these coefficients we define the following vectors and matrices:

- ATST = [$atst_j$] - row vector (1×n) for coefficients of taxes (net of subsidies) on inputs;
- AMT= [amt_j] - row vector (1×n) for coefficients of imported inputs;
- diag (ATST) – diagonal matrix (n×n) with the elements of ATST in the main diagonal;
- diag (AMT) – diagonal matrix (n×n) with the elements of AMT in the main diagonal.

We also define the following column vector (n×1):

- FB = [FTB_{iF}] - Final Demand of category F (total flows) by products, at basic prices.

5.2 Direct contents

In this case we define the following square matrices (n×n) representing direct unit contents of final demand (total flows at basic prices) for domestic output (QNFB) and imports (QMFB), for the various types of final demand. These matrices are calculated in order to meet the following identities:

(2b) $FN = QNFB \times FB$

(3b) $FM = QMFB \times FB$

QNFB and QMFB are diagonal matrices with the respective diagonal elements representing the shares of domestic output (at basic prices) and of imports (CIF), on final demand for the corresponding product i, calculated as follows:

$qnfb_{ii} = PN_{iF}/FTB_{iF}$

$qmfb_{ii} = M_{iF}/FTB_{iF}$

Note that, in this case, $qnfb_{ii} + qmfb_{ii} = 1$, and, therefore:

$QNFB + QMFB = I$ (with I representing the *unit* or *identity* matrix) and

$FN+FM=FB$

5.3 Indirect and total contents and summary table

Following steps similar to those described in section 3 for the calculation of primary inputs contents of final demand at purchaser's prices, we can derive the formulas for value added contents and for indirect contents of imports and of taxes (net of subsidies).

Table 2 presents a summary of matrices (n×n) of direct, indirect and total contents of final demand of type F at basic prices, by types of contents. The element of order (i,j) of each one of these matrices (generically denoted by UC), uc_{ij}, represents a certain type of content (imports, taxes, domestic output, GVA), direct, indirect or total, of product i, per unit of final demand of type F, addressed to product j.

Table 2 – Matrices (n×n) **of unit contents of final demand of category F, at basic prices**

Type of contents :	**Direct**	**Indirect**	**Total**
Imports	QMFB	$diag(AMT)(I\text{-}AN)^{-1}$ QNFB	QMFB+ $diag(AMT)\ (I\text{-}AN)^{-1}$ QNF
Taxes, net of subsidies, on products	-	$diag(ATST)\ (I\text{-}AN)^{-1}$ QNFB	$diag(ATST)\ (I\text{-}AN)^{-1}$ QNFB
Domestic Output	QNFB		
Gross Value Added (GVA)			$diag(AV)\ (I\text{-}AN)^{-1}$ QNFB