Measuring the factor content of trade in a context of factor intensity ambiguities

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Abstract. Many of the tests of the Heckscher-Ohlin-Vanek Theorem found in the literature suffer from a lack of generality due to inconsistent definitions of factor content, incorrect calculations when trade in inputs is considered, or paucity of data concerning countries, industries, and factors. Defining factor content in a consistent way, and drawing from the multi-year data available in the World Input-Output Database, it is possible to test the HOV Theorem with two alternative definitions of factor content for 7 factors, 56 industries, and 40 countries plus a composite 'country' comprising the rest of the world. Overall, the tests performed are consistent with the main conclusions of the literature. Therefore, if the definition of factor content based on domestic techniques is considered, then the HOV Theorem must be rejected due to the systematic occurrence of factor intensity reversals. On the other hand, if the 'actual factor content' definition is considered, then a modified version of the HOV Theorem cannot be rejected. However, there is insufficient evidence to state that this is a proof in favour of trade arising from differences in factor-endowments.

Keywords. Heckscher-Ohlin-Vanek Theorem, factor content of trade based on domestic techniques, actual factor content of trade, factor endowments, input-output tables.

Classification codes. D57, F11, F14.

1. Introduction

The Factor Abundance Theory implies the insight that commodity trade is equivalent to trade in factor services, insofar as these are embodied in commodities (Mundell, 1957). Based on certain assumptions relative to production functions, preferences, and some equilibrium conditions, this theory has been summarized as the so-called Heckscher-Ohlin-Vanek (HOV) Theorem: every country is a net exporter of its relatively abundant factor services and a net importer of its relatively scarce factor services (Vanek, 1968). That simple and intuitive prediction, being so consistent with general equilibrium theory, has made the HOV Theorem one of the main test targets in economic research for the past 70 years.

Testing the prediction of the HOV Theorem requires estimating the factor content of an arbitrary bundle of commodities, but this is no trivial task. As long as factor proportions are variable and factor prices differ internationally, there are a variety of techniques that can plausibly be used to measure both the factor content of trade and the factor intensity of any given commodity. In this context of factor intensity ambiguities, two definitions of factor content have been highlighted by scholars. One, which might be called 'factor content based on domestic techniques', consists of defining factor content as those factors that would have to be employed if domestic production techniques were used everywhere for the production of a given commodity. The other, called 'actual factor content', consists of defining factor content as those factors actually employed in the production of a commodity wherever the production of that commodity or its inputs took place.

From Leontief (1953; 1956), initial tests of the Factor Abundance Theory were carried out considering the definition of factor content based on domestic techniques in order to estimate the comparative cost of imported commodities in terms of primary inputs. The main target of these first studies was to test the null hypothesis that factor-endowments were not a good predictor of the opportunity cost of trade in terms of factor requirements (Tatemoto & Ichimura, 1959; Wahl, 1961; Stolper & Roskamp, 1961; among others). Later, it was shown that the method used in these early studies was not consistent with the main formal developments that gave rise to the HOV Theorem (Leamer, 1980; Deardorff, 1982). This criticism – along with the results of several studies that found no support for the HOV Theorem (Maskus,1985; Bowen et al., 1987) – led many researchers to abandon the previous definition of factor content in favour of the 'actual factor content' definition.

Abandonment of the earlier definition of factor content also led to a change in the formulation of the HOV Theorem itself. Indeed, a new HOV model was considered with a continuum of commodities and without factor price equalization, where chains of comparative advantage can be well-defined so as to lead to full trade specialization (Dornbusch, Fischer & Samuelson, 1980). The pioneering work of Davis and Weinstein (2001), in which the actual factor content of trade was estimated econometrically due to the paucity of data, was followed by numerous works in which the actual factor content was calculated using the data available from input-output matrices (Reimer, 2006; Trefler & Zhu, 2010; Artal-Tur et al., 2011; Nishioka, 2012; Bernhofen & Brow, 2016; among others). As a result of this change in the definition of factor content, former empirical failures were replaced by successful tests as many researchers considered this definition and the modified version of the HOV Theorem more accurate for cases where neither techniques nor factor prices are internationally equalized.

However, the different estimates of factor content (according to one or another definition) are not always consistent or comprehensive with the available data. In fact, when considering the definition of domestic factor content, it has been common to estimate domestic factor content using the techniques employed by a single country to impute factor content in all countries (e.g., Bowen et al., 1987). Since production techniques differ internationally, this single-country method does not allow for estimating the opportunity cost of trade in terms of resources, as presumed (Deardorff, 1982). Likewise, when considering the definition of domestic factor content, it is common to estimate the factor content without considering trade of inputs or, if these are considered, to incur double counting when imputing the factors used (e.g., Reimer, 2006; Trefler & Zhu, 2010). In this way, a bias is frequently introduced in the estimates of factor content, and this bias is not easy to control unless the trade in inputs is itself considered and double accounting is avoided (Reimer, 2011; Ito et al., 2016). Finally, in both cases it is common to find studies where actual content of trade is estimated regarding one or two factors, a single year, and a not-very-high level of disaggregation by sector (e.g., Trefler & Zhu, 2010; Nishioka, 2012). In this sense, it is not clear to what extent the results found in the literature can be generalized for a case in which several factors are considered, or to what extent the results are stationary over time.

The aim of the present study is to test the HOV Theorem as implied by the two main definitions of factor content and using the World Input-Output Database, which includes multi-year information on 7 factors, 56 industries, and 40 countries plus a composite ‘country’ comprising the rest of the world. In this way, by comparing the different tests, it is possible to evaluate the extent to which the main conclusions found in the literature on the HOV Theorem can be regarded as definitive, despite errors and limitations of calculation. In order to achieve this target, the following procedure has been undertaken. First, the two main definitions of factor content are stated in a way that is consistent with the HOV model, and with each other, and also comprehensive with the data. Second, the hypotheses derived from considering each definition are tested both factor-by-factor and country-by-country, according to five criteria: a slope test, a variance test, a sign test, a Kendall’s rank test, and a pairwise rank test.

Overall, the tests performed are consistent with the main conclusions of the literature. Specifically, if the definition of factor content based on domestic techniques is considered, then the HOV Theorem must be rejected. This is the case even though support is found for the assumption of identical preferences, so that the failure must be related to factor intensity reversals. On the other hand, if the definition of actual factor content is considered, then the modified version of the HOV Theorem cannot be rejected. However, due to factor intensity ambiguities, this is not sufficient evidence to state that the bulk of world factor content of trade arises from differences in factor-endowments, as the theorem assumes.

2. Defining factor content in a consistent way

The main prediction of the HOV Theorem can be deduced starting from the identity expressing full-employment equilibrium in an open economy. According to that identity, the factor content of trade of a country i can be expressed as the difference between its factor-endowments and the factor content of its expenditure:

$$\mathbf{f_i} = \mathbf{e_i} - \mathbf{A_i c_i} \tag{1}$$

where $\mathbf{f_i}$ is a vector reflecting country-i’s net factor trade, $\mathbf{e_i}$ is the vector of country-i’s factor-endowments, and $\mathbf{c_i}$ is the vector of country-i’s final expenditure. $\mathbf{A_i}$ is a mapping, typically a matrix, which maps a commodity vector into a vector reflecting the primary inputs necessary for its production. It is assumed, then, that $\mathbf{A_i}$ is defined in such a way that the pair formed by the vector reflecting the factor content of the expenditure and the vector reflecting the commodities that make up the expenditure belongs to country-i’s net production possibility set, $\mathcal{H}_i$:

$$(\mathbf{A_i c_i}, \mathbf{c_i}) \in \mathcal{H}_i \tag{2}$$

This is the same as assuming that if factor-endowments were reduced by the amounts of factors embodied in exports, and increased by the amounts needed to produce the imports, it would become possible to supply the same expenditure vector $\mathbf{c_i}$. In this sense, $\mathbf{f_i}$ is assumed to reflect the opportunity cost of trade in terms of resources.[1]

[1] Assumption (2) is necessary to conclude that there are gains from trade. Indeed, trade is regarded as better than autarky when the factors needed to produce the trade expenditure bundle would have cost more, at autarky factor prices, than the endowment of factors available (Deardorff, 1982).

From identity (1) an empirically testable prediction can be obtained assuming further statements. Thus, assuming that preferences are identical and homothetic and that commodity prices are equalized worldwide, it is possible to express country-i's expenditure vector as proportional to the world expenditure vector:

$$\mathbf{c_i} = s_i \mathbf{c_w} \tag{3}$$

where s_i is country-i's share on world income. The term $\mathbf{c_w}$ is the world expenditure vector and also the world net-output vector, given that the world economy is, by definition, a closed economy. Substituting (3) into (1), the following expression is obtained:

$$\mathbf{f_i} = \mathbf{e_i} - s_i \mathbf{A_i} \mathbf{c_w} \tag{4}$$

So, if technologies are everywhere identical, it turns out that:

$$\mathbf{A_i} \mathbf{c_w} = \mathbf{e_w} \tag{5}$$

where $\mathbf{e_w}$ is a vector reflecting world factor-endowments. Substituting this last expression in (1), an exact relationship between the factor content of trade and the factor-endowments results:

$$\mathbf{f_i} = \mathbf{e_i} - s_i \mathbf{e_w} \tag{6}$$

This is the main prediction of the HOV Theorem.

The problem then arises of how to define **A** in such a way that it is consistent with the assumptions of the HOV model and that can be constructed from the available data. Specifically, **A** must be such that assumption (2) and assumption (5) are fulfilled. Also, it must be able to be constructed, typically, from the data that can be obtained from input-output accounting databases.

Often, the following data set is available from input-output tables and underlying data, for $p, q = 1, \dots, N$ industries, $i, j = 1, \dots, C$ countries and $r, s = 1, \dots, F$ factors.

A CxC matrix $\mathbf{Z}$, made up of NxN matrices $\mathbf{Z_{ij}}$, and whose elements $[Z_{pi,qj}]$ represent the value of interindustry sales by sector p of country i to sector q of country j:

$$\mathbf{Z} = \begin{pmatrix} \mathbf{Z_{11}} & \cdots & \mathbf{Z_{1C}} \\ \vdots & \ddots & \vdots \\ \mathbf{Z_{C1}} & \cdots & \mathbf{Z_{CC}} \end{pmatrix}$$

A $Cx1$ column vector $\mathbf{x}$, made up of $Nx1$ column vectors $\mathbf{x_i}$, and whose elements $[x_{pi}]$ represent the value of gross output of sector p of country i:

$$\mathbf{x} = \begin{pmatrix} \mathbf{x_1} \\ \vdots \\ \mathbf{x_c} \end{pmatrix}$$

A CxC matrix C, made up of $Nx1$ column vectors $\mathbf{c_{ij}}$, and whose elements $[c_{pi,j}]$ represent the value of total final demand of country *i* for product *p* of country *j*:

$$\mathbf{C} = \begin{pmatrix} \mathbf{c_{11}} & \cdots & \mathbf{c_{1C}} \\ \vdots & \ddots & \vdots \\ \mathbf{c_{C1}} & \cdots & \mathbf{c_{CC}} \end{pmatrix}$$

Sometimes it is useful to look at the **C** matrix as made up of $NCx1$ vectors $\mathbf{c_{\blacksquare i}}$, each of which represent country-*i*'s spending on commodities made in the different countries:

$$\mathbf{C} = (\mathbf{c_{\blacksquare 1}} \quad \cdots \quad \mathbf{c_{\blacksquare C}})$$

Finally, an FxC matrix **E**, made up of $1xN$ vectors $\mathbf{e_{ri}}$, and whose elements $\left[e_{pr,i}\right]$ represent the amount of factor *r* employed in sector *p* of country *i*.

$$\mathbf{E} = \begin{pmatrix} \mathbf{e_{11}^T} & \cdots & \mathbf{e_{1C}^T} \\ \vdots & \ddots & \vdots \\ \mathbf{e_{K1}^T} & \cdots & \mathbf{e_{KC}^T} \end{pmatrix}$$

It is also useful to look at the **E** matrix as made up of FxN matrices $\mathbf{E_{\blacksquare i}}$, each of which represents the factors employed by industries of country *i*:

$$\mathbf{E} = (\mathbf{E_{\blacksquare 1}} \quad \cdots \quad \mathbf{E_{\blacksquare C}})$$

So, under an assumption of full employment, country-*i*'s factor-endowments can be defined as the sum by columns of country-*i*'s factor employment matrix:

$$\mathbf{e_i} = \mathbf{E_{\blacksquare i}} \mathbf{i}$$

where **i** is a $Nx1$ unitary column vector, so that $[i_i] = 1$. The world factor-endowments vector results from the sum of each country's factor-endowment vectors:

$$\mathbf{e_w} = \sum_{\mathbf{i}} \mathbf{e_i}$$

Also, from this primary dataset, it is possible to define two important coefficient matrices. First, from **Z** and **x** the world matrix of input-output coefficients can be defined as:

$$\mathbf{B} = \mathbf{Z}\hat{\mathbf{x}}^{-1} \qquad (7)$$

where its elements $b_{pi,qj}$ reflect the amount of sector-*p*'s output from country *i* used as input per unit of sector-*q*'s output in country *j*. Second, from **e** and **x** the world matrix of direct factor input coefficients can be defined as:

$$\mathbf{D} = \mathbf{e}\hat{\mathbf{x}}^{-1} \qquad (8)$$

where its elements $[d_{pr,i}]$ reflect the amount of factor r employed per unit of sector-p's output in country i. Sometimes, it is useful to look at matrix $\mathbf{D}$ as made up of FxN matrices $\mathbf{D_i}$, each of which represents the direct factor input coefficients matrix of each country i:

$$\mathbf{D} = (\mathbf{D_1} \quad \dots \quad \mathbf{D_C})$$

From these data sets, $\mathbf{A}$ can be constructed in two ways.

First, $\mathbf{A_i}$ can be constructed as a matrix that maps a vector of commodities to a vector of factor content based on country-i's techniques – that is, a vector reflecting the factors that would have to be employed if country-i's production techniques were used everywhere. In order to obtain such a linear mapping, it is necessary to assume that commodities p produced in countries $i = 1, \dots, N$ are roughly perfect substitutes. Then, according to the assumption of commodity price equalization, $\mathbf{A_i}$ can be obtained by adding the input-output coefficients over both domestic and foreign supply:

$$\widetilde{\mathbf{B}}_\mathbf{i} = \sum_\mathbf{j} \mathbf{B_{ji}}$$

and, then:

$$\mathbf{A_i} = \mathbf{D_i}\left(\mathbf{I} - \widetilde{\mathbf{B}}_\mathbf{i}\right)^{-1} \tag{9}$$

So, $\mathbf{A_i}$ is a FxN matrix whose elements $\left[a_{r,i}\right]$ represent factor-r's direct plus indirect employment in sector p if country-i's techniques were employed at every stage of production. In this way, the factor content of country-i's expenditure based on country-i's techniques is:

$$\mathbf{A_i c_i} \tag{10}$$

where $\mathbf{c_i}$ is a $Nx1$ vector that can be obtained by adding country-i's expenditure over both domestic and foreign supply:

$$\mathbf{c_i} = \sum_\mathbf{j} \mathbf{c_{ji}}$$

Substituting (10) in (1) would result in the classic HOV Theorem, in which same-industry commodities are considered perfect substitutes, giving rise to different factor-endowments and to partial trade specialization (Vanek, 1967).

Defining the factor content and formulating the HOV Theorem in this way has some advantages, as well as an obvious drawback. If the assumption about the homogeneity of each industry's world output is roughly correct, then matrix (9) allows us to calculate the opportunity cost of trade in terms of factors, so that assumption (2) of the HOV Theorem is satisfied. In this way, to the extent that the factor intensity of exports and imports is measured at the same factor prices, then it is ensured that the factor content of expenditure belongs to each country's net-production possibility set. However, to the extent that factor prices are not equalized worldwide, it is to be expected that, in general, assumption (5) will not be fulfilled. Actually, it can be ruled out in advance that the HOV Theorem holds exactly and can only be expected to find a sign and rank correlation between factor abundance and factor content of trade if factor intensity reversals are not too great.

Despite these difficulties, this definition of factor content is nevertheless much more consistent with the assumptions of the Theorem than the single-matrix definition of factor content, very common in literature. That definition implies the method of using the production techniques employed by a single country to impute factor content in all countries. In this way, $\mathbf{A_i}$ is assumed to be the same for all countries and equal to the matrix of input-output coefficients of a specific country, typically the U.S. Thus defined, **A** does not fulfil assumption (5) for the same reasons as the domestic factor content matrix. Moreover, it does not meet assumption (2). In actuality, it is likely to impute a factor content to expenditure such that if resources were reduced by the amount exported and increased by the amount imputed to imports, then it would not be possible for country *i* to supply the same expenditure vector in full-employment conditions, due to either excess or shortage of resources. Thus, the single-matrix definition of factor content does not fulfil either assumption (2) or assumption (5) and, therefore, it is not surprising that testing the HOV Theorem with this method offers so little support to the theory (e.g., Bowen et al., 1987).

In any case, **A** can also be constructed as a matrix that maps a vector of commodities to a vector of actual factor content – that is, a vector reflecting the factors actually embodied in the production of a commodity wherever the production of that commodity or its inputs took place. Thus, from the input-output and factor-input matrices, one can immediately obtain the matrix that maps an arbitrary commodity vector to a vector reflecting the factors embodied worldwide in its production:

$$\mathbf{A} = \mathbf{D}(\mathbf{I} - \mathbf{B})^{-\mathbf{1}} \tag{11}$$

A is a $FxCN$ matrix whose elements $[a_{r,pi}]$ represent factor-*r*'s direct plus indirect employment in sector *p* of country *j*. Thus defined, matrix **A** satisfies condition (5) in a trivial way since, under an assumption of full employment, the actual factor content of world net output equals the world factor-endowments:

$$\mathbf{Ac_w} = \mathbf{e_w}$$

where:

$$\mathbf{c_w} = \sum_{\mathbf{j}} \mathbf{c_{\blacksquare j}}$$

with $\mathbf{c_w}$ being a $CNx1$ vector whose elements $[c_{pj}]$ represent country-*j*'s net output of product *p*.

Nevertheless, matrix **A** satisfies assumption (2) only hypothetically. Indeed, as long as factor price equalization fails, imputing factor content in this way means that the factor intensities of exports and imports are measured at different factor prices. So, if industry-*p*'s output were roughly homogeneous worldwide, its factor intensity could not be stated in an unambiguous way. In this case, if country-*i*'s factor-endowments were reduced by the amount of factors embodied in exports and increased by the amount of factors embodied in imports, then, in general, it would not be possible to supply the same expenditure vector in full-employment conditions due to excess or shortage of resources. The only way to get around this difficulty is to assume that commodity *p* from country *i* and commodity *p* from country *j* are not perfect substitutes but in fact different commodities (or 'varieties' of the same commodity). Thus, the assumption about preferences should encompass this world of many commodities, assuming that country *i* demands a share s_i of the net output produced by every country (Trefler & Zhu, 2010). Then, the factor content of country-*i*'s expenditure can be computed as:

$$\mathbf{A_i c_i} \tag{12}$$

where $\mathbf{c_i}$ is a $CNx1$ vector whose elements $[c_{pj}]$ represent the expenditure of country *i* in product *p* of sector *j*. Substituting (12) in (1) would result a modified version of the HOV Theorem in which the dissimilarity in factor-endowments leads to price equalization failure. As long as there are many commodities, such unequal factor prices yield a natural ordering of commodities based on factor intensities, and this ordering of commodities defines a chain of comparative advantage that leads to full trade specialization (Dornbusch, Fischer & Samuelson, 1980).

Accounting the factor content from (1) and according to (9) or (11) is the same as accounting the factor content of trade based on the factor content of expenditure, as Leamer (1980) originally proposed in order to avoid some difficulties arising from Leontief-type methods. In addition to the advantages of this method highlighted by Leamer (1980) himself, there is an important additional advantage in the event of trade in inputs. In particular, accounting the factor content of trade based on the factor content of expenditure allows taking into account the trade in inputs without incurring double counting. Indeed, the factor content of trade is measured as the difference between a country's factor-endowments, given by the data, and the factor content of the country's absorption. The latter, in turn, results from imputing to the final demand all the factors that may have taken part in its production, even through the trade in inputs. In this way, a double counting is avoided, as many methods present in the literature imply (e.g., Reimer, 2006; Trefler & Zhu, 2010; among others). As long as these methods measure factor content from a net trade vector, traded inputs are counted twice: as part of the net-trade vector and also as part of the input matrix (Reimer, 2011; Dietzenbacher & Los, 2011; Stehrer, 2012; Ito et al., 2016). By measuring the factor content of trade from the factor content of expenditure, this miscalculation is avoided.

3. Testing the HOV Theorem with alternative definitions

In order to test the prediction of the alternative formulations of HOV Theorem, the calculations presented in Section 2 were carried out taking as data those provided by the 2016 Release of the World Input-Output Database (WIOD). This database is, to date, one of the most complete in relation to the number of industries, countries, and factors. WIOD Release 2016 (and the underlying data) covers 56 sectors (classified according to ISIC Rev. 4) and 43 countries for the period 2000-2014. The underlying data of that Release – included within the so-called 'Socio Economic Accounts' – provide data on the use of labour (in thousands of persons engaged) and on the use of capital in current USD.

Elsewhere, the Joint Research Centre of the European Commission has published data on net energy use in TJ and carbon-dioxide emissions in tonnes by industry and country for 2000-2016, fully consistent with the 2016 Release of the WIOD.[2]

Moreover, the underlying data of the earlier 2013 Release of the WIOD also provided information on the use of cropland, pastures, forests, and mineral resources for the period 1995-2009, covering 35 sectors classified according to ISIC Rev. 3. Considering the equivalence between sectors of ISIC Rev. 3 and ISIC Rev. 4., the data available in the 2013 Release can be used when all 56 sectors are regarded.

Data on cropland, pastures, forests, mineral resources, energy, and emissions provide information on either 40 or 43 countries (depending on the Release) as well as on the rest of the world. This is not true for the cases of labour and capital. To bridge this gap, it is possible to use data available from the Penn World Table version 10.0, assuming that all sectors in the rest-of-the-world composite 'country' employ the same amount of labour and capital per unit of income. Regarding the approximate nature of the factor usage by the rest of the world, the estimates of factor content corresponding to this composite 'country' cannot be considered as evidence for or against the HOV Theorem predictions. However, it is of great importance to take this approximation into account in order to estimate with plausibility the actual factor abundance of each country. Indeed, failure to consider the factors of the rest of the world would introduce a bias that would be difficult to control, even though this omission is quite common in the literature (e.g., Trefler & Zhu, 2010).

In this way, the present study regards a world economy made up of 41 countries that exchange final and intermediate commodities produced by any of the 56 industries. These 40 countries are those of the 2013 Release of the WIOD plus one composite 'country' comprising the rest of the world. The 56 industries are those of the 2016 Release of the WIOD. Analysis regards the employment of seven resources: labour, capital, cropland and pastures, forest land, mineral resources, energy, and emissions.

Taking this data set into account, it is possible to test the main prediction of the HOV Theorem. However, the Theorem assumes no inter-temporal trade and, therefore, a null trade balance for all countries; but this is not the case in the actual world. In order to overcome this difficulty, it is convenient to adjust the amount of factor content under a null-trade-balance assumption. To do this, it must be taken into account that country-*i*'s share on world expenditure s_i can be expressed as:

$$s_i = \frac{y_i - b_i}{y_w} \tag{13}$$

where y_i is country-*i*'s GDP, b_i is country-*i*'s trade balance, and y_w is world GDP. Therefore, the assumption on preferences (3) can be expressed as:

$$\mathbf{c_i} + \frac{b_i}{y_w}\mathbf{c_w} = \frac{y_i}{y_w}\mathbf{c_w} \tag{14}$$

[2] As long as pollutants are by-products of consuming natural resources, there is little difference between accounting the consumption of natural resources or the emission of pollutants. In this way, recording the emission of pollutants is roughly the same as accounting for the use of clean air as a productive factor (ten Raa, 2005).

where the left side of the equation denotes the consumption of country *i* in the case of its trade balance being null. So, the adjusted factor content of trade can be calculated as:

$$\mathbf{f_i^A} = \mathbf{e_i} - \mathbf{A_i}\left(\mathbf{c_i} + \frac{b_i}{y_w}\mathbf{c_w}\right) \tag{15}$$

And, according to the prediction of the Theorem, (15) this is expected to be equal to:

$$\mathbf{f_i^A} = \mathbf{e_i} - \frac{y_i}{y_w}\mathbf{e_w} \tag{16}$$

Then, (15) reflects the factor content as revealed by the data while (16) reflects the predicted factor content of trade as results from revealed factor abundance. A typical *k*-th element of (16) can be written as:

$$\frac{f_{ki}^A}{e_{ki}} = 1 - \frac{y_i/y_w}{e_{ki}/e_{kw}} \tag{17}$$

Thus, testing the HOV Theorem is the same as testing the conformity of the adjusted net factor export data $[f_{ki}^A/e_{ki}]$ with the factor abundance data $\left[1 - \frac{y_i/y_w}{e_{ki}/e_{kw}}\right]$. Typically, this conformity can be tested according to four criteria.

First, testing the proposition that factor content of trade is equal in amount to that predicted by factor abundance data. This can be done through a slope test, regressing factor content on factor abundance data without an intercept.

Second, testing the statement that the amount of the observed factor content of trade is of the same order as the predicted one. This can be checked through a variance test, computing the variance of factor content over the variance of factor abundance.

Third, testing the proposition of conformity in sign between factor content of trade and factor abundance. For this, a sign test can be conducted, obtaining the proportion of sign matches between the observed and predicted factor content.

Fourth, testing the proposition of the conformity between the factor content of trade and relative factor abundance through a rank test. This test can be performed either by computing rank correlation (typically Kendall's) between factor content and factor abundance ratios, or by computing the proportion of correct rankings when compared pairwise.

Each of these tests can be carried out factor-by-factor or country-by-country, as it is assumed in expression (17) that *k* remains constant while *i* varies, or vice versa. However, the slope test and the variance test were not carried out country-by-country, since for each year there are only as many observations as there are productive factors.

In order to test these four implications implied by the HOV Theorem, it is necessary to calculate both the factor content of trade and the factor abundance, as these result from expressions (15) and (16). Expression (15) can be calculated in two ways: according to the definition of factor content as based on domestic techniques, or according to the definition of actual factor content. According to the definition of domestic factor content, there are 41 56x56 $\mathbf{A_i}$ matrices and $\mathbf{c_i}$ is a 56x1 vector. According to the actual factor content definition, there is a single 2296x2296 $\mathbf{A}$ matrix and $\mathbf{c_i}$ is a 2296x1 vector. The temporal mean of the ratio of both factor content estimates to national factor-endowments – that is, the left side of (17) – can be seen in Table 1 and Table 2. In addition, the temporal mean of the ratio of factor abundance (14) to national factor-endowments – that is, the right side of (17) – can be seen in Table 3.

Table 1. Ratio of adjusted factor content of trade to national factor-endowments (temporal mean). Domestic factor content definition.

	Labour	**Capital**	**Crop.&Past.**	**Forest**	**Minerals**	**Energy**	**Emissions**
AUS	-3.1	4	24.2	-13	48.5	-5.4	-2.5
AUT	-0.6	-2.3	-17.8	3.7	-241.6	1.1	3.9
BEL	-1.4	-3.3	-19.6	-30.3	-957.5	15.1	-6.3
BGR	4.8	5.5	14.7	-4.6	-238.8	-22.4	-9.1
BRA	2.5	0.7	20.4	6.3	11.3	4.2	5.7
CAN	−2.2	4	14.3	17.2	44.6	10.2	8.9
CHN	1.6	-1.9	0.8	-8.5	-27.2	-0.3	-0.7
CYP	-9.3	5.9	-32.8	5.7	-24.6	-17	-28.5
CZE	0.1	-1.9	-14.9	23	-136.1	-16.6	-6
DEU	-2.8	-4	-45.7	-11.4	-355.5	1.4	2.5
DNK	-2.9	-0.4	31.1	-8.5	17.8	46.7	43
ESP	0.1	1.6	9.3	-8.8	-309.3	1.7	1.3
EST	0.2	4.9	-13.2	63.6	-63.1	-33.3	2.2
FIN	-4.5	-2.2	-29.4	31.5	-250.3	22.4	8.9
FRA	0.2	-1.8	5	-15.8	-654.6	-1.5	-2.9
GBR	-0.7	-0.8	-37.5	-21.2	-17.1	-5.4	-6.9
GRC	-1.8	6.3	1.3	-21.2	-359.1	29.4	7.2
HUN	-1.2	-2.2	17	16.8	-760.9	-9.9	-13.5
IDN	-0.4	-6.6	0.6	22	30.9	0.7	1.3
IND	3.2	-1.8	4.3	-3	-112.4	-11.5	-10.7
IRL	-17.7	-16.1	33.6	-20.8	-199.8	-54.4	-27.9
ITA	1.3	-1.7	-9.5	-29.4	-370.6	-3.5	-1.2
JPN	-1.5	-1.3	-31.8	-46	-623.8	-6	-8.1
KOR	-4.1	-0.2	-33.7	-59.5	-1861.1	-14.9	-28.4
LTU	0.7	0.5	-1.6	24.3	-930.7	-13.1	-3.8
LUX	-16.2	-17.6	-252.9	-82.8	-435	11.3	16
LVA	-3.5	4.1	-29.9	58	-169.8	-287.9	-0.6
MEX	-0.8	-8.6	-4.5	-19.3	27	-8.8	-6.1
MLT	-4.3	-7.2	-46.5	−	-103.7	-32.1	-28.7
NLD	-3.8	-5.1	30	-108.9	14.6	24.6	15.8
POL	1.1	0.4	0.7	18.1	-41.2	-8.9	-3.6
PRT	-3	2.4	-37.9	15.9	-448.1	-8.2	-6
ROU	0.1	-0.3	-3.1	13.3	-110.5	-24.2	-13.6
RUS	-14.3	0.8	-12.1	-12.1	79.6	6.9	11.3
SVK	-0.6	0.1	-16	16.4	-566.3	-1.2	9.8
SVN	-4.1	-4.6	-37.2	26.7	-145.1	-7.8	-1.5
SWE	-1.8	-2.7	-65.1	28.2	-150.3	12.9	1.4
TUR	2.7	0.1	5.5	-14.5	-103.8	-6.8	0.1
TWN	-8.6	-1.2	-68.5	−	-3141.7	-227.2	-266.5
USA	0.6	0.2	9.6	-20.7	-53.2	-3.8	-1.8
ROW	-	-	7.9	31.4	48.1	-5.2	-3

Table 2. Ratio of adjusted factor content of trade to national factor-endowments (temporal mean). Actual factor content definition.

	Labour	Capital	Crop.&Past.	Forest	Minerals	Energy	Emissions
AUS	-94	3.2	36.7	-39.5	68.6	-20.5	-11
AUT	-73.9	2.7	-87.9	-65.4	-170.8	-46.7	-50.4
BEL	-132.7	-1.4	-828.8	-955.9	-269.2	-24.6	-56.8
BGR	13.9	4.2	18.9	19.8	13.9	2	12.7
BRA	1.7	0.6	22	17.8	-0.6	3.6	-7.6
CAN	-66.7	-9.4	-6.7	40.3	23.8	19.4	-0.4
CHN	11.6	-1.8	-9.3	-57.7	4.8	11.5	15.8
CYP	-39.1	3.2	-280.2	-104.6	-80.5	-87.4	-38
CZE	-10.2	5	-34	-26.9	13.1	10.1	8.9
DEU	-83.9	-2.3	-436	-421.8	-33.5	-45.9	-31.4
DNK	-120.5	2.8	-106.4	-1143.2	-127.8	-3.9	7
ESP	-36	-0.2	-44.1	-56.1	-61.9	-16.9	-14.2
EST	-8.3	0.8	-13.9	65.9	21.3	5.1	12.9
FIN	-83.3	-3	-141.6	28	-43	-10.7	-16.6
FRA	-59.5	-3	-103	-102.1	-226.1	-32.5	-57.1
GBR	-73.2	-5.1	-275.6	-762.4	-170.7	-47.1	-37.9
GRC	-17.6	2	-12.6	-36	-543.5	5.7	-5.4
HUN	-5.2	6.4	13.7	-49.8	-80.4	-8.2	-22.1
IDN	9.2	-6.6	-37.2	19.1	41.5	1.3	-0.2
IND	7.6	1.1	-11.5	-37.8	13.9	2.5	5.6
IRL	-196.9	-20.7	-40.6	-555.5	-42.3	-116.7	-92.9
ITA	-57.7	-0.8	-205.2	-258.2	-235.4	-38	-40
JPN	-69.8	1.6	-2089.9	-275.2	-679.4	-14.5	-21.5
KOR	-64.1	4.4	-1233.7	-375.5	-265.1	14.1	0.6
LTU	-8.5	-2.9	4.5	-0.6	-380.4	-34	-50.2
LUX	-300.8	-25.9	-653	-691.3	-3239.3	-59.5	-69.7
LVA	-0.8	8.5	-1.6	53.8	-139.4	-31.3	-34.4
MEX	-5.2	-2.7	0.9	17.3	1.2	-14.9	-8
MLT	-22.4	9.7	-972.1	-	-1267888.7	-21.5	-1.7
NLD	-123.9	-5.2	-990.1	-4645.9	-468.8	54.8	-37.8
POL	0	-16.2	-20.7	-11	22.7	-2.4	5.9
PRT	-6.7	2.9	-73.2	-22.1	-42.3	-5.4	-11.8
ROU	11.5	1.4	-5.4	-0.5	1.8	1.6	8
RUS	-6.9	1.9	-15.5	5.3	73.9	30.9	28.7
SVK	-11.2	10.2	-30.8	-11.3	-210.5	9.3	1.3
SVN	-29.8	4.7	-169	-11.6	-51.9	-38.8	-20.8
SWE	-101.3	-0.1	-199.9	17.5	-65.9	-13.7	-92.7
TUR	-6.5	-2	-18.5	-46.7	-32.6	-15	-12.1
TWN	-23.6	-3.8	-1776.2	-	-306.7	22.9	15.6
USA	-48.6	-0.3	-12.6	-87.7	-30.1	-16.2	-13.8
ROW	-	-	15.2	39.4	31.9	7.5	6.7

Table 3. Ratio of predicted factor content of trade to national factor-endowments (temporal mean).

	Labour	Capital	Crop.&Past.	Forest	Minerals	Energy	Emissions
AUS	-365	10.2	81.9	-181.8	82.7	-56.9	-21.9
AUT	-345.7	27.5	-573.1	-325.2	-252	-122.1	-188.6
BEL	-420.5	1.6	-1861.2	-2680.8	-529.1	-44.8	-140.1
BGR	46.4	15.7	55.8	45.2	72.1	47.5	62.7
BRA	24.5	6.2	55.8	77.4	-44.3	10.8	-76.4
CAN	-317.8	-48.6	-105.1	75.6	-7.6	11.1	-43.6
CHN	70.8	-6.2	29.7	-36.9	65.3	58.6	70.2
CYP	-189.2	4.3	-747	-402	-100.5	-124.1	-47.8
CZE	-68.8	35.8	-143.4	-217	50.9	16.6	22.3
DEU	-303	14	-1404.7	-1546.5	-73.9	-97.8	-98.2
DNK	-415.1	16.7	-567.3	-3496.9	-360.3	-85.4	-73.5
ESP	-237.1	6	-253.4	-455	-174.3	-92.9	-128.6
EST	-41	2.8	-5.5	69.1	68.1	30.8	53.7
FIN	-364.6	10.9	-519.8	59.4	-97.1	-4.3	-74.5
FRA	-360.5	9.8	-597	-534.3	-593.4	-129.5	-301
GBR	-342.1	-21.5	-999.5	-3808.6	-500.7	-159.6	-166.4
GRC	-186.2	-11	-168.7	-315.6	-801.5	-42.6	-25.2
HUN	-38.9	34.2	-30.9	-266.1	-41.8	1.2	-27
IDN	77.5	0.1	13.9	56.5	67.6	35.8	33.3
IND	86.8	-9.5	-39.9	-12	63.1	52.5	56.9
IRL	-473.7	-16.8	-230.2	-1208	-89.8	-224.5	-208.2
ITA	-315.3	11.3	-927	-1278.8	-459	-126.5	-173.7
JPN	-335.7	4.6	-7841.5	-773.8	-1112.2	-93.3	-123.2
KOR	-141.5	23.2	-3065.7	-692.2	-209.4	25.7	15.6
LTU	-21.2	9.2	32.4	17.5	-123.5	17.6	-11.8
LUX	-584.2	-27.9	-1806	-1885	-4127.6	-91.9	-167.7
LVA	-16.4	-5.4	28	59.7	-153.7	-1	-33
MEX	-47.9	-8.9	17.5	31.8	-52	-30.2	-27.7
MLT	-129.7	3.9	-4254.4	–	-1337003.6	-28.5	-4.7
NLD	-356.1	7.1	-2397.3	-11485.6	-830.4	43.3	-134.2
POL	-30.8	-76.7	-84.4	-148	55.8	11.2	36.9
PRT	-118.6	1.6	-267	-185.1	-77.1	-50.3	-94.5
ROU	29.2	8.8	34	-59.6	56.4	21.1	25.1
RUS	28.2	-5.8	47.4	89.1	63.7	68.3	66.2
SVK	-58.4	42.8	-80.9	-129.6	-97.4	28.2	5.6
SVN	-118.2	18	-382.5	-82.4	-44.8	-49.5	-42.8
SWE	-401.7	3.7	-778.6	-1.9	-212.6	-69.9	-306
TUR	-22	-33.8	-20.3	-138.1	-12	-6.8	-15.3
TWN	-24.3	-19	-3226.4	–	-265.7	26.2	25.7
USA	-427.6	1.2	-253	-190.9	-62.8	-55.5	-56.4
ROW	56	-10.8	73.6	62.7	39.1	37.8	35.8

Looking at the data shown in Table 1 and Table 2, significant differences seem to emerge depending on whether one definition or another of factor content is taken into account. Consider the case of the U.S. According to the domestic factor content definition and under the assumption of trade equilibrium, the U.S. seems to export 0.6\% of its labour and 0.2\% of its capital, so that it seems more abundant in labour than in capital. However, according to actual factor content data, the U.S. appears to import labour amounting to 48.6\% of its labour force and to import capital amounting to 0.3\% of its capital stock, so that a contrary relative abundance appears to be present. On the other hand, according to data on factor abundance as predicted by the HOV Theorem, the U.S. should import labour amounting to more than 4 times its labour force and should export 1\% of its capital; in this way, according to the HOV Theorem's assumptions, the U.S. is much more abundant in capital than in labour. From this example, several characteristics of data on factor content can be shown. First, it is observed that inequality in factor prices and variety of production techniques seem to imply factor intensity reversals in a systematic way. Second, it is observed that the predicted amount of factor trade is much less than the observed one, resulting in a huge amount of 'missing trade'.

The domestic factor content definition was the first such definition to receive attention from scholars (Leontief, 1953; Tatemoto & Ichimura, 1959; Wahl, 1961; etc.), whose main aim was to test the null hypothesis that factor abundance was not a good predictor of the opportunity cost of trade in terms of resources. Thus, it was considered that the definition of factor content based on domestic techniques allows for measurement of the factor intensities of exports and imports at the same factor prices. By means of this definition, one expectation was to estimate how much the factor-endowments should be reduced by the amount of exported factors, and how much they should be increased by the amount of imported factors, so that the same commodity vector that is absorbed with trade can be supplied domestically. Thus, reproducing the logic of these first scholars, it is also possible, given the current data, to test the null hypothesis that factor abundance is not a good predictor of the factor content of trade based on domestic techniques. The temporal mean of the main tests of this hypothesis can be seen in Table 4 and Table 5.

Table 4. Slope, Variance, Sign, and Rank tests, factor-by-factor (temporal mean). Domestic factor content definition.

	Slope Test			Variance Test	Sign Test	Rank Test		
		Std. error	R^2				p-value	pairwise
Labour	0.012	0.003	0.174	0.001	0.737	0.284	0.013	0.642
Capital	0.015	0.041	0.068	0.065	0.45	-0.009	0.544	0.496
Cropland&Pastures	0.008	0.003	0.153	0	0.61	0.345	0.002	0.672
Forest land	0.01	0.002	0.404	0	0.647	0.472	0	0.736
Mineral resources	0	0	0.227	0	0.703	0.317	0.008	0.659
Energy	0.013	0.158	0.041	2.414	0.51	-0.088	0.452	0.456
Emissions	-0.012	0.061	0.037	0.22	0.477	-0.102	0.373	0.449

Table 5. Sign and Rank tests, country-by-country (temporal mean). Domestic factor content definition.

	Sign Test	Rank Test		
		Kendall	p-value	pairwise
AUS	0.943	0.743	0.026	0.871
AUT	0.514	0.057	0.844	0.529
BEL	0.771	0.429	0.247	0.714
BGR	0.386	-0.705	0.068	0.148
BRA	0.657	0.152	0.715	0.576
CAN	0.486	0.495	0.248	0.748
CHN	0.571	0.143	0.782	0.571
CYP	0.886	0.181	0.636	0.590
CZE	0.243	-0.438	0.252	0.281
DEU	0.657	0.114	0.820	0.557
DNK	0.343	0.362	0.342	0.681
ESP	0.586	0.133	0.797	0.567
EST	0.486	0.076	0.872	0.538
FIN	0.629	0.648	0.057	0.824
FRA	0.514	0.010	0.778	0.505
GBR	0.914	0.495	0.186	0.748
GRC	0.571	0.733	0.032	0.867

HUN	0.457	-0.371	0.347	0.314
IDN	0.743	0.448	0.230	0.724
IND	0.514	-0.181	0.688	0.410
IRL	0.843	-0.114	0.845	0.443
ITA	0.671	0.295	0.457	0.648
JPN	0.829	0.619	0.072	0.810
KOR	0.629	0.467	0.220	0.733
LTU	0.543	0.238	0.595	0.619
LUX	0.714	0.648	0.057	0.824
LVA	0.557	0.371	0.366	0.686
MEX	0.571	-0.629	0.079	0.186
MLT	0.600	0.520	0.242	0.760
NLD	0.500	0.229	0.595	0.614
POL	0.086	-0.752	0.026	0.124
PRT	0.771	0.010	0.955	0.505
ROU	0.271	-0.590	0.172	0.205
RUS	0.500	0.048	0.886	0.524
SVK	0.586	0.029	0.955	0.514
SVN	0.700	0.019	0.955	0.510
SWE	0.557	0.352	0.353	0.676
TUR	0.586	0.200	0.510	0.600
TWN	0.650	-0.053	0.972	0.473
USA	0.643	-0.048	0.977	0.476
ROW	0.600	0.267	0.519	0.633

As shown in Table 4, the proposition about the amount of domestic factor content being equal to that predicted receives little support when tested for each factor. The estimated slope is close to zero and the fit is generally bad except for 1 factor (forest land). Furthermore, the variance of the observed factor content is usually one or two orders lower than the variance of the predicted factor content, except for 2 factors (energy and emissions). Thus, there seems to be a large amount of missing trade. Moreover, the proposition of conformity in sign between domestic and predicted factor content of trade receives little support when tested both factor-by-factor (Table 4) and country-by-country (Table 5). Actually, the proportion of sign matches is 50% or greater for 5 of the 7 factors, but it is 70% or greater for only 2 of the 7 factors (labour and mineral resources). In addition, as shown in Table 5, the proportion of sign matches is 70% or greater for 10 of the 40 countries. Moreover, the proposition of factor trade revealing relative abundance of resources also receives little support, as can be seen from Kendall's rank tests, especially when tested country-by-country. The hypothesis of a zero-rank correlation is rejected (95% level) for 4 of the 7 resources (labour, cropland and pastures, forest land, and mineral resources) but for only 3 of the 40 countries. When the comparisons are made pairwise, somewhat greater support is found. Here the proportion of correct orderings exceeds 50% for 4 of the 7 resources and for 30 of the 40 countries, and it exceeds 70% for 1 of the 7 resources and for 11 of the 40 countries.

Overall, the results of the tests for the definition of factor content based on domestic techniques offer little support for the classic version of the HOV Theorem. This may be due either to the assumptions about preferences (3) or the assumptions about production techniques (5) not being fulfilled. In order to determine which of these two inconsistencies of the HOV Theorem may be the main reason for its empirical failure, it is possible to perform the following experiment. Assume that preferences are identical and homothetic and that commodity prices are equalized worldwide, and refrain from making any assumption about techniques. So, according to (3) and (11), the predicted factor content of trade turns out to be equal to:

$$\mathbf{f}_\mathbf{i}^\mathbf{A} = \mathbf{e}_\mathbf{i} - \frac{y_i}{y_w}\mathbf{A}_\mathbf{i}\mathbf{c}_\mathbf{w} \qquad (18)$$

Testing whether this prediction holds is the same as testing whether the prediction of the HOV Theorem holds by considering not the real factor-endowments but rather virtual ones resulting from an assumption that country-i's production techniques are used everywhere. Some scholars have seen this sort of exercise as a way to test the HOV Theorem by measuring world factor-endowments in terms of country-i's efficiency units (Trefler, 1993; Fisher & Marshall, 2016). However, since (18) expresses a simple identity arising only from the assumption of identical preferences and commodity price equalization, there seems to be no need to make such an assumption. Then, (18) can be used to test the null hypothesis that the predicted factor content of trade that would result from only assuming identical preferences is not a good predictor of the measured domestic factor content of trade. The temporal mean of the main tests of this hypothesis can be seen in Table 6 and Table 7.

Table 6. Slope, Variance, Sign, and Rank tests, factor-by-factor (temporal mean). Domestic factor content with 'virtual' endowments.

	Slope Test			Variance Test	Sign Test	Rank Test		
		Std. error	R^2				p-value	pairwise
Labour	0.424	0.079	0.499	0.371	0.765	0.702	0	0.771
Capital	0.336	0.058	0.412	0.302	0.687	0.529	0.015	0.690
Cropland&Pastures	0.446	0.058	0.586	0.326	0.625	0.478	0.003	0.677
Forest land	0.211	0.041	0.390	0.117	0.874	0.808	0	0.818
Mineral resources	0.899	0.038	0.905	0.833	0.960	0.941	0	0.912
Energy	0.730	0.046	0.812	0.669	0.758	0.711	0	0.768
Emissions	0.761	0.043	0.861	0.672	0.693	0.570	0.001	0.711

Table 7. Sign and Rank tests, country-by-country (time average). Domestic factor content with 'virtual' endowments.

	Sign Test	Rank Test		
		Kendall	p-value	pairwise
AUS	0.900	0.743	0.051	0.871
AUT	0.886	0.848	0.019	0.924
BEL	1	0.971	0.001	0.986
BGR	0.500	0.305	0.449	0.652
BRA	0.914	0.619	0.105	0.810
CAN	0.657	0.305	0.445	0.652
CHN	0.643	0.390	0.375	0.695
CYP	0.614	0.086	0.468	0.543
CZE	0.771	0.810	0.022	0.905
DEU	0.971	0.819	0.013	0.910
DNK	0.743	0.610	0.102	0.805
ESP	0.614	0.543	0.126	0.771
EST	0.743	0.676	0.071	0.838
FIN	0.757	0.895	0.004	0.948
FRA	0.514	0.267	0.518	0.633
GBR	0.900	0.762	0.038	0.881
GRC	0.714	0.629	0.089	0.814
HUN	0.900	0.695	0.056	0.848
IDN	0.786	0.600	0.090	0.800
IND	0.586	0.486	0.192	0.743
IRL	0.857	0.657	0.072	0.829
ITA	0.814	0.924	0.003	0.962
JPN	0.943	0.743	0.052	0.871
KOR	0.986	0.886	0.007	0.943
LTU	0.471	0.390	0.345	0.695
LUX	0.929	0.895	0.005	0.948
LVA	0.657	0.543	0.200	0.771
MEX	0.529	0.610	0.097	0.805
MLT	0.800	0.573	0.242	0.787
NLD	0.886	0.743	0.094	0.871

POL	0.614	0.543	0.157	0.771
PRT	0.743	0.800	0.016	0.900
ROU	0.571	0.648	0.069	0.824
RUS	0.514	0.067	0.886	0.533
SVK	0.600	0.371	0.396	0.686
SVN	0.700	0.571	0.210	0.786
SWE	0.829	0.867	0.007	0.933
TUR	0.814	0.752	0.049	0.876
TWN	0.983	0.987	0.004	0.993
USA	0.714	0.314	0.417	0.657
ROW	0.529	0.581	0.125	0.790

As seen in Table 6, the proposition about the amount of measured factor content being equal to that predicted receives some support when tested for each factor. The estimated slope is close to one for 3 of 7 factors (mineral resources, energy, and emissions), and the fit is very good except, perhaps, for 2 of 7 factors (capital and forest land). Furthermore, the variance of the observed factor content is no more than one order lower than the variance of the predicted factor content. Thus, it seems that there is still trade missing, although in a much smaller amount than when the actual factor-endowments were considered. Moreover, the proposition of conformity in sign between observed and predicted factor content of trade receives greater support when tested both factor-by-factor (Table 6) and country-by-country (Table 7). In fact, the proportion of sign matches is 60% or greater for all factors and 70% or greater for 4 of the 7 factors (labour, forest land, mineral resources, and energy). In addition, the proportion of sign matches is 70% or greater for 26 of the 40 countries (Table 7). Furthermore, the proposition of factor trade revealing relative abundance of resources also receives greater support, as can be seen from Kendall’s rank tests, especially when tested factor-by-factor. Therefore, the hypothesis of a zero-rank correlation is rejected for all of the factors (Table 6) and for 13 of the 40 countries (Table 7). When the comparisons are made pairwise, great support is also found. Here the proportion of correct orderings exceeds 65% for all resources and countries, and it exceeds 70% for 5 of the 7 resources and for 31 of the 40 countries.

Comparisons between the tests of Tables 4 and 5 and the tests of Tables 6 and 7 seem to indicate that the main reason why factor abundance is not a good predictor of factor content based on domestic techniques is that production techniques differ internationally. As long as factor proportions vary and factor prices are unequal, a variety of techniques exist, and there will be factor intensity reversals. Such reversals are responsible for the HOV Theorem not being fulfilled, even though the preferences are neither too different nor excessively non-homothetic. Thus, the following paradox arises, already highlighted in tests in which the factor content was estimated by the single-matrix method (Bowen et al., 1987). Under the assumption that each industry’s output is roughly homogeneous worldwide, the opportunity cost of trade in terms of factors does not seem to be related to the factor-endowments of the different countries. This is despite the fact that preferences are sufficiently similar and homothetic so that, if production techniques were the same, the hypothesis that the HOV Theorem is not fulfilled would not be rejected. This paradox may be considered as a generalization of Leontief’s paradox and must be related to the existence of factor intensity reversals.

The failure of tests for the HOV Theorem for a case in which the definition of factor content based on domestic techniques was considered led scholars to regard the alternative definition of actual factor content. This definition indicates that, factor prices being unequal, the factor intensities of exports and imports are measured at different factor prices. In order to overcome this difficulty, the HOV Theorem must be reformulated in such a way that it is assumed that there are NxC different commodities and that the identity of preferences encompass this variety of commodities, assuming that country i demands a share s_i of the commodities produced by every country. Thus, the focus changes to testing the hypothesis that factor abundance is not a good predictor of the actual factor content of trade. The temporal mean of the main test of this null hypothesis can be seen in Table 8 and Table 9.

Table 8. Slope, Variance, Sign and Rank tests, factor-by-factor (temporal mean). Actual factor content definition.

	Slope Test			Variance Test	Sign Test	Rank Test		
		Std. error	R^2				p-value	pairwise
Labour	0.280	0.020	0.720	0.116	0.942	0.786	0	0.893
Capital	0.211	0.040	0.432	0.106	0.698	0.442	0	0.721
Cropland&Pastures	0.314	0.016	0.880	0.110	0.780	0.809	0	0.905
Forest land	0.383	0.011	0.966	0.158	0.908	0.809	0	0.904
Mineral resources	1.130	0	1	1.336	0.935	0.774	0	0.887
Energy	0.369	0.042	0.640	0.218	0.845	0.596	0	0.798
Emissions	0.268	0.026	0.689	0.101	0.878	0.669	0	0.835

Table 9. Sign and Rank tests, country-by-country (temporal mean). Actual factor content definition.

	Sign Test	Rank Test		
		Kendall	p-value	pairwise
AUS	0.971	0.990	0.001	0.995
AUT	1	0.629	0.068	0.814
BEL	0.929	0.886	0.005	0.943
BGR	0.814	-0.019	0.358	0.490
BRA	0.829	0.676	0.055	0.838
CAN	0.786	0.762	0.030	0.881
CHN	0.771	0.638	0.067	0.819
CYP	0.986	0.657	0.078	0.829
CZE	0.929	0.705	0.046	0.852
DEU	0.871	0.857	0.007	0.929
DNK	0.829	0.895	0.009	0.948
ESP	0.957	0.686	0.042	0.843
EST	0.786	0.838	0.013	0.919
FIN	0.771	0.924	0.002	0.962
FRA	0.857	0.781	0.018	0.890
GBR	1	0.914	0.004	0.957
GRC	0.743	0.838	0.016	0.919
HUN	0.757	0.276	0.497	0.638
IDN	0.771	0.695	0.043	0.848
IND	0.671	0.705	0.036	0.852
IRL	1	0.800	0.015	0.900
ITA	0.857	0.886	0.007	0.943
JPN	0.986	0.981	0.001	0.990
KOR	0.929	0.943	0.002	0.971
LTU	0.457	0.324	0.423	0.662
LUX	1	0.857	0.012	0.929
LVA	0.543	0.486	0.242	0.743
MEX	0.814	0.352	0.360	0.676
MLT	0.883	0.853	0.035	0.927
NLD	0.857	1	0	1
POL	0.857	0.762	0.022	0.881
PRT	0.900	0.571	0.105	0.786
ROU	0.786	0.371	0.365	0.686
RUS	0.743	0.200	0.651	0.600
SVK	0.886	0.533	0.240	0.767
SVN	0.986	0.390	0.375	0.695
SWE	0.843	0.838	0.010	0.919
TUR	0.786	0.114	0.730	0.557

TWN	0.967	0.893	0.016	0.947
USA	0.971	0.514	0.146	0.757
ROW	0.886	0.448	0.245	0.724

As shown in Table 8, the proposition about the amount of actual factor content being equal to that predicted receives some support when tested for each factor. The estimated slope is close to one for 1 of 7 factors (mineral resources), and the fit is excellent except, perhaps, for 1 of 7 factors (capital). Moreover, the variance of the observed factor content is no more than one order lower than the variance of the predicted factor content. Thus, it seems that there is still trade missing, but in a much smaller amount than in the case of domestic factor content (Table 4). Additionally, the proposition of conformity in sign between actual factor content and factor abundance receives greater support when tested both factor-by-factor (Table 8) and country-by-country (Table 9). In this case, the proportion of sign matches is 70% or greater for all factors except one (capital) and for 38 of the 40 countries. Also, the proposition of factor trade revealing relative abundance of resources receives strong support from Kendall's rank tests, especially when tested factor-by-factor. Thus, the hypothesis of a zero-rank correlation is rejected for all the factors (Table 8) and for 23 of the 40 countries (Table 9). Great support is also found when the comparisons are made pairwise. Thus, the proportion of correct orderings exceeds 70% for all resources (Table 8) and for 33 of the 40 countries, the proportion being 50% or greater for all countries (Table 9).

Overall, the results of the tests shown in Tables 8 and 9 offer strong support for the hypothesis that factor abundance is a good predictor of the actual factor content of trade. Such a systematic correlation can be seen as evidence in favour of the many-commodities, no-factor-price-equalization version of the HOV Theorem. Indeed, this is the conclusion of tests that have regarded this definition of factor content using less complete data sets and even committing mistakes in computation (e.g., Davis & Weinstein, 2001; Reimer, 2006; Trefler & Zhu, 2010; etc.). According to this version of the HOV Theorem, there are a large number of commodities that can be ordered naturally according to their factor intensities. This ordering of commodities and differences in factor-endowments shape a chain of comparative advantage, implying full trade specialization. This means that a country i abundant in factor r produces exclusively factor-r-intensive subsets of commodities from p to q. Only one set of commodities are produced by several countries: these are the so-called 'borderline commodities' found at the limits of the chains of comparative advantage – that is, commodities $p-1$ and $q+1$ (Dornbusch, Fischer & Samuelson, 1980).

However, systematic correlation between factor abundance and factor content of trade can also arise from factor substitution. Thus, to the extent that unit inputs are function of factor prices and factor price equalization fails, it is to be expected that a country i, being abundant in factor r, will use that factor more intensively in the production of the same commodities than another country j where the same factor is scarce. Assume a single-sector model like that of Fadinger (2011) where all trade is due to preferences for variety, given an aggregate scale effect where the larger the amount of varieties used in production, the greater the global efficiency. Here each country is expected to absorb a share s_i of the commodities produced by every country, s_i being proportional to the size of the importing country, just like in the modified HOV model. Then, even if factor abundance were not a reason for trade, countries would be net exporters of their abundant factor, as they use that factor more intensively in all industries.

Here the question arises as to whether actual factor trade is due to full trade specialization, as assumed, or else a mirage caused by factor intensity reversals. In order to shed light on this point, it is possible to perform the following experiment similar to one proposed by Nishioka (2012) with a different aim. Consider the case in which differences in the use of factors between industries of the same country are not regarded, but where instead it is assumed that country-i's industries use the same amount of factors per unit of income. This is the same as assuming that, instead of N industries, there is only one sector in each country. So, if the assumptions of the modified version of the HOV Theorem are true, it can be expected that factor abundance should be a worse predictor of the factor content arising from a one-sector assumption than of the actual content of trade. This is so because, according to this version of the Theorem, the factor content of trade is explained by trade arising from full specialization in chains of comparative advantage. As differences in factor intensities between subsets of commodities disappears, part of the factor trade would be missed. In this regard, Table 10 and Table 11 show the temporal mean of certain tests of the null hypothesis that factor abundance is not a good predictor of the factor content of trade that would result if there were no differences in factor usage between the industries of each country.

Table 10. Slope, Variance, Sign and Rank tests, factor-by-factor (temporal mean). Actual factor content definition without considering industries.

	Slope Test			Variance Test	Sign Test	Rank Test		
		Std. error	R^2				p-value	pairwise
Labour	0.291	0.018	0.780	0.113	0.963	0.814	0	0.907
Capital	0.301	0.016	0.900	0.102	0.938	0.856	0	0.928
Cropland&Pastures	0.343	0.019	0.864	0.132	0.890	0.904	0	0.952
Forest land	0.371	0.009	0.976	0.144	0.892	0.819	0	0.910
Mineral resources	0.325	0	1	0.106	0.970	0.816	0	0.908
Energy	0.288	0.022	0.788	0.100	0.902	0.779	0	0.889
Emissions	0.260	0.019	0.782	0.081	0.908	0.761	0	0.881

Table 11. Sign and Rank tests, country-by-country (temporal mean). Actual factor content without considering industries.

	Sign Test	Rank Test		
		Kendall	p-value	pairwise
AUS	0.971	0.981	0.001	0.990
AUT	1	0.952	0.002	0.976
BEL	0.957	0.943	0.002	0.971
BGR	0.643	0.552	0.122	0.776
BRA	0.900	0.914	0.004	0.957
CAN	1	0.981	0.001	0.990
CHN	0.900	0.914	0.003	0.957
CYP	0.986	0.971	0.001	0.986
CZE	0.957	0.914	0.004	0.957
DEU	1	0.962	0.002	0.981
DNK	1	1	0	1
ESP	0.986	0.933	0.002	0.967
EST	0.829	0.810	0.020	0.905
FIN	0.986	0.962	0.001	0.981
FRA	1	0.962	0.002	0.981
GBR	1	0.943	0.002	0.971
GRC	0.957	0.848	0.009	0.924
HUN	0.871	0.771	0.023	0.886

IDN	0.829	0.895	0.004	0.948
IND	0.829	0.857	0.007	0.929
IRL	1	0.990	0.001	0.995
ITA	0.986	1	0	1
JPN	0.986	1	0	1
KOR	0.871	0.886	0.006	0.943
LTU	0.529	0.190	0.610	0.595
LUX	1	0.990	0.001	0.995
LVA	0.614	0.571	0.131	0.786
MEX	0.986	0.876	0.011	0.938
MLT	0.833	0.907	0.012	0.953
NLD	1	1	0	1
POL	0.914	0.952	0.002	0.976
PRT	0.929	0.724	0.122	0.862
ROU	0.871	0.810	0.016	0.905
RUS	1	0.981	0.001	0.990
SVK	0.957	0.829	0.016	0.914
SVN	0.986	0.905	0.006	0.952
SWE	0.929	0.962	0.001	0.981
TUR	0.786	0.543	0.180	0.771
TWN	0.983	0.947	0.011	0.973
USA	0.929	0.952	0.002	0.976
ROW	1	0.962	0.001	0.981

If the tests of Table 8 and Table 10 are compared, then it can be seen that the predicted factor content resulting from the reformulated HOV Theorem is nearly as good a predictor of ‘fake’ factor content as it was of actual factor content of trade. Indeed, regarding the estimated slope and the variance test, there seems to be significantly more missing trade for only 2 of the 7 factors (mineral resources and energy). The variance and slope tests are almost the same for the remaining factors, whether or not the differences between industries are considered. So, there is a bit more missing trade for some factors when such differences are not considered (forest land and emissions), while for other factors the case is the opposite (labour, capital, and cropland). Furthermore, regarding the sign test, when considering differences between industries, this improves only for one factor (forest land); regarding the rank tests, it does not improve for any factor. In addition, if the tests of Table 9 and Table 11 are compared, then it can be concluded that factor abundance is a better predictor of the ‘fake’ factor content than it is of actual factor content. Actually, the tests improve for only 8 of the 40 countries if differences between industries are considered, while for the remaining countries the tests worsen or remain the same.

In this way, comparing the tests in Tables 8 and 9 and the tests in Tables 10 and 11, it can be seen that, except in the case of a few factors, the tests remain the same or worsen when differences between industries within countries are considered. This outcome would be compatible with the assumptions of the modified HOV Theorem only if it is concluded that each country specializes in segments of a chain of comparative advantage that are very far from each other, so that ‘borderline commodities’ have little or no relevance in international trade. However, an outcome like that shown when comparing Tables 8/9 with Tables 10/11 could very nearly be reached without assuming that trade arises from factor abundance. Indeed, as commodity trade embodies factor services, countries would always be net exporters of their abundant factors, as they use such factors more intensively in all industries. In this way, although factor abundance seems to be a good predictor of the actual factor content of trade, there does not seem to be sufficient evidence to assume that such a correlation is due to full trade specialization arising from differences in factor-endowments. Indeed, such a correlation may be a spurious relationship due to factor substitution.

Conclusions

Given that factor prices and production techniques differ internationally, two definitions of factor content have been highlighted by scholars: factor content based on domestic techniques, and actual factor content. These definitions give rise to two alternative versions of the HOV model, and both have been tested in several studies. However, many of the tests of the HOV Theorem in the literature suffer from a lack of generality due to inconsistent definitions of factor content, incorrect calculations when inputs are considered, or insufficient data on countries, industries, and factors. Defining factor content in a consistent way, and regarding the multi-year data available from the World Input-Output Database, it is possible to test the HOV Theorem with alternative definitions of factor content for 7 factors, 56 industries, and 40 countries plus a composite 'country' comprising the rest of the world.

Overall, the tests performed are consistent with the main conclusions of the literature. If the definition of factor content based on domestic techniques is considered, then the HOV Theorem must be rejected and, therefore, the opportunity costs of trade in terms of factors do not seem to be related to factor abundance. This is despite the fact that preferences seem sufficiently similar and homothetic, so that if production techniques were universally the same, some support could be found for the HOV Theorem. This empirical failure may be considered a generalization of Leontief's paradox, and it must be related to the systematic occurrence of factor intensity reversals.

On the other hand, if the 'actual factor content' definition is considered, a modified version of the HOV Theorem which assumes full trade specialization cannot be rejected. However, if the factor content of trade is calculated according to this definition but without considering differences between industries within countries, test results remain roughly the same. Such an outcome can be explained by concluding that each country specializes in segments of a chain of commodities that are very far from each other, according to the assumptions of the modified version of the HOV Theorem. Nevertheless, an outcome such as this can be compatible with alternative models where trade does not arise from factor abundance. Thus, there is insufficient evidence from actual factor content tests to state that trade arises from factor-endowment differences.

Taking into account the main conclusions of this study and relating them to others in the literature, it seems that research on the HOV Theorem using input-output databases and performing the customary factor content tests will lead to the same point, no matter how much the approaches are refined or how much data is regarded. Therefore, further studies are necessary to shed light on the already well-established results of such factor content tests. Specifically, if the empirical failure of the classic version of the HOV Theorem is considered, then it is necessary to inquiry into the causes of that failure by resorting to more general theoretical models, in the manner of Fisher (2011) or Morrow and Trefler (2017). On the other hand, if the modified version of the HOV Theorem is regarded, it is necessary to design an experiment that allows for testing the hypothesis that trade arises from differences in factor-endowments (and not from other aspects), perhaps in a similar way to Fadinger (2011) but considering alternative specifications of the HOV Theorem. Ultimately, it seems that due to the existence of ambiguities in factor intensity, the Factor Abundance Theory remains an unsolved riddle.

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