GRASING A MATRIX WITH LOW PRECISION ARITHMETIC

Topic: Input-Output Theory and Methodology (2)
Author: Juan Manuel Valderas Jaramillo

(1) Research Question

Biproportional techniques have been used in inputâ€“output analysis since Wassily Leontief introduced a biproportional procedure in 1941 to analyze intertemporal changes in inputâ€“output tables. Over time, these methods evolved into the RAS family and related extensions. One major development within this framework is the GRAS method, first proposed by GÃ¼nlÃ¼k-Senesen and Bates (1988) and subsequently formalized and refined by other authors. GRAS has become widely used for balancing and updating inputâ€“output tables, particularly those containing both positive and negative entries.

While the standard GRAS method operates under two-dimensional constraintsâ€”such as the row and column totals of national inputâ€“output tablesâ€”it has been extended to multidimensional settings through the nD-GRAS method (Valderas-Jaramillo and Rueda-Cantuche, 2021), thereby enabling applications in more complex frameworks.

Methods belonging to the GRAS family are highly popular for updating, regionalizing, and extending inputâ€“output frameworks and social accounting matrices (SAMs). Their appeal lies mainly in the quality of the results and the relative ease of implementation, as the underlying algorithm relies on the standard quadratic equation. Some limitations of this algorithm have been addressed in the literatureâ€”for example, Temurshoev et al. (2013) proposed a modification allowing the algorithm to operate in cases where vectors contain no positive entries.

In this paper, we introduce a new modification of the GRAS algorithm designed to address a common empirical situation characterized by low-precision numerical computations.

(2) Methodology

The standard quadratic formula may become numerically unstable when implemented in finite-precision arithmetic. In the context of the GRAS matrix-updating procedureâ€”particularly in large-scale multi-regional inputâ€“output tables (MRIOs), supply-and-use tables (SUTs), or social accounting matrices (SAMs), whether in two-dimensional or multidimensional frameworksâ€”the quadratic equation must be solved repeatedly throughout the iterative updating process.

Under such conditions, finite-precision arithmetic may lead to numerical inaccuracies, including round-off errors and divisions by quantities approaching zero. These issues can result in low-accuracy root calculations and, in some cases, affect the convergence and overall performance of the algorithm.

This paper systematically examines the conditions under which such numerical instabilities arise and proposes algorithmic modifications to mitigate these problems and improve computational robustness.

(3) Data

The proposed method is illustrated using simulated matrices. Both the standard and the modified algorithms are implemented in order to compare their performance in terms of convergence properties, solution quality, and computational speed along the trajectory toward the final solution.

If feasible, the method will also be applied to a real-world multi-regional inputâ€“output (MRIO) matrix to assess its empirical relevance and practical performance.

(4) Novelty and Contribution

This paper refines the algorithmic procedure underlying the GRAS method to prevent situations in which low-precision computations degrade numerical performance and compromise convergence. Although such cases are practically relevant, they are largely overlooked in existing discussions of the analytical solution of GRAS and its associated algorithm.

Given that this quadratic-based procedure constitutes the computational core of several GRAS-related updating methodsâ€”such as SUT-RAS, KRAS, and nD-GRASâ€”its numerical robustness is crucial for empirical applications of matrix-balancing techniques.

To the best of our knowledge, the modification proposed in this paper has neither been explicitly discussed nor implemented in the inputâ€“output literature.