As the Door Closes: Non-Tariff Measures, Export Fixed Costs and Cascading Effects in the Global Production Network

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This paper develops a multi-country, multi-sector model with heterogeneous firms and endogenous entry to study how export fixed-cost shocks propagate through global production networks and affect aggregate welfare. We characterize equilibrium propagation using two endogenous network statistics, supplier centrality and consumer centrality, which summarize upstream and downstream transmission across the joint production-trade system. The model delivers three main results. First, once endogenous firm selection and production networks are jointly considered, standard sufficient-statistics results are no longer sufficient for welfare analysis because welfare changes additionally depend on endogenous adjustments in consumer centrality. Second, export fixed-cost shocks reshape the extensive margin of trade and generate cascading effects across countries and sectors through input-output linkages. Third, welfare effects can be decomposed into a negative variety effect caused by the contraction in firm mass and a positive selection effect driven by improvements in weighted productivity. To quantify these mechanisms, we study China's rare-earth export restrictions during 2006–2014. Difference-in-differences estimates show that downstream sectors more exposed to rare-earth inputs experienced significant declines in export participation and export values, while upstream sectors exhibited positive responses.

I. Motivation

Non-tariff measures (NTMs) are used in trade settings more and more frequently when conventional tariffs have steadily declined. Figure 1 shows that the number of Sanitary and Phytosanitary (SPS) and

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Technical Barriers to Trade (TBT) notifications submitted by WTO members has increased sharply since the mid-1990s. While heterogeneous in form, a key feature of these NTMs is that they directly operate through fixed export costs (Calì et al., 2026). Compliance with product standards, certification requirements, technical regulations, and inspection procedures typically entails sizable sunk or fixed expenditures that firms must incur before exporting to a given destination. NTMs not only directly affect firms' entry and exit decisions in destination markets, but also influence adjustments in export decisions across other destinations and the domestic production decision (Melitz and Redding, 2014; Crowley, Meng and Song, 2018; Cui and Li, 2023).

These adjustments also propagate beyond their own industries through input-output linkages (Baqaee, 2018). Focusing on the closed-economy production network, Baqaee (2018) shows that ignoring the extensive margin can substantially underestimate their aggregate effects in production networks. Although some studies examine the effects of trade liberation or tariff reduction in multinational production networks (Caliendo and Parro, 2015; Baqaee and Farhi, 2024), little is known about how fixed-cost shocks propagate across countries through production and trade linkages.

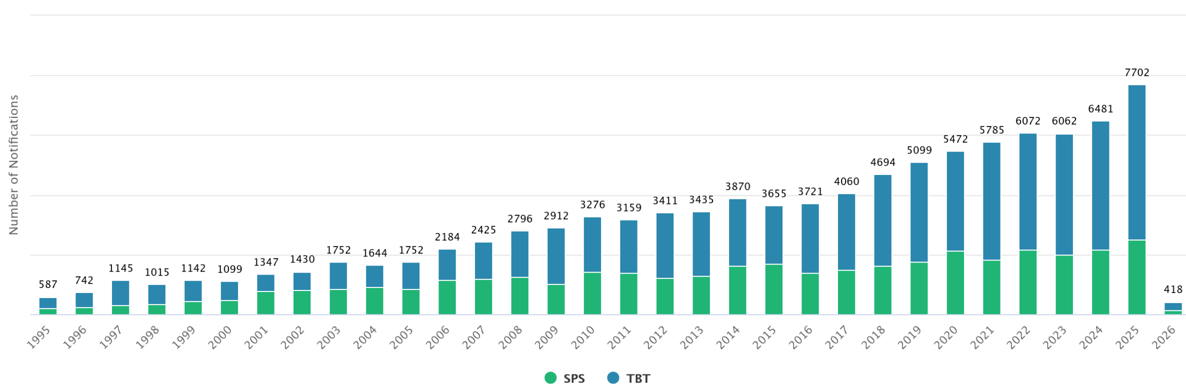


Figure 1. The number of SPS and TBT notifications from 1995 to 2025

Notes: these data come from https://www.epingalert.org/en/FactsAndFigures/Notifications

In this paper, we model NTMs as fixed costs in our model and explore the general feedback of adjusting fixed costs in a multi-country production network. We characterize the consumer and supplier centralities in a multi-national setting, which implies that production networks of all the countries interacting with the global trade network have an influence on the supplier and consumer centrality of all the countries and industries. The consumer and supplier centralities reflect the importance of one sector acting as a consumer and supplier, respectively. The former measures the upstream effect and the latter measures the downstream effect. Three production structures, including ordinary input-output network, horizontal economy as well as vertical economy, are discussed to clarify the propagation channels of shocks. We illustrate the responses of two centralities to the changes in trade share and mass of firms under two industries and countries. Our numerical results show that the propagation of trade shocks is not an inherent feature of shocks themselves, but is entirely governed by the structure of production and trade. We also rewrite the formula of export values, import values and domestic market size.

Next, we decompose welfare into three parts, i.e. changes in consumer centralities, tariffs, and cumulative market shares of exporting firms. Changes in consumer centralities capture responses of production networks to shocks, and changes in the cumulative market shares of export firms reflect the firm reallocation effects. Our results are complementary to the existing sufficient-statistics literature. When firm selection and network structure are exogenous, our framework collapses to the familiar sufficient-statistics results. When they are endogenous, welfare analysis necessarily requires accounting for entry and network effects. We also compare our results with Baqaee (2018), and find that the effects of fixed costs on welfare face a trade-off between changes in varieties and productivity reallocation.

Finally, we quantify the effect of China's rare-earth export restrictions (CREER). Our descriptive results show that CREER substantially reduces China's rare-earth export values and reshapes the composition of exporters. It also generates price responses larger than those typically associated with tariffs. Reduced-form results show that CREER generates substantial spillover effects on the extensive and intensive trade margins across sectors through input-output linkages, highlighting the importance of production networks in shaping the aggregate consequences of trade restrictions.

The first strand of literature related to our paper is about production network. Acemoglu et al. (2012) explain the origin of aggregate fluctuation from the production network. Acemoglu, Akcigit and Kerr (2016) construct a multi-sector general equilibrium model under the perfect competition to build up the micro foundation of traditional production network (Leontief, 1936). Baqaee (2018) introduces the monopoly competition and love of variety into multi-sector general equilibrium model and find that entry and exit of firms increase the aggregate fluctuations. However, these models are single-country and ignore the effect of trade network. The most related to our paper is Baqaee and Farhi (2024), and we still have two major differences: one is that Baqaee and Farhi (2024) construct a global network from the perspective of national accounts by using the heterogenous-agent input–output, but we explicitly show the roles of trade network and production network in propagation and reconstruct the inter-national input-output table by using a general equilibrium model. The other is they focus on the impact of tariffs and other wedges on the aggregate measures, such as GDP, GNI as well as welfare, while we pay more attention to the impact of adjustment of the export fixed costs on the welfare and are likely to describe the propagation channel of shocks through production network and trade network.

The second strand of literature related to our paper is about the quantitative trade literature (Eaton and Kortum, 2002; Melitz, 2003), which concentrate on the effect of tariffs (i.e. trade liberation) on the aggregate outcomes and welfare but ignore the role of firm entry or exit responding to these changes. Eaton and Kortum (2002) and Caliendo and Parro (2015) focus on inter-industrial trade based on the Ricardian comparative advantage theory, while Krugman (1992) and Melitz (2003) emphasize the importance of intra-industrial trade, which is extended to a multi-industry model by Hsieh and Ossa (2016), in order to explore the effect of Chinese technological progress on the global welfare. While a large literature examines the effects of tariff changes on aggregate outcomes, our paper complements this line of research by focusing on export fixed costs.

Another body of work related to our paper is empirically evaluating the effect of NTMs or fixed cost on economic consequences (Brandt, Kambourov and Storesletten, 2026; Calì et al., 2026; Calì, Le Moglie and Presidente, 2024; Cali and Montfaucon, 2021). Brandt, Kambourov and Storesletten (2026) shows entry barriers are the regional differences of wage and productivity, and the reform of state-owned firms removed entry barriers and promoted convergence of economic outcomes among regions. Cali and Montfaucon (2021) find that NTMs reduce the survival of firms in export markets as well as the intensive and extensive margins of their exports, which leads to more negative effect than import tariffs. Calì et al. (2026) show that NTMs raise the cost of imported intermediate inputs, limiting firms' access to cheaper inputs from China and thereby reducing their export sales and resilience to shocks. Some literature explore the effect of CREER on cumulative abnormal return (Müller, Schweizer and Seiler, 2016), export outcomes of downstream sectors (Chen, Hu and Li, 2021) and direct technological change(Alfaro and Chor, 2025). Although there are abundant literature empirically exploring the effects of NTMs on economic outcomes, they ignore the role of input-output network. Our paper explores the effect of CREER not only on downstream sectors but also the upstream sectors through input-output linkages, and we provide a unified conceptual framework investigating the relationship between NTMs and international trade.

The remainder of our paper is structured as follows: Section II shows our basic conceptual framework. Section III derives the centrality measures. Section IV introduces welfare analysis, and compares our results with previous literature. Section V takes China's rare-earth export restrictions as an empirical example.

II. Benchmark Model

In this section, we illustrate the basic multi-national and industrial general equilibrium model, which includes the optimal behaviors of economic agents in our model and clarifying the equilibrium. The world economy of N countries (indexed by i, j). Each country has S sectors (indexed by t, s), and is endowed with L_i units of labors and a representative household . There are M_{is} firms in sector s facing monopolistic competition and constant markup rate. Here, we adopt the following notation. Subscripts index industries, while superscripts index countries. For example, α^i_{ts} denotes the input requirement of industry s used in the production of industry t in country i. When multiple indices appear in subscripts or superscripts, their ordering reflects the direction of flows, from origin to destination, regardless of whether the indices refer to countries or industries. This convention follows the input–output literature but differs from the standard in macroeconomics. For example, π^{hi}_t denotes the trade share of industry t goods exported from country h to country i. The same convention applies to industry indices. For variables that are specific to a given country–industry pair, we use subscripts to index both dimensions. For instance, φ_{is} denotes the productivity of a firm in industry s in country i.

A. Production

In the rest of paper, we refer φ_{jt} to a single firm to illustrate simply. For the firm φ_{is}, the production function is:

$$q_{is}(\varphi_{is}) = \left[(\alpha^L_{is})^{\frac{1}{\epsilon}} L_{is}(\varphi_{is})^{\frac{\epsilon-1}{\epsilon}} + \sum_{t\in S} (\alpha^i_{ts})^{\frac{1}{\epsilon}} x^i_{ts}(\varphi_{is})^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

Here, α_{is}^{L} and α_{ts}^{i} are the share parameters for how intensively sector s use labors and inputs from sector t, respectively. $L_{is}(\varphi_{is})$ and $x_{ts}^{i}(\varphi_{is})$ are labors and composite inputs from sector t used by the firm with productivity φ_{is} in sector s, respectively. $\epsilon > 1$ is the elasticity of substitution among production factors. The composite input good, $x_{ts}^{i}(\varphi_{is})$, is given by:

$$x_{ts}^{i}(\varphi_{is}) = \left(\sum_{j=1}^{N} \int_{\varphi_t^{ji*}}^{+\infty} M_t^{ji} x_{ts}^{ji}(\varphi_{jt}, \varphi_{is})^{\frac{\sigma_t - 1}{\sigma_t}} \, dG(\varphi_{jt}) \right)^{\frac{\sigma_t}{\sigma_t - 1}} . \tag{2}$$

where $x_{ts}^{ji}(\varphi_{jt}, \varphi_{is})$ is input good from firm φ_{jt} used by the firm φ_{is}, M_t^{ji} is the mass of firms exporting to country i in sector t. $\sigma_t > 0$ is the elasticity of substitution among goods in sector t. $G(\varphi_{jt}) = 1 - \left(\frac{b_{jt}}{\varphi_{jt}}\right)^{\theta_t}$ is the CDF (Cumulative Distribution Function) of productivity in sector t, which means that the productivity is drew from a Pareto Distribution. For simplicity, we choose $b_{jt} = 1$ in this illustrative case, but we allow b_{Ft} to be different from 1 to capture the gap of GDP between two countries. Producers enter market and charge the Dixit–Stiglitz markup $\frac{\sigma_s}{\sigma_s - 1}$ over its marginal cost. The marginal cost of sector s is

$$mc_{is} = \left[\alpha_{is}^{L} w_{is}^{1-\epsilon} + \sum_{t \in S} \alpha_{ts}^{i} \tilde{p}_{it}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{3}$$

where $\tau_s^{ji} \geq 1$ is iceberg cost of goods transport from country i to country j. $\tilde{p}_{is}$ is the open-economic price of industry s that household face within country i, the specific form is given by

$$\tilde{p}_{is} = \left(\sum_{j=1}^{N} \left(\tau_s^{ji} p_s^{ji} \right)^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}} \tag{4}$$

where p_s^{ji} is the aggregate price given by sector s in country j for exporting to country i. We set the wage as the numerate. And the amount of labor is set to 1.

FIRM ENTRY AND EXIT. — We apply the same logic of entry and exit of Melitz (2003).Take sector s in country i as an example to describe the process of firm entry and exit. There are large number of potential entrants

M_{is}^e. Firms hire f_{is}^e units of labor as sunk cost to draw their initial productivity φ_{is} from a distribution $g(\varphi_{is})$, corresponding to the CDF $G(\varphi_{is})$. However, the firm also need to pay fixed costs to decide whether production for domestic market (f_{is}) and foreign market (f_s^{ij}) or not. When some entrants draw a relatively low productivity from $g(\varphi_{is})$, they decide to stop to produce or exit. However, firms with high productivity level can start to produce goods to earn $\pi_{is}(\varphi_{is})$. There is a productivity cutoff level φ_{is}^*, below which firms exit the market. $1 - G(\varphi_{is}^*)$ is the ex-ante probability of successful entry, and the mass of surviving firms is,

$$M_{is} = M_{is}^e \left(1 - G(\varphi_{is}^*)\right).$$

For successful entrants, their conditional productivity distribution is $\nu(\varphi_{is})$:

$$\nu(\varphi_{is}) = \begin{cases} \dfrac{g(\varphi_{is})}{1 - G(\varphi_{is}^*)}, & \text{if } \varphi_{is} \geq \varphi_{is}^* \\ 0, & \text{o.w.} \end{cases} \tag{5}$$

Finally, M_s^{ij} denotes the mass of exporting firms serving destination j. Similar to Melitz (2003),

$$M_{is}^E = \frac{\gamma_{is} L_i}{\left[1 + \frac{s_{is}^L}{\mu_s} \frac{\sigma_s \theta_s}{\theta_s - \sigma_s + 1}\right] \sum_j f_s^{ij} \left(\frac{\varphi_s^{ij*}}{\varphi_{is}^*}\right)^{-\theta_s} + f_{is}^E}. \tag{6}$$

Different from Melitz and Redding (2014) and Bai, Jin and Lu (2024), the term $\frac{s_{is}^L}{\mu_s} \frac{\theta_s}{\theta_s - \sigma_s + 1}$ capture the variable inputs of labor. We define the export selection intensity as:

$$\chi_s^{ij} \triangleq \frac{\tilde{\varphi}_{is}}{\tilde{\varphi}_s^{ij}} = \frac{\varphi_{is}^*}{\varphi_s^{ij*}} \tag{7}$$

The above equation implies that the higher weighted export productivity is, the smaller the selection intensity is. The second equality use the relationship between cutoff productivity and the weighted export productivity. This equation also reflects productivity reallocation effects.

Optimal input for individual firms. — According to Shephard's Lemma, the optimal labors $L_{is}^{*}(\varphi_{is})$ and intermediate input $x_{st}^{ij*}(\varphi_{is})$ are given by

$$x_{st}^{ij*}(\varphi_{is},\varphi_{jt}) = \left(\frac{mc_{jt}}{\tilde{p}_{js}}\right)^{\epsilon}\left(\frac{\tilde{p}_{js}}{\tau_s^{ij}p_{is}(\varphi_{is})}\right)^{\sigma_s}\alpha_{st}^{j}q_{jt}(\varphi_{jt}) \tag{8}$$

$$l_{jt}^{*}(\varphi_{is}) = \left(\frac{mc_{jt}}{w}\right)^{\epsilon} q_{jt}\left(\varphi_{jt}\right)\alpha_{jt}^{L} = \left(mc_{jt}\right)^{\epsilon}\alpha_{jt}^{L}q_{jt}\left(\varphi_{jt}\right) \tag{9}$$

We aggregate all the optima inputs of firms in industry t and country j, and the optimal intermediate input $x_{st}^{ij*}(\varphi_{is})$ is given by

$$x_{st}^{ij*}(\varphi_{is}) = \left(\frac{mc_{jt}}{\tilde{p}_{js}}\right)^{\epsilon}\left(\frac{\tilde{p}_{js}}{\tau_s^{ij}p_{is}(\varphi_{is})}\right)^{\sigma_s}\alpha_{st}^{j}q_{jt} \tag{10}$$

B. Demand

There is a representative household in every country, whose utility function is

$$U_i = \left(\sum_s \beta_{is}^{\frac{1}{\epsilon}} c_{is}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} - \mathcal{L} \tag{11}$$

where c_{is} is the amount of composite consumption of sector s. β_{is} is the preference parameter, which satisfies $\sum_s \beta_{is} = 1$. $\mathcal{L}$ denotes the disutility of labor, which is expressed by $\mathcal{L} = \Pi_s L_{is}^{\gamma_{is}}$. In the disutility function, γ_{is} denotes the allocation parameter among sectors and satisfies $\sum_s \gamma_{is} = 1$. c_{is} is given by:

$$c_{is} = \left(\sum_{j=1}^{N}\int_{\varphi_s^{ji*}}^{+\infty}\frac{M_s^{ji}}{1-G(\varphi_s^{ji*})}c_s^{ji}(\varphi_{js})^{\frac{\sigma_s-1}{\sigma_s}}\,dG(\varphi_{js})\right)^{\frac{\sigma_s}{\sigma_s-1}} \tag{12}$$

The household's budget constraint is

$$\sum_s \tilde{p}_{is}c_{is} = w_i L_i \tag{13}$$

Importantly, due to endogenous export selection, the set of exporting firms differs across markets, so that p_s^{ji} embeds the cutoff productivity associated with bilateral trade. The optimal demand of the household in country i to firm in country j and industry s with productivity is given by

$$c_s^{ji*}(\varphi_{js}) = \left(\frac{P_i}{\tilde{p}_{is}}\right)^{\epsilon} \left(\frac{\tilde{p}_{is}}{\tau_s^{ji} p_{js}(\varphi_{js})}\right)^{\sigma_s} \beta_{is} C_i \tag{14}$$

DEFINITION 1: *From cost minimization under CES preferences, bilateral trade shares are given by*

$$\pi_t^{hi} = \frac{\left(\tau_t^{hi} p_t^{hi}\right)^{1-\sigma_t}}{\sum_{j=1}^{N} \left(\tau_t^{ji} p_t^{ji}\right)^{1-\sigma_t}} \tag{15}$$

Equation 15 illustrates the bilateral expenditure share implied by CES preferences. However, this representation alone does not make explicit the endogenous margins through which trade shares respond to exogenous shocks. In particular, the bilateral price term p_t^{hi} is not a primitive object in heterogeneous-firm trade models. In models such as Melitz (2003), bilateral prices depend endogenously on exporter selection, cutoff productivity, firm entry, and fixed export costs. Therefore, although equation 15 is useful for characterizing demand-side expenditure allocation, it is not sufficient for isolating the structural channels through which fixed-cost shocks affect trade shares. For this reason, the trade literature usually rewrites bilateral expenditure shares in a gravity form, which makes explicit how productivity, trade costs, and selection jointly determine bilateral trade flows. In particular, the domestic expenditure share is

$$\pi_t^{ii} = \frac{\left(p_t^{ii}\right)^{1-\sigma_t}}{\sum_{j=1}^{N} \left(\tau_t^{ji} p_t^{ji}\right)^{1-\sigma_t}} \tag{16}$$

C. Equilibrium

Now we define the general equilibrium as follows:

Definition 1. This general equilibrium consist of a collection of price $p_s^{ij}(\varphi_{is}), w_i$ and quantities such that:

1) Given the price, the representative household choose the amount of consumption to maximize

utility subject to budget constraint.

2) Given the price, the firm choose the optimal inputs of labor and intermediate goods to minimize firm's cost.

3) Labor market clear such that

$$(mc_{is})^{\epsilon-\sigma_s}\mu_s^{-\sigma_s}\mathcal{D}_{is}M_{is}^e\left(\varphi_{is}^*\right)^{-\theta_s}\alpha_{is}^L\widetilde{\varphi}_{is}^{\sigma_s-1}+\sum_{j=1}^N M_{is}^e\left(\varphi_s^{ij*}\right)^{-\theta_s}f_s^{ij}+M_{is}^e f_{is}^e=\gamma_{is}L_i \tag{17}$$

where $\mathcal{D}_{is}=\left[\sum_{j=1}^N\left(\frac{P_j}{\tilde{p}_{js}}\right)^{\epsilon}\left(\frac{\tilde{p}_{js}}{\tau_s^{ij}}\right)^{\sigma_s}\beta_{js}C_j+\sum_{j=1}^N\sum_{t=1}^S\left(\frac{mc_{jt}}{\tilde{p}_{js}}\right)^{\epsilon}\left(\frac{\tilde{p}_{js}}{\tau_s^{ij}}\right)^{\sigma_s}\alpha_{st}^j y_{jt}\right]$.

4) if we extend to multinational framework, each country keeps trade balance. The trade deficit S_i is given by:

$$\sum_{s=1}^K\sum_{j=1}^N M_{is}^E\frac{\sigma\theta_s}{\theta_s-\sigma_s+1}w_i f_s^{ij}\left(\varphi_s^{ij*}\right)^{-\theta_s}-D_i=\sum_{s=1}^K\sum_{j=1}^N M_{js}^E\frac{\sigma\theta_s}{\theta_s-\sigma_s+1}w_j f_s^{ji}\left(\varphi_s^{ji*}\right)^{-\theta_s} \tag{18}$$

In our model, we set S_i to be 0.

III. Centrality measures

A. The Supplier and Consumer Centrality

DEFINITION 2: *Similar to Baqaee (2018), we also define the supplier centrality $\widetilde{\beta}$ and the consumer centrality $\widetilde{\alpha}$,*

$$\widetilde{\beta}=\left[I-\left[\hat{\pi}\circ\tau^{-\epsilon}\circ\hat{\chi}\right]A\Phi^{\epsilon}\tilde{M}^{\epsilon}\mathcal{C}^{\epsilon}\mu^{-\epsilon}\right]^{-1}\left[\hat{\pi}\circ\tau^{-\epsilon}\circ\hat{\chi}\right]\beta \tag{19}$$

$$\widetilde{\alpha}=\left[\mu^{\epsilon-1}\Phi^{1-\epsilon}\tilde{M}^{1-\epsilon}\mathcal{C}^{1-\epsilon}-\frac{1}{N}A'\left[\hat{\pi}^{\,1-\epsilon}\circ\tau^{1-\epsilon}\circ\hat{\chi}^{1-\epsilon}\right]'\right]^{-1}\alpha^L \tag{20}$$

where β and α^L denote $NK\times 1$ vectors of taste parameters and labor shares. A is a $NK\times NK$ matrix denoting the input-output relationship among industries, which is composited by N countries' direct consumption coefficient matrix. μ is the diagonal matrix consisting of industrial markup rate. Ξ is given by $\mathbf{1} \times \mathbf{1}' \otimes I$. $\mathbf{1}$ is a $N \times 1$ identity vector and I is the $K \times K$ identity matrix. Although iceberg costs explicitly entry two centralities, we mainly focus on the impact of fixed costs on aggregated outcomes. Iceberg costs are set to be exogeneous. $\circ$ denotes Hadamard product.

PROOF:

The details of proof are shown in appendix A.A2. □

Table 1 summaries the matrix representations underlying the two centrality measures. These centralities are concerned with four economic objects. First, masses of firms capture extensive margin of industries. Second, trade shares determine the allocation of product flows across countries. The third economic object is export selection intensity, measured by the ratio of export productivity and domestic productivity. The last one is absolute level of domestic productivity.

Table 1— Matrices and Their Elements

Economic Object	Matrix	Element	Matrix Structure
Mass of Firms	$\tilde{M}$	$(M_{jt}^{E})^{\frac{1}{\sigma_t - 1}}$	$NK \times NK$ diagonal matrix
Trade Share	$\tilde{\pi}$	$(\pi_s^{ij})^{\frac{1}{1-\sigma_s}}$	$NK \times NK$ matrix, block-diagonal structure
	$\hat{\pi}$	$(\pi_s^{ij})^{\frac{\epsilon-\sigma_s}{1-\sigma_s}}$	$NK \times NK$ matrix, block-diagonal structure
Export Selection Intensity	$\tilde{\chi}$	$(\chi_s^{ij})^{\frac{\sigma_s - 1 - \theta_s}{\sigma_s - 1}}$	$NK \times NK$ matrix, same structure as π
	$\hat{\chi}$	$(\chi_s^{ij})^{(\sigma_s - \epsilon)\frac{\sigma_s - \theta_s - 1}{\sigma_s - 1}}$	$NK \times NK$ matrix, same structure as π
Productivity	Φ	$\tilde{\varphi}_{jt}^{\frac{\sigma_t - \theta_t - 1}{\sigma_t - 1}}$	$NK \times NK$ diagonal matrix
Scaler term	$\mathcal{C}$	$\left[\frac{\theta_t}{1 - \sigma_t + \theta_t}\right]^{\frac{\theta_t}{(\sigma_t - 1)^2}}$	$NK \times NK$ diagonal matrix

To simplify our notation, we define:

$$
\begin{aligned}
H &= \left[\hat{\pi} \circ \tau^{-\epsilon} \circ \hat{\chi}\right] \\
G &= \left[\hat{\pi}^{\,1-\epsilon} \circ \tau^{1-\epsilon} \circ \hat{\chi}^{1-\epsilon}\right] \\
D &= \Phi^{\epsilon}\tilde{M}^{\epsilon}\mathcal{C}^{\epsilon}\mu^{-\epsilon} = diag\left(d_{11}, d_{12}, \cdots, d_{1K}, \cdots, d_{N1}, \cdots, d_{NK}\right) \\
E &= \mu^{\epsilon-1}\Phi^{1-\epsilon}\tilde{M}^{1-\epsilon}\mathcal{C}^{1-\epsilon} = diag\left(e_{11}, e_{12}, \cdots, e_{1K}, \cdots, e_{N1}, \cdots, e_{NK}\right)
\end{aligned}
$$

Therefore,

$$\widetilde{\beta} = [I - HAD]^{-1} H\beta \tag{21}$$

$$\widetilde{\alpha} = \left[E - \frac{1}{N}A'G'\right]^{-1} \alpha^{L} \tag{22}$$

These two centralities imply the importance of one industry as a supplier and consumer respectively. Supplier centrality $\tilde{\beta}$ captures how shocks propagate upstream through production linkages. It reflects how changes in costs or markups affect the supply side via input–output connections and trade channels. Consumer centrality $\tilde{\alpha}$ captures how shocks propagate through final demand and import channels. It reflects how changes in income shares and trade structure affect aggregate demand transmission.

Economic variables enter the two centralities with different powers, and therefore influence them through distinct channels. To illustrate this asymmetry, consider the markup matrix μ. It appears with exponent $-\epsilon$ in the supplier centrality, but with exponent $1-\epsilon$ in the consumer centrality. This example highlights that economic variables may affect upstream and downstream propagation in non-identical ways, underscoring the role of market structure in shaping network amplification.

We define

$$s = H\beta \circ [P^{\epsilon}C \otimes \mathbf{1}_{K\times 1}] + HADs$$

where $P^{\epsilon}C$ is a $N \times 1$ vector,s is a $NK \times 1$ vector and we get,

$$s = \widetilde{\beta} \circ [P^{\epsilon}C \otimes \mathbf{1}_{K\times 1}]$$

The equilibrium of two centralities is similar to Baqaee (2018),

$$\tilde{\beta}_{ik} = \frac{p_{ik}^{\epsilon} y_{ik}}{P_i^{\epsilon} C_i}, \qquad \text{and} \qquad \tilde{\alpha}_{ik} = \left(\frac{p_{ik}}{w_i}\right)^{1-\epsilon}$$

If we keep the homogeneous productivity assumption under the international trade setting, productivity matrix Φ degenerate in to a $K \times K$ equality matrix. The adjusted export selection intensity $\tilde{\chi}$ and $\hat{\chi}$ also collapse to a diagonal matrix with diagonal element equals to either 1 or 0, which depends on whether $f_{is} = f_s^{ij}$. If $f_s^{ij} > f_{is}$, all the firms choose to only serve the domestic market instead of exporting. The intuition is that under homogeneous productivity, all firms earn zero profits in equilibrium. Consequently, there is no additional surplus that can cover the export fixed cost f_s^{ij}. Two centralities are rewritten by

$$\tilde{\beta} = \left[I - \left[\hat{\pi} \circ \tau^{-\epsilon} \circ \hat{\chi}\right] A \tilde{M}^{\epsilon} \mathcal{C}^{\epsilon} \mu^{-\epsilon}\right]^{-1} \left[\hat{\pi} \circ \tau^{-\epsilon} \circ \hat{\chi}\right] \beta \tag{23}$$

$$\tilde{\alpha} = \left[\mu^{\epsilon-1} \tilde{M}^{1-\epsilon} \mathcal{C}^{1-\epsilon} - \frac{1}{N} A' \left[\tilde{\pi}^{\,1-\epsilon} \circ \tau^{1-\epsilon} \circ \tilde{\chi}^{1-\epsilon}\right]\right]^{-1} \alpha^{L} \tag{24}$$

B. *Special Production Structures*

To illustrate the economic mechanisms embedded in the centrality system, we consider two production structures and give the analytical solution of two centralities under 2 industries and 2 countries. One is a horizontal economy in which sectors produce only for final consumption and do not use intermediate inputs from other sectors. In this case the production network degenerates and the direct input coefficient matrix degenerates into zero matrix. The other structure is a vertical economy, where production follows a sequential chain: the output of sector 1 serves as an intermediate input for sector 2, while the final sector produces goods consumed by households. Horizontal and vertical economies represent two polar cases of production structures. In practice, production networks can be viewed as combinations of these two extremes.

First, we discuss a horizontal economic network, which is shown in figure 2. Labor is the only product factor of all the sectors and sectors produce only for final consumption and do not use intermediate inputs from other sectors. In this case the production network degenerates and the direct input coefficient matrix become zero. Two centralities are rewritten by

$$\tilde{\beta} = H\beta \tag{25}$$

$$\tilde{\alpha}' = E^{-1}\alpha^L \tag{26}$$

For the horizontal economy with 2 industries and 2 countries, the supplier centrality is given by

$$\tilde{\beta} = \begin{bmatrix} h_1^{11}\beta_{11} + h_1^{12}\beta_{21} \\ h_2^{11}\beta_{12} + h_2^{12}\beta_{22} \\ h_1^{21}\beta_{11} + h_1^{22}\beta_{21} \\ h_2^{21}\beta_{12} + h_2^{22}\beta_{22} \end{bmatrix}$$

And the consumer centrality is:

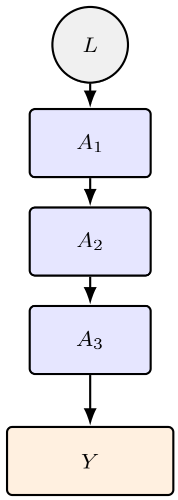
$$\tilde{\alpha}' = \begin{bmatrix} \mu_{11}^{1-\epsilon}\Phi_{11}^{\epsilon-1}\tilde{M}_{11}^{\epsilon-1}\alpha_{11}^L & \mu_{12}^{1-\epsilon}\Phi_{12}^{\epsilon-1}\tilde{M}_{12}^{\epsilon-1}\alpha_{12}^L & \mu_{21}^{1-\epsilon}\Phi_{21}^{\epsilon-1}\tilde{M}_{21}^{\epsilon-1}\alpha_{21}^L & \mu_{22}^{1-\epsilon}\Phi_{22}^{\epsilon-1}\tilde{M}_{22}^{\epsilon-1}\alpha_{22}^L \end{bmatrix}$$

The above equations imply that there is no effect of changes in trade shares on the consumer centrality, and the supplier centrality is also not influenced by domestic sectoral state outcomes, i.e. markup rate, productivity and mass of firms.

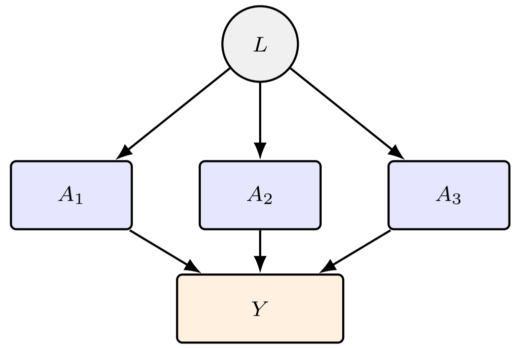
Next, we consider a vertical production network where production follows a sequential chain: the output of sector A1 serves as an intermediate input for sector A2, while the final sector produces goods consumed by households. Labor is the only production factor of sector A1.

For the vertical economy with 2 industries and 2 countries, the direct input coefficient matrix degenerates into

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$



(a) Vertical Economy



(b) Horizontal Economy

Figure 2. Comparison between Vertical and Horizontal Economies

Note: L denotes the labor production factor. A_1, A_2 and A_3 denote three sectors. Y stands for the final consumption.

Similarly, the supplier and consumer centrality are given by,

$$\tilde{\beta} = \begin{bmatrix} h_1^{11} d_{12} \left(h_2^{11} \beta_{12} + h_2^{12} \beta_{22} \right) + h_1^{12} d_{22} \left(h_2^{21} \beta_{12} + h_2^{12} \beta_{22} \right) \\ 0 \\ h_1^{21} d_{12} \left(h_2^{11} \beta_{12} + h_2^{12} \beta_{22} \right) + h_1^{22} d_{22} \left(h_2^{21} \beta_{12} + h_2^{22} \beta_{22} \right) \\ 0 \end{bmatrix}$$

and

$$\tilde{\alpha}' = \left[\frac{1}{e_{11}}, \frac{g_2^{11}}{2e_{11}e_{12}}, \frac{1}{e_{21}}, \frac{g_1^{12}}{2e_{11}e_{22}} + \frac{g_1^{22}}{2e_{21}e_{22}} \right] \tag{27}$$

C. Numerical Illustration of Network Propagation

Although the analytical expressions derived above characterize the equilibrium values of supplier and consumer centralities, their economic implications are not always transparent from the closed-form formulas. In particular, the centrality system depends simultaneously on trade shares, firm mass, productivity, and the input–output structure. As a result, comparative statics derived from the matrix expressions may obscure the underlying propagation mechanism. On the other hand, how the effect of shocks on centralities to be significant.

We construct a simple numerical example with two symmetric countries and two sectors without iceberg costs and export selection. The average domestic productivity is normalized to 1. The across-industry substitute elasticity parameter is set to $\epsilon = 5$. And the intra-industry substitute elasticities are set to 4, which means the markups of industries are $4/3$. We mainly focus on the effects of changes in trade shares and mass of firms on the two centralities. The input–output structure is fixed and given by

$$A = \begin{bmatrix} 0.2 & 0.1 \\ 0.5 & 0.3 \end{bmatrix}$$

which ensures that sectors are technologically interdependent while avoiding symmetry that would trivialize the comparative statics. Taste parameters and labor shares are set to

$$\beta = (0.4, 0.6)$$

and

$$\alpha^L = (0.3, 0.6),$$

respectively, so that sectors differ in both demand importance and primary factor intensity. Within this environment we examine how two types of shocks affect the equilibrium centralities: changes in bilateral trade shares and changes in the mass of firms.

Trade share shock. — We study the response of centralities to changes in bilateral trade shares to emphasize the role of allocation rather than technological efficiency. Figure 7 plots the derivative of supplier centrality with respect to the bilateral trade share of sector 1 exports from country 1 to country 2. We emphasize two key comparisons across the three economic structures.

We first compare the horizontal economy with the benchmark economy to highlight the role of production networks. In the horizontal economy without input–output linkages, changes in bilateral trade shares affect supplier centrality only through direct reweighting of sourcing patterns. As a consequence, the response is limited and does not propagate across sectors. By contrast, in the benchmark economy, the presence of production linkages allows trade shocks to transmit across sectors and countries. This generates richer cross-sectoral responses and amplifies the impact of trade-share changes on centrality.

Second, comparing the vertical economy with the benchmark economy isolates the role of network feedback. Both structures feature production linkages, but differ in whether these linkages form feedback loops. In the vertical economy, production flows are unidirectional and do not feed back to upstream sectors. Consequently, trade-share shocks induce a monotonic reallocation of supplier importance: as country 2 shifts its sourcing toward country 1, the supplier centrality of country 2's sector 1 increases, while that of country 1's sector 1 decreases. In contrast, in the benchmark economy, the presence of feedback loops implies that shocks propagate and return through the network. This feedback mechanism generates amplification and, importantly, non-monotonic responses in centrality, as observed in the figure.

Taken together, these results show that the effects of trade-share shocks are not solely determined by the magnitude of trade reallocation, but critically depend on the structure of the production network. In particular, while production linkages enable the transmission of shocks, it is the presence of feedback loops that fundamentally alters their qualitative effects, giving rise to amplification and sign reversals in centrality.

COROLLARY 1: *Two centralities under the Cobb-Douglas-styled product functions and utility functions, $\epsilon \to 1$, can be expressed by*

$$\tilde{\beta} \to \left[I - \left(\pi \circ \tau^{-1} \circ \hat{\chi}\right) A\Phi\tilde{M}\mathcal{C}\mu^{-1}\right]^{-1} \left[\pi \circ \tau^{-1} \circ \hat{\chi}\right] \beta \tag{28}$$

$$\tilde{\alpha}' \to \alpha^{L\prime} \left[I - \frac{1}{N} A'\right]^{-1} \tag{29}$$

Equation implies that

$$y_{is}p_{is} = \left[I - \left(\pi \circ \tau^{-1} \circ \hat{\chi}\right) A\Phi\tilde{M}c\mu^{-1}\right]^{-1} \left[\pi \circ \tau^{-1} \circ \hat{\chi}\right] \beta PC$$

where PC is the $N \times 1$ vector of nominal GNE, which stands for the total final expenditure of all the

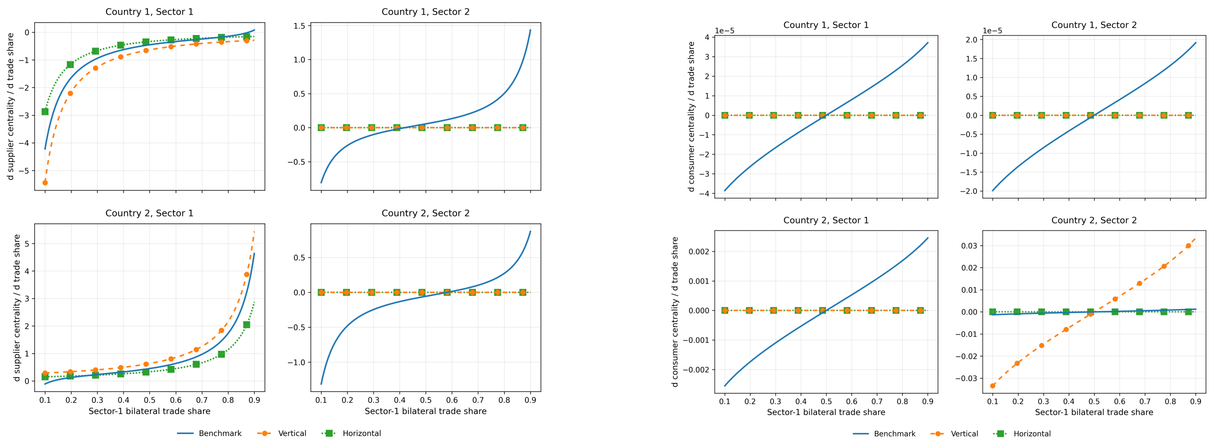


Figure 3. Numerical examples of three economic structure

Notes: The left subfigure and the right subfigure shows the results of the feedback of supplier and consumer centrality of two industries in two countries to the trade share of industry 1 exporting from country 1 to country 2, respectively. There is no the general equilibrium feedback of the supplier centrality of both two sectors in two countries in horizontal and vertical structures, and no the general equilibrium feedback of the consumer centrality of both two sectors in two countries in horizontal and vertical structures.

countries. And we find that the trade matrix without any power, which has the same effect on aggregate outcomes with the direct input coefficient matrix. In the consumer centrality, the only factor is the direct input coefficient matrix, which means that under CD-styled functions, the prices of sectors are affected by the input-output relationship.

D. *Measure Exports and Domestic Market Size*

In this section, we measure the export values and domestic market size by using consumer and supplier centrality. We derive a decomposition of nominal sales into final demand and intermediate demand components, which allows us to characterize the comparative statics of export revenue with respect to export fixed costs. The total expenditure of industry s in country j is given by

$$E_{is} = FE_{is} + ME_{is} \tag{30}$$

Where E_{is}^{F} is the final expenditure used for final good and E_{is}^{M} denotes the final expenditure used for intermediate good.

PROPOSITION 1: *The total expenditure of country i for industry s is given by*

$$FE_i = \tilde{\alpha}_i \left(\tilde{\pi}_i^D\right)^{\epsilon-1} \beta_i P_i^\epsilon C_i \tag{31}$$

$$ME_i = \tilde{\alpha}_i \left(\tilde{\pi}_i^D\right)^{\epsilon-1} A_i \mu^{-\epsilon} \Phi_i^\epsilon \tilde{M}_i^\epsilon \mathcal{C}^\epsilon \tilde{\beta}_i P_i^\epsilon C_i \tag{32}$$

Combining the two components, total expenditure vector can be expressed as

$$E_i = FE_i + ME_i = \hat{\tilde{\alpha}}_i \left(\tilde{\pi}_i^D\right)^{\epsilon-1} \left[\beta_i + A_i \mu^{-\epsilon} \Phi_i^\epsilon \tilde{M}_i^\epsilon \mathcal{C}^\epsilon \tilde{\beta}_i\right] P_i^\epsilon C_i \tag{33}$$

PROOF:

The details of proof are shown in appendix A.A3. □

IV. Welfare Analysis

In this section, we employ a streamlined representation to simplify the above model and prepare for quantifying. In appendix, we provide a more general framework with more than 2 countries. There are two countries, simply denoted by H and F, which are referred to home country and foreign country. The exogenous shock originates from the adjustment of f_s^{HF}, which is expressed by $\hat{f}_s^{HF}$, and other fixed and iceberg costs keep constant. In equilibrium, real consumption and the aggregate price index are given by

$$C_i = \frac{w_i L_i}{P_i}, i \in \{H, F\} \tag{34}$$

where $P_i = \left[\sum_{s=1}^{K} \beta_{is} \widetilde{p}_{is}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$

PROPOSITION 2: *The general feedback of welfare in country i is given by*

$$dln\left(C_i\right) = \sum_{j \in \{H.F\}} \sum_{s=1}^{S} \Omega_s^{ji} \left[\frac{1}{1-\epsilon} dln\left(\widetilde{\alpha}_{js}\right) + dln\left(\tau_s^{ij}\right) + \frac{1}{1-\sigma_s} dln\left(\varsigma_s^{ji}\right)\right] \tag{35}$$

where Ω_s^{ji} denotes the adjusted preference weight under CES utility function, which is unrelated to export countries, while the second term denotes the bilateral trade share, which is given by

$$\Omega_s^{ji} = \frac{\beta_{is}\tilde{p}_{is}^{1-\epsilon}}{\sum_{h=1}^{K}\beta_{ih}\tilde{p}_{ih}^{1-\epsilon}}\pi_s^{ij} \tag{36}$$

In equation 35, $dln\left(\tau_s^{ij}\right)$ captures the change in iceberg costs and tariffs, which is most conventional among previous literature. Same as Melitz and Redding (2015), ς_s^{ji} denotes the cumulative domestic market share of firms exporting to country i in country j, which is $\varsigma_s^{ji} = \left(\frac{\varphi_{js}^*}{\varphi_s^{ji*}}\right)^{\theta_s-\sigma_s+1} = \left(\chi_s^{ji}\right)^{\theta_s-\sigma_s+1}$. Equation 35 also illustrates that once production networks are taken into account, shocks to sectoral consumer centrality also affect welfare outcomes.

A. Relation to the Sufficient Statistics Literature

Before analyzing comparative statics in response to changes in export fixed costs, we first compare proposition 2 with previous literature. A large literature shows that, under CES preferences and monopolistic competition, welfare changes induced by trade liberalization can be summarized by a set of sufficient statistics—most prominently the trade elasticity and domestic expenditure shares (Arkolakis, Costinot and Rodríguez-Clare, 2012). The sufficient-statistics result crucially hinges on the assumption that policy shocks do not alter the set of active firms. Under this condition, changes in real income are fully captured by the change in the domestic expenditure share. Melitz and Redding (2015) challenges this logic. Relaxing the homogenous productivity and allowing firms to make export decisions, trade elasticity and domestic expenditure shares are not enough to describe the change in welfare, instead of including domestic market shares of exporter. Our framework further extends this literature by incorporating production networks and endogenous entry simultaneously. In this environment, policy shocks affect not only export selection but also the propagation of demand across sectors through the production network. As a result, welfare changes depend on an additional sufficient statistic: the change in consumer centrality. This term captures how shocks to one sector reallocate demand weights across the network and therefore alter the general equilibrium transmission of trade shocks.

An implication of this result is that ignoring the production network may lead to biased welfare evaluations. The direction of the bias depends on how policy shocks originated from some sectors reshape the network position of all the sectors due to cascading effects. The key mechanism lies in how strongly policy shocks alter sectoral consumer centrality. If a policy shock induces a large change in the centrality of a sector, the resulting adjustment in demand weights across the network may either amplify or offset the direct effects captured by the other two terms in the welfare decomposition. In other words, changes in consumer centrality act as a general equilibrium adjustment margin: large shifts in centrality can reinforce the welfare impact of trade policies, while in other cases they may partially compensate for the direct trade and selection effects.

Finally, our result nests existing frameworks as special cases(Arkolakis, Costinot and Rodríguez-Clare, 2012; Melitz and Redding, 2015). If the production network channel is shut down, equation 35 collapse to Melitz and Redding (2015),

$$dln\left(C_i\right) = \sum_{j\in\{H.F\}} \pi^{ji}\left[dln\left(\tau^{ij} + \frac{1}{1-\sigma}dln\left(\varsigma^{ji}\right)\right)\right] \tag{37}$$

Including productivity homogeneity allows us to exclude extensive margin of export ς^{ji}, and equation 37 degenerates into

$$dln\left(C_i\right) = \sum_{j\in\{H.F\}} \pi^{ji} dln\tau^{ij} = \frac{1}{\epsilon-1}dln\pi^{ii} \tag{38}$$

which is the same as Arkolakis, Costinot and Rodríguez-Clare (2012).

B. *Compare with Baqaee (2018)*

Now, we consider the closed economy with multi-sector, and compare the result with Baqaee (2018). The closed and multi-sectoral setting allow us to isolate the effect of production network with markup and endogenous firm selection from the international trade reallocations. In the closed economy, the trade network disappears, and the consumer centrality and supplier centrality are

$$\tilde{\beta} = \left[I - A\Phi^{\epsilon}\tilde{M}^{\epsilon}\mathcal{C}^{\epsilon}\mu^{-\epsilon}\right]^{-1} \tag{39}$$

$$\tilde{\alpha}^{l} = \left[\mu^{\epsilon-1}\Phi^{1-\epsilon}\tilde{M}^{1-\epsilon}\mathcal{C}^{1-\epsilon} - A'\right]^{-1}\alpha^{L} \tag{40}$$

and the effect of fixed cost f_t on the welfare is given by

$$(41)\qquad \frac{d\ln(C_i)}{d\ln f_t} = \frac{1}{1-\epsilon}\frac{\beta_s'\Lambda_2^M\left[I-\Gamma\right]^{-1}}{\beta'\tilde{\alpha}+\beta'\Lambda_2^M\left[I-\Gamma\right]^{-1}\mathbf{1}}e_t + \frac{1}{\epsilon-1}\frac{\beta_s'\left[\Lambda_2^M\left[I-\Gamma\right]^{-1}\epsilon\iota+\Lambda_2^\Phi\right]}{\beta'\tilde{\alpha}+\beta'\Lambda_2^M\left[I-\Gamma\right]^{-1}\mathbf{1}}\frac{e_t}{\theta_t}$$

PROOF:

The details of proof are shown in appendix. □

Equation 41 implies that under the assumption of heterogenous productivity, the effect of fixed cost on the welfare decomposes into two parts: one is the extensive margin of firm(negative), the other is productivity change(positive). And the sign of $\frac{d\ln(C_i)}{d\ln f_t}$ is no longer theoretically determinate, which depends on the magnitude of two mechanism. The intuition behind the formula is that when the fixed cost firms face increase, the mass of variety will decrease disproportionately because of the existence of production network. However, the exit of firms also rise up the weighted average of productivity, which can partially offset, or even dominate, the welfare loss from the reduction in varieties.

V. Quantitative Analysis

In this section we focus on China's rare-earth export restrictions (CREER) implemented from 2006 to 2014, a prominent policy episode illustrating how NTMs reshape market consequences (Alfaro and Chor, 2025; Chen, Hu and Li, 2021; Müller, Schweizer and Seiler, 2016), motivating the quantitative framework developed in Section II. We represent several facts on the relationship between CREER and export outcomes. We design an identification framework to explore the spillover effect of CREER on trade extensive and intensive margin across industries.

A. Background of CREER

As key inputs for advanced manufacturing industries, including military equipment, electric vehicles and semiconductors, the production and processing of REEs have significant implications for the global economy. This episode also provides a clear example of NTMs, as the policy relied on export quotas and exporter qualification requirements rather than tariffs. Moreover, detailed customs transaction-level data allows us to observe export quantities, exporter participation, and export prices, making it possible to examine the market effects of NTMs more directly.

Since the early 2000s, China has been the dominant supplier in the global REE industry due to abundant rare-earth resources and strong capacity of mining, production and processing(Alfaro and Chor, 2025). From 1991 to 2004, China's rare earth exports increased from 8,204 tons to 53,300 tons, while domestic consumption rose from 8,286 tons to 33,400 tons. During the same period, rare earth production expanded from 48,000 tons in 1995 to 95,000 tons in 2004. As a result, China's rare earth reserves declined from 43 million tons in 1995 to 27 million tons in 2002 . The rapid expansion of the REE industry created excess supply, pushing prices downward and reducing firms' value added. Meanwhile, extensive mining and processing activities generated severe environmental pollution (Müller, Schweizer and Seiler, 2016).

B. *Data sources and process*

To provide empirical evidence, we assemble multiple micro-level and macro-level data sets. Here we briefly describe the main data sources.

Production network. — We construct the production network based on the inter-countries input-output table of 2000 and 2005, which is from World Input-Output Database (WIOD). The data set contains a time-series input-output tables from 2000 to 2014 among 56 sectors and 40 countries and economies (Timmer et al., 2015). We drop sectors with 0 total output, and collapse these service sector to one sector. Finally, we get 27 sectors. Based on the 2000 table, we calculate the direct input coefficient to measure China's production network. we use data in 2005 to construct data moments for estimating parameters.

Trade data. — We use the administrative data from China's customs agency from 2010 to 2014, which includes source or destination country, import (output) values, quantity, cif price, and the HS-8 code of the product. Observations with missing import (output) values, quantity and HS code are dropped. In spirit of Imbert et al. (2022), we match HS4 commodity code with the input-output sector of WIOD.

Firm Register Data. — Firm-level data are obtained from the National Industrial and Commercial Enterprise Registration Database covering the period 2000–2015, which provides the detailed information including firm names, registration dates, registered capital, industry and location. Observations without key information, such as firm names, registration dates, or registered capital, are dropped. These records are matched with Chinese customs transaction-level export data using firm names, and then identify firms exporting rare-earth elements (REEs) and their chemical compounds to describe the import and export dynamics of mass of REEs-related firms from 2000 to 2015.

FIRM LEVEL DATA. — We use the other firm-level data from the Annual Survey of Industrial Firms (ASIF) from the National Bureau of Statistics. This dataset provides detailed information on a firm's industry, address, ownership, output, and financial information and covers all industrial firms with annual revenue above 5 million RMB (approximately $604,000). We re-classify their industry information according to the Industrial Classification for National Economic Activities (GB/T 4754-2002), and then match with the input-output code of WIOD followed by Imbert et al (2022). Then we combine the ASIF data with trade data to measure the ratio of exporting firms. We also use WIOD to obtain the technology parameters and taste parameters and the trade share between China and United States.

C. The impact of CREER on the rare-earth export

Fact 1. NTMs substantially reduce China's rare-earth export volume. Beginning in 2006, the Chinese government tightened the regulatory framework governing rare-earth production and exports in order to better manage the industry (Cali and Montfaucon, 2021). In particular, the Ministry of Commerce introduced a qualification system for rare-earth exporters, requiring firms to meet eligibility criteria to obtain export licenses and quota allocations. This qualification system imposed explicit scale thresholds for exporters. For example, manufacturing firms were required to have exported at least 1,500 tons of rare-earth products in 2004 or to have achieved export revenues exceeding 25 million RMB. In addition, exporters had to satisfy a broad set of compliance requirements—covering quality certification (e.g., ISO9000), environmental facilities and verified pollution-control standards. These requirements substantially increased the fixed costs of market access for exporters and restricted export participation to firms that satisfied the eligibility and compliance requirements. Following the introduction of the qualification system, China's rare-earth export volumes began to decline, as shown in Figure 4.

Fact 2. NTMs reshape the composition of exporting firms. In July 2010, China further drastically reduced the REE export quota for the second half of the year by 72%, with a stated motive of combating

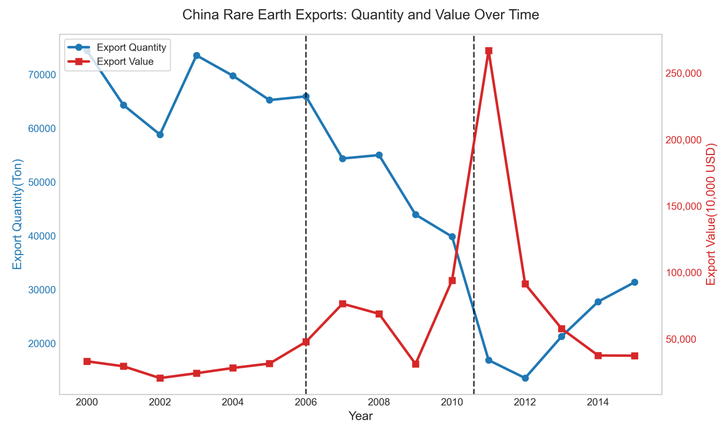


Figure 4. China's Rare Earth Export Quantity and Value

Notes: Rare-earth export quantity and value data are constructed from Chinese customs transaction-level data. We identify rare-earth products by selecting HS6 records whose product names refer to rare-earth elements, including Sc, Y, La, Ce, Pr, Nd, Pm, Sm, Eu, Gd, Tb, Dy, Ho, Er, Tm, Yb, and Lu, as well as their corresponding compounds. Export quantities and values are aggregated at the annual level. The blue line represents export quantities, and the red line represents export values. The two dashed lines indicate the introduction of export restrictions in 2006 and the quota reduction in July 2010.

illegal mining and environmental pollution (Müller, Schweizer and Seiler, 2016). The aggregate export quota was allocated across firms according to their historical export performance. Following the quota reduction, export volumes declined sharply. At the same time, the composition of exporting firms also changed. As shown in Figure 5, the number of exporting firms decreased substantially, while the average registered capital of exporters increased significantly, suggesting that the exporter population shifted toward larger and more established firms. These changes indicate a significant restructuring of the export market.

Fact 3. NTMs generate price responses larger than those typically associated with tariffs. The impact of these regulations was also reflected in REE export prices. However, compared with restriction imposed by 2010, the effect of the initial restriction imposed in 2006 is not obvious. The reason is that the early regulatory measures were relatively weak and their enforcement remained limited, therefore, illegal mining and smuggling activities allowed rare-earth products to continue entering international markets (Chen, Hu and Li, 2021), which dampened the price response. However, the quota adjustment in 2010 caused REE prices to increase by over 100%, and in some cases nearly 200%. As shown in Figure 6, export prices for several rare-earth products rose dramatically following the quota reduction. The magnitude of this price response far exceeds the typical price adjustments generated by conventional

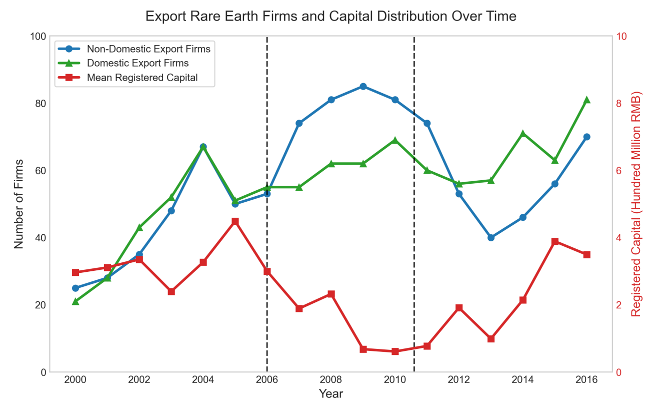


Figure 5. Rare Earth Export Firms and Capital Distribution from 2000 to 2016

Notes: Rare-earth export quantity and value data are constructed from Chinese customs transaction-level data. We identify rare-earth products by selecting HS6 records whose product names refer to rare-earth elements, including Sc, Y, La, Ce, Pr, Nd, Pm, Sm, Eu, Gd, Tb, Dy, Ho, Er, Tm, Yb, and Lu, as well as their corresponding compounds. Export quantities and values are aggregated at the annual level. The blue line represents export quantities, and the red line represents export values. The two dashed lines indicate the introduction of export restrictions in 2006 and the quota reduction in July 2010.

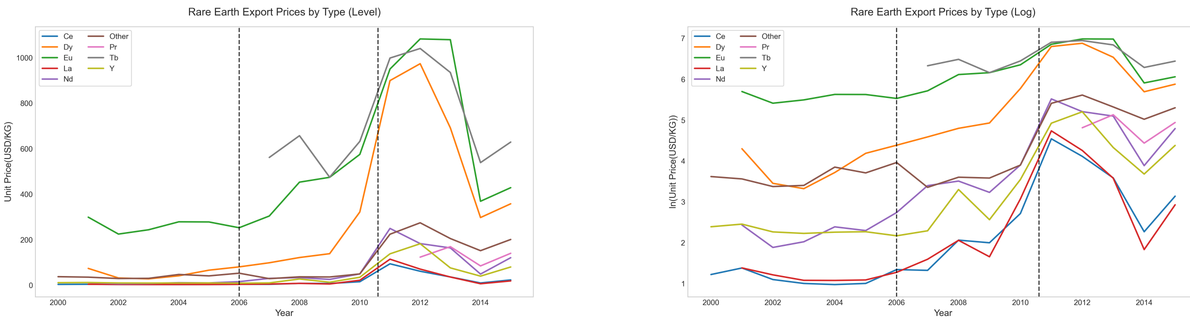
tariffs.

D. *Reduced form evidences of the propagation impact of CREER*

To provide empirical evidence for the propagation mechanism of rare-earth export restrictions (CREER), we examine how this policy shock affects trade outcomes across sectors through the production network. Rare-earth elements serve as important intermediate inputs in a wide range of downstream industries, including electronics, renewable energy equipment, and advanced manufacturing. Consequently, restrictions on rare-earth exports may propagate through input-output linkages, affecting firms' trade decisions and pricing behavior in both upstream and downstream sectors. We document two types of propagation effects. First, CREER influences firms' extensive margin of trade, namely whether a product is exported or imported. Second, the policy also affects the intensive margin, including export and import prices and values. To test the propagation effect along the production network, we estimate the following generalized difference-in-differences specification:

(42)

$$\mathbb{I}(Value_{it} > 0) = \alpha_i + \beta_1 \mathbb{I}(Year > 2006)_t \times DownExpo_s + \beta_2 \mathbb{I}(Year > 2006)_t \times UpExpo_s + \eta_t + \varepsilon_{it}$$



(a) Level Prices of Rare Earth Export

(b) Ln Prices of Rare Earth Export

Figure 6. Rare Earth Export Prices by Product Type

Notes: The figure plots export unit prices of several rare-earth elements and their logarithms using data from Chinese customs transaction records. Differences across destination countries are abstracted from because the export restrictions apply broadly across markets. Export prices are therefore averaged across destinations at the aggregate level. Each line represents the average export price of one rare-earth element and its corresponding chemical compounds over time.

where $\mathbb{I}(Value_{it} > 0)$ is an indicator function, which denotes whether the commodity i exports (imports) or not. $\mathbb{I}(Year > 2006)_t$ denotes that if the year is after 2016, the value equals to 1, otherwise, 0. $a_{j,mini}$ denotes the dependent degree of sector j to the mining and quarrying sector, which is measured by normalized direct input coefficient. Similarly, we use $a'_{j,mini}$ to measure the dependent degree of the mining and quarrying sector to sector j. α_i and η_t are HS8 commodity fixed effect and year fixed effect, respectively. β_1 and β_2 are the coefficients of interest. We cluster at the HS6 level.

Column (1) of table 2 reports the effect of CREER on the export extensive margin. We find that a one-standard-deviation increase in downstream exposure to the mining sector reduces the probability of exporting by 1.23%. In contrast, sectors that are more strongly connected upstream with the mining sector exhibit a higher probability of exporting.

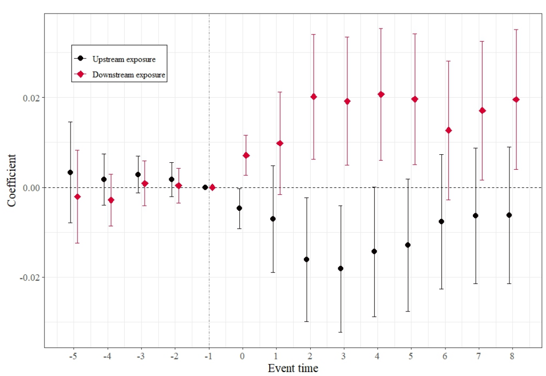
Figure 7 shows the pre-trend test of two outcomes. Before 2006, the coefficients are insignificant for both export and import extensive margin.

We next examine the intensive margin of trade by considering export and import values as well as prices. The sample is restricted to the top 20 trading partners of China based on total imports in 2000. We estimate the following regression,

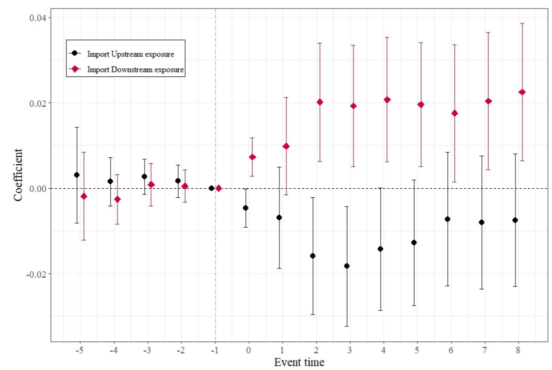
$$y_{ict} = \alpha_i + \beta_1 \mathbb{I}(Year > 2006)_t \times DownExpo_s + \beta_2 \mathbb{I}(Year > 2006)_t \times UpExpo_s + \eta_{ct} + \varepsilon_{it} \tag{43}$$

Table 2— The impact of REEs Restriction on extensive margin

	(1) Export Dummy	(2) Import Dummy
$\mathbb{I}(Year > 2006) \times DownsExpo$	-0.0123**	-0.0124**
	(0.0061)	(0.0061)
$\mathbb{I}(Year > 2006) \times UpExpo$	0.0170***	0.0181***
	(0.0060)	(0.0060)
HS8 FEs	✓	✓
Year FEs	✓	✓
N	137144	137746
Adj. R^2	0.4712	0.4582



(a) Level Prices of Rare Earth Export



(b) Ln Prices of Rare Earth Export

Figure 7. event study outcome of export dummy and import dummy

Notes: Confidence intervals are at the 95% level. In the subfigure 5a and 5b, the red lines stand for the intensity difference model of upstream exposure, and the black lines denote the intensity difference model of downstream exposure.

where y_{ict} is the outcome variable, including export (import) price and value. η_{ct} is the country-by-year fixed effect to absorb the time-varying unobservable factors of trade countries. Other variables are same as the above equation.

Table 3 reports the results. Columns (1) and (2) show that sectors with stronger downstream dependence on rare-earth inputs experience a significant decline in export values following the restriction. Specifically, a one-standard-deviation increase in downstream exposure reduces export value by approximately 7.36%. In contrast, sectors that are more strongly linked upstream with the mining sector experience a significant increase in export values, suggesting that upstream sectors may benefit from the reallocation of demand following the restriction. Columns (3) and (4) indicate that the price effects are mainly concentrated in upstream sectors. Downstream sectors show limited price adjustments, suggesting that the primary impact operates through trade quantities rather than prices. For imports, we find that downstream sectors experience little change in import values, although import prices decline slightly. In contrast, stronger upstream linkages are associated with higher import values and prices, reflecting adjustments in intermediate input sourcing.

Table 3— The impact of REEs Restriction on intensive margin

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Export				Import			
	Ln(Value)		Ln(Price)		Ln(Value)		Ln(Price)	
$\mathbb{I}(Year > 2006) \times DownsExpo$	-0.0736***	-0.0605***	-0.0104	-0.0110	0.0038	0.0073	-0.0568***	-0.0577***
	(0.0164)	(0.0167)	(0.0075)	(0.0075)	(0.0178)	(0.0190)	(0.0080)	(0.0080)
$\mathbb{I}(Year > 2006) \times UpExpo$	0.1956***	0.1864***	0.0232**	0.0227**	0.2660***	0.2645***	0.0752***	0.0723***
	(0.0151)	(0.0154)	(0.0095)	(0.0095)	(0.0187)	(0.0196)	(0.0099)	(0.0099)
HS8 FEs	✓	✓	✓	✓	✓	✓	✓	✓
Year FEs	✓		✓		✓		✓	
Country-Year FEs		✓		✓		✓		✓
N	1280752				1186949			
Adj. R^2	0.4799	0.5452	0.8684	0.8698	0.3540	0.4369	0.8417	0.8463

Overall, these results provide reduced-form evidence that rare-earth export restrictions generate substantial spillover effects across sectors through input-output linkages. Both the extensive and intensive margins of trade respond to the policy shock, highlighting the importance of production networks in shaping the aggregate consequences of trade restrictions. These empirical patterns suggest that shocks to export regulations can propagate across sectors through production linkages. However, the magnitude and direction of such propagation depend on the interaction between firm selection and production-network amplification. Understanding these mechanisms requires a structural framework.

E. Structural form evidences of the propagation impact of CREER

CALIBRATION. — The parameters set of a specific sector is $\left\{\sigma_s, \theta_s, b_i, f_s^{ij}\right\}_{s=1}^{K}$. The estimators of σ_s are obtained from Su et al. (2026). Following with Huang, Ju and Yue (2026), we estimate the Pareto shape parameters using the firm-level sales distribution. Under the assumption of a Pareto distribution, the upper tail of firm sales satisfies a rank-size relationship. For each sector, we estimate:

$$lnRank_{is} = a_s - \frac{\theta_s}{\sigma_s - 1} lnSales_{is} + \varepsilon_{is} \tag{44}$$

where a_s is sectoral fixed effects and ε_{is} is the error term. We measure the rank based on the revenues of firms and keep observations with over median sales. Then we run this regression sector by sector to obtain the estimators $\frac{\theta_s}{\sigma_s-1}$.

As for other parameters concerned with fixed costs and productivity, we estimate them by SMM. Now we introduce the basic procedure of solving equilibrium. Mass of firms and cutoff productivity are central to our work. First, based on the zero profit conditions and the cutoff export productivity, lemma 1 shows the relationships among cutoff productivity.

LEMMA 1: *According to zero profit conditions, the cutoff export productivity is expressed as*

$$\varphi_s^{ij*} = \tau_s^{ij} \frac{mc_{is}}{mc_{js}} \left(\frac{f_s^{ij}}{f_{js}}\right)^{\frac{1}{\sigma_s - 1}} \varphi_{js}^* \triangleq B_s^{ij} \varphi_{js}^* \tag{45}$$

where $B_s^{ii} = 1, \quad \forall i \in \{H, F\}$. *According to the free entry conditions,*

$$\left(\varphi_s^{iH*}\right)^{-\theta_s} + \left(\varphi_s^{iF*}\right)^{-\theta_s} = \frac{\theta_s - \sigma_s + 1}{\sigma_s - 1} f_{is}^e \tag{46}$$

According to lemma 1, cutoff productivity can by expressed as follows,

$$(\varphi_{Hs}^{*})^{-\theta_s} = \frac{\frac{\theta_s-\sigma_s+1}{\sigma_s-1}\left[f_{Hs}^{e} - \left(B_s^{HF}\right)^{-\theta_s} f_{Fs}^{e}\right]}{1-\left(B_s^{FH}B_s^{HF}\right)^{-\theta_s}}$$
$$(\varphi_{Fs}^{*})^{-\theta_s} = \frac{\frac{\theta_s-\sigma_s+1}{\sigma_s-1}\left[f_{Fs}^{e} - \left(B_s^{FH}\right)^{-\theta_s} f_{Hs}^{e}\right]}{1-\left(B_s^{FH}B_s^{HF}\right)^{-\theta_s}}$$

Table 4 shows the estimated elasticity of substitute , shape parameters and fixed costs.

VI. Conclusion

This paper develops a multi-country, multi-sector model with heterogeneous firms and endogenous entry to study how export fixed-cost shocks propagate through global production networks and affect aggregate welfare. Under CES preference and technology, we need two centralities to capture the importance of industry as supplier and consumer, respectively. Standard sufficient-statistics results are no longer sufficient for welfare analysis, because consumer centralities, tariff and cumulative market share jointly influence welfare. We study the effect of China's rare-earth export restrictions during 2006–2014. Our empirical results show that export fixed-cost shocks reshape the extensive margin of trade and generate cascading effects across countries and sectors through input-output linkages.

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Table 4— Estimated Sectoral Parameters and Fixed Costs

Sector	σ_s	θ_s	f_s^{HF}	f_s^{H}	f_s^{FH}	f_s^{F}
4	5.5152	6.1615	0.0013	0.0001	0.0049	0.0004
5	5.6754	6.5785	0.0011	0.0006	0.0055	0.0009
6	3.6985	3.9887	0.0028	0.0002	0.0057	0.0060
7	4.0183	4.5273	0.0005	0.0001	0.0021	0.0005
8	3.2709	3.2997	0.0005	0.0002	0.0027	0.0002
9	3.3856	3.2954	0.0001	0.0002	0.0012	0.0000
10	4.1858	5.0138	0.0011	0.0002	0.0043	0.0005
11	2.8500	2.4513	0.0019	0.0008	0.0097	0.0006
12	3.0635	2.8652	0.0006	0.0000	0.0016	0.0007
13	3.0792	3.0808	0.0014	0.0001	0.0034	0.0024
14	2.6298	2.4365	0.0018	0.0002	0.0061	0.0017
15	3.4262	3.1591	0.0024	0.0009	0.0120	0.0008
17	2.2868	1.7398	0.0050	0.0004	0.0116	0.0092
18	2.2960	1.7474	0.0021	0.0002	0.0057	0.0029
19	2.7120	2.2783	0.0020	0.0008	0.0099	0.0008
20	3.0533	2.6516	0.0016	0.0001	0.0047	0.0018
22	2.5117	2.1241	0.0008	0.0001	0.0018	0.0015
24	2.6404	2.3498	0.0033	0.0008	0.0153	0.0001
25	3.4159	2.7775	0.0001	0.0000	0.0002	0.0000

Notes: This table reports the estimated sector-level substitution elasticities (σ_s), Pareto shape parameters (θ_s), and fixed costs. f_s^{HF} and f_s^{FH} denote bilateral export fixed costs, while f_s^{H} and f_s^{F} denote domestic fixed costs in Home and Foreign countries, respectively.

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Details of Proof

A1. Mass of Firms

Given the cutoffs, the measure of firms from country i in sector s serving market j is

$$M_{is}^{ij} = M_{is}^{E} \left(\varphi_{is}^{ij*}\right)^{-\theta_s}. \tag{A1}$$

Define the survival probability as

$$\omega_{is}^{ij} \equiv \left(\varphi_{is}^{ij*}\right)^{-\theta_s}, \tag{A2}$$

so that

$$M_{is}^{ij} = M_{is}^{E} \omega_{is}^{ij}. \tag{A3}$$

For a firm with productivity φ_{is}, variable profit is

$$\frac{r_s^{ij}(\varphi_{is})}{\sigma_s}. \tag{A4}$$

Under the zero-cutoff-profit condition,

$$\frac{r_s^{ij}(\varphi_s^{ij*})}{\sigma_s} = w_i f_s^{ij}. \tag{A5}$$

Firm revenue satisfies

$$r_s^{ij}(\varphi_{is}) = r_s^{ij}(\varphi_s^{ij*}) \left(\frac{\varphi_{is}}{\varphi_s^{ij*}}\right)^{\sigma_s - 1}. \tag{A6}$$

Hence expected revenue generated in market j by one entrant is

$$\int_{\varphi_s^{ij*}}^{\infty} r_s^{ij}(\varphi_{is}) dG_i(\varphi_{is}) = \sigma_s w_i f_s^{ij} \int_{\varphi_s^{ij*}}^{\infty} \left(\frac{\varphi_{is}}{\varphi_s^{ij*}}\right)^{\sigma_s - 1} dG_i(\varphi_{is}). \tag{A7}$$

With Pareto productivity,

$$\int_{\varphi_s^{ij*}}^{\infty} \left(\frac{\varphi}{\varphi_s^{ij*}}\right)^{\sigma_s - 1} dG_i(\varphi) = \frac{\theta_s}{\theta_s - \sigma_s + 1} \left(\varphi_s^{ij*}\right)^{-\theta_s}. \tag{A8}$$

Therefore total revenue generated by country-i, sector-s entrants is

$$R_{is} = \frac{\sigma_s \theta_s}{\theta_s - \sigma_s + 1} w_i \sum_j M_s^{ij} f_s^{ij} \left(\varphi_s^{ij*}\right)^{-\theta_s} = M_{is}^E \frac{\sigma_s \theta_s}{\theta_s - \sigma_s + 1} w_i \sum_j f_s^{ij} \left(\frac{\varphi_s^{ij*}}{\varphi_{is}^*}\right)^{-\theta_s}. \tag{A9}$$

Variable cost is a fraction $1/\mu_s$ of revenue, where

$$\mu_s = \frac{\sigma_s}{\sigma_s - 1}. \tag{A10}$$

Let s_{is}^L denote the labor share in variable production. Variable labor demand in country i, sector s is

$$L_{is}^{var} = \frac{s_{is}^L}{w_i} \frac{R_{is}}{\mu_s}. \tag{A11}$$

Substituting the expression for R_{is},

$$L_{is}^{var} = M_{is}^{E}\frac{s_{is}^{L}}{\mu_s}\frac{\sigma_s\theta_s}{\theta_s-\sigma_s+1}\sum_j f_s^{ij}\left(\frac{\varphi_s^{ij*}}{\varphi_{is}^{*}}\right)^{-\theta_s}. \tag{A12}$$

Fixed operating and exporting labor demand is

$$L_{is}^{fix} = M_{is}^{E}\sum_j f_{is}^{ij}\left(\frac{\varphi_s^{ij*}}{\varphi_{is}^{*}}\right)^{-\theta_s}. \tag{A13}$$

Entry labor demand is

$$L_{is}^{entry} = M_{is}^{E} f_{is}^{E}. \tag{A14}$$

We impose sector-level labor-market clearing:

$$L_{is}^{var} + L_{is}^{fix} + L_{is}^{entry} = \gamma_{is}L_i. \tag{A15}$$

Substituting the above expressions yields

$$M_{is}^{E}\left[\frac{s_{is}^{L}}{\mu_s}\frac{\sigma_s\theta_s}{\theta_s-\sigma_s+1}\sum_j f_s^{ij}\left(\frac{\varphi_s^{ij*}}{\varphi_{is}^{*}}\right)^{-\theta_s} + \sum_j f_s^{ij}\left(\frac{\varphi_s^{ij*}}{\varphi_{is}^{*}}\right)^{-\theta_s} + f_{is}^{E}\right] = \gamma_{is}L_i. \tag{A16}$$

Hence entrant mass is

$$M_{is}^{E}=\frac{\gamma_{is}L_{i}}{\left[1+\frac{s_{is}^{L}}{\mu_{s}}\frac{\sigma_{s}\theta_{s}}{\theta_{s}-\sigma_{s}+1}\right]\sum_{j}f_{s}^{ij}\left(\frac{\varphi_{s}^{ij*}}{\varphi_{is}^{*}}\right)^{-\theta_{s}}+f_{is}^{E}}. \tag{A17}$$

Finally, bilateral firm masses are

$$M_{is}^{ij}=M_{is}^{E}\left(\frac{\varphi_{is}^{ij*}}{z_{i}}\right)^{-\theta_{s}}. \tag{A18}$$

A2. Proof of Definition 2

PROOF:

The flow of product of firm φ_{is} in country i and industry s is given by

$$y_{is}(\varphi_{is})=\sum_{j=1}^{N}c_{s}^{ij}(\varphi_{is})+\sum_{i=1}^{N}\sum_{t=1}^{S}x_{st}^{ij}(\varphi_{is}) \tag{A19}$$

This implies that part of the output is consumed by households in each country, while the remainder is used as intermediate inputs in production.

According to the optimal consumption and input,

$$c_{s}^{ij}(\varphi_{is})=\left(\frac{P_{j}}{\tilde{p}_{js}}\right)^{\epsilon}\left(\frac{\tilde{p}_{js}}{\tau_{s}^{ij}p_{is}(\varphi_{is})}\right)^{\sigma_{s}}\beta_{js}C_{j} \tag{A20}$$

and

$$x_{st}^{ij}(\varphi_{is})=\left(\frac{mc_{jt}}{\tilde{p}_{js}}\right)^{\epsilon}\left(\frac{\tilde{p}_{js}}{\tau_{s}^{ij}p_{is}(\varphi_{is})}\right)^{\sigma_{s}}\alpha_{st}^{j}y_{jt} \tag{A21}$$

So,

(A22)
$$y_{is}(\varphi_{is}) = \left[\sum_{j=1}^{N}\left(\frac{P_j}{\tilde{p}_{js}}\right)^{\epsilon}\left(\frac{\tilde{p}_{js}}{\tau_s^{ij}}\right)^{\sigma_s}\beta_{js}C_j + \sum_{i=1}^{N}\sum_{t=1}^{S}\left(\frac{mc_{jt}}{\tilde{p}_{js}}\right)^{\epsilon}\left(\frac{\tilde{p}_{js}}{\tau_s^{ij}}\right)^{\sigma_s}\alpha_{st}^{j}y_{jt}\right] p_{is}(\varphi_{is})^{-\sigma_s} \triangleq \mathcal{D}_{is}p_{is}(\varphi_{is})^{-\sigma_s}$$

which means that

(A23)
$$\widetilde{y}_{is} = \mathcal{D}_{is}\left(p_{is}\right)^{-\sigma_s}$$

Now we substitute in

(A24)
$$\left(\frac{\tilde{p}_{js}}{\tau_s^{ij}p_{is}(\varphi_{is})}\right)^{\sigma_s} = \left(\frac{\tau_s^{ij}p_{is}(\varphi_{is})}{\tau_s^{ij}p_s^{ij}}\right)^{-\sigma_s}\left(\frac{\tau_s^{ij}p_s^{ij}}{\tilde{p}_{js}}\right)^{-\sigma_s} = \left(\frac{\tilde{\varphi}_s^{ij}}{\varphi_{is}}\right)^{-\sigma_s}(\pi_s^{ij})^{\frac{\sigma_s}{1-\sigma_s}}(M_s^{ij})^{\frac{\sigma_s}{1-\sigma_s}}$$

The second equality use the definition of bilateral trade share. We also obtain the product of single-firm's output and the domestic average price with power ϵ

(A25)
$$\begin{aligned} y_{is}(\varphi_{is})p_{is}^{\epsilon} =& \sum_{j=1}^{N}(\chi_s^{ij})^{-\frac{\sigma_s-1-\theta_s}{\sigma_s-1}\epsilon}(\pi_s^{ij})^{\frac{\epsilon-\sigma_s}{1-\sigma_s}}(\tau_s^{ij})^{-\epsilon}\left(\frac{\tilde{\varphi}_s^{ij}}{\varphi_{is}}\right)^{-\sigma_s}(M_s^{ij})^{\frac{\sigma_s}{1-\sigma_s}}\beta_{js}P_j^{\epsilon}C_j \\ &+\sum_{j=1}^{N}\sum_{t=1}^{S}(\chi_s^{ij})^{-\frac{\sigma_s-1-\theta_s}{\sigma_s-1}\epsilon}(\pi_s^{ij})^{\frac{\epsilon-\sigma_s}{1-\sigma_s}}(\tau_s^{ij})^{-\epsilon}\left(\frac{\tilde{\varphi}_s^{ij}}{\varphi_{is}}\right)^{-\sigma_s}(M_s^{ij})^{\frac{\sigma_s}{1-\sigma_s}}\tilde{\varphi}_{jt}^{\epsilon}M_{jt}^{\frac{\epsilon}{\sigma_t-1}}\mu_t^{-\epsilon}\alpha_{st}^{j}p_{jt}^{\epsilon}y_{jt} \end{aligned}$$

Then aggregate output following by

(A26)
$$y_{is} = \left[\frac{1}{1-G(\varphi_{is}^*)}\int_{\varphi_{is}^*}^{+\infty} M_{is}y_{is}(\varphi_{is})^{\frac{\sigma_s-1}{\sigma_s}}\,dG(\varphi_{is})\right]^{\frac{\sigma_s}{\sigma_s-1}}$$

The product of industrial output and the domestic average price with power ϵ is given by

$$\begin{aligned} p_{is}^{\epsilon} y_{is} = & \sum_{j=1}^{N} (\chi_s^{ij})^{\frac{-\theta_s \sigma_s}{\sigma_s - 1}} \widetilde{\varphi}_{is}^{\sigma_s} \left[(\chi_s^{ij})^{-\frac{\sigma_s - 1 - \theta_s}{\sigma_s - 1}\epsilon} (\pi_s^{ij})^{\frac{\epsilon - \sigma_s}{1-\sigma_s}} (\tau_s^{ij})^{-\epsilon} \left(\tilde{\varphi}_s^{ij}\right)^{-\sigma_s} \beta_{js} P_j^{\epsilon} C_j \right] \\ & + \sum_{j=1}^{N} \sum_{t=1}^{S} (\chi_s^{ij})^{\frac{-\theta_s \sigma_s}{\sigma_s - 1}} \widetilde{\varphi}_{is}^{\sigma_s} \left[(\chi_s^{ij})^{-\frac{\sigma_s - 1 - \theta_s}{\sigma_s - 1}\epsilon} (\pi_s^{ij})^{\frac{\epsilon - \sigma_s}{1-\sigma_s}} (\tau_s^{ij})^{-\epsilon} \left(\tilde{\varphi}_s^{ij}\right)^{-\sigma_s} \tilde{\varphi}_{jt}^{\epsilon} M_{jt}^{\frac{\epsilon}{\sigma_t - 1}} \mu_t^{-\epsilon} \alpha_{st}^{j} p_{jt}^{\epsilon} y_{jt} \right] \end{aligned} \tag{A27}$$

We group terms related to productivity, and get

$$(\chi_s^{ij})^{\frac{-\theta_s \sigma_s}{\sigma_s - 1}} \widetilde{\varphi}_{is}^{\sigma_s} (\chi_s^{ij})^{-\frac{\sigma_s - 1 - \theta_s}{\sigma_s - 1}\epsilon} (\tilde{\varphi}_s^{ij})^{-\sigma_s} = (\chi_s^{ij})^{(\sigma_s - \epsilon)\frac{\sigma_s - \theta_s - 1}{\sigma_s - 1}} \tag{A28}$$

And

$$\begin{aligned} M_{jt}^{\frac{\epsilon}{\sigma_t - 1}} & = \left(M_{jt}^{E} (\varphi_{jt}^{*})^{-\theta_t} \right)^{\frac{\epsilon}{\sigma_t - 1}} \\ & = (M_{jt}^{E})^{\frac{\epsilon}{\sigma_t - 1}} (\varphi_{jt}^{*})^{-\frac{\epsilon \theta_t}{\sigma_t - 1}} \\ & = (M_{jt}^{E})^{\frac{\epsilon}{\sigma_t - 1}} (\tilde{\varphi}_{jt})^{-\frac{\epsilon \theta_t}{\sigma_t - 1}} \left[\frac{\theta_t}{1 - \sigma_t + \theta_t} \right]^{\frac{\epsilon \theta_t}{(\sigma_t - 1)^2}} \end{aligned} \tag{A29}$$

Where the third equality uses

$$\tilde{\varphi}_{jt} = \left[\frac{\theta_t}{1 - \sigma_t + \theta_t} \right]^{\frac{1}{\sigma_t - 1}} (\varphi_{jt}^{*}) \tag{A30}$$

Therefore, $p_{is}^{\epsilon} y_{is}$ is as follows,

$$\begin{aligned} p_{is}^{\epsilon} y_{is} = & \sum_{j=1}^{N} \left[(\chi_s^{ij})^{(\sigma_s - \epsilon)\frac{\sigma_s - \theta_s - 1}{\sigma_s - 1}} (\pi_s^{ij})^{\frac{\epsilon - \sigma_s}{1-\sigma_s}} (\tau_s^{ij})^{-\epsilon} \beta_{js} P_j^{\epsilon} C_j \right] \\ & + \sum_{j=1}^{N} \sum_{t=1}^{S} \left[(\chi_s^{ij})^{(\sigma_s - \epsilon)\frac{\sigma_s - \theta_s - 1}{\sigma_s - 1}} (\pi_s^{ij})^{\frac{\epsilon - \sigma_s}{1-\sigma_s}} (\tau_s^{ij})^{-\epsilon} \tilde{\varphi}_{jt}^{\frac{\sigma_t - \theta_t - 1}{\sigma_t - 1}} \left[\frac{\theta_t}{1 - \sigma_t + \theta_t} \right]^{\frac{\epsilon \theta_t}{(\sigma_t - 1)^2}} (M_{jt}^{E})^{\frac{\epsilon}{\sigma_t - 1}} \mu_t^{-\epsilon} \alpha_{st}^{j} p_{jt}^{\epsilon} y_{jt} \right] \end{aligned} \tag{A31}$$

Recall that the marginal cost is given by

$$mc_{is} = \left[\alpha_{is}^{L} w_{is}^{1-\epsilon} + \sum_{t \in S} \alpha_{ts}^{i} \tilde{p}_{it}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{A32}$$

According to the trade-share, we obtain

$$\tilde{p}_{it} = \left(\frac{1}{\pi_t^{hi}} \times (\tau_t^{hi} p_t^{hi})^{1-\sigma_t}\right)^{\frac{1}{1-\sigma_t}} = (\pi_t^{hi})^{\frac{1}{\sigma_t - 1}} (\tau_t^{hi} p_t^{hi}), \quad \forall h = 1, 2, \ldots, N \tag{A33}$$

The marginal cost of industry s in country i is rewritten by

$$mc_{is}^{1-\epsilon} = \alpha_{is}^{L} w_{is}^{1-\epsilon} + \frac{1}{N} \sum_{t=1}^{S} \sum_{h=1}^{N} \alpha_{ts}^{i} (\pi_t^{hi})^{\frac{1-\epsilon}{\sigma_t - 1}} (\tau_t^{hi} p_t^{hi})^{1-\epsilon} \tag{A34}$$

Now substitute in

$$p_t^{hi} = p_{ht} \left(\frac{\tilde{\varphi}_{ht}}{\tilde{\varphi}_t^{hi}}\right)^{\frac{\sigma_t - 1 - \theta_t}{\sigma_t - 1}} \triangleq p_{ht} \left(\chi_t^{ij}\right)^{1 - \frac{\theta_t}{\theta_t - 1}} = p_{ht} \widetilde{\chi}_t^{ij} \tag{A35}$$

Therefore, the general feedback of domestic average price is also given by

$$p_{is}^{1-\epsilon} \tilde{\varphi}_{is}^{1-\epsilon} M_{is}^{\frac{1-\epsilon}{\sigma_s - 1}} \mu_s^{\epsilon - 1} = \alpha_{is}^{L} w_{is}^{1-\epsilon} + \frac{1}{N} \sum_{t=1}^{S} \sum_{h=1}^{N} \alpha_{ts}^{i} (\pi_t^{hi})^{\frac{1-\epsilon}{\sigma_t - 1}} (\tau_t^{hi})^{1-\epsilon} (\tilde{\chi}_t^{hi})^{1-\epsilon} (p_{ht})^{1-\epsilon} \tag{A36}$$

Similarly, we substitute into

$$M_{is}^{\frac{1-\epsilon}{\sigma_s - 1}} = (M_{is}^{E})^{\frac{1-\epsilon}{\sigma_s - 1}} (\tilde{\varphi}_{is})^{-\frac{\theta_s (1-\epsilon)}{\sigma_s - 1}} \left[\frac{\theta_s}{1 - \sigma_s + \theta_s}\right]^{-\frac{\theta_s (1-\epsilon)}{(\sigma_s - 1)^2}} = (M_{is}^{E})^{\frac{1-\epsilon}{\sigma_s - 1}} (\varphi_{is}^{*})^{-\theta_s \frac{1-\epsilon}{\sigma_s - 1}} \tag{A37}$$

and get

$$p_{is}^{1-\epsilon}(M_{is}^E)^{\frac{1-\epsilon}{\sigma_s-1}}(\tilde{\varphi}_{is})^{\frac{\sigma_s-\theta_s-1}{\sigma_s-1}(1-\epsilon)}\left[\frac{\theta_s}{1-\sigma_s+\theta_s}\right]^{-\frac{\theta_s(1-\epsilon)}{(\sigma_s-1)^2}}\mu_s^{\epsilon-1} = \alpha_{is}^L w_{is}^{1-\epsilon} + \frac{1}{N}\sum_{t=1}^{S}\sum_{h=1}^{N}\alpha_{ts}^i(\pi_t^{hi})^{\frac{1-\epsilon}{\sigma_t-1}}(\tau_t^{hi})^{1-\epsilon}(\tilde{\chi}_t^{hi})^{1-\epsilon}(p_{ht})^{1-\epsilon} \tag{A38}$$

□

LEMMA 2: *Consider two industries k and l in the closed-economy benchmark such that*

$$\alpha_{kj} = \alpha_{lj}, \quad j = 1, \ldots, K, \qquad \mu_k = \mu_l,$$

and

$$\Phi_k^{1-\epsilon}\tilde{M}_k^{1-\epsilon} = \Phi_l^{1-\epsilon}\tilde{M}_l^{1-\epsilon}.$$

Then the supplier centralities of the two industries are identical,

$$\tilde{\beta}_k = \tilde{\beta}_l.$$

PROOF:

The result then follows from the same induction argument used in the proof of Proposition 4 in Baqaee (2018). □

LEMMA 3: *Consider two industries k and l in the closed-economy benchmark such that*

$$\alpha_{jk} = \alpha_{jl}, \quad j = 1, \ldots, K, \qquad \mu_k = \mu_l,$$

and

$$\Phi_k^{1-\epsilon}\tilde{M}_k^{1-\epsilon} = \Phi_l^{1-\epsilon}\tilde{M}_l^{1-\epsilon}.$$

Then the consumer centralities of the two industries are identical,

$$\tilde{\alpha}_k = \tilde{\alpha}_l.$$

PROOF:

The result then follows from the same induction argument used in the proof of Proposition 5 in Baqaee (2018). □

Two lemmas show that, once productivity heterogeneity is introduced, industry centrality depends on the productivity-adjusted firm mass ΦM rather than the number of firms alone. Intuitively, an industry with fewer but more productive firms can generate the same effective supply capacity as an industry with many less productive firms. Productivity heterogeneity generates the trade-off between productivity and mass of firms.

A3. Proof of Proposition 1

PROOF:

According to the optimal demand and input, the expenditure of final spending and intermediate to firms with productivity φ_{js} in industry s of country j is given by

$$FE_s^{ji}(\varphi_{js}) = \tau_s^{ji} p_{js}(\varphi_{js}) \left(\frac{P_i}{\tilde{p}_{is}}\right)^{\epsilon} (\pi_s^{ji})^{-\frac{\sigma_s}{1-\sigma_s}} \left(\frac{p_s^{ji}}{p_{js}(\varphi_{js})}\right)^{\sigma_s} \beta_{is} C_i \nu(\varphi_{js}) M_s^{ji}$$

and

$$ME_{st}^{ji}(\varphi_{js}) = \tau_s^{ji} p_{js}(\varphi_{js}) \left(\frac{mc_{it}}{\tilde{p}_{is}}\right)^{\epsilon} (\pi_s^{ji})^{-\frac{\sigma_s}{1-\sigma_s}} \left(\frac{p_s^{ji}}{p_{js}(\varphi_{js})}\right)^{\sigma_s} \alpha_{st}^{i} y_{it} \nu(\varphi_{js}) M_s^{ji}$$

We take a sum of survival firms in industry s of country j,

$$FE_s^{ji} = \pi_s^{ji}(\tilde{p}_{is})^{1-\epsilon}\beta_{is}P_i^{\epsilon}C_i \tag{A39}$$

$$ME_s^{ji} = \sum_{t=1}^{K}\pi_s^{ji}(\tilde{p}_{is})^{1-\epsilon}\alpha_{st}^{i}mc_{it}^{\epsilon}y_{it} \tag{A40}$$

Then we substitute $\tilde{p}_{is}$ into $p_{is}(\pi_s^{ii})^{\frac{1}{\sigma_s-1}}$ and mc_{it} into $\mu_t^{-1}\tilde{\varphi}_{it}p_{it}M_{it}^{\frac{1}{\sigma_t-1}}$ and obtain FE_s^{ji} and ME_s^{ji}.

$$FE_s^{ji} = p_{is}^{1-\epsilon}\pi_s^{ji}\left(\pi_s^{ii}\right)^{\frac{1-\epsilon}{\sigma_s-1}}\beta_{is}P_i^{\epsilon}C_i \tag{A41}$$

$$ME_s^{ji} = \sum_{t=1}^{K}p_{is}^{1-\epsilon}\pi_s^{ji}\left(\pi_s^{ii}\right)^{\frac{1-\epsilon}{\sigma_s-1}}\alpha_{st}^{i}\mu_t^{-\epsilon}\tilde{\varphi}_{it}^{\epsilon}M_{it}^{\frac{\epsilon}{\sigma_t-1}}p_{it}^{\epsilon}y_{it} \tag{A42}$$

where FE_s^{ji} is the final expenditure in country i to industry s in country j. Similarly, ME_s^{ji} is the expenditure in country i for buying goods from industry s in country j for production.

$$M_{it}^{\frac{\epsilon}{\sigma_t-1}} = (\varphi_{it}^{*})^{\frac{-\theta_t}{\sigma_t-1}\epsilon}\left(M_{it}^{E}\right)^{\frac{\epsilon}{\sigma_t-1}} = (\widetilde{\varphi}_{it})^{\frac{-\theta_t}{\sigma_t-1}\epsilon}\left[\frac{\theta_t}{1-\sigma_t+\theta_t}\right]^{\frac{\theta_t\epsilon}{(\sigma_t-1)^2}}\left(M_{it}^{E}\right)^{\frac{\epsilon}{\sigma_t-1}}$$

where the third equality uses

$$\tilde{\varphi}_{jt} = \left[\frac{\theta_t}{1-\sigma_t+\theta_t}\right]^{\frac{1}{\sigma_t-1}}(\varphi_{jt}^{*})$$

$\square$

The bilateral trade value is given by

(A43)

$$E_s^{ji} = p_{is}^{1-\epsilon}\pi_s^{ji}\left(\pi_s^{ii}\right)^{\frac{1-\epsilon}{\sigma_s-1}}\beta_{is}P_i^{\epsilon}C_i + \sum_{t=1}^{K}p_{is}^{1-\epsilon}\pi_s^{ji}\left(\pi_s^{ii}\right)^{\frac{1-\epsilon}{\sigma_s-1}}\alpha_{st}^{i}\mu_t^{-\epsilon}\left(\widetilde{\varphi}_{it}\right)^{\frac{\sigma_t-1-\theta_t}{\sigma_t-1}\epsilon}\left[\frac{\theta_t}{1-\sigma_t+\theta_t}\right]^{\frac{\theta_t\epsilon}{(\sigma_t-1)^2}}\left(M_{it}^{E}\right)^{\frac{\epsilon}{\sigma_t-1}}p_{it}^{\epsilon}y_{it}$$

We define a common factor Υ_{is} as follow,

$$\Upsilon_{is} = \beta_{is} P_i^{\epsilon} C_i + \sum_{t=1}^{K} \alpha_{st}^{i} \mu_t^{-\epsilon} \left(\widetilde{\varphi}_{it}\right)^{\frac{\sigma_t - 1 - \theta_t}{\sigma_t - 1}\epsilon} \left[\frac{\theta_t}{1 - \sigma_t + \theta_t}\right]^{\frac{\theta_t \epsilon}{(\sigma_t - 1)^2}} \left(M_{it}^{E}\right)^{\frac{\epsilon}{\sigma_t - 1}} p_{it}^{\epsilon} y_{it}$$

and E_s^{ji} is rewritten

$$E_s^{ji} = p_{is}^{1-\epsilon} \pi_s^{ji} \left(\pi_s^{ii}\right)^{\frac{1-\epsilon}{\sigma_s - 1}} \Upsilon_{is}$$

Now we turn FE_{is} and ME_{is} into matrix forms.

$$FE_i' = \begin{bmatrix} FE_{i1} & FE_{i2} & \cdots & FE_{iK} \end{bmatrix}_{1\times K}$$

$$ME_i' = \begin{bmatrix} ME_{i1} & ME_{i2} & \cdots & ME_{iK} \end{bmatrix}_{1\times K}$$

Using the centrality representation derived above, the two expenditure vectors can be written as follows:

$$FE_i = \tilde{\alpha}_i \left(\tilde{\pi}_i^{D}\right)^{\epsilon - 1} \beta_i P_i^{\epsilon} C_i$$

and

$$ME_i = \tilde{\alpha}_i \left(\tilde{\pi}_i^{D}\right)^{\epsilon - 1} A_i \mu^{-\epsilon} \Phi_i^{\epsilon} \tilde{M}_i^{\epsilon} \mathcal{C}^{\epsilon} \tilde{\beta}_i P_i^{\epsilon} C_i$$

where $\tilde{\alpha}_i$ denotes the block matrix of consumer centrality corresponding to country i, and $\tilde{\alpha}_i$ stands for diagonal matrix of $\tilde{\alpha}_i$. $\tilde{\pi}_i^{D}$ is a $K \times K$ diagonal matrix with elements $(\pi_s^{ii})^{\frac{1-\epsilon}{\sigma_s - 1}}$. β_i is the vector of taste parameters in country i. A_i denotes the direct input–output coefficient matrix of country i. $\tilde{\Phi}_i$ is the domestic production matrix of country i. These formulas imply that supplier centrality influences expenditure on intermediate inputs used in production, but does not directly enter the expression for final expenditure.

Combining the two components, total expenditure vector can be expressed as

$$E_i = FE_i + ME_i = \hat{\tilde{\alpha}}_i \left(\tilde{\pi}_i^{D}\right)^{\epsilon - 1} \left[\beta_i + A_i \mu^{-\epsilon} \Phi_i^{\epsilon} \tilde{M}_i^{\epsilon} \mathcal{C}^{\epsilon} \tilde{\beta}_i\right] P_i^{\epsilon} C_i \tag{A44}$$

According to accounting equality, the aggregate revenue of industry s in country i, R_{is}, equals to the total expenditure to varieties in industry s and country i of all the countries, which means that

$$R_{is} = \sum_{j=1}^{N} E_s^{ij} = \sum_{j=1}^{N} \pi_s^{ij} E_{js}$$

The second equality uses $E_s^{ij} = \pi_s^{ij} E_{js}$. And because sunk fixed costs absorb all the profits of surviving firms, the income of household in every country is the total wage, i.e. 1. The final expenditure is

$$\sum_{s=1}^{K} FE_{is} = w_i L_i = 1,$$

which means that

$$\tilde{\alpha}_i' \left(\tilde{\pi}_i^D\right)^{\epsilon-1} \beta_i P_i^\epsilon C_i = 1 \quad \Rightarrow \quad P_i^\epsilon C_i = \frac{1}{\tilde{\alpha}_i' \left(\tilde{\pi}_i^D\right)^{\epsilon-1} \beta_i} \tag{A45}$$

A4. Welfare decomposition of the closed economy with multi-sector

The feedback of the mass of firms is,

$$\frac{d\ln(M)}{d\ln f_v^{pq}} = [I-\Gamma]^{-1}\left[(1-\epsilon)\frac{d\ln(\mathbf{1}_K \otimes C)}{d\ln f_v^{pq}} + \epsilon\iota\frac{d\ln\Phi^e}{d\ln f_v^{pq}} - e_{is}\frac{f_s^{pq}}{\sum_{j=1}^{N} f_s^{ij}}\right] \tag{A46}$$

And

$$\begin{aligned}\Gamma = {} & \tilde{\pi}\frac{\partial vec(\ln\pi)}{\partial\ln(M)} \\ & + \text{diag}(\tilde{\alpha})^{-1}\left[\frac{\partial\ln\tilde{\alpha}}{\partial\ln(M)} + \frac{\partial\ln\tilde{\alpha}}{\partial vec(\ln\pi)}\frac{\partial vec(\ln\pi)}{\partial\ln(M)}\right] \\ & + \frac{\partial vec(\ln\pi^D \otimes \mathbf{1}_N)}{\partial\ln(M)} \\ & + \tilde{\pi}\iota\,\text{diag}(\tilde{\beta})^{-1}\left[\frac{\partial\ln\tilde{\beta}}{\partial\ln(M)} + \frac{\partial\ln\tilde{\beta}}{\partial vec(\ln\pi)}\frac{\partial vec(\ln\pi)}{\partial\ln(M)}\right]\end{aligned} \tag{A47}$$

Under the single-country setting,

$$\tilde{\pi} \to 1,$$

and the trade channel is closed,

$$\frac{\partial vec(\ln \pi)}{\partial \ln(M)} = \frac{\partial \ln \tilde{\alpha}}{\partial vec(\ln \pi)} \frac{\partial vec(\ln \pi)}{\partial \ln(M)} = \frac{\partial \ln \tilde{\beta}}{\partial vec(\ln \pi)} \frac{\partial vec(\ln \pi)}{\partial \ln(M)} = 0 \tag{A48}$$

Therefore,

$$\Gamma = \text{diag}(\tilde{\alpha})^{-1} \frac{\partial \ln \tilde{\alpha}}{\partial \ln(M)} + \iota \, \text{diag}(\tilde{\beta})^{-1} \frac{\partial \ln \tilde{\beta}}{\partial \ln(M)} \tag{A49}$$

and

$$\frac{d \ln(M)}{d \ln f_v} = [I - \Gamma]^{-1} \left[(1-\epsilon) \frac{d \ln(C)}{d \ln f_v} + \epsilon \iota \frac{d \ln \Phi}{d \ln f_v} - e_v \frac{f_v}{\sum_{j=1}^N f_s} \right] \tag{A50}$$

Similarly, the consumer centrality is given by

$$\frac{d \ln(\tilde{\alpha})}{d \ln f_t^{hi}} = \text{diag}(\tilde{\alpha})^{-1} \Lambda_2^M \frac{d \ln(M)}{d \ln f_t^{hi}} + \text{diag}(\tilde{\alpha})^{-1} \Lambda_2^\Phi \frac{d \ln(\Phi)}{d \ln f_t^{hi}} \tag{A51}$$

Now we substitute into $\dfrac{d \ln(M)}{d \ln f_v}$, and get

$$\begin{aligned} \frac{d \ln(\tilde{\alpha})}{d \ln f_t^{hi}} &= \text{diag}(\tilde{\alpha})^{-1} \Lambda_2^M [I - \Gamma]^{-1} \left[(1-\epsilon) \frac{d \ln(C)}{d \ln f_v} + \epsilon \iota \frac{d \ln \Phi}{d \ln f_v} - e_v \frac{f_v}{\sum_{j=1}^N f_s} \right] \\ &\quad + \text{diag}(\tilde{\alpha})^{-1} \Lambda_2^\Phi \frac{d \ln(\Phi)}{d \ln f_t^{hi}} \end{aligned} \tag{A52}$$

The general feedback of welfare degenerates into

$$d \ln(C_i) = - \sum_{s=1}^K \sum_{j=1}^N \beta_{is}^{ji} \frac{1}{1-\epsilon} d \ln \tilde{\alpha}_{js} \tag{A53}$$

Where

$$\begin{aligned}\Omega_s^{ji} &= \frac{\beta_{is}\tilde{p}_{is}^{1-\epsilon}}{\sum_{h=1}^{K}\beta_{ih}\tilde{p}_{ih}^{1-\epsilon}}\pi_s^{ji}\\ &= \frac{\beta_{is}\tilde{p}_{is}^{1-\epsilon}}{\sum_{h=1}^{K}\beta_{ih}\tilde{p}_{ih}^{1-\epsilon}} = \frac{\beta_s\tilde{\alpha}_s}{\beta'\tilde{\alpha}}\end{aligned} \tag{A54}$$

$$\begin{aligned}\beta'\tilde{\alpha}\frac{d\ln(C_i)}{d\ln f_t^{hi}} &= -\frac{1}{1-\epsilon}\sum_{s=1}^{K}\beta_s d\tilde{\alpha}_s\\ &= -\frac{\beta_s'}{1-\epsilon}\Lambda_2^M[I-\Gamma]^{-1}\left[(1-\epsilon)\frac{d\ln(C)}{d\ln f_v} + \epsilon\iota\frac{d\ln\Phi}{d\ln f_v} - e_t\frac{f_t}{\sum_{j=1}^{N}f_s}\right]\\ &\quad + \beta_s'\Lambda_2^{\Phi}\frac{d\ln(\Phi)}{d\ln f_t^{hi}}\end{aligned} \tag{A55}$$

$$\begin{aligned}\beta'\tilde{\alpha}\frac{d\ln(C_i)}{d\ln f_t^{hi}} &= -\beta_s'\Lambda_2^M[I-\Gamma]^{-1}\frac{d\ln(C)}{d\ln f_v}\\ &\quad - \frac{\beta_s'}{1-\epsilon}\Lambda_2^M[I-\Gamma]^{-1}\epsilon\iota\frac{d\ln\Phi}{d\ln f_v}\\ &\quad + \frac{\beta_s'}{1-\epsilon}\Lambda_2^M[I-\Gamma]^{-1}e_t + \beta_s'\Lambda_2^{\Phi}\frac{d\ln(\Phi)}{d\ln f_t^{hi}}\end{aligned} \tag{A56}$$

$$\begin{aligned}\left(\beta'\tilde{\alpha} + \beta_s'\Lambda_2^M[I-\Gamma]^{-1}\mathbf{1}\right)\frac{d\ln(C_i)}{d\ln f_t^{hi}} &= -\beta_s'\left[\frac{1}{1-\epsilon}\Lambda_2^M[I-\Gamma]^{-1}\epsilon\iota + \Lambda_2^{\Phi}\right]\frac{d\ln\Phi}{d\ln f_t^{hi}}\\ &\quad + \frac{\beta_s'}{1-\epsilon}\Lambda_2^M[I-\Gamma]^{-1}e_t\end{aligned} \tag{A57}$$

Therefore,

$$\frac{d\ln(C_i)}{d\ln f_t^{hi}} = \frac{1}{1-\epsilon}\frac{\beta_s'\Lambda_2^M[I-\Gamma]^{-1}}{\beta'\tilde{\alpha}+\beta_s'\Lambda_2^M[I-\Gamma]^{-1}\mathbf{1}}e_t + \frac{1}{\epsilon-1}\frac{\beta_s'\left[\Lambda_2^M[I-\Gamma]^{-1}\epsilon\iota+\Lambda_2^{\Phi}\right]}{\beta'\tilde{\alpha}+\beta_s'\Lambda_2^M[I-\Gamma]^{-1}\mathbf{1}}e_t\frac{1}{\theta_t} \tag{A58}$$

Where we use

$$\frac{d\ln\Phi}{d\ln f_t} = \frac{1}{\theta_t} \tag{A59}$$

which comes from the single-country free-entry condition,

$$\mathcal{J}(\varphi_t^*)f_t = f_t^e \tag{A60}$$

where

$$\mathcal{J}(\varphi_t^*) = (\varphi_t^*)^{-\theta_s}.$$