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An Input-Output approach for assessing sustainability transitions in complex social-ecological systems: Conceiving a SESIO framework

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Abstract: Achieving the urgently needed global transition to sustainability is humanity's greatest challenge. Addressing this challenge requires an understanding of the complex economic-social-ecological dynamics in which we are embedded. While Input-Output analysis has proven to be a useful tool for assessing our complex economic systems, including accounting for the social and environmental impacts of economic activities across intricate production networks, it falls short in capturing key aspects of emerging sustainability paradigms. These aspects include social and ecological boundaries, as well as the interconnectedness and dynamics of Social-Ecological Systems, where the economy, society, and environment interact with each other through direct and indirect connections and feedback effects. This article discusses the applicability of Input-Output frameworks for assessing sustainability transitions within such systems. It then proposes a new framework—illustrated through a hypothetical example—that better captures the logic of Nested Sustainability and Social-Ecological Systems. In doing so, the study aims to contribute to the discussion and development of robust analytical frameworks capable of guiding decision-making in our increasingly complex world.

Keywords: Extended input-output models, hybrid input-output models, Nested Sustainability, Triple Bottom Line, Social-Ecological Systems, sustainability transitions.

1. Background

1.1. Sustainability concepts and Social-Ecological Systems

The multifaceted nature of sustainability has led to the adoption of a multidimensional approach for its assessment. Conventionally, this has involved combining indicators related to the "three pillars" of sustainable development—economic, social, and environmental dimensions—in sustainability analyses. These pillars, which date back to at least the 1980s (*e.g.*, see IUCN, UNEP, WWF, 1980), were discussed at the 2002 UN World Summit on Sustainable Development (UN, 2002) and gained further prominence at the 2012 UN Conference on Sustainable Development, also known as "Rio+20" (UN, 2012). In line with this concept, the "Triple Bottom Line" (TBL) framework was introduced in the 1980s and 1990s (*see* Spreckley, 1981; and Elkington, 1997), advocating that businesses integrate social and environmental dimensions into their economic accounting. Namely, to complement the "bottom line" of their financial statements, which reflects net profits over time, with social and environmental "bottom lines" measuring their impacts on society and the environment, thus showing companies' performance in terms of "people, planet, and profit" (Purvis *et al.*, 2019). This simple concept, often illustrated as three overlapping circles in a Venn diagram—where sustainability lies at the intersection of the three dimensions (Figure 1-left)—became the dominant perspective on sustainability (*e.g.*, UNDP, 2012), and is still widely employed in businesses' sustainability frameworks.

However, the TBL conceptual model has been criticized for portraying the economy, society, and the environment as largely independent dimensions that merely intersect at certain points, and by classifying

anything that considers the three dimensions as "sustainable". In response to such critique, an alternative conceptual model known as the "Nested Sustainability"—or "Nested Dependencies" model (Figure 1-centre)—was proposed, emphasizing the dependency relations among the three dimensions. This model depicts the economy as nested within society, and society as nested within the environment (Giddings *et al*., 2002). Situating the economy within society implies that the economy constitutes a subset of human society (*i.e*., encompassing only certain human interactions) and is dependent on it (*i.e.*, reliant on a functioning social system). Likewise, positioning society within the environment implies that human society is a subset of the environment and is dependent on it (*i.e*., reliant on suitable environmental conditions and ecosystem services). Arguably, this view implies that the system is also constrained by societal and ecological *limits*. Therefore, the sustainable nature of any economic activity is to be determined by its impact on social and environmental conditions, and their corresponding limits, a concept more recently embraced in the "planetary boundaries" framework (*see* Rockström *et al*., 2009; Steffen *et al*. 2015; and Richardson *et al*., 2023).

Lastly, the concept of "Social-Ecological Systems" (SES) has emerged, among other things, as a critique of the notion that clear boundaries exist between human systems (the economy and society) and natural systems (the environment). SES emphasize the intertwined interconnections and interdependencies between human and natural systems, where human and non-human components (or "agents") influence and react to each other through a network of connections and feedback effects (Figure 1–right), ultimately co-evolving as a single, complex adaptive system (Biggs *et al*., 2021). According to this perspective, sustainability assessments must account for limits, as well as for systemic interconnections and feedbacks.

1.2. Sustainability transitions' assessments

Sustainability transitions refer to processes that bring about structural changes in key parts of our social and economic systems (*e.g.*, transformations in energy and food systems, including shifts in the actors involved, their behaviours, and their relationships), ultimately leading to a more sustainable human society. Accordingly, assessing sustainability transitions requires a multidimensional approach that integrates economic, social, and environmental factors and focuses on systemic changes. This entails that evaluating whether an activity supports or hinders a sustainability transition should be based on an assessment of its economic, social, and environmental impacts; on whether those impacts strengthen or degrade other parts of the broader system (*e.g.*, economic stability, social cohesion, or ecological integrity); and on how the system restructures itself as a result. This rationale thus aligns with key features of the TBL, Nested Sustainability, and SES frameworks. Namely, the consideration of a) economic, social, and environmental dimensions, b) societal and ecological limits, c) systemic interconnections, and d) feedback effects.

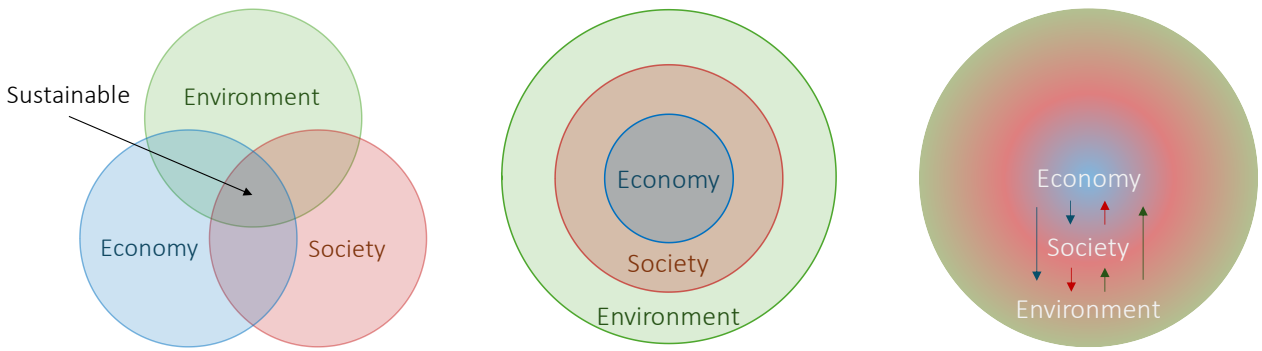

Fig. 1: Two common conceptual models of sustainability: The "Triple Bottom Line" (left) and the "Nested Sustainability" (centre); and a depiction of the notion of Social-Ecological Systems (right)

2. Approach

Against this background, how can an analytical tool such as Input-Output (IO) analysis be used to assess sustainability transitions? This question can be explored, first, by examining the extent to which current IO models account for key aspects of sustainability transitions, and second, by considering how IO frameworks might be adapted to incorporate essential features of sustainability concepts that are more aligned with the assessment of such transitions.

In this article, the answer to this question reflects the author's perspective, drawing on an overview of the rapidly expanding IO literature and the author's ideas for leveraging the strengths of IO modelling while addressing some of its current limitations. To illustrate these points, a simple hypothetical example is presented.

3. Discussion

3.1. To what extent are key aspects of sustainability transitions considered in current IO models?

IO analysis has proven to be a powerful tool for capturing and understanding economic complexity. By linking production and consumption sectors across the economy, IO models enable the assessment of the direct and indirect effects of economic activity along entire supply chains. In line with this logic, IO models have been extended beyond purely economic applications since their inception. For instance, Leontief himself explored their use for environmental impact analyses (Leontief, 1970). Coupling IO models with data on environmental and social indicators has led to the creation of "extended" economic models, suited for analysing environmental impacts (*e.g.*, Lenzen *et al.*, 2012 & 2013) and social impacts (*e.g.*, Gómez-Paredes *et al.*, 2015 & 2016). This capability, combined with the flexibility of IO modelling, has made such models widely used in sustainability science research (*e.g.*, Galli *et al.*, 2012; Wiedmann and Lenzen, 2018). However, while the simultaneous consideration of economic, social, and environmental dimensions is useful for revealing potential synergies and trade-offs associated with different activities—thus aligning IO frameworks with "nexus" analyses (*e.g.*, Liu *et al.*, 2018)—such an approach follows only the logic of the TBL concept (Figure 1-left). It does not account for interdependencies, limits, or feedback effects between these dimensions.

Traditional IO models do not include transactions beyond the economic sphere; that is, they consider only the workings of the economy as represented in the central circle of the Nested Sustainability conceptual model (Figure 1-centre), while overlooking non-economic transactions within the social and environmental spheres that underpin the functioning of the economy (the outer circles). Notable exceptions to this are the early frameworks proposed by Isard *et al.* (1968) and Daly (1968), which sought to incorporate elements of ecological systems into IO arrangements. Isard *et al.* (1968) and Isard (1969) aimed to include food chains, while Daly discussed the integration of "ecological commodities" that serve as inputs to the economy, natural sinks that absorb the outputs of economic activity, and purely ecological transactions occurring within non-human sectors—all of which he regarded as the "biophysical foundations of economics" (Daly, 1968, p. 401). However, these pioneering efforts, which attempted to link the economic and environmental spheres—the inner and outer circles of the Nested Sustainability model—have largely been overlooked in the subsequent development of IO models and their applications to this day.

With regard to limits or constraints, some models have incorporated supply and demand constraints in the context of natural disasters or extraordinary events (*e.g.*, COVID-19; *see* Pichler and Farmer, 2022). Yet, no IO model appears to account for the social and environmental constraints that arise from the normal functioning of

SES. Additionally, traditional IO modelling tends to omit feedback effects resulting from changes within the broader economy–society–environment complex adaptive system when such limits are exceeded.

Consequently, even extended IO models fail to capture the complexity of SES and thus remain limited in their capacity to assess sustainability transitions.

3.2. How can IO frameworks be adapted to incorporate essential features of sustainability concepts more aligned with sustainability transition assessments?

It should be clear at this point that simply continuing to extend IO models through a series of "satellite accounts" incorporating new sets of economic, social, and environmental indicators will not suffice to make them suitable for assessing sustainability transitions within complex SES. What is needed is to situate the economic system within society, and society within the environment, following the Nested Dependency conceptual model. Then, it is essential to incorporate social and ecological limits, systemic interconnections (transactions occurring) beyond the economic sphere, and feedback effects within the entire system (*e.g.*, when limits are transgressed).

3.2.1. SESIO framework

Building on the practice of extending IO models to enhance their assessment capabilities, but moving beyond the mere addition of satellite accounts, an IO framework can be developed that follows the rationale of the Nested Sustainability conceptual model and the logic of SES. This proposed framework would incorporate nine submatrices recording transactions across the economy, society, and environment dimensions. Furthermore, this holistic approach—referred to here as the "Social-Ecological Systems Input-Output" (SES-IO) framework (*see* Figure 2)—would introduce limits or boundaries on transactions to reflect social and environmental constraints and integrate additional feedback effects beyond those inherent to IO analysis.

In more detail, the components of the SESIO framework would include the following submatrices and vectors:

	Economy	Society	Environment	*f*	*x*	*l*
Economy	***EE*** Economy to Economy	***ES*** Economy to Society	***EN*** Economy to Nature	Exogenous demand	Total outputs	Limits
Society	***SE*** Society to Economy	***SS*** Society to Society	***SN*** Society to Nature			
Environment	***NE*** Nature to Economy	***NS*** Nature to Society	***NN*** Nature to Nature			

Fig. 2: Proposed SESIO framework

EE (Economy-Economy): This corresponds to the conventional IO transaction matrix, which records transactions between economic sectors within the economy. This matrix may take the form of an industry-by-industry, commodity-by-commodity, or commodity-by-industry table.

ES (Economy-Society): This matrix captures the outputs of the economy that serve as inputs to society—for example, the goods and services consumed by households. Including households in this matrix effectively "closes" the model with respect to households, while treating all other components of final demand as exogenous variables in the final demand vector $\boldsymbol{f}$. However, depending on the specific purpose of constructing a model under the SESIO framework, a different component of final demand could be included instead of households, or even all final demand sectors could be incorporated, thereby creating a fully closed model for the purpose of assessing the structure of the entire system and comparing total outputs $\boldsymbol{x}$ with the corresponding limits $\boldsymbol{l}$.

EN (Economy-Environment/Nature): This matrix represents the economic inputs into the environment, such as pollution and waste generated by production processes. It may also account for positive environmental contributions, such as those resulting from regenerative agriculture, reforestation, or environmental remediation.

SE (Society-Economy): This matrix captures society's contributions to the economy, such as labour and other human inputs essential for economic production.

SS (Society-Society): This matrix accounts for transactions among sectors of society that occur outside the formal economy—and are therefore not recorded in the ***EE*** matrix—such as domestic labour, informal employment[1], voluntary work, community reciprocity,[2] and barter. Given their non-market nature, these transactions are more appropriately represented in non-monetary units, as in a hybrid social matrix[3].

SN (Society-Environment/Nature): This matrix includes outputs from human beings that are inputs into the environment—namely, those resulting from human metabolism and the consumption of products, such as household-generated sewage discharges, solid waste, and atmospheric emissions from fuel combustion, as well as emissions from respiration[4].

NE (Environment/Nature-Economy): This matrix represents the environment's inputs into the economy—namely, natural resources.

NS (Environment/Nature-Society): This matrix considers inputs from the environment directly into human societies—namely, ecosystem services[5].

NN (Environment/Nature-Environment/Nature): This matrix describes ecological interactions and natural processes that underpin the functioning of ecosystems and the broader Earth System.

Limits: Lastly, the SESIO framework includes a vector $\boldsymbol{l}$ of social and environmental limits and/or thresholds, which serves as the basis for generating a vector $\boldsymbol{w}$ of "budgets and deficits" used to readjust the model.

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[1] Informal employment is estimated to include up to 2 billion workers, representing over 60% of the global employed population. While the rate is below 40% in developed countries, it averages 69.6% in developing and emerging economies (Bonnet, Vanek, & Chen, 2019).

[2] Reciprocal favours between neighbours—common in cohesive communities and social networks—often compensate where the formal economy fails or falls short in meeting people's needs (Leonard, 2000).

[3] Although conventional IO tables record transactions in monetary terms, "[i]n essence, money does not matter for an input-output table" (Moss, 2013, p. 224).

[4] Due to their negligible impact, respiratory emissions may be excluded from SESIO models.

[5] These are benefits of nature to households and communities (Boyd & Banzhaf, 2007) that are not considered in conventional economic accounting systems (Cato, 2009).

In this framework, the disaggregation and scope of the row and column sectors in all submatrices will depend on the knowledge of interactions/transactions, data availability, and the specific research questions being addressed. Consequently, a variety of models can be constructed based on this framework. For illustrative purposes, the following is a simple hypothetical example demonstrating how a SESIO model would function.

3.2.2. Hypothetical example

Consider a small rural community located within a river basin, whose economy is based on agriculture (including fisheries) and the manufacturing of goods (including fish-based products). Assume a hybrid matrix of annual transactions, ***Z***, as presented in Table 1. In ***Z***, the economy submatrix ***EE*** comprises two highly aggregated sectors: *Agriculture* and *Manufacturing*. The society-related submatrices ***ES, SE,*** *and* ***SS*** account for the employed population from two income groups: low-income households (*Low*) and middle-income households (*Middle*), along with a sector accounting for the total employed members of the community (*Community*). The environment-related submatrices ***EN, SN, NE, NS,*** *and* ***NN*** include atrazine load (*Pesticides*), water dilution capacity for atrazine[6] ($Water_{DC}$), the river's volume of water (*Water*), the annual fish catch from the river (*Fish catch*), and the river's fish stock (*Fish*).

In this framework, the underlined row sectors *Community*, *Water*, and *Fish* are *total resource sectors*, as they aggregate the corresponding resource transactions for comparison between their total outputs ***x*** and their respective limits ***l***, as further explained below. From Table 1, we observe that to meet a final demand of $8 million, both sectors must generate a total output of $23 million. Conventionally, the ***EE*** submatrix describes the intersectoral transactions in monetary units.

Surrounding ***EE***, the *social* submatrices describe the following relationships: To the right of ***EE***, the ***ES*** submatrix indicates that low-income workers consume $4 million annually in agricultural products and $2 million in manufactured goods. Middle-income workers consume $6 million in agricultural products and $7 million in manufactured goods. Since this partially closes the model with respect to households, the vector ***f*** in this example refers to other components of final demand (*e.g.*, the unemployed population, government, investment, and net exports). Below ***EE***, the ***SE*** submatrix shows labour inputs into both economic sectors. Specifically, 6,000 low-income workers are employed in agriculture, while 2,000 low-income workers and 6,000 middle-income workers are employed in manufacturing. To the right, the ***SS*** submatrix indicates that low-income individuals informally employ 2,000 workers[7]. Finally, the *total resource* row sector "*Comm*unity" shows that, overall, 10,000 low-income individuals and 6,000 middle-income individuals are employed. This information thus corresponds to the transpose of the vector formed by the total outputs of the "*Low*" and "*Middle*" sectors. The corresponding limits indicate that the community has a total workforce of 20,000 people, evenly divided between low- and middle-income workers. Comparing the total outputs with their corresponding limits, we can infer a community unemployment rate of 20%, which is concentrated among middle-income workers, since the entire low-income workforce is (formally plus informally) employed.

Surrounding the social and economic submatrices, the environmental submatrices convey the following information: The ***EN*** submatrix indicates that 5 tons of atrazine from pesticide runoff on agricultural lands enter

[6] Water's dilution capacity is defined as the effective volume of water required to dilute a given effluent to an environmentally acceptable concentration (UKMPA, 2013). In this case, it refers to the volume needed to dilute atrazine discharges to a concentration that does not harm aquatic life.

[7] The challenge of accounting for informal labour in terms of wages is circumvented by expressing it in terms of the number of people employed.

the river. However, this value creates a conflict of units in the agriculture row sector, thereby violating the consistency of row sums in hybrid IO analysis. Thus, following the approach used in economic-ecologic models (*see* Miller and Blair, 2009), the data from the ***EN*** submatrix is transposed and positioned adjacent to the ***NE*** submatrix, resulting in Table 2. In this adjusted configuration, the transaction between the row sector "*Pesticides*" and "*Agriculture*" can be interpreted as the amount of atrazine that needs to be emitted—under the current agricultural production process—for the agricultural sector to generate its output[8].

Continuing with Table 2, no transactions are recorded in the ***SN*** submatrix. Below ***SE***, the ***NE*** submatrix shows that the agricultural sector utilizes a net 120 million m^3 of water from the river annually, while the manufacturing sector uses 50 million m^3. The agricultural sector also captures 100 tons of fish annually. To the right, the ***NS*** submatrix indicates that low-income individuals withdraw 200,000 m^3 of water annually directly from the river (not via a water-supply industry, which would correspond to a transaction in the ***EE*** submatrix). It is assumed that workers from middle-income households obtain their water from a different source and through a water-supply industry, which is not included in the ***EE*** matrix, as it is not considered relevant to this system. Similarly, low-income individuals consume 180 tons of fish annually, obtained directly from the river or through informal employment (not via the formal market)[9]. The ***NN*** submatrix describes a series of environmental interactions. The transaction from the row sector "*$Water_{DC}$*" to the column sector "*Pesticides"* represents the water dilution capacity for atrazine—namely, 515.46 million m^3 of water required to dilute 5 tons of atrazine to a concentration that prevents negative impacts on aquatic organisms (*i.e.,* 9.7 tons/km^3)[10]. The water used for dilution is recorded in the transaction between "*Water*" and "*$Water_{DC}$*"[11]. The river's remaining unimpacted volume of water (14.314 km^3) is "used" by the aquatic ecosystem to sustain the river's organisms, including fish. Finally, the 100 tons of fish captured by the agricultural sector and the 180 tons consumed by low-income individuals are drawn from the row sector "*Fish*", recorded in the transaction between "*Fish*" and "*Fish catch*".

Limits for these environmental row sectors are the river's total annual water flow (15 km^3, or 15,000 million m^3) and the fisheries' maximum sustainable yield (300 tons of fish per year). In this example, different forms of limits are presented. The "*Community*" limit is disaggregated into its components, while the other two limits are aggregated. Furthermore, the "*Community*" limit is an *expandable limit*, whereas the limit of the row sector "*Water*", corresponding to the river's total water flow, is considered here to be a *fixed limit* that cannot be expanded or exceeded. In contrast, the limit of the row sector *"Fish"* represents a *threshold* that cannot be expanded but can be overshot.

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[8] This differs from the total amount of atrazine used by the agricultural sector. Therefore, the *Pesticides* sector could be disaggregated into *Pesticides Used* and *Pesticides Emitted*. Additional, further rows could be added for other inputs to the agricultural sector—such as phosphorus—along with corresponding *total resource* row sectors (*e.g.*, "*Phosphates*", similar to "*Water*" and "*Fisheries*"), which could be compared with related limits (*e.g.*, total phosphate reserves or a threshold such as "peak phosphorus"; *see* Cordell and White, 2011). However, this is omitted here for the sake of simplicity.

[9] Note that water and fish are common ecosystem services used by poor rural households (*see* Cavendish, 2000).

[10] The United States Environmental Protection Agency (EPA) recommends a maximum concentration of 9,7 µg/L of atrazine in water bodies to prevent adverse effects on fish, aquatic plants, invertebrates, and amphibians (EPA, 2024).

[11] The transactions "*$Water_{DC}$*"-"*Pesticides*" and "*Water*"-"*$Water_{DC}$*" could be avoided by directly using a transaction "*Water*"-"*Pesticides*". However, both transactions are included here to make the relationship explicit.

Table 1. SESIO model's matrix of transactions

Z		Economy		Society			Environment					*f*	*x*	*l*	< units >	
		Agriculture	Manufacturing	Low	Middle	*Community*	Pesticides	$Water_{DC}$	*Water*	Fish catch	*Fish*					
Economy	Agriculture	1	4	4	6		5*					8	23		$1x10^6$ $	*T. of atrazine
	Manufacturing	3	3	2	7							8	23		$1x10^6$ $	
Society	Low	6	2	2									10	10	$1x10^3$ people	
	Middle		6										6	10	$1x10^3$ people	
	Community			10	6								**16**	**20**	$1x10^3$ people	
Environment	Pesticides															
	$Water_{DC}$															
	Water															
	Fish catch															
	Fish															

Table 2. Matrix of transactions with *EN* submatrix transposed

Z (adjusted)		Economy		Society			Environment					*f*	*x*	*l*	< units >	
		Agriculture	Manufacturing	Low	Middle	*Community*	Pesticides	$Water_{DC}$	*Water*	Fish catch	*Fish*					
Economy	Agriculture	1	4	4	6							8	23		$1x10^6$ $	
	Manufacturing	3	3	2	7							8	23		$1x10^6$ $	
Society	Low	6	2	2									10	10	$1x10^3$ people	Disaggre-gated limit
	Middle		6										6	10	$1x10^3$ people	
	Community			10	6								**16**	**20**	$1x10^3$ people	
Environment	Pesticides	5											5		*T. of atrazine*	
	$Water_{DC}$						515.46						515.46		$1x10^6$ m^3	
	Water	120	50	0.20				515.46			14314.34		**15,000**	**15,000**	$1x10^6$ m^3	→ *Fixed limit*
	Fish catch	100		180									280		*T. of fish*	
	Fish									280			**280**	**300**	*T. of fish*	→ *Threshold*

Note: Empty cells in the submatrices and vectors represent zeros and are omitted for ease of readability

Following traditional IO modelling, the corresponding technical coefficients matrix, ***A***, can be derived from equation (1) and is presented in Table 3. The coefficients in the ***EE*** submatrix are the traditional economic technical coefficients, also known as direct input coefficients, which represent the amount of inputs required from each row sector by each column sector per monetary unit of total output. Outside the ***EE*** submatrix, the coefficients can be interpreted as follows: The coefficients in the ***ES*** submatrix represent the annual per capita expenditure (in thousand $) on agricultural and manufactured products by low- and middle-income workers. In the ***SE*** submatrix, the coefficients indicate the number of low- and middle-income workers required per thousand $ of total output. The "*Low*"-"*Low*" coefficient in the ***SS*** submatrix represents the number of low-income individuals employed per low-income worker. The remaining two coefficients are logically unitary, as they simply aggregate employment data by income segment[12].

Next, the coefficients in the ***NE*** submatrix represent the tons of atrazine entering the river per million $ of total output (or g/$); the m^3 of water required by the agricultural and manufacturing sectors per monetary unit of total output; and the tons of fish captured from the river per million $ of total output (or g/$). The coefficients in the ***NS*** submatrix correspond to the per capita annual consumption of water (in thousand m^3 per low-income worker)[13] and fish (in kg per low-income worker)[14]. Lastly, the coefficients in the ***NN*** submatrix represent the amount of water needed per unit of pesticide to reach the environmentally safe concentration of atrazine (*i.e.*, 103.09 million m^3 per ton of atrazine; or 103.09 L/μg),[10] corresponding to the transaction *"Water$_{DC}$"-"Pesticides"*; the origin of the water required to cover this dilution capacity (transaction *"Water"-"Water$_{DC}$"*)[15]; the volume of water that supports fish (transaction *"Water"-"Fish",* in million m^3 per ton of fish, or m^3/g); and the amount of fish per fish caught from the river (transaction *"Fish"-"Fish catch"*)[16].

The corresponding Leontief inverse matrix, ***L***, is obtained from the IO identity described in equation (2), where ***I*** is the identity matrix. The resulting matrix, which includes *total requirements*, is presented in Table 4. These coefficients capture both direct and indirect input requirements across sectors within the entire SES.

Now, let us consider the effect that the growth of final demand would have on the whole system. Suppose the final demand for manufactured products increases by $3 million. This results in two vectors: a vector Δf, representing the change in final demand, and a vector f^*, representing the new final demand (equation 3). We

$$A = Z\hat{x}^{-1} \tag{1}$$

$$L = (I - A)^{-1} \tag{2}$$

$$\Delta f = \begin{bmatrix} 0 \\ 3 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad ; \quad f^* = \begin{bmatrix} 8 \\ 11 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3}$$

[12] These coefficients reflect the self-evident fact that each employee is an individual drawn from the community's workforce.

[13] This corresponds to 20 m^3 per person per year, a volume comparable to the amount recommended by the United Nations to meet basic individual needs for drinking, cooking and cleaning (UNWATER, 2013).

[14] This corresponds to 18 kg per person per year, which aligns with the global average for fish consumption (FAO, 2012).

[15] This value is 1, as all the water required for the dilution of the pesticide is sourced from the river.

[16] This value is also 1, being consistent with the fact that all the fish caught come from the river.

Table 3. SESIO model's technical coefficients matrix

A		Economy		Society			Environment				
		Agriculture	Manufacturing	Low	Middle	*Community*	Pesticides	$Water_{DC}$	*Water*	Fish catch	*Fish*
Economy	Agriculture	0.04	0.17	0.40	1.00						
	Manufacturing	0.13	0.13	0.20	1.17						
Society	Low	0.26	0.09	0.20							
	Middle		0.26								
	Community			1.00	1.00						
Environment	Pesticides	0.22									
	$Water_{DC}$						103.09				
	Water	5.22	2.17	0.02				1.00			51.12
	Fish catch	4.35		18.00							
	Fish									1.00	

Table 4. SESIO model's Leontief inverse matrix

$L = (I-A)^{-1}$		Economy		Society			Environment				
		Agriculture	Manufacturing	Low	Middle	*Community*	Pesticides	$Water_{DC}$	*Water*	Fish catch	*Fish*
Economy	Agriculture	1.53	1.35	1.10	3.10						
	Manufacturing	0.55	2.32	0.86	3.26						
Society	Low	0.56	0.69	1.70	1.37						
	Middle	0.14	0.61	0.22	1.85						
	Community	0.70	1.30	1.93	3.22	1.00					
Environment	Pesticides	0.33	0.29	0.24	0.67		1.00				
	$Water_{DC}$	34.27	30.16	24.68	69.46		103.09	1.00			
	Water	897.32	977.68	1843.36	2037.95		103.09	1.00	1.00	51.12	51.12
	Fish catch	16.70	18.30	35.43	38.05					1.00	
	Fish	16.70	18.30	35.43	38.05					1.00	1.00

Note: Empty cells in the submatrices represent zeros and are omitted for ease of readability

can then calculate the change in total output Δx, and the new total output vector x^*, using the corresponding IO equations shown in (4). The resulting vectors Δx and x^* (equation 5) show the additional total outputs required to meet the change in, and the new final demand[17]. According to these results, the agricultural sector would need to generate an additional $4 million in total output, while the manufacturing sector would need to produce $7 million more. This, in turn, would require approximately 2,000 new jobs in each income group, thereby increasing total employment (both formal and informal) to 20,000 people in the community. The environmental repercussions would consist of the release of an additional ton of pesticide (atrazine) into the river, requiring 90 million m^3 of additional water for its dilution, bringing the total dilution requirement to 606 million m^3. Altogether, this results in an extra water demand of 2,933 million m^3, raising the total water use to 17,933 million m^3. Lastly, the fish catch would increase by 55 tons, reaching a total of 335 tons of fish removed from the river annually.

$$\Delta x = (I - A)^{-1}\Delta f \quad ; \quad x^* = (I - A)^{-1} f^* \tag{4}$$

$$\Delta x = \begin{bmatrix} 4 \\ 7 \\ 2 \\ 2 \\ 4 \\ 1 \\ 90 \\ 2933 \\ 55 \\ 55 \end{bmatrix} ; \; x^* = \begin{bmatrix} 27 \\ 30 \\ 12 \\ 8 \\ 20 \\ 6 \\ 606 \\ 17933 \\ 335 \\ 335 \end{bmatrix} ; \; l = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 10 \\ 20 \\ 0 \\ 0 \\ 15000 \\ 0 \\ 300 \end{bmatrix} \tag{5}$$

From here, to assess the impact of these increases, we turn to the vector of limits, l (also shown in Equation 5). Let us consider a reduced version of this vector, l^R, containing only the limit-related values, along with a corresponding reduced version of the new vector of total outputs x^{*R}. Subtracting x^{*R} from l^R yields a vector of "*budgets and deficits*", w (equation 6).

$$\begin{bmatrix} 10 \\ 10 \\ 20 \\ 15000 \\ 300 \end{bmatrix} - \begin{bmatrix} 12 \\ 8 \\ 20 \\ 17933 \\ 335 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \\ -2933 \\ -35 \end{bmatrix} \tag{6}$$

$$l^R \quad - \quad x^{*R} \quad = \quad w$$

[17] All values are rounded for ease of explanation.

In this vector, the positive values (budgets) represent the remaining resources before reaching their respective limits, while the negative values (deficits) indicate the extent to which the limits have been exceeded. Thus, from the disaggregated "*Community*" limit, we observe that although the generated labour would suggest full employment of the community's workforce, there is, in fact, a deficit in the number of low-income workers and a similar budget in the number of middle-income workers. This implies that the labour requirement could not have been met by the existing workforce alone, but rather through the immigration of 2,000 low-income workers into the community[18]. It also means that the unemployment rate now stands at 9%. Next, the deficit related to the "*Water*" limit indicates that the new total output exceeds the available water supply—an impossibility, given that we have assumed it to be a *fixed limit* (this issue is addressed in the next step). Lastly, regarding "Fish," the deficit of 35 tons per year reflects the extent of exploitation beyond the maximum sustainable yield.

To assess the changes in the SES, equation 7 can be used to calculate a new matrix of transactions, $\boldsymbol{Z^*}$ (Table 5). Subsequently, *expandable limits* should be updated to eliminate deficits, *fixed limits* should be adjusted by subtracting deficits from one or more of the corresponding row sector transactions, and *thresholds* should be maintained to reveal overshoots.

$$Z^* = A\hat{x}^* \tag{7}$$

This procedure yields Table 6. Note that the *expandable limit* associated with the "*Community*" row sector now reflects the growth in the community's workforce. The *fixed limit* corresponding to the "Water" row sector has been adjusted by subtracting the relevant deficit from the river's remaining uncompromised volume of water, which sustains the aquatic ecosystem and supports the river's fish population[19]. The reduction in this volume reflects the increasing pressure on the aquatic ecosystem and the associated risk to its integrity and functionality as it approaches its given environmental flow. It is important to note that this feedback in the system would be overlooked without the consideration of limits and the corresponding correction of deficits. Finally, the total output exceeding the *threshold* for the "*Fish*" row sector indicates the overshoot.

Thus, the adjusted matrix of transactions (Table 6), incorporating the updated column vector of limits, captures all the changes and feedback effects resulting from a purely economic variation—namely, increased consumption of manufactured products—providing a more comprehensive assessment of impacts and a clearer understanding of the complexity of the SES. This matrix can subsequently be used to evaluate further changes in the system.

[18] This assumes that none of the new low-income jobs were filled by available middle-income workers.

[19] If we were to calculate a new matrix of technical coefficients, $A^* = Z^*\widehat{x^*}^{-1}$, we would observe that the amount of uncompromised water per ton of fish decreased from 51.12 million m^3 (in $\boldsymbol{A}$) to 42.36 million m^3 (in $\boldsymbol{A^*}$).

Table 5. New matrix of transactions

Z (new)*		Economy		Society			Environment					*f*	*x*	*l*	*< units >*	
		Agriculture	Manufacturing	Low	Middle	*Community*	Pesticides	$Water_{DC}$	*Water*	Fish catch	*Fish*					
Economy	Agriculture	1	5	5	8							8	27		$1x10^6$ $	
	Manufacturing	4	4	2	9							11	30		$1x10^6$ $	
Society	Low	7	3	2									12	12	$1x10^3$ *people*	*Disaggregated limit*
	Middle		8										8	10	$1x10^3$ *people*	*Disaggregated limit*
	Community			12	8								**20**	**22**	$1x10^3$ *people*	*Disaggregated limit*
Environment	Pesticides	6											6		*T. of atrazine*	
	$Water_{DC}$						605.94						605.94		$1x10^6$ m^3	
	Water	141	65	0.24				605.94			17120.65		**17,933**	**15,000**	$1x10^6$ m^3	*Fixed limit*
	Fish catch	118		217									335		*T. of fish*	
	Fish									335			**335**	**300**	*T. of fish*	*Threshold*

Table 6. Adjusted new matrix of transactions

Z (adjusted)*		Economy		Society			Environment					*f*	*x*	*l*	*< units >*	
		Agriculture	Manufacturing	Low	Middle	*Community*	Pesticides	$Water_{DC}$	*Water*	Fish catch	*Fish*					
Economy	Agriculture	1	5	5	8							8	27		$1x10^6$ $	
	Manufacturing	4	4	2	9							11	30		$1x10^6$ $	
Society	Low	7	3	2									12	12	$1x10^3$ *people*	*Disaggregated limit*
	Middle		8										8	10	$1x10^3$ *people*	*Disaggregated limit*
	Community			12	8								**20**	**22**	$1x10^3$ *people*	*Disaggregated limit*
Environment	Pesticides	6											6		*T. of atrazine*	
	$Water_{DC}$						605.94						605.94		$1x10^6$ m^3	
	Water	141	65	0.24				605.94			14187.60		**15,000**	**15,000**	$1x10^6$ m^3	*Fixed limit*
	Fish catch	118		217									335		*T. of fish*	
	Fish									335			**335**	**300**	*T. of fish*	*Threshold*

Note: Empty cells in the submatrices and vectors represent zeros and are omitted for ease of readability

4. Concluding remarks

Notions of sustainability have evolved through the recognition of interconnectedness among its three pillars—economic, social, and environmental—and of their interdependencies, constraints, and feedback effects, which collectively constitute a single SES. Consequently, methodological tools for assessing sustainability transitions must also evolve toward more comprehensive frameworks.

Building upon the established usefulness of IO frameworks in capturing economic complexity, this article proposes a framework more closely aligned with the rationale of the Nested Sustainability concept and the notion of SES—termed the SESIO framework—by incorporating nine matrices that account for economy-society-environment transactions and by adding output limits for corresponding feedbacks. The simple subnational hypothetical example presented here illustrates what a SESIO model might look like and how it can capture interactions across an economy-society-environment system that conventional IO analysis—even when extended to include social and environmental accounts—would fail to capture.

The nine matrices, which constitute submatrices within the SESIO framework, comprise economy-economy (***EE***), economy-society (***ES***), economy-environment/nature (***EN***), society-economy (***SE***), society-society (***SS***), society-environment/nature (***SN***), environment/nature-economy (***NE***), environment/nature-society (***NS***), and environment/nature-environment/nature (***NN***) transactions. In this way, the framework extends traditional IO modelling to the social and environmental domains. From this foundation, multiple SESIO models can be constructed depending on the research question and the availability of data.

Developing such models is undoubtedly a challenging endeavour. Nonetheless, conventional IO analysis has already incorporated ***EE*** interrelationships along with social and environmental transactions similar to those proposed for ***ES***, ***EN***, ***SE***, and ***NE*** submatrices (*see* Miller and Blair, 2009). Furthermore, as previously mentioned, pioneering work by Daly (1968) and Isard *et al*. (1968) initiated discussions on combining the ***EE*** submatrix with ***EN***, ***NE***, and ***NN*** type submatrices. In the words of Daly:

> *"How does one integrate the world of commodities into the larger economy of nature? […] Leontief's input-output model has proved useful in dealing with phenomena of interdependence, and it may offer the most promising analytical framework within which to consider the above question. Just as the annual flow of gross national product, or final commodities, requires a supporting matrix of flows of intermediate commodities, so does the annual flow of all economic commodities (final and intermediate) require a supporting matrix of flows of physical things which carry no price tag but nonetheless are necessary complements to the flows of those things which do carry price tags"* (Daly, 1968, p.400).

Of the nine submatrices, the most difficult to construct will likely be the ***SS*** and ***NN*** submatrices, as non-market activities (***SS*** submatrix) are not systematically recorded in national statistics, and ecological—and more broadly, Earth System—interconnections (***NN*** submatrix) are complex and not yet fully understood. This suggests that developing such submatrices for a comprehensive and accurate representation of the workings of society and nature within SESIO models may be virtually impossible. However, it is not necessary to strive for a perfect model, as no model can ever fully represent reality. Citing Daly again:

> *"Certainly no single classification would give a complete representation of the exquisitely tangled web of physical life relations—but then the usual input-output model is also a very incomplete*

picture of economic relations. Different classifications can be used to serve different limited purposes." (Daly, 1968, p.404).

Accordingly, in these models we may simply account for "all inputs and outputs […] relevant for the problem being examined" (Isard, 1969, p.93).

Another important issue concerns the underlying assumptions of IO modelling, particularly the linear relationships among inputs and outputs. Can such linearity be assumed in the social and environmental realms? Once again, the answer depends on what is included in the model. While labour per unit of output, food and water consumption per individual, the volume of water required to dilute pollutants, and other relationships described in the hypothetical example presented in this article may reasonably be assumed to be largely linear, many others clearly are not. Daly also reflected on this issue:

"Leontief's basic assumption of constant (slowly changing) technology over time seems to be much closer to the facts […], since in the non-human economy technical change (evolution) is much slower than in the human economy. Linearity [or constant coefficients…] would seem to be at least equally appropriate as a first approximation […] since biological populations grow by adding identical units-hence input-output relations of biological populations are more likely to be proportional to scale (linear) than are such relations for populations of firms (that is, industries) in which new members are never such close replicas of old members." (Daly, 1968, p.404).

On the other hand, non-linear relationships are clearly present in SES (*e.g., see* Mathias *et al.*, 2020). In such cases, however, SESIO models may be paired with system dynamics models (as in Cordier *et al*., 2017) or with other nonlinear modelling approaches and methodologies (as also suggested by Isard *et al.*, 1968).

Overall, the contribution of the SESIO framework to the evolution of IO analysis lies in its conceptual design and the integration of all nine submatrices into a single analytical structure, one that incorporates total output limits for additional feedback effects, features that are largely absent from conventional IO applications. These elements are fundamental for developing more holistic models capable of capturing the complexity of SES and for assessing, in that light, the effectiveness of sustainability transition strategies.

Much time has passed since the early proposals of Daly (1968) and Isard *et al*. (1968), with virtually no progress made in developing their integrative IO models. The main obstacle has likely been the difficulty of processing large volumes of data and organizing them in a manner consistent with IO analysis. In this regard, the present article also serves as an invitation to renew the conversation around constructing such models in the current era, when data processing capacities are significantly greater and rapidly expanding. It may now be timely to consider advocating for the System of National Accounts to set the standards, and for national statistical agencies to generate the data needed to make SESIO models feasible.

Lastly, constructing models based on the proposed SESIO framework and applying them to real-world case studies will likely require collaboration across multiple academic disciplines—bringing together economists, social scientists, ecologists, and others. While challenging, this is a valuable feature, as this kind of transdisciplinary effort is precisely what is required to understand and model SES. Further research should therefore focus on refining and developing the framework within transdisciplinary groups, as well as on building SESIO models and applying them to evaluate plans and actions aiming at realising the global sustainability transition upon which humanity increasingly depends.

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