INTERRELATION OF DEVELOPMENT OF TOOLS AND ANALYTICAL CAPABILITIES OF THE INPUT-OUTPUT METHOD[1]

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The tools of the input-output method in the article refers to its mathematical and statistical apparatus. The relationship between tools improvements and input-output analytical capabilities is shown in the case of endogenization options for final demand elements in the input-output model. Comparative analysis of endogenization methods is accompanied by calculations based on the regional input-output tables.

Keywords: open and closed models, endogenization, regional input-output models

Introduction. Input-output models do not belong to the category of economic and mathematical models, the development of which is carried out mainly on the basis of improving the mathematical apparatus, i.e. aimed at "modeling for the sake of modeling." The input-output model is the result of a generalization of statistical material processed as balance tables. Writing these tables in the form of a mathematical model allowed the IO method not only to rise to a new analytical level, but also to become a predictive tool. The improvement of the mathematical and statistical apparatus in the process of developing the method made it possible to solve new problems of macrostructural analysis and forecasting, to give quantitative certainty to many theoretical ideas about structural shifts and economic growth. Thus, the mathematical toolkit of the method at the dawn of its formation made it possible to obtain total requirement coefficients that have deep economic meaning, due to the determination of national economic costs necessary to obtain a final product. Based on the development and improvement of the mathematical apparatus and the economic content of the model, it was possible to obtain a quantitative assessment of the intersectoral dependence of prices. In modern conditions of globalization, it was the development of the mathematical tools and statistical base of the method that made it possible to obtain a quantitative assessment of the participation of national economies in global value added chains. There is hardly a need to continue the list of analogous examples: the entire history of research in the field of "input-output" is a sample of mutually complementary processes for improving the tools and expanding the analytical capabilities of the method.

Taking into account the above, in the opinion of the author of the article, a fairly detailed presentation of mathematical calculations concerning the methods of endogenization of elements of final demand in the input-output model, undoubtedly having applied significance, is permissible. The correct mapping of the mechanisms of interaction between income and demand of economic agents within the framework of the input-output model, implemented through the endogenization of elements of final demand, is a necessary element in assessing the prospects for economic dynamics. The comparative analysis of the methods of endogenization of elements of final demand (or "closure" of the input-output model by elements of final demand), performed in the article, is directly related to the search for directions for intensifying economic growth through an increase in demand. The endogenization of the final demand elements in the input-output model, i.e. the evaluation of the final demand elements as a result of model calculations, allows to take into account not only the cyclical effects of the initial impulses, but also more subtle moments in the mechanisms of the relationship between income and demand. For example, the very structure of household consumption is a function of differentiation of household incomes, which at the same time also determines the size of the multiplier effect of population income growth. Of no less interest is the endogenization of the final demand of other sectors, for example, federal or regional government bodies, and the measurement of the degree of influence of government spending growth on the level and dynamics of GDP. The correct mapping of the dependence of income and the final demand of economic agents, which is translated into the input-output model constructs as the dependence of the elements of the second and third quadrants, is one of the conditions for conducting reasonable macrostructural calculations.

This article traces the traditional interpretation of the concept of "endogenous and exogenous variables" of the economic and mathematical model. Variables whose values are given as scenario conditions for calculations using the model in question are considered exogenous (or "entries") for the model. Variables whose values are determined by calculations under given scenario conditions are called endogenous (or "outlets"). As a rule, there is no rigid fixation of endo- or exogenicity of variables in models: depending on the problem being solved, variables can act in both capacities.

Regarding the input-output model, we note that classical task statements assume: 1) exogenicity of the output vector and endogenicity of the final demand vector; 2) exogenicity of the final demand vector and endogenicity of the output vector; 3) the exogenicity of individual elements of the final demand vector and the remaining elements of the output vector and, accordingly, the endogenicity of unknown elements of the final demand vectors and outputs (the so-called "mixed" setting of the problem). In the context of the goal set in the article, the second setting of the task is considered. It is meant that despite the exogenous assignment of the final demand vector, the so-called "endogenization" of individual functional elements of the final demand, for example, final household consumption, public administration expenditures or gross accumulation, is possible. The economic meaning of endogenizing final demand elements is to simultaneously evaluate balanced values of output vectors, endogenizable final demand element and other

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final demand. By other finite demand is meant a vector obtained as a difference between the finite demand vector of the classical input-output model and the vector of the endogenizable element of the finite demand.

Approaches to endogenizing functional elements of final demand. Methods for endogenizing functional elements of final demand are based on two different approaches. *The first approach* involves "embedding" ratios in the classic "input-output" model, establishing the dependence of the endogenizable element of final demand on output volumes. Technically, the use of such a model boils down to solving a system of n equations regarding n unknown output values (n is the number of sectors) with a given vector of other final demand. As a rule, such calculations are performed in a dynamized version. For example, for the input-output model with the investment block, the dependence of the yearly volumes of gross accumulation for the entire forecast period on the corresponding sector outputs is established. In such a model, at a given value of the vector of other final demand (for this case, better known as "net final demand"), the corresponding yearly values of the volumes of sector outputs and the yearly provision of gross accumulation by the fund-creating industries are determined. To endogenize the final household consumption vector, a relationship is established between it and elements of the third quadrant, in particular, based on industry demand functions. Through the shares of elements of the third quadrant in the outputs, said dependence eventually becomes the dependence of the industry consumption of households on the outputs. Calculations according to such a model, as a rule, are also carried out in a dynamized version: the growth of final demand imposes appropriate requirements for the output of sectors - outputs also grow, and the incomes of the population increase accordingly. The increased incomes of the population, in turn, increase household consumption, which leads to an increase in output up to the "attenuation" of the process. Initial momentum is possible in elements of the third quadrant, in final household consumption, or in outputs.

The second approach of endogenizing elements of final demand involves expanding the dimension of the input-output model by introducing an additional sector (or sectors), conditionally considered as productive sector. For example, it is possible to separate the household sector from the final demand column and the salary row and place this data in the first quadrant, thereby presenting households as one of the endogenous sectors, as shown in Table 1. A similar operation is known as a closing of the model in relation to households. Although this approach is included in well-known foreign textbooks (see, for example, [1]), this approach is uncommon in studies of our country. Since in the literature of our country according to the input-output method, endogenization is mostly carried out on the basis of building a dependence between the elements of the second and third quadrants and cyclic calculations based on the original n-industry model, the question arises: how much do the results of calculations according to the extended model and cyclic calculations based on the original n-industry model differ. The answer to this question is necessary, if only for the reason that the extended (closed) model, despite its simplified nature, also has some advantages (for example, the relative simplicity of the content), which does not allow it to be excluded from the arsenal of endogenization tools for elements of final demand.

Table 1

Input-output table with households endogenous sector

Sector	Sector					Other final demand	Output
	1	2	3	…	$n+1$		
1	x_{11}	x_{12}	x_{13}	…	$x_{1,n+1}$	Y^*_1	X_1
2	x_{21}	x_{22}	x_{23}	…	$x_{2,n+1}$	Y^*_2	X_2
3	x_{31}	x_{32}	x_{33}	…	$x_{3,n+1}$	Y^*_3	X_3
…	…	…	…	…	…	…	…
	Quadrant I					Quadrant II	
$n+1$	$x_{n+1,1}$	$x_{n+1,2}$	$x_{n+1,3}$	…	$x_{n+1,n+1}$	Y^*_{n+1}	X_{n+1}
Other added value	Z^*_1	Z^*_2	Z^*_3	…	Z^*_{n+1}		
	Quadrant III						
Output	X_1	X_2	X_3	…	X_{n+1}		

In Table 1, the (n + 1) column of the first quadrant corresponds to the final consumption of households, the (n + 1) row corresponds to wages previously included in the third quadrant. Accordingly, the column "Other final demand" is equal to the final demand minus the final consumption of households, the row "Other added value" - added value minus wages. The transition to the closed IO model is carried out, as in the case of the classic static IO model: to estimate the direct cost coefficients (n + 1), the row is divided into a row of sector outputs along with the other rows. The "output" of a household sector is defined as the total cost of labor services provided to various sectors of the economy. The direct cost ratios of the (n + 1) column, representing the share of the output of sectors consumed by households in the total cost of labor services, are similarly calculated. In general, the elements at the intersection of the (n + 1) row and the (n + 1) column and the (n + 1) row and the "Other Final Demand" column are non-zero. The first element represents household labor costs (e.g. household services), the second element may mean, for example, payments to public servants.

Thus, if we denote Y^* - a vector of other final demand; x_{n+1} - "output" of households; $x_{n+1,j}$, where $j = 1,..., n + 1$ - labor remuneration in sectors, then the system of equations of the IO model

$$x_i - \sum_{j=1}^{n} a_{ij} x_j = y_i,\ i=1,\dots,n, \tag{1}$$

after closure is transformed to:

$$x_i - \sum_{j=1}^{n} a_{ij} x_j - a_{i,n+1} x_{n+1} = y_i^*,\ \ i=1,\dots,n, \tag{2}$$

$$x_{n+1} - \sum_{j=1}^{n} a_{n+1,j} x_j - a_{n+1,n+1} x_{n+1} = y_{n+1}^*. \tag{3}$$

In matrix form (2) - (3) are written as:

$$\overline{X} - \overline{A}\,\overline{X} = \overline{Y^*} \tag{4}$$

where $\overline{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} X \\ x_{n+1} \end{bmatrix}$, $\overline{Y^*} = \begin{bmatrix} y_1^* \\ \vdots \\ y_{n+1}^* \end{bmatrix} = \begin{bmatrix} Y^* \\ y_{n+1}^* \end{bmatrix}$, $\overline{A} = \begin{bmatrix} A & h_l \\ v_l & h \end{bmatrix}$, A – technological matrix of the initial model (1),

$v_l = \lfloor a_{n+1,1},\dots, a_{n+1,n} \rfloor$, $h_l = \begin{bmatrix} a_{1,n+1} \\ \vdots \\ a_{n,n+1} \end{bmatrix}$, $h = a_{n+1,n+1}$.

Using block partition of the matrix, (4) can be written as:

$$\begin{bmatrix} I-A & -h_l \\ -v_l & (1-h) \end{bmatrix} \overline{X} = \begin{bmatrix} Y^* \\ y_{n+1}^* \end{bmatrix} \tag{5}$$

Solution (4) is written as:

$$\overline{X} = (I-\overline{A})^{-1}\overline{Y^*} = \overline{L}\,\overline{Y^*} \tag{6}$$

The same result for x_{n+1} as based on (6) can be obtained using the base model (1). Then the balanced values of labor remuneration V_l ("output" of households) and other final demand according to the n-sector model must obey the ratio:

$$V_l = v_l (I-A)^{-1}(h_l V_l + Y^*) \tag{7}$$

Hereof

$$V_l = [1 - v_l (I-A)^{-1} h_l]^{-1} v_l (I-A)^{-1} Y^* \tag{8}$$

$$\Delta V_l = [1 - v_l (I-A)^{-1} h_l]^{-1} v_l (I-A)^{-1} \Delta Y^* \tag{9}$$

Based on (6) we obtain

$$\begin{bmatrix} X \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} I-A & -h_l \\ -v_l & (1-h) \end{bmatrix}^{-1} \begin{bmatrix} Y^* \\ y_{n+1}^* \end{bmatrix} \tag{10}$$

Then, according to the algebra of block matrices[2], if there is an original matrix $\begin{bmatrix} E & F \\ G & H \end{bmatrix}$, we can also denote the matrix inverse to it by $\begin{bmatrix} S & T \\ U & V \end{bmatrix}$, in which case $S = E^{-1}(I - FU)$, $T = -E^{-1}FV$, $U = -VGE^{-1}$, $V = (H - GE^{-1}F)^{-1}$.

Assuming

$$\begin{bmatrix} I-A & -h_l \\ -v_l & (1-h) \end{bmatrix}^{-1} = \begin{bmatrix} S & T \\ U & V \end{bmatrix} \tag{11}$$

while h =0, $U = [1 - v_l (I-A)^{-1} h_l]^{-1} v_l (I-A)^{-1}$.

Thus, for IO model with (n+1) sectors at $y_{n+1}^* = 0$

$$x_{n+1} = UY^* = [1 - v_l (I-A)^{-1} h_l]^{-1} v_l (I-A)^{-1} Y^*, \tag{12}$$

Which coincides with (8), and

$$x_{n+1} = V_l \tag{13}$$

at the given $\overline{Y^*}$.

From (12) it follows that

$$\Delta x_{n+1} = [1 - v_l (I-A)^{-1} h_l]^{-1} v_l (I-A)^{-1} \Delta Y^*. \tag{14}$$

[2] *For more information on block matrices see, for example, [1].*

Formula (14) shows that the labor remuneration increase of an (n + 1) -dimensional model due to momentum in other final demand can be calculated based on an n-dimensional model.

As noted above, when considering model (1) over time with respect to household income and consumption, i.e. assuming the continuation of the initial impulse ΔY^*, the following calculation algorithm can be applied. Let's say the initial impulse for other final demand is ΔY^*. Accordingly, the labor remuneration for the first iteration equals to

$$V_l^1 = v_l (I-A)^{-1}(Y^* + V_l h_l + \Delta Y^*) \tag{15}$$

for the second iteration:

$$V_l^2 = v_l (I-A)^{-1}(Y^* + V_l h_l + \Delta V_l^1 h_l + \Delta Y^*), \tag{16}$$

where $\Delta V_l^1 = V_l^1 - V_l$ and so on.

Accordingly for *k*-iteration:

$$V_l^k = v_l (I-A)^{-1}(Y^* + V_l h_l + h_l \sum_{i=1}^{k-1} \Delta V_l^i + \Delta Y^*) \tag{17}$$

where $\Delta V_l^i = V_l^i - V_l^{i-1}$.

$V_l^k - V_l$ shows the entire increase in labor remuneration at the k-th step due to the initial impulse ΔY^*. Let's prove that $V_l^k - V_l \to \Delta x_{n+1}$ while $k \to \infty$, where Δx_{n+1} is defined according to (14), i.e. using an expanded (closed) model.

For convenience of presentation, we will introduce additional designations:

$H = (h_1, ..., h_n)^T$ – final household consumption,

$Z = (z_1, ..., z_n)$ – cost vector on labor remuneration.

Then, in the new designations, the total labor remuneration costs

$$V_l = \sum_{i=1}^{n} z_i,$$

$$v_l = \left(\frac{z_1}{x_1}, ..., \frac{z_n}{x_n}\right),$$

$$h_l = \left(\frac{h_1}{x_{n+1}}, ..., \frac{h_n}{x_{n+1}}\right)^T = \left(\frac{h_1}{\sum_{i=1}^{n} z_i}, ..., \frac{h_n}{\sum_{i=1}^{n} z_i}\right)^T = \frac{1}{\sum_{i=1}^{n} z_i}(h_1, ..., h_n)^T.$$

The wage budget increase for each iteration can be written as:

$$\Delta V_l^1 = v_l (I-A)^{-1} \Delta Y^*$$

$$\Delta V_l^2 = v_l (I-A)^{-1} h_l \Delta V_l^1 = v_l (I-A)^{-1} h_l v_l (I-A)^{-1} \Delta Y^*$$

$$\Delta V_l^3 = v_l (I-A)^{-1} h_l \Delta V_l^2 = [v_l (I-A)^{-1} h_l]^2 v_l (I-A)^{-1} \Delta Y^*$$

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$$\Delta V_l^{k+1} = v_l (I-A)^{-1} h_l \Delta V_l^k = [v_l (I-A)^{-1} h_l]^k v_l (I-A)^{-1} \Delta Y^*$$

Then:

$$\begin{aligned} V_l^{k+1} - V_l &= \Delta V_l^1 + \Delta V_l^2 + ... + \Delta V_l^{k+1} = v_l(I-A)^{-1}\Delta Y^* + v_l(I-A)^{-1} h_l v_l (I-A)^{-1}\Delta Y^* + \\ &+ [v_l(I-A)^{-1}h_l]^2 v_l (I-A)^{-1}\Delta Y^* + \ldots + [v_l(I-A)^{-1}h_l]^k v_l(I-A)^{-1}\Delta Y^* = \\ &= [1 + v_l(I-A)^{-1}h_l] + [v_l(I-A)^{-1}h_l]^2 + [v_l(I-A)^{-1}h_l]^3 + ... + [v_l(I-A)^{-1}h_l]^k v_l(I-A)^{-1}\Delta Y^*. \end{aligned} \tag{18}$$

The right part (18) is the formal power series of the number $v_l(I-A)^{-1}h_l$, converging at $0 < v_l(I-A)^{-1}h_l < 1$. Let's prove that $v_l(I-A)^{-1}h_l < 1$.

$$\begin{aligned} v_l(I-A)^{-1}h_l &= v_l(I-A)^{-1}H\frac{1}{\sum_{i=1}^{n} z_i} = v_l(I-A)^{-1}H\frac{1}{\sum_{i=1}^{n} z_i} + v_l(I-A)^{-1}Y^*\frac{1}{\sum_{i=1}^{n} z_i} - \\ &- v_l(I-A)^{-1}Y^*\frac{1}{\sum_{i=1}^{n} z_i} = v_l(I-A)^{-1}(H + Y^*)\frac{1}{\sum_{i=1}^{n} z_i} - v_l(I-A)^{-1}Y^*\frac{1}{\sum_{i=1}^{n} z_i}. \end{aligned} \tag{19}$$

Taking into account that $(I-A)^{-1}(H+Y^*) = X$, we obtain:

$$\begin{aligned} v_l(I-A)^{-1}h_l &= v_l X\frac{1}{\sum_{i=1}^{n} z_i} - v_l(I-A)^{-1}Y^*\frac{1}{\sum_{i=1}^{n} z_i} = \left(\frac{z_1}{x_1}, ..., \frac{z_n}{x_n}\right)\cdot\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}\cdot\frac{1}{\sum_{i=1}^{n} z_i} - \\ &- v_l(I-A)^{-1}Y^*\frac{1}{\sum_{i=1}^{n} z_i} = \left(\sum_{i=1}^{n} z_i\right)\cdot\frac{1}{\sum_{i=1}^{n} z_i} - v_l(I-A)^{-1}Y^*\frac{1}{\sum_{i=1}^{n} z_i} = 1 - v_l(I-A)^{-1}Y^*\frac{1}{\sum_{i=1}^{n} z_i} \end{aligned} \tag{20}$$

$(I-A)^{-1}Y^*$ is a positive vector of outputs X^*, providing for the other final demand Y^*. Then $v_l(I-A)^{-1}Y^*\frac{1}{\sum_{i=1}^{n} z_i}$ is a positive number as the product of vectors with non-negative (not all zero) elements. Since the left part (20) is a positive number (as the product of vectors and matrices with non-negative (not all zero) elements), then $0 < v_l(I-A)^{-1}h_l < 1$.

The sum of row (18) at $k \to \infty$, equals to $[1-v_l(I-A)^{-1}h_l]^{-1}v_l(I-A)^{-1}\Delta Y^*$, which coincides with the right part (14), hence $V_l^k - V_l \to \Delta x_{n+1}$ at $k \to \infty$.

Thus, the multiplicative effect of the increase in other final demand for value-added elements can be calculated in two ways: based on the assessment of the cyclic effects of the initial impulse and on the basis of the "expansion" of the input-output model by highlighting the element of the final demand and the corresponding element of the added value in the additional sector, conditionally considered as production.

Empirical results of applying different approaches to endogenizing elements of final demand. Let's consider the advantages and disadvantages of these approaches using *the example of empirical calculations according to the regional input-output model*. For empirical calculations, the regional level was not chosen by chance. The statistical problems of the regional input-output tables are well known. Therefore, the assessment of the possibilities of applying an expanded model with less difficulties of information content is of practical interest primarily for the regional level. Experimental calculations were made on the basis of the 25-sector IO table of the Republic of Bashkortostan for 2002[3].

A comparative estimate of the multiplicative effect[4] of the growth of the regional payroll budget (including social contributions) was made on the basis of two approaches: cyclic calculations of salary increase due to the initial impulse using the classic input-output model and the expanded input-output model. As an initial impetus, a possible increase in social benefits due to transfers to the region, which amounted to 6% of the regional payroll budget, was considered. As expected, the results of the initial impulse multiplicative effect assessment for the two approaches coincided. The specified multiplicative effect was 4.7% of the payroll budget. The following are the results of cyclical calculations of the increase in wages for consecutive 9 iterations due to the initial impulse (%), which practically provide the total multiplicative effect:

ΔV_l^1	ΔV_l^2	ΔV_l^3	ΔV_l^4	ΔV_l^5	ΔV_l^6	ΔV_l^7	ΔV_l^8	ΔV_l^9
56,5	24,6	10,7	4,7	2,0	0,9	0,4	0,2	0,1

Despite the coincidence of the results, it should be noted that the approach based on cyclic multiplicative effects, in addition to estimating the total multiplicative effect, allows you to track the distribution of the multiplicative process over time, since the initial impulse can have a long-term impact on economic dynamics[5].

However, the practical implementation of the closing across the household sector of the IO model requires consideration of a number of subtleties, and the whole procedure is usually far more complicated than could be imagined in the previous discussion. The most significant assumption of an "extended" (closed) model is the "freezing" of household consumer behavior, which is reflected in the constancy of the direct cost coefficients $a_{i,\, n+1}$ (or elements of the hl vector). The non-linearity of the dependence of household consumption costs on their outputs is much more significant than for other sectors. Household consumption patterns are undoubtedly changing with their income levels changing. In addition, household spending on final consumption is formed by not only the payroll budget, but also other elements of added value. The endogenization of the household sector in the input-output model requires consideration of both factors noted and a number of others as well.

An alternative assessment of the multiplicative effect of wage growth was carried out according to the input-output model for the Republic of Bashkortostan for 2002, which includes a differentiated balance of income and consumption of the population. The household cash expenditure vector for end-use is the sum of demand vectors for decile groups of population. Sector elements of demand vectors, in turn, are constructed as nonlinear regression models based on data on spending on the purchase of goods and services by decile groups of households, distributed by income level, obtained from data from a sample survey of household budgets. At the same time, the available cash income values of the decile groups are evaluated as a function of the total available cash income value and the distribution parameter (variance of income logarithms). This approach requires a transition from salary indicators to the gross available cash income of the population. By taking these ratios into account in the model, the structure of household final consumption also becomes endogenous and depends on household income and its distribution among income groups. In this form, it is possible to more accurately track the dynamics of changes in demand (both in general and its structure) under the influence of changes in income. In addition, the input-output model, which includes a differentiated balance of income and consumption of the population, can also be used to assess the impact of changing different components of value added on the distribution of income between groups. The calculations are made in a dynamized version, which assumes estimates of the cyclic effects of the initial impulse over a certain period of time[6].

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[3] This table is based on the structural proportions of the input-output base tables for 1995 and current statistical observations. The tables for 1995, in turn, were compiled under the guidance of the author of the article following the results of the All-Russian one-time-only survey of the structure of spending.
[4] The calculations were made with the participation of V.F. Gayazov and R.S. Ishbulatov.
[5] Similar studies, for example, are performed in article [3].
[6] For more detailed description of the model see [4, 5].

The results of alternative calculations using the IO model with an integrated differentiated balance of income and consumption of the population differ significantly from the data obtained from the extended model. Thus, the initial impulse due to the growth of social benefits due to transfers to the region in the amount of 6% of the regional payroll budget shows much more "modest" results of the multiplier effect in the considered model as compared to the expanded model. It makes up 3.5% of the basic level of the payroll budget. The difference of 1.2 percent points is explained primarily by taking into account the change in the structure and volume of final demand of households depending on the level of income of decile groups. Thus, within the framework of the input-output model with an integrated differentiated balance, the dynamics of changes in demand (both in general and its structure) under the influence of changes in income is more accurately monitored. Initial and modified versions of the final household demand structure are shown in Table 2.

The data of Table 2 show that under the influence of the initial impulse of income growth, the structure of demand changes - the share of food sectors (food industry and agriculture) decreases and the share of services, as well as engineering and construction increases, i.e., with an increase in income, the population increases expenses on purchasing housing accommodations, engineering industry products (cars, household appliances, etc.) and reduces the share of foodstuff costs.

Table 2

Dynamics of final household demand structure
depending on income level, %

Sector	Modified pattern	Original pattern
Power generation	0,58	0,60
Oil extraction industry	0,00	0,00
Oil refining industry	1,81	1,82
Gas industry	0,08	0,08
Coal industry	0,00	0,00
Other fuel industries	0,00	0,00
Iron and steel industry	0,00	0,00
Non-ferrous industry	0,00	0,00
Chemical industry	2,87	2,91
Petrochemical industry	1,24	1,24
Machinery and equipment, metal fabrication	9,48	8,83
Timber and woodworking industry	2,03	1,90
Construction materials	0,25	0,25
Consumer goods industry	12,76	12,82
Food industry	31,84	32,54
Other manufacturing industries	1,68	1,69
Construction	1,70	1,56
Agriculture and forest industry	9,96	10,24
Transportation and communication	8,77	8,78
Trade, re-selling business and catering	3,30	2,93
Other types of goods and services activity	0,04	0,04
Utilities and other non-production consumer services	6,11	6,28
Public health, physical education and social welfare	4,48	4,42
Science	0,00	0,00
Government administration	1,04	1,05

In addition to changing the structure of final household demand, the multiplicative effect difference for the two approaches is also influenced by the fact that the input-output model, which includes a differentiated balance of income and consumption of the population, takes into account the impact of changes in different components of the GVA on the distribution of income between decile groups.

It should be noted that in the extended input-output model, it is also possible to take into account the dependence of the final demand of households on the income of decile groups. For example, such an approach is possible by disaggregating the "household industry" into several sub-sectors depending on total income. The theoretical basis of such replacement of v_l row and h_l column with matrices is considered in article [6]. However, such breakdown turns the advantage of the extended model, which is the simplicity of the content, into its disadvantage, since solving the problems of statistical support of such a problem causes extraordinary difficulties.

Conclusions. The examples given in the article are illustrative in nature, but their statistical base, based on real input-output tables of the region, allows us to express judgments about the advantages and disadvantages of alternative options for estimating multiplicative effects due to initial impulses in the input-output model variables. They are as follows.

The results of calculations based on the assessment of cyclic multiplicative effects using the classical input-output model and the model, expanded due to the inclusion of elements of final demand in the first quadrant, coincide.

The extended model has a clear appeal in terms of content availability and calculation transparency.

In its turn, the estimation of cyclic multiplicative effects based on the classic IO model has additional capabilities related to tracking the time slice of the propagation of the initial pulse.

More accurate predictive and analytical calculations can be made on the basis of input-output models, including specialized blocks connecting endogenizable elements of final demand with elements of the third quadrant or with outputs.

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