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A New Interregional Input-Output Model with Endogenous Self-sufficiency Rate

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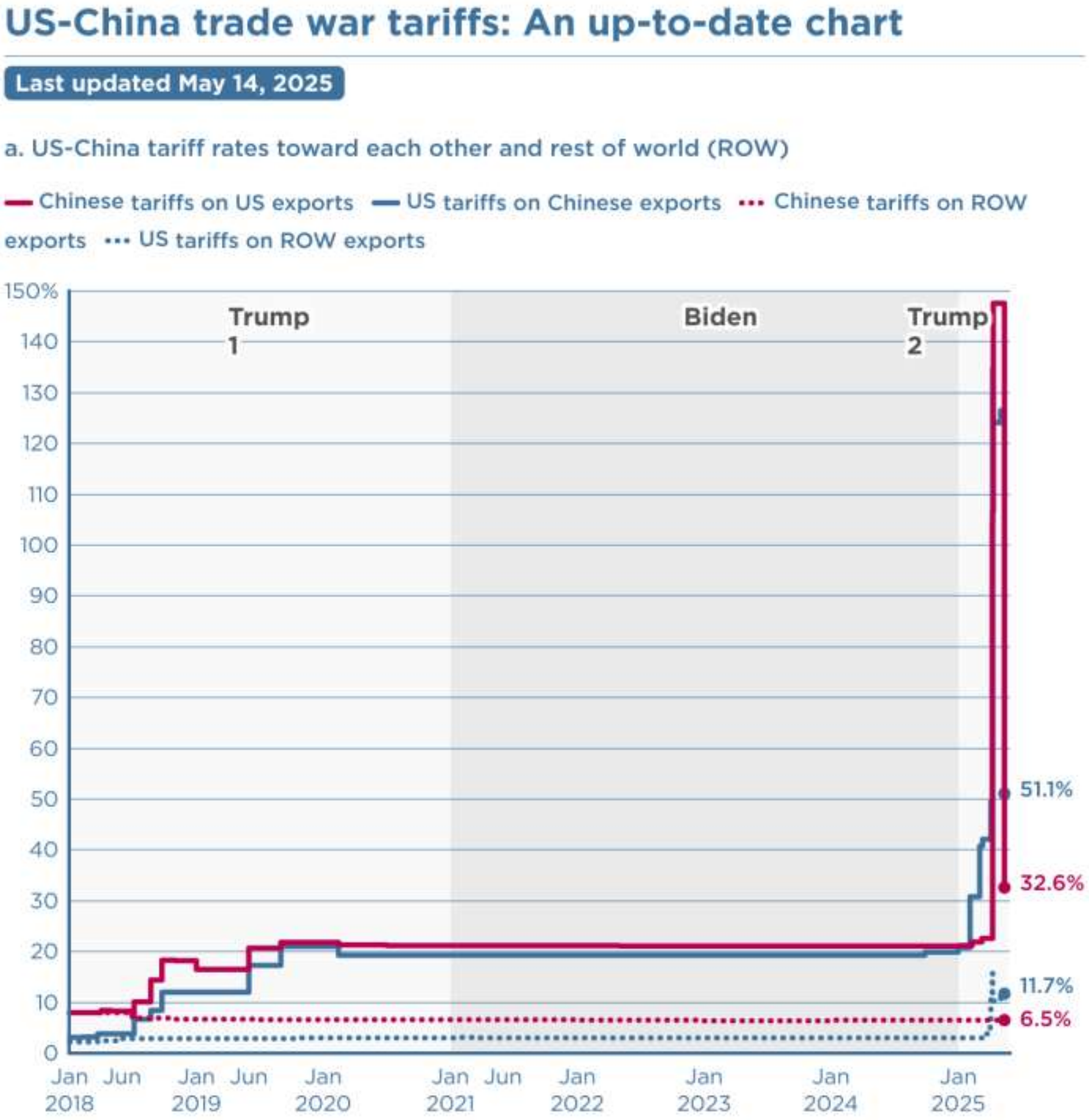
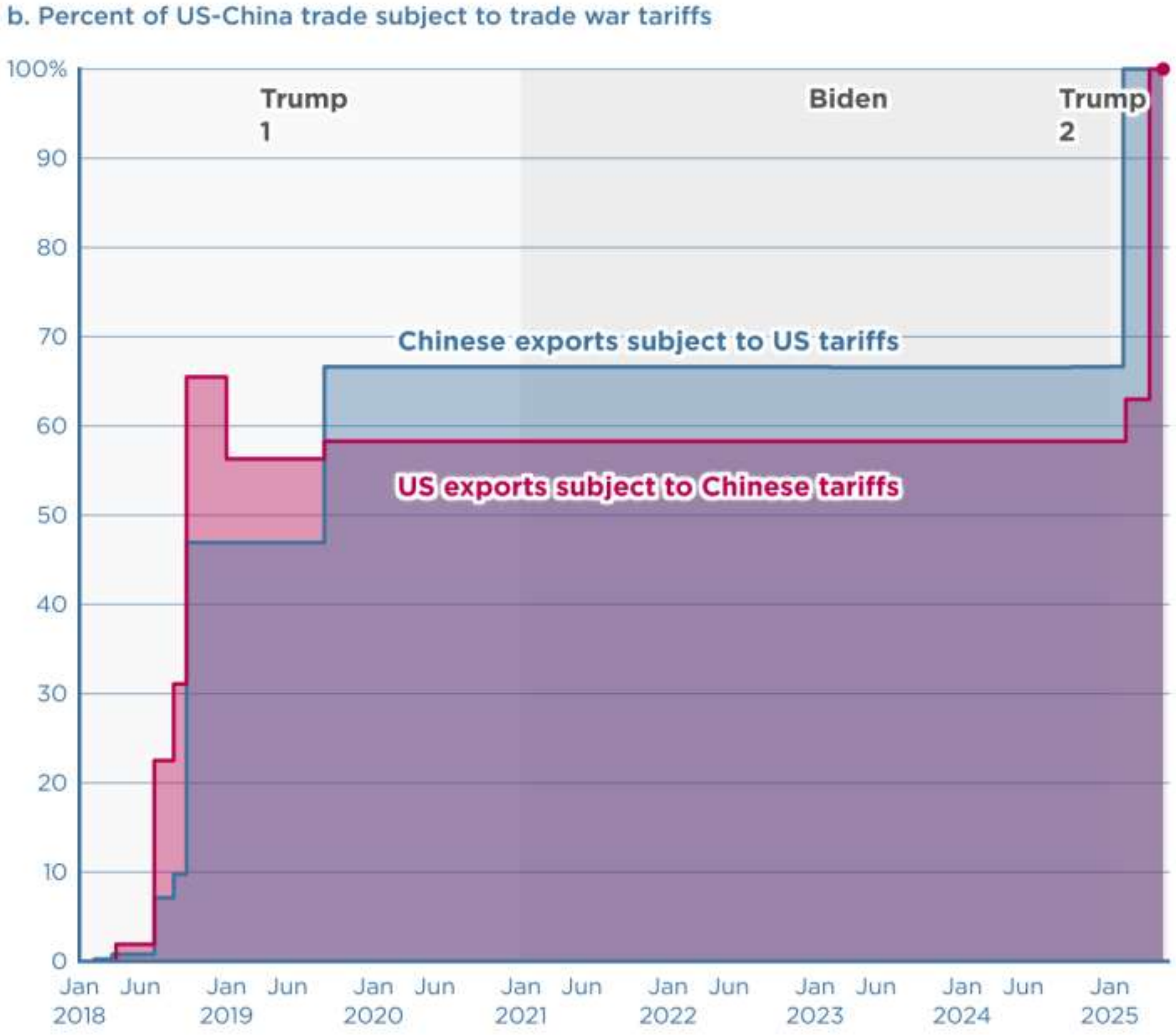
Introduction

Introduction

CHN-U.S.

Global trade is on track to hit a record $33 trillion by 2024.





Is the use of tariffs effective?

- **The direct effects** include an increase in import prices, a rise in domestic substitution demand, changes in fiscal revenue, and initial adjustments in trade structure.
- **The indirect effects**, gradually propagate through channels such as production networks, intermediate input prices, resource allocation, consumer behavior, and firm expectations.

- **Computable General Equilibrium (CGE)** models capture how markets co-adjust in multi-sector, multi-country settings. They are widely used to simulate tariff shocks on economic structure, welfare, and global value chains (Shoven & Whalley, 1984; Liu & Ma, 2022 ;Kahn et al., 2024;).Today, CGE models are standard tools for tariff evaluation by the WTO, IMF, and World Bank.
- **Structural models,** by specifying explicit causal mechanisms and structural parameters, identify how tariffs transmit through prices, output, and supply chains. They are well-suited to uncover policy transmission channels at the micro level. (**Eaton & Kortum**,2002;Caliendo & Parro ,2015; Arkolakis et al,2023;Kreuter & Riccaboni,2023)

One key challenge in modeling tariff shocks lies in measuring the numerous indirect effects they generate.

- **Input-output models,** as a framework to analyze the interdependence of industries , have been a critical tool for measuring the production network effect.
- (Feenstra, 1994; Broda and Weinstein, 2006).
- (Zhang et al., 2020; Fontagné et al., 2022)/(Dietzenbacher and van der Linden, 1997; Los et al., 2016).
- (Tian et al., 2021; Giammetti et al., 2020; Wu and Luo, 2016; Sonis et al., 2000).

In trade theory, **the Armington specification** assume that product varieties from different places of production are imperfect substitutes.(**Armington, 1969).**

Extensive empirical research has tested this assumption(Saito,2004;Feenstra. et al,2018; Bajzik et al.2020; Caliendo and Parro, 2022; Guerra and Sancho,2024).Their demand for different varieties will depend on the so-called Armington elasticity.

$$\frac{M_i}{D_i} = \left(\frac{p_i^M}{p_i^D}\right)^{-\sigma_i}\left(\frac{1-\delta_i}{\delta_i}\right)^{\sigma_i}$$

Armington Elasticity (Elasticity of Substitution between domestic products and imports) are non-zero

change in Relative price → *change in M/D(in self-sufficiency level)* → *the impact of external shocks*

IRIO assume that Armington Elasticity are zero

change in Relative price *self-sufficiency level are constant*

How to address the limitations of classic methodologies?

Route 1: Estimate the Price Elasticity of Demand and apply the results to the IRIO model. Considering the substitution of imported goods from the perspective of final demand.

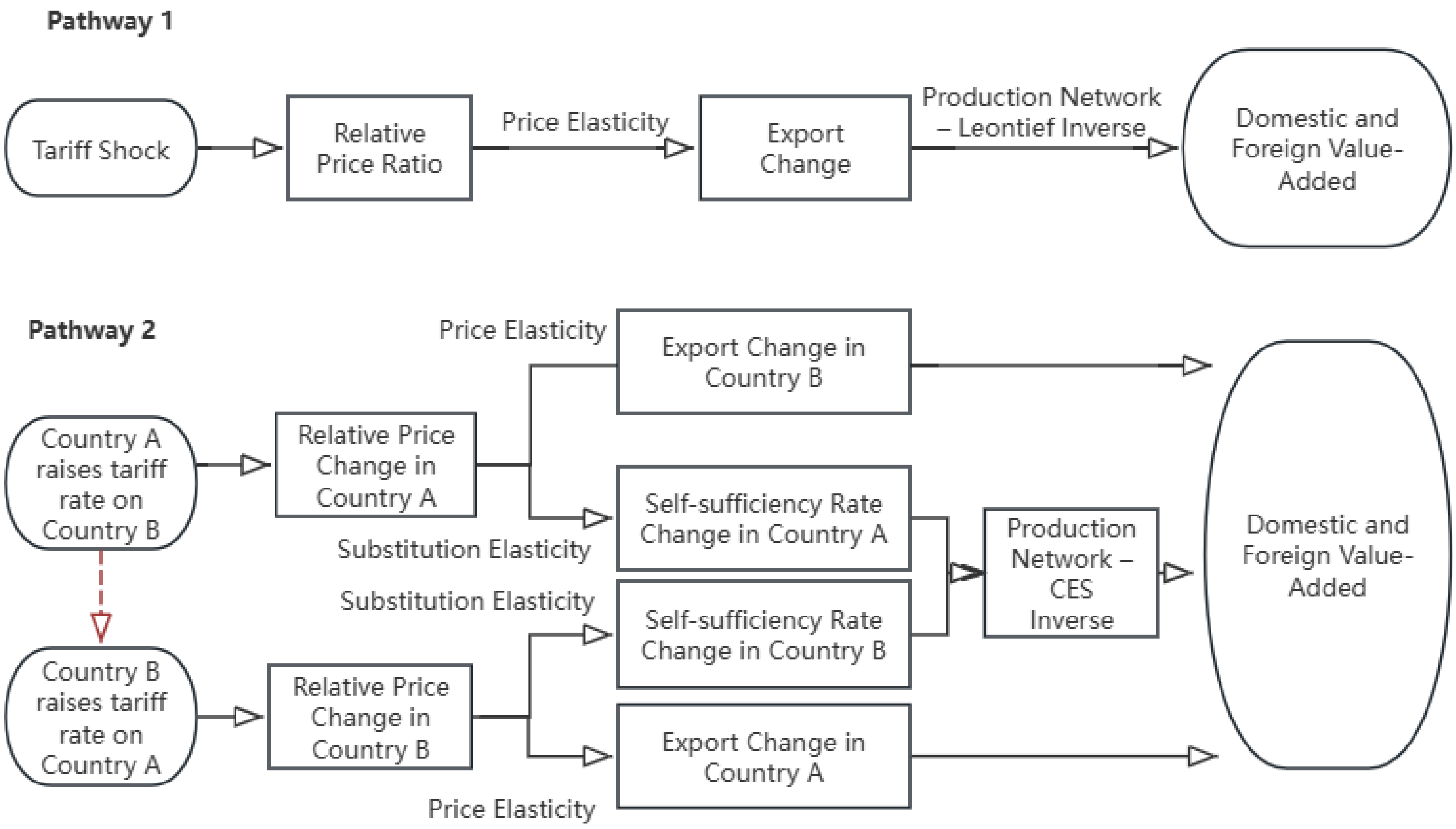
$$\ln Q_X = c + \alpha \ln \frac{R}{P_D} + \boldsymbol{\beta} \frac{P_X}{P_D} + \varepsilon$$

Limitations: When analyzing the propagation of shocks through production networks, the substitution effects between imported and domestic products are still not taken into account.

Route 2: **The Hypothetical Extraction Method(HEM)** addresses the issue of partial substitution by adjusting the coefficients of the A matrix. （Giammett,2020)）

$$\mathbf{A}^* = \begin{bmatrix} a_{GG} & \boldsymbol{a_{GE}}^* & a_{GR} \\ \boldsymbol{a_{EG}}^* & a_{EE} & a_{ER} \\ a_{RG} & a_{RE} & a_{RR} \end{bmatrix} \qquad i\dot{m}_i = \varepsilon_{Di}(\gamma_i + \omega_i)$$

Limitations: The issue of substitution when shocks propagate through the production network has not yet been thoroughly addressed.


Pathway 1
Tariff Shock
Relative Price Ratio
Price Elasticity
Export Change
Production Network – Leontief Inverse
Domestic and Foreign Value-Added
Pathway 2
Price Elasticity
Export Change in Country B
Country A raises tariff rate on Country B
Relative Price Change in Country A
Self-sufficiency Rate Change in Country A
Substitution Elasticity
Production Network – CES Inverse
Domestic and Foreign Value-Added
Substitution Elasticity
Self-sufficiency Rate Change in Country B
Country B raises tariff rate on Country A
Relative Price Change in Country B
Export Change in Country A
Price Elasticity

Micro-foundations

Background of the Story

- *Consider a static perfectly competitive economy with two countries, n industries*
- Each industry produces a single product.
- Product varieties from different places of production are differentiated, with different price ($p_{\mathrm{i}}^{\mathrm{r}}, p_i^s$)

	Country r		Country s	
Type	Physical Quantity	Price	Physical Quantity	Price
Product 1(r)	z_1^{r}	p_1^{r}	z_1^s	p_1^s
	……	……	……	……
Product n(r)	$z_{\mathrm{n}}^{\mathrm{r}}$	p_n^{r}	z_{n}^s	p_n^s

Micro-foundations

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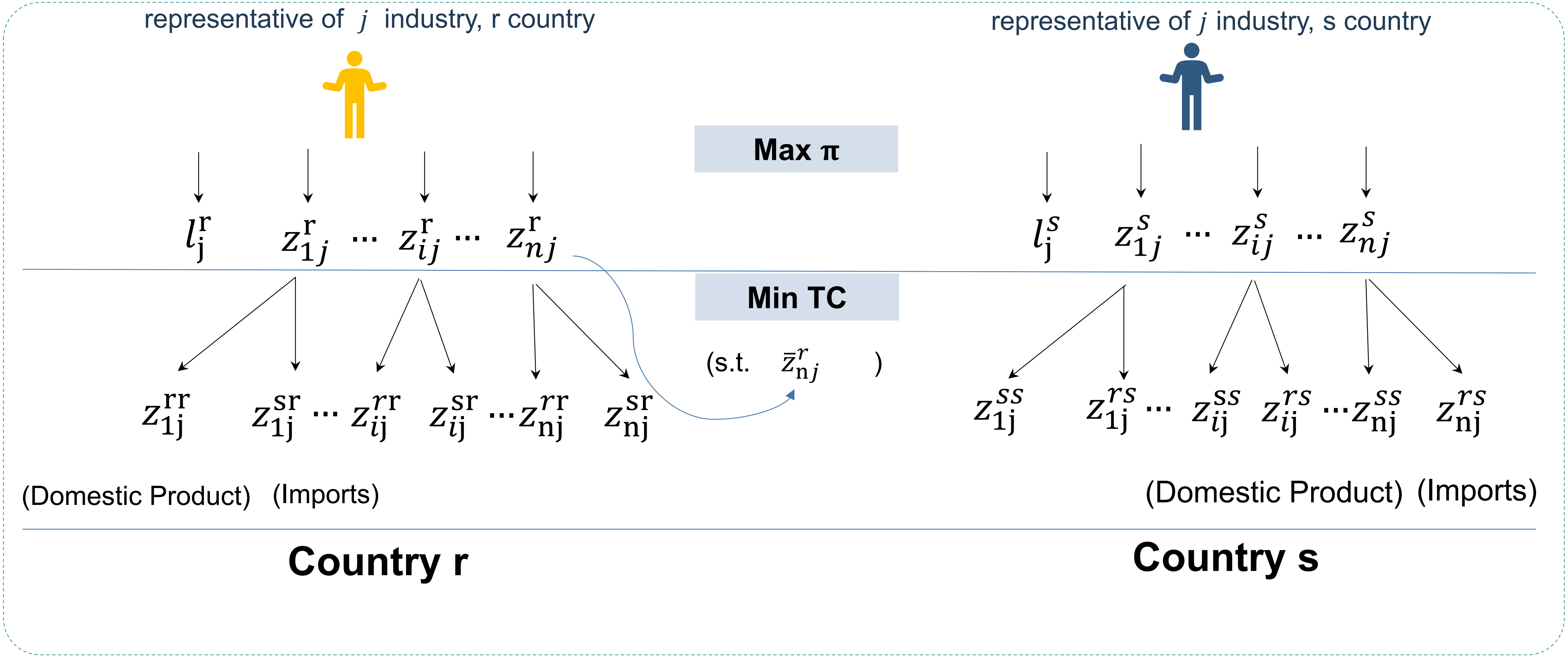
Decision of representative producer



Micro-foundations

CD Function (Acemoglu et al.2012，2016),

$$z_j^t = l_j^{t\,\alpha_{lj}^t} \prod_{i=1}^{n} z_{ij}^{t\;a_{ij}^t} \qquad t = (r, s)$$

Intermediate Inputs (CES aggregator)

$$z_{ij}^r = \left[z_{ij}^{r,r\,\frac{\varepsilon_{ir}-1}{\varepsilon_{ir}}} + z_{ij}^{s,r\,\frac{\varepsilon_{ir}-1}{\varepsilon_{ir}}} \right]^{\frac{\varepsilon_{ir}}{\varepsilon_{ir}-1}} \qquad z_{ij}^s = \left[z_{ij}^{r,s\,\frac{\varepsilon_{is}-1}{\varepsilon_{is}}} + z_{ij}^{s,s\,\frac{\varepsilon_{is}-1}{\varepsilon_{is}}} \right]^{\frac{\varepsilon_{is}}{\varepsilon_{is}-1}}$$

z_{ij}^t : *aggregate inputs made up of domestic and imported inputs.*

ε_{it} : *Armington elasticity*

The first stage

MAX Profit: $\pi^t = z_j^t p_j^t - w^t l_j^t - \sum_{i=1}^{n} \boxed{p_i^{ct}} z_{ij}^t$

$$p_i^{ct} = \left((p_i^r)^{1-\varepsilon_{it}} + (p_i^s)^{1-\varepsilon_{it}} \right)^{\frac{1}{1-\varepsilon_{it}}} \qquad \text{(Dixit and Stiglitz, 1977)}$$

$$\downarrow$$

$$\boxed{p_i^{ct} z_{ij}^t = p_i^r z_{ij}^{rt} + p_i^s z_{ij}^{st}}$$

The first-order condition, $a_{ij}^t = \dfrac{p_i^{ct} z_{ij}^t}{p_j^t z_j^t}$

$$\boxed{z_{ij}^{optimum} = z_{ij}^{observed}}$$

Output elasticity of intermediate inputs = Intermediate input coefficient(Single-Region Model of competitive type)

Micro-foundations

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The second stage

- MIN TC: $TC_{ij}^{t} = p_i^r z_{ij}^{rt} + p_i^s z_{ij}^{st}$

$$s.t.\ \bar{z}_{ij}^{t} = \left[z_{ij}^{rt\,\frac{\varepsilon_{it}-1}{\varepsilon_{it}}} + z_{ij}^{st\,\frac{\varepsilon_{it}-1}{\varepsilon_{it}}} \right]^{\frac{\varepsilon_{it}-1}{\varepsilon_{it}}}$$

$\bar{z}_{ij}^{t}$:The fixed aggregate intermediate inputs which has been decided at the first stage

- The first-order condition,

$$\frac{p_i^r}{p_i^s} = \left(\frac{z_{ij}^{rt}}{z_{ij}^{st}} \right)^{-\frac{1}{\varepsilon_{it}}} = p_i \qquad t = (r, s)$$

Relative price among countries

Self-sufficiency Rate

$$\theta_{ij}^{t} = \frac{p_i^t z_{ij}^{tt}}{p_i^r z_{ij}^{rt} + p_i^s z_{ij}^{st}}$$

$$\theta_i^r = \frac{1}{1 + p_i{}^{\varepsilon_{ir}-1}}$$

$$\theta_i^s = \frac{1}{1 + p_i{}^{1-\varepsilon_{is}}}$$

VS

$$\theta_{ij}^{r} = \frac{a_{ij}^{rr}}{a_{ij}^{rr} + a_{ij}^{sr}}$$

$$\theta_{ij}^{s} = \frac{a_{ij}^{ss}}{a_{ij}^{rs} + a_{ij}^{ss}}$$

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A comparative static analysis of parameters

- $\varepsilon_{ir} > 1, \varepsilon_{is} > 1$, $p_i \uparrow (p_i^r \uparrow, \overline{p_i^s})$
  - $\theta_i^r \downarrow$:Country r has heightened reliance on imports, leading to deepening structural dependence on Country s.
  - $\theta_i^s \uparrow$:Country s has intensified its reliance on domestic products, correspondingly diminishing its dependence on Country r.
- $\varepsilon_{ir} < 1, \varepsilon_{is} < 1$, $p_i \uparrow (p_i^r \uparrow, \overline{p_i^s})$
  - $\theta_i^r \uparrow$:Country r has intensified its reliance on domestic products, correspondingly diminishing its dependence on Country s.
  - $\theta_i^s \downarrow$:Country r has heightened reliance on imports, leading to deepening structural dependence on Country r.
- $\varepsilon_{ir} > 1, \varepsilon_{is} < 1$, $p_i \uparrow (p_i^r \uparrow, \overline{p_i^s})$
  - $\theta_i^r \downarrow / \theta_i^s \downarrow$:The production networks of the two countries are converging towards structural integration
- $\varepsilon_{ir} < 1, \varepsilon_{is} > 1$, $p_i \uparrow (p_i^r \uparrow, \overline{p_i^s})$
- $\theta_i^r \uparrow / \theta_i^s \uparrow$:The production networks of the two countries are undergoing structural decoupling.
- $\varepsilon_{ir} = \varepsilon_{is} = 1$

  Relative price has no longer affect self-sufficiency rate.

ICIO Model Restructuring

ICIO Model Restructuring

Balanced relationship：

$$\sum_{j=1}^{n} x_{ij}^{rr} + \sum_{j=1}^{n} x_{ij}^{rs} + y_i^r = x_i^r$$
$$\sum_{j=1}^{n} x_{ij}^{sr} + \sum_{j=1}^{n} x_{ij}^{ss} + y_i^s = x_i^s$$

$$\sum_{j=1}^{n} p_i^r z_{ij}^{rr} + \sum_{j=1}^{n} p_i^r z_{ij}^{rs} + y_i^r = p_i^r z_i^r$$
$$\sum_{j=1}^{n} p_i^s z_{ij}^{sr} + \sum_{j=1}^{n} p_i^s z_{ij}^{ss} + y_i^s = p_i^s z_i^s$$

Substituting output elasticity of intermediate inputs and self-sufficiency rate.

$$\begin{aligned} p_i^r z_{ij}^{rr} &= \theta_i^r p_i^{cr} z_{ij}^r \\ &= \theta_i^r a_{ij}^r p_j^r z_j^r \\ &= \theta_i^r a_{ij}^r x_j^r \end{aligned}$$

$$\theta_{ij}^t = \frac{p_i^t z_{ij}^{tt}}{p_i^r z_{ij}^{rt} + p_i^s z_{ij}^{st}}$$

$$a_{ij}^t = \frac{p_i^{ct} z_{ij}^t}{p_j^t z_j^t}$$

$$\begin{aligned} p_i^r z_{ij}^{rs} &= (1-\theta_i^s) p_i^{cs} z_{ij}^s \\ &= (1-\theta_i^s) a_{ij}^s p_j^s z_j^s \\ &= (1-\theta_i^s) a_{ij}^s x_j^s \end{aligned}$$

$$\begin{aligned} p_i^s z_{ij}^{sr} &= (1-\theta_i^r) p_i^{cr} z_{ij}^r \\ &= (1-\theta_i^r) a_{ij}^r p_j^r z_j^r \\ &= (1-\theta_i^r) a_{ij}^r x_j^r \end{aligned}$$

$$\begin{aligned} p_i^s z_{ij}^{ss} &= \theta_i^s p_i^{cs} z_{ij}^s \\ &= \theta_i^s a_{ij}^s p_j^s z_j^s \\ &= \theta_i^s a_{ij}^s x_j^s \end{aligned}$$

ICIO Model Restructuring

Balanced equations

$$\sum_{j=1}^{n} \theta_i^r a_{ij}^r x_j^r + \sum_{j=1}^{n} (1-\theta_i^s) a_{ij}^s x_j^s + y_i^r = x_i^r$$

$$\sum_{j=1}^{n} (1-\theta_i^r) a_{ij}^r x_j^r + \sum_{j=1}^{n} \theta_i^s a_{ij}^s x_j^s + y_i^s = x_i^s$$

The matrix form:

$$\hat{\theta}^r A^r X^r + \left(1-\hat{\theta}^s\right) A^s X^s + Y^r = X^r$$

$$\left(1-\hat{\theta}^r\right) A^r X^r + \hat{\theta}^s A^s X^s + Y^s = X^s$$

It could be expressed as in the relationship of the change in variables:

$$\begin{bmatrix} \Delta X^r \\ \Delta X^s \end{bmatrix} = \begin{bmatrix} I - \hat{\theta}^r A^r & -\left(1-\hat{\theta}^s\right) A^s \\ -\left(1-\hat{\theta}^r\right) A^r & I - \hat{\theta}^s A^s \end{bmatrix}^{-1} \begin{bmatrix} \Delta Y^r \\ \Delta Y^s \end{bmatrix}$$

	Country r			Country s		
Country r	$\theta^r_1 a^r_{11}$	$\cdots$	$\theta^r_1 a^r_{1n}$	$(1-\theta^s_1)a^s_{11}$	$\cdots$	$(1-\theta^s_1)a^s_{1n}$
	$\cdots$		$\cdots$	$\cdots$		$\cdots$
	$\theta^r_n a^r_{n1}$	$\cdots$	$\theta^r_n a^r_{nn}$	$(1-\theta^s_n)a^s_{n1}$	$\cdots$	$(1-\theta^s_n)a^s_{nn}$
Country s	$(1-\theta^r_1)a^r_{11}$	$\cdots$	$(1-\theta^r_1)a^r_{1n}$	$\theta^s_1 a^s_{11}$	$\cdots$	$\theta^s_1 a^s_{1n}$
	$\cdots$		$\cdots$	$\cdots$		$\cdots$
	$(1-\theta^r_n)a^r_{n1}$	$\cdots$	$(1-\theta^r_n)a^r_{nn}$	$\theta^s_n a^s_{n1}$	$\cdots$	$\theta^s_n a^s_{nn}$

$$\theta^r_1 a^r_{11} = \frac{p^r_1 z^{rr}_{11}}{p^r_1 z^r_1} = a^{rr}_{11}$$

Country r	a^{rr}_{11}	$\cdots$	a^{rr}_{1n}	a^{rs}_{11}	$\cdots$	a^{rs}_{1n}
	$\cdots$		$\cdots$	$\cdots$		$\cdots$
	a^{rr}_{n1}	$\cdots$	a^{r}_{nn}	a^{rs}_{n1}	$\cdots$	a^{rs}_{nn}
Country s	a^{sr}_{11}	$\cdots$	a^{sr}_{1n}	a^{ss}_{11}	$\cdots$	a^{ss}_{1n}
	$\cdots$		$\cdots$	$\cdots$		$\cdots$
	a^{sr}_{n1}	$\cdots$	a^{sr}_{nn}	a^{ss}_{n1}	$\cdots$	a^{ss}_{nn}

The impact of tariff

Trade cost

Based on the benchmark model, we extend the two-country framework to a multi-country model.

Trade costs are incorporated by introducing trade markups.

$$\tilde{\tau}_i^{s,t} = \tau_i^{s,t} d_i^{s,t}$$

$$\text{FOB} \xrightarrow{d_i^{s,t}:\, iceburg\ transport\ cost} \text{CIF} \xrightarrow[\tau_i^{s,t}:\, tariff]{} \text{CFR}$$

The composite price in country t can be rewritten as

$$\tilde{p}_i^{ct} = \left(\left(\tilde{\tau}_i^{1,t} p_i^1\right)^{1-\varepsilon_i^t} + \left(\tilde{\tau}_i^{2,t} p_i^2\right)^{1-\varepsilon_i^t} + \cdots + \left(\tilde{\tau}_i^{s,t} p_i^t\right)^{1-\varepsilon_i^t} + \cdots + \left(\tilde{\tau}_i^{m,t} p_i^m\right)^{1-\varepsilon_i^t} \right)^{\frac{1}{1-\varepsilon_i^t}}$$

Country s's share in country t's intermediate input consumption.

$$\tilde{\theta}_{ij}^{s,t} = \frac{\left(\tilde{\tau}_i^{s,t} p_i^s\right)^{1-\varepsilon_i^t}}{\left(\tilde{\tau}_i^{1,t} p_i^1\right)^{1-\varepsilon_i^t} + \left(\tilde{\tau}_i^{2,t} p_i^2\right)^{1-\varepsilon_i^t} + \cdots + \left(\tilde{\tau}_i^{s,t} p_i^t\right)^{1-\varepsilon_i^t} + \cdots + \left(\tilde{\tau}_i^{m,t} p_i^m\right)^{1-\varepsilon_i^t}}$$

The production network will undergo structural changes.

$$\begin{bmatrix} X^1 \\ X^2 \\ \vdots \\ X^m \end{bmatrix} = \begin{bmatrix} I - \hat{\theta}^{1,1} A^1 & -\hat{\theta}^{1,2} A^2 & \cdots & -\hat{\theta}^{1,m} A^m \\ -\hat{\theta}^{2,1} A^1 & I - \hat{\theta}^{2,2} A^2 & & -\hat{\theta}^{2,m} A^m \\ \vdots & & \ddots & \vdots \\ -\hat{\theta}^{m,1} A^1 & -\hat{\theta}^{m,2} A^2 & \cdots & I - \hat{\theta}^{m,m} A^m \end{bmatrix}^{-1} \begin{bmatrix} Y^1 \\ Y^2 \\ \vdots \\ Y^m \end{bmatrix}$$

Empirical Analysis